# *High precision tests of the weak equivalence principle in space have begun. Much higher precision is within reach.*

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*656th Wilhelm und Else Heraeus-Seminar*
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*The WEP & its relevance to fundamental physics and cosmology*

# *The Weak Equivalence Principle (WEP)*

- *In a gravitational field all bodies fall with the same acceleration regardless of their mass and composition.*
  *WEP also known as the Universality of Free Fall (UFF), or the equivalence of inertial and gravitational mass.*

  **Eötvös parameter quantifying violation:**

  $$\eta = \frac{\Delta a}{a}$$ $a$ **average free-fall acceleration ('driving signal')**

- **No violation found to:**

- $10^{-13}$ in the field of the Earth *Eöt-Wash group/Rotating Torsion Balance (RTB)*
- $\simeq 10^{-13}$ in the field of the Sun *Eöt-Wash/RTB & Lunar Laser Ranging (LLR)*
- a few parts in $10^4$ for self-gravitation energy in the field of the Sun *LLR*
- a few parts in $10^5$ in the field of Dark Matter in our galaxy *Eöt-Wash/RTB*

# *WEP and General Relativity*

- *WEP is the foundation pillar of GR: gravity is a long-range interaction which couples in the same way to all forms of matter-energy*

  Dicke in 1964, after testing WEP for Au and Al analyzed the physical properties of the two atoms in great detail:

  ***. . . "We would conclude that in most physical aspects gold and aluminum atoms differ substantially from each other and that the equality of their accelerations represents a very important condition to be satisfied by any theory of gravitation."***

- **Such universal coupling makes gravity different from all known forces of nature described by the Standard Model of particle physics, and can be tested to very high precision by WEP experiments**

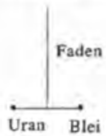
# *Einstein and the equivalence of inertial and gravitational mass (.. an 'experimentum crucis')*

***1912: Einstein writes to Wilhelm Wien***

Mit den besten Grüssen Ihr

A. Einstein.

Post-Scriptum. Es kam mir nachträglich eine viel empfindlichere Methode in den Sinn, um eine nicht genaue Proportionalität der trägen und der schweren Masse von Uran und Blei zu konstatieren, falls es eine solche gibt. Es wäre nämlich in diesem Falle die auf die Körper infolge der Erddrehung wirkende Zentrifugalkraft nicht für alle Körper der Schwere proportional. Die scheinbare Lotrichtung eines Uran-Lotes und eines Blei-Lotes müssten voneinander abweichen. Es müsste ferner eine Drehwage, an deren Balken ein Uranstück




bezw. Bleistück angebracht ist, ein Drehmoment erfahren, wenn der Wagebalken in die West-Ost-Richtung gebracht wird, welches Drehmoment bei Kommutieren der Wage um 180° sein Vorzeichen ändern würde.[3] Dieser Effekt wäre, wie ich mich durch Rechnung überzeugte *ganz bequem* messbar. Vielleicht hätten Sie die Güte, diesen einfachen Versuch ausführen zu lassen, der die Bedeutung eines experimentum crucis hätte.

Post Scriptum. **Subsequently there occurred to me a much more sensitive method for ascertaining an inexact proportionality between inertial and gravitational mass of uranium and lead, if such exists.** For in this case the centrifugal force exerted upon bodies on account of Earth's rotation would not be proportional for all bodies. The apparent plumb directions of a uranium plumb and a lead plumb would have to deviate from each other. Further, **a torsion balance with a piece of uranium and a piece of lead on its beam would have to experience a torque when the beam is brought into the west east direction, with the torque changing its sign when the balance is commutated by** 180°. As I established through calculation this effect should be *quite easy* to measure. Would you, perhaps, be so kind as to have this simple experiment **–which would have the significance of an *experimentum crucis*–** carried out?

*The Collected Papers of Albert Einstein*

# *Einstein and the Eötvös' experiment*

***1916: "The foundation of the General Theory of relativity"***
***§ 2 The need for an extension of the postulate of relativity*.**
*Einstein writes:*

"This view is made possible for us by the teaching of experience as to the existence of a field of force, namely the gravitational field, which possesses the remarkable property of imparting the same acceleration to all bodies.[1)]

Footnote [1)] ***Eötvös has proved experimentally that the gravitational field has this property in great accuracy."***

Diese Auffassung wird dadurch ermöglicht, daß uns die Erfahrung die Existenz eines Kraftfeldes (nämlich des Gravitationsfeldes) gelehrt hat, welches die merkwürdige Eigenschaft hat, allen Körpern dieselbe Beschleunigung zu erteilen.[1)]

1) Daß das Gravitationsfeld diese Eigenschaft mit großer Genauigkeit besitzt, hat Eötvös experimentell bewiesen.

# *Gravitational wave polarization & WEP*

PRD, 2009

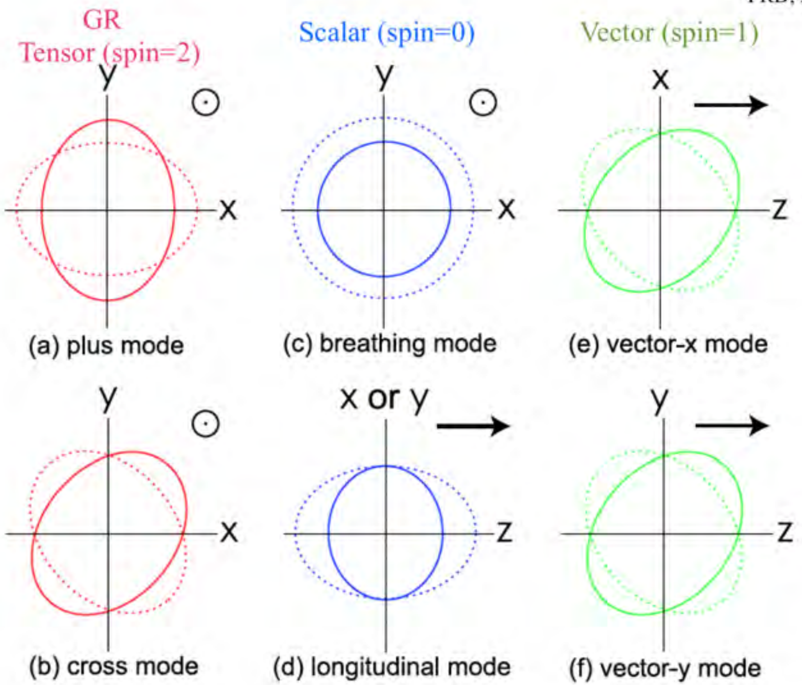



FIG. 1 (color online). Six GW polarizations in a general metric theory of gravitation. The two ellipses (or circles) show the effect of a GW with each polarization on test masses arranged on a circle at the moments of different phases by $\pi$. The circled dot and the arrow represent the propagating direction of the GW.

