Galileo Galilei−GG: Post ESA-M5 Interview Note

submitted by the interviewed scientists
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on behalf of the GG team

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Premise: We take the opportunity kindly offered by Olivier Grasset, Co-Chair of SARP-M5 panel, to Suresh Doravari −after he had to interrupt him because of lack of time− to submit a written Note. Sec. I concerns the point Suresh was trying to make, by describing how the violation signal is recovered undiminished after up-conversion from orbital to spin frequency, while any noise on the rotor with a component at spin frequency (such as creep noise) would be attenuated by the weak suspensions. Sec. II compares the required force sensitivity and the load in GG to those of other ground based detectors, and Sec. III addresses the use of cold gas thrusters at 1 Hz, both issues raised during the Interview.

I. 2D VERSUS 1D OSCILLATORS

The dynamical system of two concentric test cylinders which constitute the GG differential accelerometer is, in essence, a mechanical oscillator with 2 degrees of freedom in the plane perpendicular to the spin/symmetry axis (PRL 2011, Fig. 1).

The natural (resonance) frequency of the oscillator is $\nu_{n_{GG}} \simeq 1.9\,\mathrm{mHz}$ (stiffness $k \simeq \mu\omega_n^2$, reduced mass μ) while the rotation frequency is $\nu_{spin_{GG}} \simeq 1\,\mathrm{Hz}$. In this super-critical regime each cylinder spins around its own axis and the two axes are very well centered on each other, any offset by construction (fixed on the rotor by definition) being reduced by the factor $\chi_{selfcentering} \simeq (\nu_{n_{GG}}/\nu_{spin_{GG}})^2 \simeq 1/2.9 \times 10^5$ (PRL 2011, Eq. (21), first term; measured with the GGG demonstrator in the lab: see Bremen talk 2017, slide 34). An intuitive way to imagine these two weakly coupled cylinders in rapid rotation is to consider that a rotor in super-critical rotation is the closest possible approximation to an ideal free rotor, whose rotation axis would obviously pass through its center of mass.

If one cylinder is attracted by the Earth more than the other, because the Weak Equivalence Principle (WEP) is violated or because there is a new composition-dependent force, then, as the GG satellite orbits the Earth at $\nu_{orb_{GG}} \simeq 0.17\,\mathrm{mHz}$, there is a differential force between the two cylinders $\vec{F}_{WEP_{GG}}$ pointing to the Earth's center of mass, of constant size (for a circular orbit), hence changing direction at frequency $\nu_{orb_{GG}}$.

Being at a frequency below the resonance, this force is not attenuated by the oscillator, which responds with a displacement vector between the centers of mass of the two cylinders $\vec{r}_{WEP_{GG}} \simeq \vec{F}_{WEP_{GG}}/k$ (PRL 2011, Eq. (21), second term), also an Earth-pointing vector of constant size ($\Delta r_{WEP_{GG}} \simeq 0.6\,\mathrm{pm}$), which is what we measure with the laser gauge. Since the laser gauge co-rotates with the system at 1 Hz, it simply reads this displacement at frequency $\nu_{WEP_{GG}} \simeq (\nu_{spin_{GG}} \pm \nu_{orb_{GG}}) \simeq \nu_{spin_{GG}} \simeq 1\,\mathrm{Hz}$. Thus, in GG the low frequency force signal is not attenuated because it is below the resonance; nonetheless it is up-converted by a very large factor to 1 Hz, and this is a well known advantage.

An experimental test carried out with GGG in the lab has demonstrated that a low frequency signal in the non rotating frame is up-converted to the (higher) spin frequency without attenuation (CQG 2012. Fig 2).

It is therefore apparent that in GG there is no "physical" sensitive axis flipping every half a second: the centers of mass of the two cylinders have the liberty to move in the sensitive plane so that their relative displacement vector $\vec{r}_{WEP_{GG}}$ is always Earth-pointing like the differential force $\vec{F}_{WEP_{GG}}$ which generates it.

Other oscillators, such as the rotating torsion balance, the Virgo/LIGO mirrors and the Microscope test cylinders, are all 1D oscillators. This uniqueness of GG sometimes gives rise to misunderstandings.

