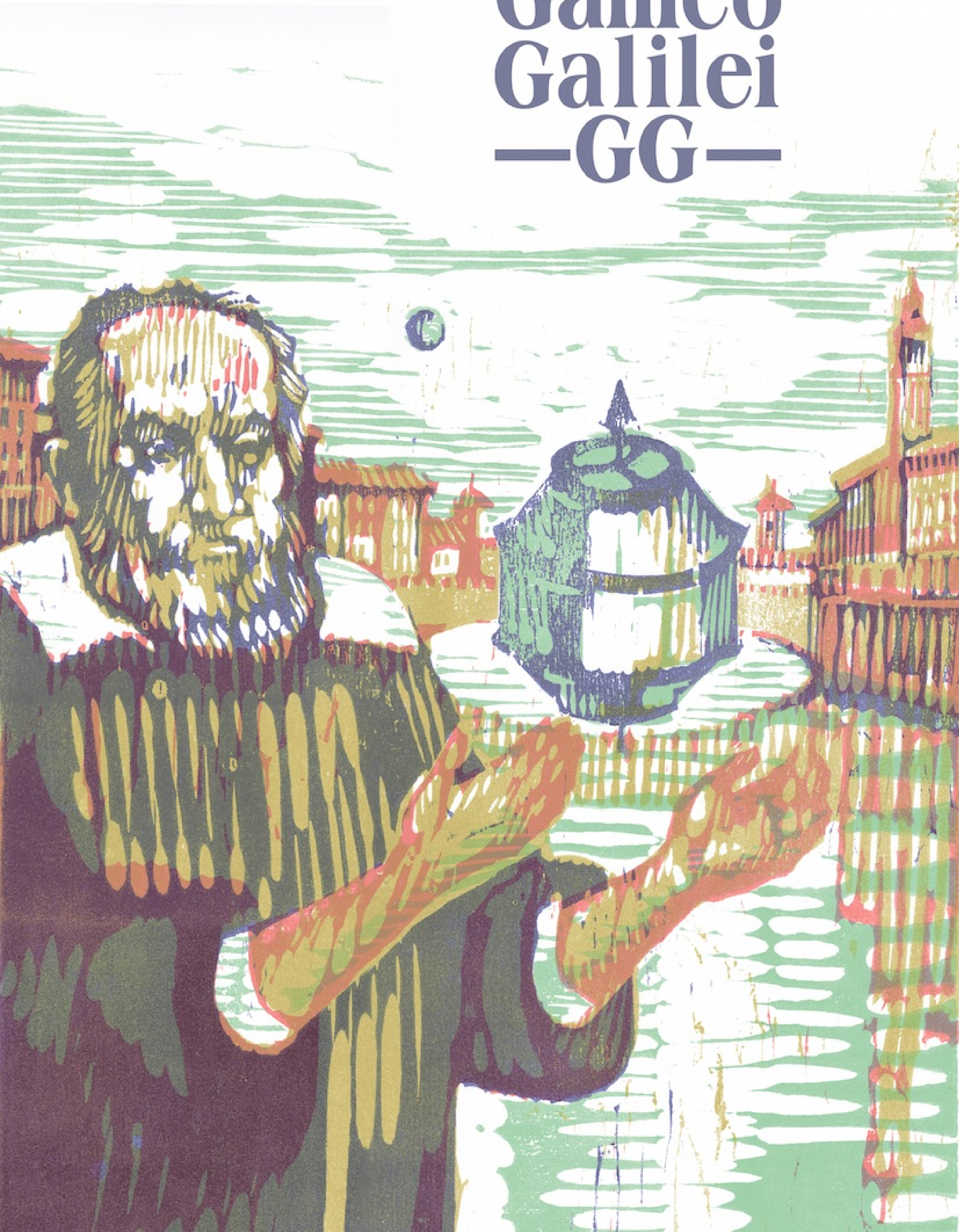
Galileo Galilei

—GG—



Galileo Galilei (GG): a mission to test the founding pillar of General Relativity to 10^{-17}

Submitted 5 October 2016 in response to "Call for a medium-size mission opportunity in ESA's Science Programme (M5)"

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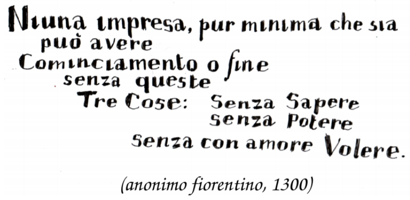
(The lead proposer agrees to dedicate at least 20% of her time to support the study activities throughout the study period)



Niuna impresa, pur minima che sia
può avere
Cominciamento o fine
senza queste
Tre Cose: Senza Sapere
senza Potere
senza con amore Volere.