- ***First Ligo-Virgo coincidence event GW170814 provides information on polarization: preliminary check of tensor-only (GR) vs scalar-only and vector-only. Tensor-only is favored, as expected***

  ***PRL, 6 October 2017***

- ***Evidence of an additional polarization would favor theories of gravity different from GR, but still metric***

  ***→ WEP must hold in all cases***

  ***… but still be violated in the presence of a new long-range interaction …***

# *WEP and Dark Matter*

- *Basic to our understanding of the cosmos is the assumption that the required non luminous DM interacts with ordinary matter only by the gravitational interaction and there is no new long-range interaction*

  $\Rightarrow$ **this assumption should be tested by the most sensitive possible experiments**

- **Do test bodies made of ordinary matter fall with the same acceleration toward DM in our galaxy?**

  *Candidate DM particles are typically new particles, not included in the Standard Model (SM), which would generate a long-range composition-dependent scalar interaction*

- **WEP experiments set limits which a new long-range interaction MUST obey**

  *RTBs rule out an interaction other than gravity between DM and ordinary matter to a few parts in* $10^5$

*Eöt-Wash/RTB: CQG 2012, first test PRD 1993*

# *WEP and Dark Energy*

- *Evidence that the accelerated expansion of the Universe requires the existence of DE is so strong that ESA is building the **Euclid satellite** to establish the nature of DE*
- **A major objective of Euclid is to discriminate between DE as cosmological constant and dynamical DE. Most theories envisage dynamical DE as a new long-range scalar field** (in addition to the pure tensor long-range gravitational field)
- Unless the new field couples only to DM (in which case evidence can be found only at large scale) **its coupling to ordinary matter is subject to test by WEP experiments which can place limits or rule out its existence** $\Rightarrow$

– need for time evolution, or screening mechanisms (e.g. 'chamaleons' ), for the new field to be reconciled with WEP tests

– WEP tests in orbit, unlike those on ground, would avoid screening!

**Microscope can already settle this issue...**

***"Cosmology and Fundamental Physics with the Euclid Satellite", Living Review 2016***

# *WEP and the fine structure constant*

- $\alpha = \dfrac{1}{4\pi\varepsilon_\circ}\dfrac{e^2}{\hbar c}$

  *A time variation of $\alpha$ would imply the existence of a scalar particle which inevitably couples to nucleons, through the $\alpha$ dependence of their masses and therefore mediates a new composition-dependent long-range force*

  $\Downarrow$

  *WEP violation*

- WEP experiments with RTBs and improvements in space can test this effect at levels of interest (according to large scale observations, e.g. absorption lines in high redshift quasars)

***Dvali & Zaldarriaga, PRL 2002***

*How can WEP experiments probe*
*fundamental physics & cosmology so deeply?*

# *Key properties of WEP experiments*

- WEP experiments test interactions for **composition-dependence**, which is a key issue for both fundamental physics & cosmology
- Tests of WEP are **null experiments** (if sensor is differential like signal): no violation, no effect $\rightarrow$ most precise physics experiments
- Tests of WEP **are not absolute measurements**

  $\Rightarrow$ can reach very high precision

  Absolute measurements: a measured signal must be compared with a theoretical prediction to yield the physical quantity of interest (measurements of big $G$, the gravitational redshift, PPN parameter $\gamma$, Lense-Thirring effect ... ). Consider the measurement of gravitational redshift by GP-A in 1976:

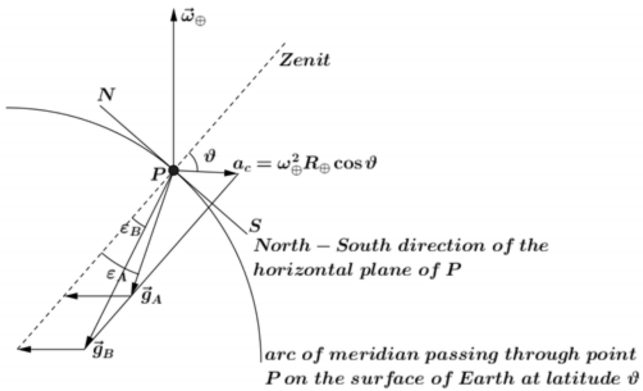
$$\left(\frac{\Delta\nu}{\nu}\right)_{GP-A} = [1 + (2.5 \pm 70) \cdot 10^{-6}] \cdot \left(\frac{\varphi_s - \varphi_e}{c^2} - \frac{|\vec{v}_s - \vec{v}_e|^2}{c^2} - \frac{\vec{r}_{se} \cdot \vec{a}_e}{c^2}\right)$$

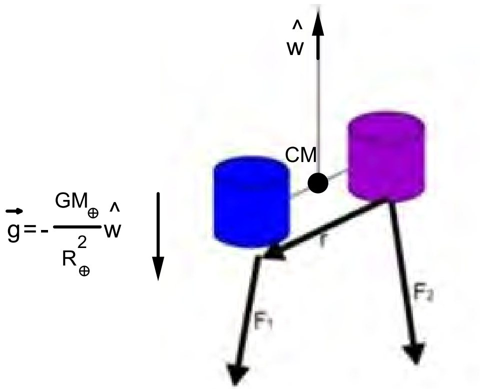
  $\Rightarrow$ hard to reach very high precision and accuracy. The experiment lasted 2 hours, the paper was published 4 years later...

- ***Both on ground and in space WEP experiments are small and controllable***

## *WEP experiments sensitivity: past milestones*

# *Torsion balance versus pendulums & mass-dropping (I)*



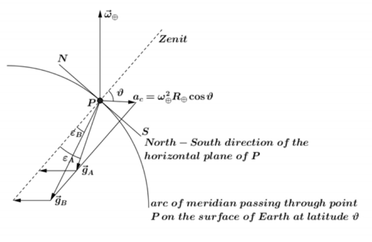

- If WEP is violated plumb lines with bobs of different material are deflected differently toward South
- TB is extremely sensitive (low torsional stiffness, low dissipation) & intrinsically differential **(unlike individual pendulums)**
- TB is sensitive only to forces on the test masses **not parallel** to each other (as in WEP violation)

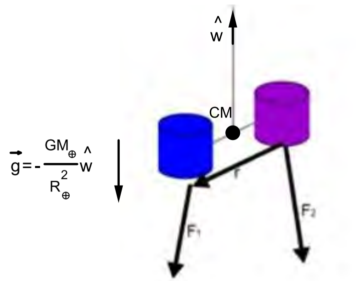
$$T_w = \frac{\vec{r} \cdot \vec{F}_1 \times \vec{F}_2}{|\vec{F}_1 + \vec{F}_2|}$$

$\Rightarrow$ ***can reach relative precisions in the measurement of WEP violating torques much better than its manufacturing tolerances (high rejection of common mode effects)***

*Note that this property holds at 1-g, but is lost at 0-g!*

# *Torsion balance versus pendulums & mass-dropping (II)*




- Thermal noise is lower at higher frequencies

  *Saulson, PRD 1990*

  ⇒ Earth's rotation (passive) makes signal from the Sun at diurnal frequency

  *Dicke; Braginsky*

  ⇒ rotation of the balance (motor & bearings) up-converts the signal from any source body to a higher frequency where thermal noise is even lower

  *Eöt-Wash RTBs*

- Equilibrium position of TB under external torque does not depend on initial conditions
  ... unlike mass dropping tests: **initial condition (release) errors coupled to Earth's gravity gradient mimic a violation signal and are a major limitation (despite a driving signal** 600 **times stronger than for pendulums&TBs)**
  **– mass dropping experiments test WEP only in the field of the Earth**

# *Past milestones (I)*

Progress has come in bursts, each time driven by a new idea/technique:

- Pedulums instead of mass dropping – source body: Earth; signal DC

  ***Galileo & Newton***

- TB instead of pendulums – source body: Earth; signal DC

  ***Eötvös***

- TB in the field of the Sun – signal at $\nu_{day} \simeq 1.16 \times 10^{-5}$ Hz provided by Earth's (passive) diurnal rotation