The torsion balance has a natural (resonance) frequency $\nu_{n_{TB}} \simeq 1\,\mathrm{mHz}$. The signal of WEP violation from Earth is fixed in the North-South direction. In order to up-convert it from zero to higher frequency, you must *physically* rotate the arm of the balance in the horizontal plane of the lab, by placing the balance on a turntable rotating at $\nu_{tt_{TB}}$. As a result, the signal flips every half rotation period passing through two zero values every period. This is a 1D mHz oscillator *forced* at the frequency $\nu_{tt_{TB}}$ of the turntable. It is well known that if the forcing frequency is below the resonance, the forcing term is not attenuated, while above resonance it is attenuated as $\chi_{attenuation_{TB}} \simeq (\nu_{n_{TB}}/\nu_{tt_{TB}})^2$. The Eöt-Wash balance rotates at $\nu_{tt_{TB}} < \omega_{n_{TB}}$ so that the signal is not attenuated.

The Virgo/LIGO mirrors are pendulums with $\nu_{n_{mirror}} \simeq 0.5\,\mathrm{Hz}$. For the tiny ripples of spacetime caused by a gravitational wave passing by to show up above residual noise, the mirrors must be maintained as much as possible as *undisturbed test masses*. It is the job of the multi-stage superattenuators to kill local seismic noise for each mirror/pendulum. And they do so very effectively above about 10 Hz, because any seismic noise term at frequency $\nu_{seismic} \gg \nu_{n_{mirror}}$ will be attenuated by each stage as $\chi_{attenuation_{mirror}} \simeq (\nu_{n_{mirror}}/\nu_{seismic})^2$; the farther the disturbance frequency is from the pendulum frequency, the better its effect is attenuated. With a pendulum frequency of about 0.5 Hz, it is not surprising that the sensitivity plots of Virgo/LIGO typically start only at 10-20 Hz, never at 1 Hz.

It is worth noting that any noise other than seismic noise, with frequency above the resonance, will be attenuated too; attenuation is a property of the dynamical system, not of the forces it is subjected to. This is the case of creep noise in the suspensions, which was mentioned during the Interview as a possible noise that in GG might turn out to be larger than thermal noise.

Creep has been investigated by Virgo scientists over the last 20 years. Each mirror is suspended from a multistage system for noise attenuation. Each mechanical stage supports the weight of the stages below it by means of a set of steel cantilever blade springs. The stress from the load acting on the blades has been found to induce a drooping of the blade tips of several microns per day, due to a series of microscopic yielding events known as micro-creep.

Anybody who has visited Virgo has certainly been impressed by the superattenuator towers, and immediately gets an idea of the huge weight to be sustained. Nonetheless, Virgo scientists have been able to reliably model the effect of creep, taking into account the attenuation factor provided by the system (Cagnoli et al., 1997, Eq. (12)), and to find appropriate material and treatments to reduce the onset of creep (Beccaria et al., 1998).

While discussing creep noise with Suresh two days after the GG Interview, on 9 November 2017 at Nikhef, in Amsterdam, Alessandro Bertolini has shown the results of an extensive experimental work, published in 2008 (Virdone et al., 2008), in which they were able to measure creep over an artificially extended period of time and finally produced a simple procedure capable of eliminating all its effects from the suspensions of advanced LIGO.

Deviations from elasticity have been investigated by Riccardo De Salvo, who has played a major role in the realization of Virgo, LIGO and TAMA noise attenuators and is a supporting scientist of GG too; he has suggested that some glassy metals might be considered because they are dislocation-free and would not show such deviations (De Salvo et al., 2011).

More recently LIGO scientists at Caltech in the group of Rana Adhikari, also a supporting scientist of GG, have noticed that “the response of elastic materials to external changing conditions can proceed through small and discrete releases of stress, rather than a continuous and smooth deformation as described by the classical elasticity theory. In a macroscopic elastic body, the sum of all those small crackling events can create a detectable displacement noise (crackling noise)” (Vajente, 2017). At Caltech they have even designed an instrument to measure crackling noise down to $10^{-15}\,\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$ starting from 10 Hz up to 1000 Hz (Vajente et al., 2016)).

The bottom line of all this work is that at the present level of sensitivity of the LIGO/Virgo detectors, which has made possible the recent astonishing detection of gravitational wave signals, there is no sign of creep or crackling noise in the range between 100 and 2000 Hz.

No creep or crackling noise has ever been reported, to our knowledge, for WEP tests with rotating torsion balances. We guess that this may be due to the much lower load as compared to LIGO/Virgo (Sec. II).

How can we reliably translate the LIGO/Virgo results at 100 Hz into GG at 1 Hz?