(anonimo fiorentino, 1300)

No matter how small an undertaking,
it cannot start or come to fruition
without knowledge,
without means,
or without loving tenacity.

(anonymous from Florence, 1300)

Cover: color xylography by Alice Milani

Annex A – European core team members and supporting scientists

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Executive summary

The General theory of Relativity (GR)[1] stands on the fundamental assumption that in a gravitational field all bodies fall with the same acceleration regardless of their mass and composition, a 'fact of nature' known as the Universality of Free Fall (UFF) or the Weak Equivalence Principle (WEP). As experimental evidence for UFF, Einstein took the results of Eötvös, who in the same years had achieved an impressive 1000-fold improvement by suspending test masses of different composition on a torsion balance rather than individual pendulums([1], Sec. 2, p. 773).

The Standard Model of particle physics and the theory of General Relativity, taken together, form our current view of the physical world. While the former governs the physics of the microcosm, the latter governs physics at the macroscopic level. Gravity couples in the same way to all forms of mass-energy, in all bodies, regardless of composition. Such universal coupling makes gravity different from all known forces of nature described by the Standard Model, and is at the heart of the fact that the two theories have so far resisted all attempts at reconciliation into a single unified picture of the physical world. This is the crossroad physics faces at the present time, which is of vital interest not only to theorists, specially given that the nature of about 95% of the matter-energy in the Universe –the so called dark matter and dark energy– is presently unknown.

An experiment capable of testing UFF to an extremely high precision can potentially break this deadlock. The situation is reminiscent of that at the end of the 19th century, when Michelson and Morley tested by very precise light interferometry the propagation of the newly discovered electromagnetic waves through the hypothetical ether[2]. Their null experimental result showed beyond question that although its existence was generally assumed, there was in fact no ether; which led to the special theory of relativity (Michelson was awarded the Nobel prize in 1907).

Should a violation of UFF-WEP be detected, this could be for one of two reasons: either GR is to be amended (possibly reconciling gravity with quantum mechanics) or a new long range composition dependent 'fifth' force of nature is at play. Either would imply a revolution in physics. Such a new force is related also to a time variation of the fine structure constant and could be detected by a very high precision test of UFF[3]. Confirmation of UFF at very high precision would be a landmark for all physical theories to comply with. There is no firm target as to the level at which violation should occur, but the higher the precision of the test, the higher the chances to find new physics. For this reason, over the last century physicists have been improving the precision of UFF tests at every possible opportunity. Rotating torsion balances[4, 5] and Lunar Laser Ranging[6–8] have confirmed UFF-WEP to 10^{-13}, and the Microscope space experiment, in a low Earth orbit since 25 April 2016, is aiming to reach 10^{-15}[9].

The GG satellite experiment[10], proposed to fly on a similar low Earth orbit as Microscope but with a different experiment design (also at room temperature), will test UFF-WEP in the field of the Earth to 10^{-17}, four orders of magnitude better than rotating torsion balances on the ground and two orders of magnitude better than Microscope in space. It is noted that measurements of the gravitational redshift (see [11]) and cold atom drop tests[12] are not competitive[13–15].

Any non-null result from Microscope will be a fundamental discovery which would call for urgent checking. GG will be able to achieve this with a precision one hundred times better.

UFF tests are composition dependent null experiments, which are by their nature among the most precise types of experiments in physics. They could directly detect dark matter through its differential effect towards the center of the galaxy[16]. By comparison, composition independent tests of GR require the absolute measurement of some physical quantity, hence are much less precise. In weak field the best solar system test is at a few times 10^{-5}[17]; in strong field the best binary pulsar tests have reached a few times 10^{-4}[18], and improve with time as more data are collected. The recent detection of a gravitational wave signal produced by merging black-holes[19] opens the era of gravitational wave astronomy. In analyzing the signal as a test of GR the authors state: *"The constraints provided by GW150914 on deviations from GR are unprecedented due to the nature of the source, but they do not reach high precision for some types of deviation, particularly those affecting the inspiral regime. A much higher SNR and longer signals are necessary for* *more stringent tests."*[20]. Calibration uncertainties are at the 10 % level and affect directly the reconstructed strain signal[21]. Not surprisingly, a comparison between composition independent tests of GR and tests of UFF shows the far superior probing power of the latter[22].