  ***Dicke & Braginsky***

- Rotating torsion balance (with motor and bearings) – source bodies: Earth, Sun, DM in the galaxy; signal at (or very close to) $\nu_{spin} \simeq 10^{-3}$ Hz
  – **Have reached the level of thermal noise and found it in agreement with theoretical prediction (at room temperature & TB rotation frequency)**
  – Rotation has been found more effective than cryogenics in achieving low thermal noise

  ***Eöt-Wash***

# *Past milestones (II)*

- **End of 19th century. First TB tests of WEP in the field of Earth:** Eötvös improves pendulum tests by more than 3 orders of magnitude to $10^{-8}$ and better... But signal is DC.. checks require manual inversion..
- **1960s - early 1970s. First evidence that up-conversion of signal frequency by rotation is crucial:** Dicke and Braginsky take the Sun as source body and exploit diurnal ('passive') Earth rotation to up-convert violation signal from Sun to diurnal frequency reaching $10^{-11}$ and $10^{-12}$ in the field of the Sun.
- **End of 20th century - 21st century. First use of rotating TBs to test WEP in the field of Earth and the Sun**. Eöt-Wash improves the old Eötvös results in the field of Earth by almost 5 orders of magnitude (to $10^{-13}$) and by almost 1 order of magnitude in the field of the Sun (to a few $10^{-13}$).

  Lunar Laser Ranging tests of WEP for Earth and the Moon in the field of the Sun are at $10^{-13}$ (Self-gravitation binding energies of Earth and Moon in the field of the Sun obey EP to a few $10^{-4}$)

  Mass dropping tests with bulk masses are more than 3 orders of magnitude behind (at about $7 \cdot 10^{-10}$) despite a driving signal from Earth 600 times stronger than on a torsion balance!

  Mass dropping tests with cold atoms are at $10^{-8}$

# Tests of WEP: the milestones

| **Scientists** | **Instrument** | **Source body: Earth** | **Source body: Sun** | **Source body: Dark matter in our galaxy** |
|---|---|---|---|---|
| **Galileo** | Individual pendulums | $\simeq 10^{-3}$ | | |
| **Newton** | Individual pendulums | $\simeq 10^{-3}$ | | |
| **Bessel** | Individual pendulums | $\simeq 10^{-5}$ | | |
| **Eötvös** | Non-rotating torsion balance | $\simeq 10^{-8}$ | | |
| **Pisa&CERN** | Mass dropping (bulk masses) | $\simeq 7 \cdot 10^{-10}$ | | |
| **Lin Zhou et al.** | Mass dropping (cold atoms) | $\simeq 10^{-8}$ | | |
| **Dicke** | Torsion balance (diurnal rotation relative to the Sun; "passive", no motor, no bearings) | | $10^{-11}$ | |
| **Braginsky** | Torsion balance (diurnal rotation relative to the Sun; "passive", no motor, no bearings) | | $10^{-12}$ | |
| **Eöt-Wash** | Rotating torsion balance (with motor and bearings) | $10^{-13}$ | a few $10^{-13}$ | a few $10^{-5}$ |
| **J.G.W. S.G.T./Müller Murphy** | Lunar laser ranging | | $\simeq 10^{-13}$ | |

*WEP experiments sensitivity:*
*the next leaps can occur only in space*

# *A WEP experiment in low Earth orbit (I)*

*Strength of driving signal for WEP experiments on ground and in Low Earth Orbit (in $m\,s^{-2}$)*

| | *Earth's field* | | *Sun's field* | |
|---|---|---|---|---|
| | *Ground* | *LEO* | *Ground* | *LEO* |
| *mass dropping* (*Galileo – like tests*) | $9.8 \xrightarrow{\text{factor 1.2 loss}}$ | $\simeq 8$ | — | — |
| *suspended masses* (*regardless of the suspension type: mechanic, electrostatic, superconducting coils...*) | $\simeq 0.016 \longrightarrow$ | $\simeq 8$ | $\simeq 0.0057$ | $\simeq 0.0057$ |

*factor 600 loss* (9.8 $\to$ $\simeq 0.016$)

*factor 2.8 loss* ($\simeq 0.016$ $\to$ $\simeq 0.0057$)

***factor 500 gain!*** ($\simeq 0.016$ $\to$ $\simeq 8$)

**Only for experiments with suspended masses:**

- ***One major plus: driving signal from Earth*** $\simeq 500$ ***times stronger***

# *A WEP experiment in low Earth orbit (II)*

**Two key advantages for any small force experiment in orbit:**

- Test masses coupling to s/c: weightlessness makes coupling very weak & losses very low
- Local noise: the 'lab' (=dedicated spacecraft) is an isolated system in space:

  – no 'terrain', no terrain tilts, no local microseismicity

**Third key advantage only for WEP expeirments in orbit:**

- Rotation: the whole 'lab' rotates (not possible on ground, motor & bearings needed):

  – rotation totally 'passive' (by angular momentum conservation - GG): no motor, no bearings
  – controlled rotation (Microscope): thrusters & propellant but no bearings (because there is no stator in space, entire 'lab' spins with TMs..

*Conditions "sine quibus non"*
*for a very high precision test of WEP in low Earth orbit*

# *Testing WEP in space: two major issues to deal with (1)*

- ***Non-gravitational effect of residual air-drag & solar radiation pressure on the outer surface of the spacecraft***

  – huge non-gravitational acceleration compared to target signal and competing with it (in GG with $\eta = 10^{-17}$ it is more than 9 orders of magnitude bigger than signal)
  – results in an equal & opposite inertial acceleration on TMs, 'ideally' the same' (common mode)
  $\Downarrow$
  – to reach very high precision **it must be compensated the hard way** (by Drag-Free Control, with thrusters and propellant) & **rejected by the TMs themselves (learn from the TB...)**

*Note: non-gravitational acceleration on LISA pathfinder in $L_1$ only about a factor 4 lower than for spacecraft to test WEP in low Erath orbit (you cannot turn off the Sun...)*

# *Testing WEP in space: two major issues to deal with (2)*

- ***Earth tidal effects (gravity gradients)***

– classical differential accelerations between the TMs competing with the signal
– **linear with the offsets between the centers of mass of the TMs**

$\Rightarrow$ the TMs must be concentric ('ideally' ...)

$\Rightarrow$ **General agreement: TMs for testing WEP in space should be concentric, co-axial cylinders**
(a cylinder can be made dynamically close to a sphere if needed..)

# *A high precision test of WEP in orbit requires...*

- A dedicated spacecraft capable of drag compensation
- TMs in the form of **concentric, coaxial cylinders**
- Capability of the TMs to reject common mode effects
- TMs weakly coupled to spacecraft (low $\omega_n$): weak coupling means high sensitivity
  - TMs must NOT be free: initial condition (release) errors too large

    ***Blaser CQG, 2001, Nobili et al. GRG, 2008***
  - Restoring force needed
- **Rotation ($\omega_{spin}$ the higher the better)** to up-convert the signal to higher frequency (where thermal and other sources of noise are known to be lower...)
  - *Cylinders rotating with $\omega_{spin} > \omega_n$ do self-center (better than they were by construction) if they spin around the symmetry axis and are weakly coupled in the plane $\perp$ to it.*

    ***Theory of rotordynamics, GGG lab demonstrations***

*Cylinders rotating with $\omega_{spin} > \omega_n$ do self-center (better than they were by construction) if they spin around the symmetry axis and are weakly coupled in the plane $\perp$ to it.*