Of course attenuation of effects at frequencies above the resonance occurs also in GG, in which case the natural (resonance) frequency $\nu_{n_{GG}} \simeq 1.9\,\mathrm{mHz}$ is 250 times lower than in LIGO/Virgo. As a result, should creep noise occur in the GG suspensions at 1 Hz, it would be largely attenuated as $(\nu_{n_{GG}}/1\,\mathrm{Hz})^2 \simeq 1/2.9 \times 10^5$, because of being very far away from the resonance frequency (we note in passing that this factor is the same as the factor $\chi_{selfcentering}$ which makes the two cylinders selfcenter by reducing the original offset by construction!). Again, this is the case not just for creep noise, but for any other noise source that might occur on the suspensions at the GG up-converted signal frequency of 1 Hz.

The final question usually asked at this point, which we have also asked ourselves since when GG was conceived is: if 1 Hz noise is so effectively killed, why is the violation signal not killed too? The answer is the following. The violation signal is at the orbital frequency in the non rotating (inertial) frame as the satellite orbits around the Earth, and is read at (up-converted to) 1 Hz by the rotating readout. Note that this is the case also for the main component of air drag at the orbital frequency, as well as for other low frequency effects, such as that of solar radiation pressure. Instead, creep or crackling noise would take place in the suspensions, which co-rotate with the whole system, and might have a component at the 1 Hz frequency of the signal.

The extremely low load that the GG suspensions will be subjected to (Sec. II) and the very small displacements (to several nanometer level at most) make this event extremely unlikely. However, should GG be selected, this issue will be investigated with the contribution of the best experts in the world who are already supporting GG and will of course be even more willing to contribute to it if the mission shall be given a chance to fly.

II. FORCE: SENSITIVITY AND LOAD COMPARISON WITH OTHER EXPERIMENTS

An important issue raised in Questions 1, as well as during the Interview, is that the mechanical suspensions of GG are required to allow the detection of a very small force ($F_{WEP_{GG}} \simeq 8 \times 10^{-17}\,\mathrm{ms}^{-2} \times 5\,\mathrm{kg} \simeq 4 \times 10^{-16}\,\mathrm{N}$, $\mu = m/2 = 5\,\mathrm{kg}$ is the reduced mass of the GG oscillator), and that has never been done before.

It is interesting to examine the forces measured with the rotating torsion balances and with LIGO/Virgo, both using mechanically suspended test masses.

Rotating torsion balances have reached, in the horizontal plane of the lab, a sensitivity to differential accelerations as small as $10^{-15}\,\mathrm{ms}^{-2}$ (Wagner et al.,CQG2012, Table 3). With 40 g test masses, it means a force resolution $F_{TB} \simeq 4 \times 10^{-17}\,\mathrm{N}$. This was shown to be the limit of thermal noise from internal damping (same paper, Fig. 4), meaning that any other noise sources (motor noise as well as creep or crackling etc..), is smaller than this. With a 70 g balance, the fiber is loaded with a force $F_{load_{TB}} \simeq 0.7\,\mathrm{N}$.

As for LIGO/Virgo, they can detect a change in distance between the mirrors of about $10^{-19}\,\mathrm{m}$ (LIGO facts). Hence, horizontal oscillation noise in the mirrors must be smaller than this, which means (with about 1 m suspension fiber and 40 kg mirrors) that horizontal forces must be smaller than $F_{mirror} \simeq 10^{-19} \times g \times 40 \simeq 4 \times 10^{-17}\,\mathrm{N}$. However, the fiber must also provide a force $F_{load_{mirror}} \simeq 400\,\mathrm{N}$ in order to sustain the mirror against local gravity.

In summary, the force resolution to be achieved with the GG suspensions is 10 times higher (hence less demanding) than both the torsion balances and the Virgo/LIGO mirrors.

More importantly, the largest force to be sustained in GG is due to drag, whose acceleration on the suspended masses is 50 million times smaller than 1-g (not even considering partial compensation by drag-free control); for a 20 kg mass of the GG balance, the largest force to be sustained is $F_{max_{GG}} \simeq 2m\,a_{drag_{max}} \simeq 20 \times 2 \times 10^{-7} \simeq 4 \times 10^{-6}\,\mathrm{N}$, which is 100 million times smaller than for the fibers of the Virgo mirrors and 170 thousand times smaller than for the torsion balance fiber ($F_{load_{mirror}} \simeq 400\,\mathrm{N}$ and $F_{load_{TB}} \simeq 0.7\,\mathrm{N}$ respectively).