Celestial bodies with different non-negligible self-gravitational binding energies allow the property of gravity itself to obey UFF to be tested. Tests of UFF for gravity are performed with Lunar Laser Ranging[6, 7], with pulsar-white dwarfs binary[23] and recently with the triple pulsar data[24]. While GR requires self gravitation to obey UFF, other metric theories of gravity do not, hence these tests can discriminate. However, since self gravitation is a very small fraction of the total mass-energy even for celestial bodies, tests of whether it obeys the UFF are much less precise.

In orbit, a 20-fold better sensitivity than torsion balances to differential accelerations is enough for GG to improve UFF-WEP tests by 10^4([25], Fig. 3). The test bodies are concentric cylinders in both Microscope and GG. Rotation is needed to up-convert the Earth's signal to higher frequency, and this is where the two experiments crucially differ. Microscope's cylinders are sensitive along the symmetry axis, hence rotation must occur around an axis perpendicular to this axis, with the problem that any such axis is unstable to small perturbations. In GG, rotation occurs around the symmetry axis (the only stable axis) while the test cylinders are sensitive in the plane perpendicular to it (the signal also possesses 2 degrees of freedom). The whole spacecraft spins by angular momentum conservation, thus ensuring passive attitude stabilization and signal up-conversion to 1 Hz (3 orders of magnitude higher frequency than Microscope). It is known that at this frequency thermal noise is much lower[26] and the integration time for GG to reach the 10^{-17} target with SNR=2 is only a few hours[27], leaving plenty of time in a 1-year mission to check for systematic errors. Microscope has an integration time of about 1.4 d to reach 10^{-15} with similar SNR.

For the very low thermal noise of GG to be exploited, the readout noise relevant for a target displacement signal of $\simeq 0.6\,\mathrm{pm}$ at 1 Hz must not be a limitation. A heterodyne laser gauge has been designed[28] for which a noise of $\simeq \frac{3\,\mathrm{pm}}{\sqrt{\mathrm{Hz}}}$ at 1 Hz has been measured[29]. The interferometer is similar to the one flown on LISA Pathfinder[30], but much less complex due to the high frequency of the signal, the small separation distance, and no need to stabilize the frequency of the laser.

A 10-fold increase in sensitivity for GG, with the same thermal noise, would require an integration time 100 times longer[27]. Thus, aiming to 10^{-18} would require 15 days –which is still feasible– and by reducing the major systematic errors, the sensitivity of GG could be further improved. A concrete possibility that strengthens the baseline 10^{-17} target.

By respecting the symmetry and physical properties of the problem, issues which are known to strongly limit these experiments, such as the radiometer effect and the spatial offset between the centers of mass of the test cylinders, are radically diminished in GG ([31, 32], [33] Appendix C).

The cylindrical symmetry naturally allows a second pair of equal composition test cylinders to be accommodated in a Russian-doll nested configuration co-centered on the center of mass of the whole system ([34], Figs. 2 and 4) for an extra zero-check in addition to systematic checks. This answers a major criticism expressed by the panel which evaluated GG during the competition for M4[35]. As for the concern of the panel that GR effects due to the proper rotation of the Earth and the test cylinders might compete with the signal, it is found that these effects are orders of magnitude too small to be taken into account, both in Microscope and in GG[36].

Other criticisms raised by the panel[35] are addressed in this proposal as well.

The GG mission has been studied at Phase A2 level with ASI funding. The Study was carried out by Thales Alenia Space (TAS). It includes a high-fidelity software simulator based on the flight proven simulator of GOCE built by TAS for ESA, and relies on the laboratory results of a full-scale demonstrator[10, 33, 37]. The entire data package from the Study is available[38].

The cost of GG is well below the M5 cost cap. A 5-yr development plan (1-yr definition study and 4-yr implementation phase) is proposed, based on a protoflight approach. GG is a science-sat where every piece of equipment is designed for and concurs in the mission scientific objective. The traditional boundary line between service module and payload module hardly applies. For these reasons, the entire development, payload as well as spacecraft, is proposed to be carried out under the responsibility of ESA.

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