***Theory of rotordynamics, GGG lab demonstrations***

*First milestone:*

*Microscope in orbit to test the WEP to* $10^{-15}$ *in the Earth's field*

# *Microscope: first milestone of WEP testing in space*

- A dedicated spacecraft capable of drag compensation *ok*
- TMs in the form of **concentric, coaxial cylinders**

  *Offset errors by construction are not reduced in orbit; partial reduction of their effects is done by a posteriori data analysis of main tidal effect*

- Capability of the TMs to reject common mode effects

  *Test cylinders are suspended individually. Common mode accelerations are reduced by matching their sensitivities with inflight calibrations*

** TMs weakly coupled to s/c: weak coupling, high sensitivity

*Very weak coupling, very high sensitivity along symmetry axis*

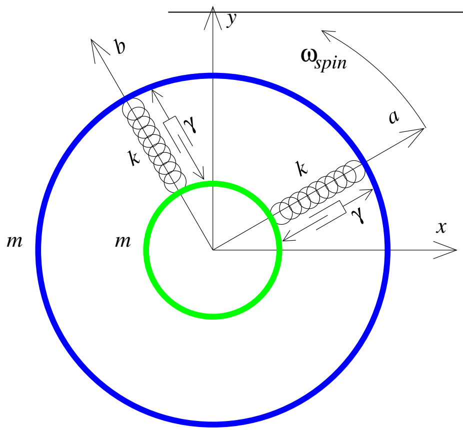
*Electrostatic spring is negative (unstable) $\rightarrow$ restoring force provided by active control*

** **Rotation ($\omega_{spin}$ the higher the better)** to up-convert the signal to higher frequency

*Rotation around axis $\perp$ to symmetry axis of test cylinders $\rightarrow$*

*Rotation of s/c actively controlled, requires thrusters and propellant but no bearings. Limited spin rate*

# *For next milestone: fulfill all requirements for a WEP test in orbit*




Test cylinders spinning around symmetry axis weakly coupled in the plane $\perp$ to it ($\omega_n = \sqrt{k/\mu}$)

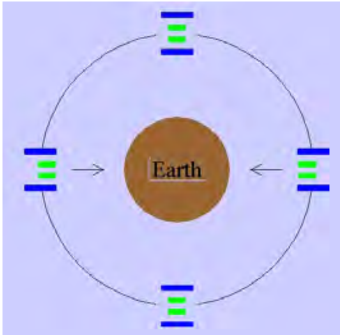
- Mechanical coupling (negative spring) provides needed restoring force. Response to a low frequency force (like violation signal): $\vec{r} \simeq \frac{\vec{F}}{k}$
- $\omega_{spin} \gg \omega_n$ ensures good reduction of offset errors by construction: offsets between the centers of mass are reduced as $\left(\frac{\omega_n}{\omega_{spin}}\right)^2$
- Signal up-converted to high spin frequency where thermal noise is much lower $\rightarrow$ very short integration time

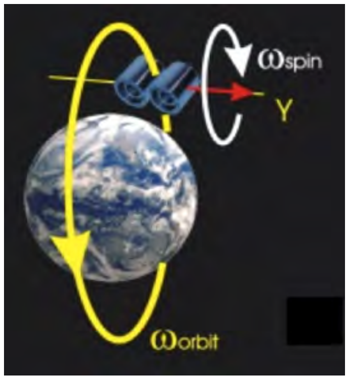
  *Saulson PRL 1990, Eöt-Wash RTBs results Pegna et al. PRL 2011, Nobili et al. PRD 2014*
- Rotation around symmetry axis & cylindrical symmetry allow s/c passive 1-axis attitude stabilization (by conservation of angular momentum after initial spin-up)
- Coupling can be arranged to allow inflight adjustments for high common mode rejection

***NOTE: Mechanical coupling is stiffer than electrostatic coupling $\Rightarrow$ lower sensitivity (smaller displacement produced by target signal ): not a problem if read-out is good enough to detect it (laser interferometry read-out to replace capacitance read-out)***

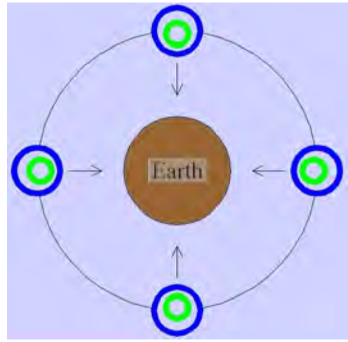
# *The change for next milestone at a glance*

***Microscope***



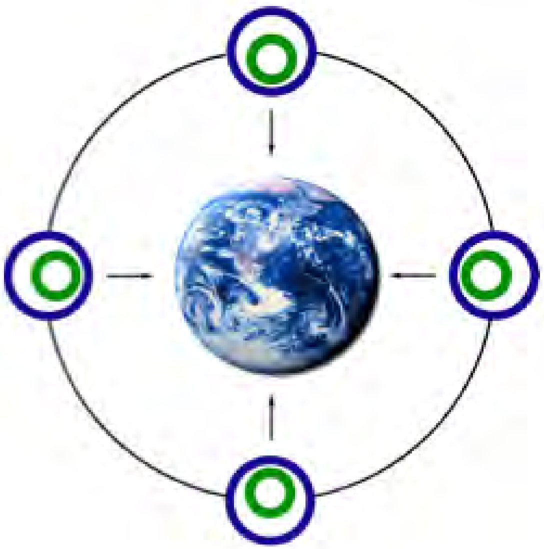



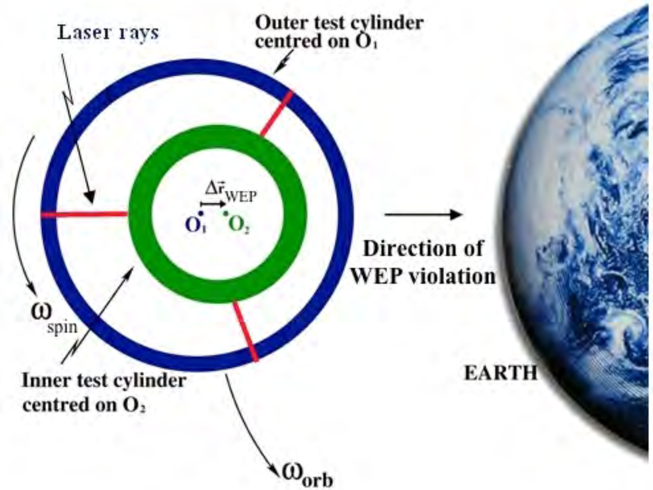

***Galileo Galilei–GG***




*GG in the M5 competition of ESA*
*aiming to test the WEP to* $10^{-17}$ *in the Earth's field*
*(10000 gain over RTBs: 500 comes for free in orbit, 20 must be gained)*

# *GG: the signal and its up-conversion to high frequency*






***If the test cylinders fall with different accelerations towards the Earth (WEP violation) there is a relative displacement vector pointing to the Earth's center of mass: violation signal is at orbital frequency***
$\nu_{orb} \simeq 1.7 \times 10^{-4}\,\mathrm{Hz}$ $(P_{orb} \simeq 5800\,\mathrm{s})$

***The violation displacement vector $\Delta\vec{r}$ points to the Earth's center of mass with frequency $\nu_{oorb}$. If the test cylinders spin around the symmetry axis, together with the laser gauge, the violation signal is read at $\nu_{WEP} = \nu_{orb} - \nu_{spin} \simeq \nu_{spin} \simeq 1\,\mathrm{Hz}$ $(P_{spin} \simeq 1\,\mathrm{s})$***
***Demonstrated by RTBs .. but $\nu_{spinTB} \simeq 10^{-3}\,\mathrm{Hz}$***