In summary, the force resolution to be achieved in orbit with the mechanical suspensions of GG is less demanding than that which has been achieved on ground, under far bigger loads, by the mechanical suspensions of the Virgo/LIGO mirrors and the torsion balance. The GG springs can be manufactured as monolithic flexures, whose stiffness and losses can be rigorously measured on ground, and this can be demonstrated by the end of the Assessment Study should GG be selected.

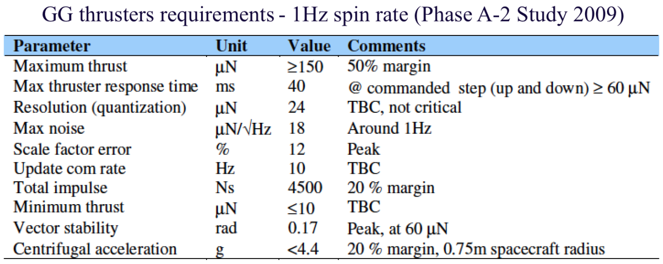
III. DRAG COMPENSATION WITH COLD GAS THRUSTERS AT 1 Hz

A concern about using cold gas thrusters at 1 Hz was raised during the Interview. It is therefore worth recalling that implementation of the drag-free and attitude control system with cold gas microthrusters has been addressed in the Phase A-2 Study funded by the Italian Space Agency (ASI) and carried out by TAS (Thales Alenia Space) Italy in 2009, and no criticality was found.

The requirements derived for the microthrusters at 1 Hz spin rate, as reported in the data package of this Study, are reproduced in the Table below.

The thrust noise requirement must not exceed $18\,\mu\mathrm{N}/\sqrt{\mathrm{Hz}}$ around 1 Hz, when the thruster is commanded at 20 Hz (the control band). At the time of the Phase A-2 study, the thrusters had already been measured on the TAS nanobalance, in steady state at different thrust levels up to 0.5 mN. The thrust noise was below $1\,\mu\mathrm{N}/\sqrt{\mathrm{Hz}}$, practically independent of the frequency. Similar measurements have been performed for Microscope, LPF and lately Euclid, with 10 times lower noise levels measured, probably on account of better isolation of the nanobalances at ONERA, where such tests take place lately. Levels on the order of $0.1\,\mu\mathrm{N}/\sqrt{\mathrm{Hz}}$ are also reported from the LPF flight results, although we have seen no dedicated publication yet.

The noise at 1 Hz when the commanded thrust is not stationary has not been measured on a nanobalance, to our knowledge. Flight data will eventually be available from Microscope which will give indications of any dependence of the noise on the thrust modulation (we expect the drag-free control band to be around a few 0.1 Hz and the attitude control band around 10 Hz). Some small impact of commanded thrust changes on noise may be expected, depending on the commanded thrust profile. However there is no reason to expect the noise to grow very significantly when the thrust is modulated, and a factor of 10 or 100 as would be required to bring the noise to a level where it could affect the experiment is absurd. There will be harmonics at multiples of the control frequency (with variations depending on the discretization scheme) but that is way above the dangerous zone, as it should be.



GG thrusters requirements - 1Hz spin rate (Phase A-2 Study 2009)

Parameter	Unit	Value	Comments
Maximum thrust	µN	≥150	50% margin
Max thruster response time	ms	40	@ commanded step (up and down) ≥ 60 µN
Resolution (quantization)	µN	24	TBC, not critical
Max noise	µN/√Hz	18	Around 1Hz
Scale factor error	%	12	Peak
Update com rate	Hz	10	TBC
Total impulse	Ns	4500	20 % margin
Minimum thrust	µN	≤10	TBC
Vector stability	rad	0.17	Peak, at 60 µN
Centrifugal acceleration	g	<4.4	20 % margin, 0.75m spacecraft radius

The micropropulsion electronics and fluidics were already found viable by the manufacturer Selex-Galileo at the time of the Phase A-2 study, for operation at 20 Hz and beyond (for GG, 2 Hz spin rate was also considered at the time).

Nowadays alternatives to the cold-gas thruster would be available, too. The ADS mini-gridded ion engines have been characterized for Euclid and are under consideration for the Next Generation Gravity Mission. LISA, too, may need ion thrusters because of the lifetime requirement lately considered by ESA (see the CDF study just released). The measured noise characteristics of these electric thrusters are in the same region as mentioned above (order of $0.1\,\mu\mathrm{N}/\sqrt{\mathrm{Hz}}$ at 1 Hz). Should GG be selected, the drag-free control issue will certainly be investigated again, taking advantage of what has been learned in the meantime with Microscope and LPF in orbit, but it is apparent from this older study that it is not going to be a showstopper for GG.