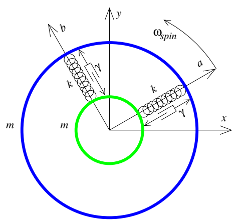
# *The equations and the solution in a nutshell...*

- **Equations of motion (non rotating frame):**

  $\mu\ddot{\vec{r}} + \gamma_{\omega_{spin}}(\dot{\vec{r}} - \vec{\omega}_{spin} \times \vec{r}) + k\vec{r} = \vec{F}$

  $\gamma_{\omega_{spin}}$ small internal damping at $\nu_{spin}$, related to loss angle $\phi_{\omega_{spin}}$ (inverse of quality factor):

  $\phi_{\omega_{spin}} \simeq \frac{\gamma_{\omega_{spin}}\omega_{spin}}{\mu\omega_n^2} = \frac{\gamma_{\omega_{spin}}\omega_{spin}}{k}$




- **There is a non-zero offset error by construction of the reduced mass from the spin axis: $\vec{\epsilon}$ (fixed on the rotor)**

- **General solution (in the non rotating frame):**

$$\vec{r}(t) \simeq -\vec{\epsilon}(\omega_{spin}t)\left(\frac{\omega_n}{\omega_{spin}}\right)^2 +$$

$$\frac{\vec{F}}{k} - \phi_{\omega_{spin}}\frac{\vec{\omega}_{spin}}{\omega_{spin}} \times \frac{\vec{F}}{k} +$$

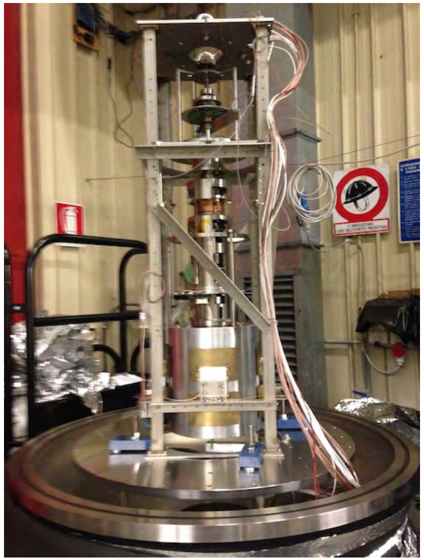
$$+A_0 e^{\phi_{\omega_{spin}}\omega_n t/2}\begin{pmatrix}\cos(\omega_n t + \varphi_A)\\ \sin(\omega_n t + \varphi_A)\end{pmatrix} +$$

$$+B_0 e^{-\phi_{\omega_{spin}}\omega_n t/2}\begin{pmatrix}\cos(-\omega_n t + \varphi_B)\\ \sin(-\omega_n t + \varphi_B)\end{pmatrix}$$

– **How do the equations & their solution tell us that the system is in orbit around the Earth?**

*They don't (precisely because in orbit the system is weightlessness conditions) until gravity gradients (tides) and other perturbations are introduced, which can all be added linearly*

# *Self-centering measured (I)*



**With spin frequency higher than normal mode frequency ($\omega_{spin} > \omega_n$ , super-critical rotation) & 2D: the offset error by construction $\vec{\epsilon}$ is reduced by physics as $\omega_n^2/\omega_{spin}^2$ (self-centering):**

$$\vec{r}(t) \simeq -\epsilon\left(\frac{\omega_n^2}{\omega_{spin}^2 - \omega_n^2}\right)\begin{pmatrix} \cos(\omega_{spin}t + \varphi) \\ \sin(\omega_{spin}t + \varphi) \end{pmatrix} \simeq -\epsilon\left(\frac{\omega_n^2}{\omega_{spin}^2}\right)\begin{pmatrix} \cos(\omega_{spin}t + \varphi) \\ \sin(\omega_{spin}t + \varphi) \end{pmatrix}$$

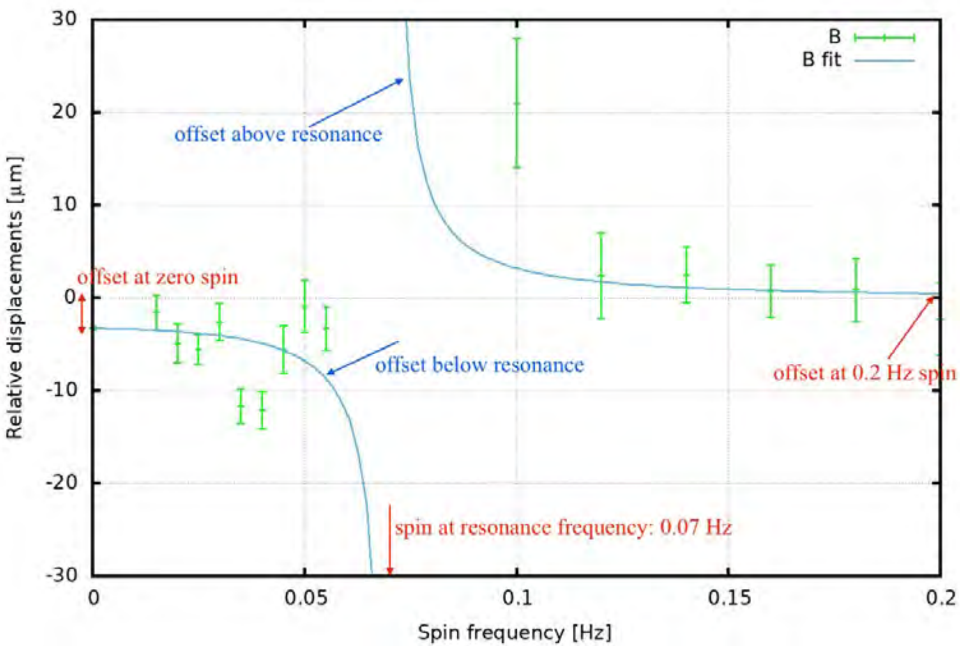
***Measured in the lab with GGG (GG on Ground) demonstrator: test cylinders spinning above the resonance are better centered on one another than they were by construction***

# *Self-centering measured (II)*

**With spin frequency higher than normal mode frequency ($\omega_{spin} > \omega_n$ , super-critical rotation) & 2D: the offset error by construction $\vec{\varepsilon}$ is reduced by physics as $\omega_n^2/\omega_{spin}^2$ (self-centering):**

$$\vec{r}(t) \simeq -\varepsilon \left( \frac{\omega_n^2}{\omega_{spin}^2 - \omega_n^2} \right) \begin{pmatrix} \cos(\omega_{spin}t + \varphi) \\ \sin(\omega_{spin}t + \varphi) \end{pmatrix} \simeq -\varepsilon \left( \frac{\omega_n^2}{\omega_{spin}^2} \right) \begin{pmatrix} \cos(\omega_{spin}t + \varphi) \\ \sin(\omega_{spin}t + \varphi) \end{pmatrix}$$

***Demonstrated by GGG prototype in the lab: test cylinders spinning above the resonance are better centered on one another than they were by construction***




**Offset between GGG test cylinders as the spin frequency increases from below the resonance, through the resonance and above the resoance: self-centering above the resonance is apparent. Experimental data agree with theoretical prediction**

# *Whirl motion and time constant of whirl growth*

- **Whirl motion occurs because of dissipation at the spin frequency. Except in cases of high dissipation, the whirl frequency (in the non-rotating frame) is the same as the natural frequency of the system when it does not spin:**

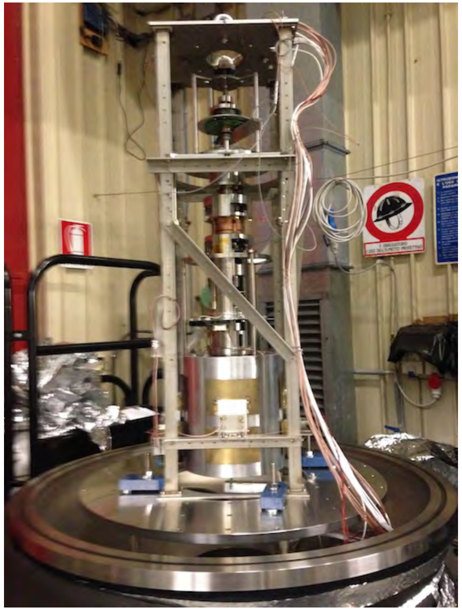
$$\vec{r}_w(t) \simeq A_0 e^{+t/\tau} \begin{pmatrix} \cos(\omega_n t + \varphi_A) \\ \sin(\omega_n t + \varphi_A) \end{pmatrix}$$
$$\simeq A_0 \Big(1 + \frac{t}{\tau}\Big) \begin{pmatrix} \cos(\omega_n t + \varphi_A) \\ \sin(\omega_n t + \varphi_A) \end{pmatrix}$$

– The time constant $\tau$ of whirl growth is proportional to the quality factor, inversely proportional to dissipation, at the spin frequency (dissipation is low at high frequency):

$$\tau = \frac{2Q_{\omega_{spin}}}{\omega_n} = \frac{Q_{\omega_{spin}}}{\pi} P_n \simeq 9.5\,\mathrm{d}$$

– Whirl is damped with capacitors as sensors & actuators (small forces required) and science data are taken in between successive damping, with whirl damping off (no disturbances on test masses)

# *Whirl motion damped*



***On ground, with GGG demonstrator:***

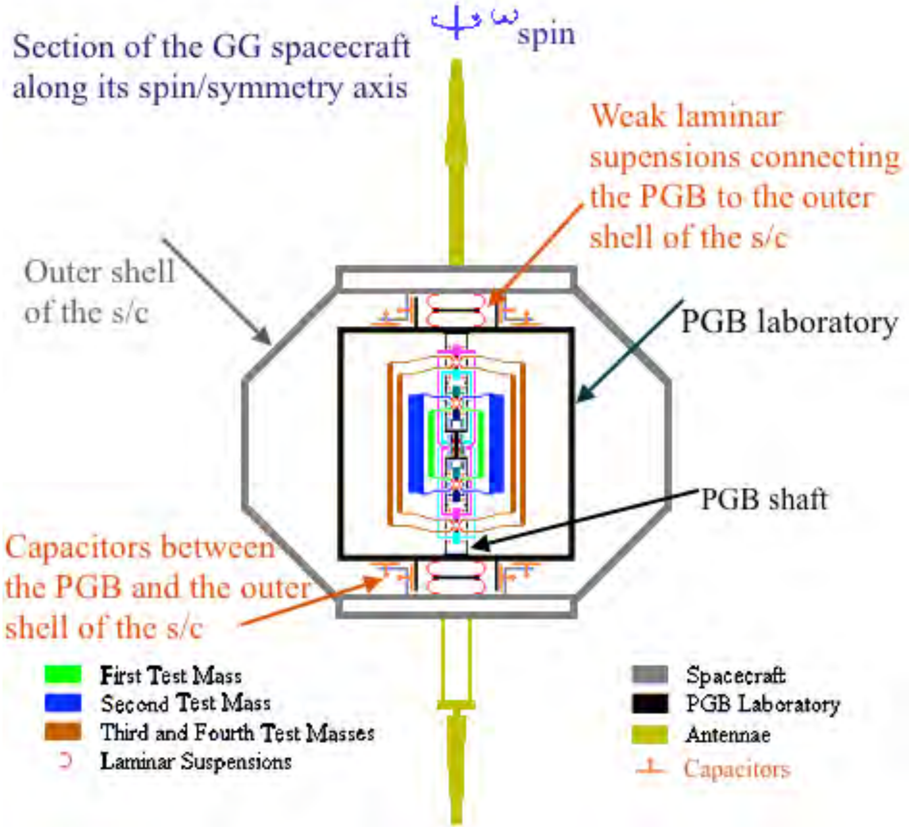
– suspensions stiffer than in space $\rightarrow$ shorter $P_{n_{GGG}}$
– motor & bearings noise (absent in space) $\rightarrow$ slower spin rate $\nu_{spin_{GGG}}$
– slower $\nu_{spin_{GGG}}$ $\rightarrow$ higher dissipation, lower $Q_{\nu_{spinGGG}}$

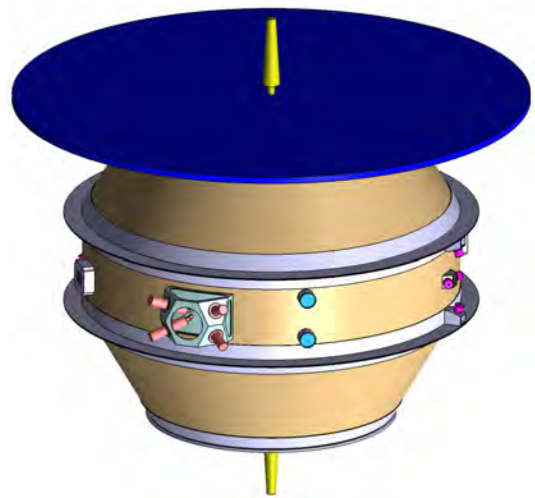
$\Rightarrow$ the time constant of whirl growth for GGG test cylinders is much shorter:

$$\tau_{GGG} = \frac{Q_{\nu_{spinGGG}}}{\pi} P_{n_{GGG}} \simeq 2.5\,\mathrm{h}$$

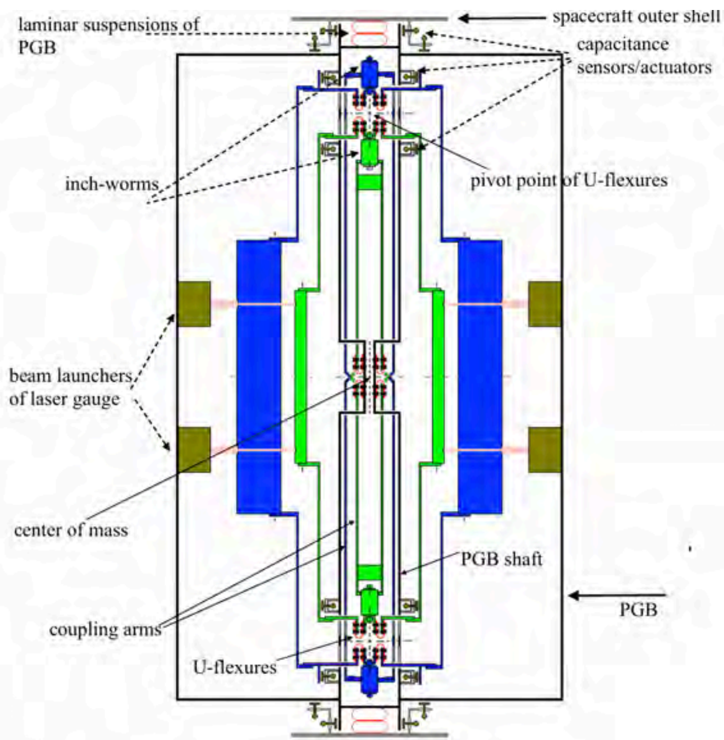
$\Rightarrow$ **whirl motion has been damped all the time during ground runs (lasting up to 1 month..)**

# GG at a glance
# with two concentric pairs of test cylinders



## *For high precision test of WEP fly a balance, not individual cylinders..*




- Effect of drag huge compared to signal but 'ideally' common mode → can be rejected by a balance
- Balancing the balance against drag much easier than balancing a balance on ground against 1-$g$ (because drag is orders of magnitude smalelr than $g$)
- PZT actuators (inch-worms) very effective in adjusting the balance arms until the spurious differential effect of drag is minimized
- Precision measurements made possible by balancing (much better than construction tolerances!!!)

(Balance animation: local)
(Balance animation: online)

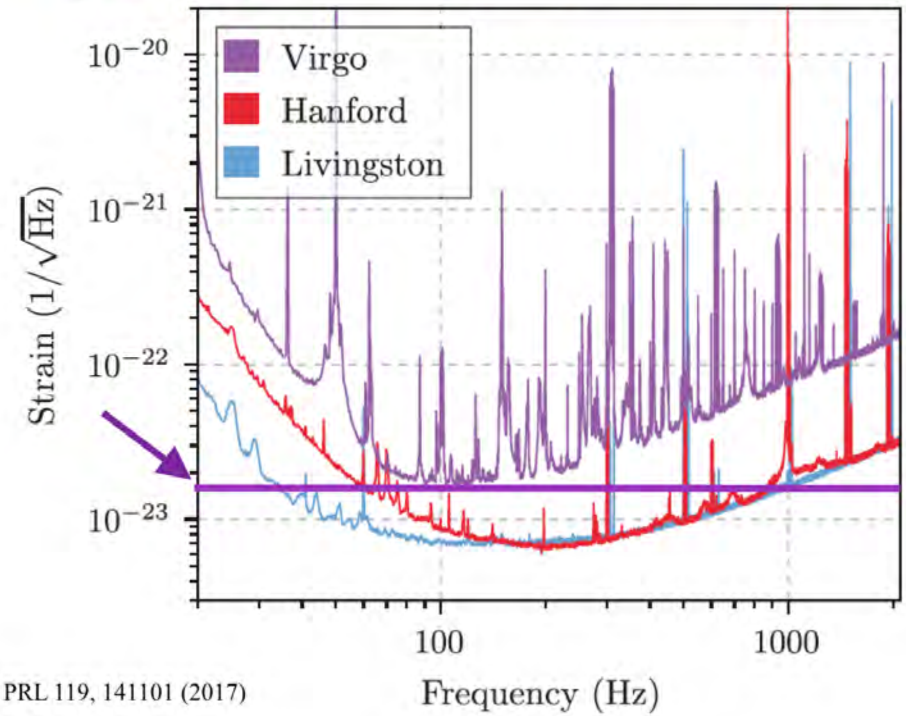
*Thermal noise: the ultimate limit*
*Thermal noise, integration time,*
*time available to check for systematic errors*

# *Thermal noise from internal damping: at which frequency?*

$$|\hat{F}_{th}(\omega_{signal})|^2 = 4k_BT(\mu\omega_n^2)\frac{\phi(\omega_{signal})}{\omega_{signal}} \qquad [\mathrm{N}^2/\mathrm{Hz}]$$

$$t_{int} = SNR^2 \cdot \frac{|\hat{F}_{th}(\omega_{signal})|^2}{F(\omega_{signal})^2}$$

Virgo: 3km-arm interferometer, metallic suspension wires, 40 kg mirrors:
@ 100 Hz displacement noise is about 6xe-20 m/root(Hz)




PRL 119, 141101 (2017)

# *Thermal noise from internal damping: evidence from rotating torsion balances*

Class. Quantum Grav. **29** (2012) 184002 T A Wagner *et al*

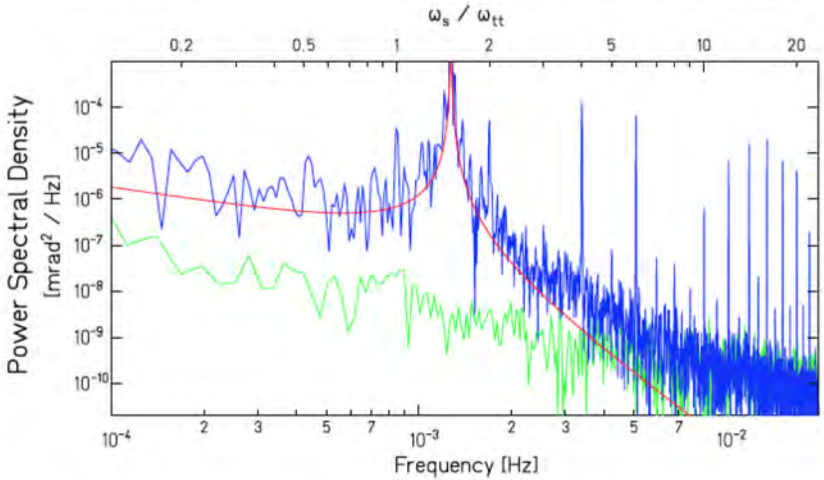



**Figure 4.** Power spectral density of the twist signal. The upper (blue) histogram shows WEP data taken with $\omega_{tt}/\omega_0 = 2/3$. The curve is the thermal noise predicted in equation (7) for a room-temperature oscillator with $Q = 6000$. The peaks at integer multiples of $\omega_{tt}$ arise from reproducible variations in $\omega_{tt}$ (see equation (9)). The small peak at $\omega_{tt}/2$ is caused by the turntable leveling system that recomputed the tilt every two turntable rotations [12]. The lower (green) histogram displays data taken with the turntable stationary and the pendulum resting on a support to show the readout noise. The low-frequency readout noise is ascribed to thermal fluctuations.

- With TB rotating at $\simeq 10^{-3}$ Hz the Eöt-Wash group has achieved: $\eta_{\oplus} = 10^{-13}$
- Improvement over the best test in the field of Earth by Eötvös (zero spin, DC signal): 5 orders of magnitude!!!
- They reached the thermal noise limit and found it in agreement with the theoretical prediction

# *Thermal noise from internal damping in GG: signal frequency up-conversion is the key*

If GG were not spinning $\Rightarrow$ signal at $\omega_{orb} \simeq \frac{2\pi}{6000\,\mathrm{s}}$

$$t_{int-zerospin} = SNR^2 \cdot \frac{4k_BT(\mu\omega_n^2)}{(\mu g_h \eta)^2} \cdot \frac{1}{Q_{\omega_{orb}}\omega_{orb}}$$

GG spinning at $\nu_{spin} = 1\,\mathrm{Hz} \Rightarrow$ signal at $\nu_{spin} \pm \nu_{orb} \simeq \nu_{spin} = \frac{2\pi}{1\,\mathrm{s}}$

$$t_{int-1Hz} = SNR^2 \cdot \frac{4k_BT(\mu\omega_n^2)}{(\mu g_h \eta)^2} \cdot \frac{1}{Q_{\omega_{spin}}\omega_{spin}}$$

$\Downarrow$

$$\frac{t_{int-1Hz}}{t_{int-zerospin}} \simeq \cdot \frac{Q_{\omega_{orb}}}{Q_{\omega_{spin}}} \cdot \frac{1}{6000} \ll 1$$

also because $Q_{\omega_{orb}} \ll Q_{\omega_{spin}}$

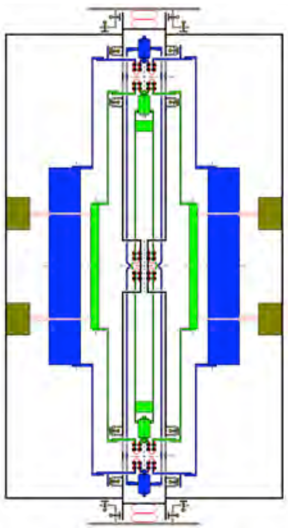
- Also, Johnson noise ($\mu$-metal shield of $B_\oplus$ by 150-200) & gas damping noise (avoid too small gaps. In GG, 2 cm gap feasible with laser gauge readout)
- Overall, SNR=2 requires only a few hours: 1 full measurement/day planned. **Plenty of time left for checking systematics**

— ***PRL 107, 200801 (2011)***
***Phys. Rev. D 89, 042005 (2014)***

*With very low thermal noise GG needs a very low-noise readout*

*(signal at* 1 Hz *makes life a lot easier...)*

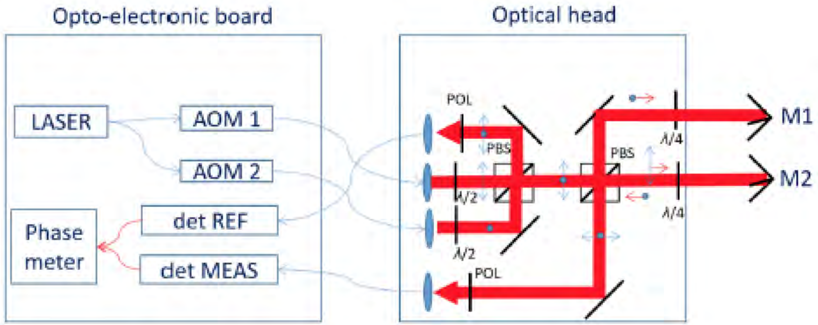
# *A laser gauge: the ideal readout to replace capacitive sensors*

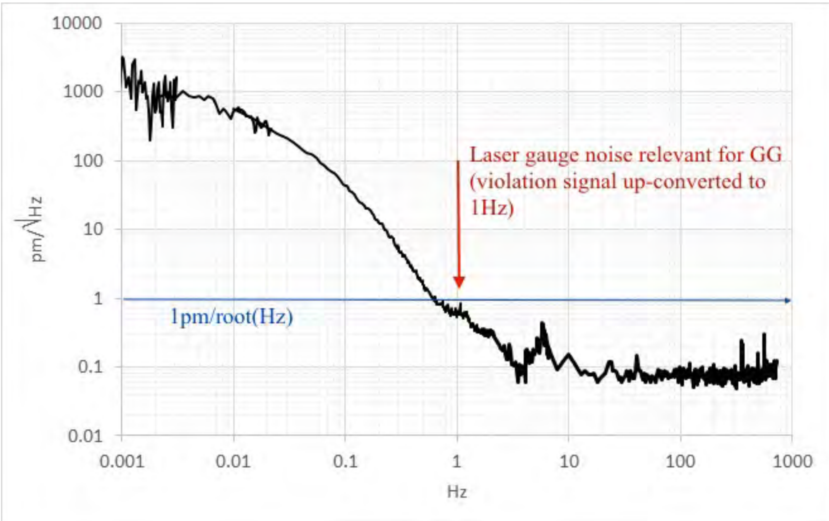


Heterodyne laser interferometer to read relative displacements of test cylinders & recover violation signal

- Inherently differential measurement
- No calibration needed (displacement given in terms of laser wavelength)
- No limitation from size of gap between cylinders (gas damping noise relevant if gap is small: $C \propto \frac{1}{d}$)
- Very low noise
- No laser frequency stabilization needed in GG, interferometer far less demanding than the one flown on LISA-PF
- Lesson from LISA-PF: interferometer noise measured in space lower than on ground (further evidence that lab environment in space much more quiet than on ground)

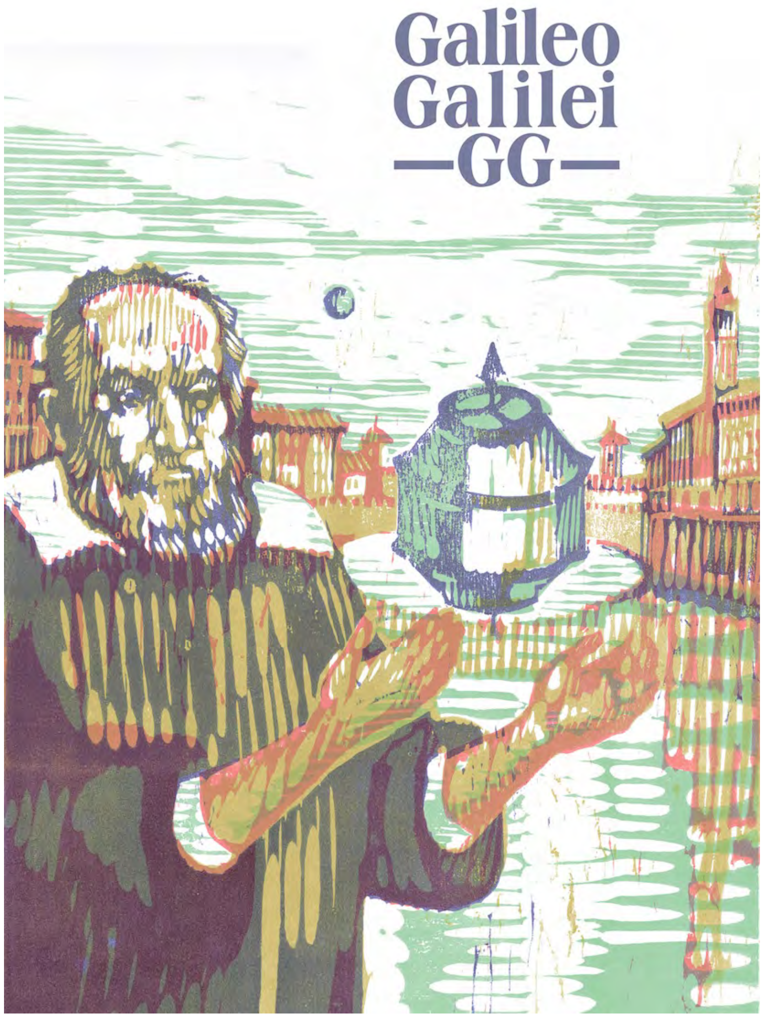
# *The laser gauge for GG at INRIM: measured displacement noise*




- GG: violation signal up-converted to 1 Hz
- Laser gauge displacement noise measured at INRIM: $0.6\,\frac{\mathrm{pm}}{\sqrt{\mathrm{Hz}}}$ @ 1 Hz (low frequency noise due to optical fibers, can be reduced)
- ... noise is 1/3 of target signal after only 10 s of integration time!!!

***Thanks to ESA for funding...***




- *GG has passed the first round of selection for next M5 mission of ESA. If successful:*

  - **GG will test the WEP to $10^{-17}$ in the field of Earth**
  - **Will test DM coupling to ordinary matter other than gravitationally 20 times better than now**
  - **Will improve current WEP tests in the field of the Sun by 20 times**

GG webpage: http://eotvos.dm.unipi.it