2. SCIENTIFIC GOAL OF THE MISSION

From an experimental point of view, a violation of the Universality of Free Fall would invalidate the Weak Equivalence Principle, hence the Einstein Equivalence Principle, thus placing a limit on the validity of the General Theory of Relativity itself. This is the physical motivation behind a continuing interest within the scientific community worldwide in performing more and more accurate experimental tests of the UFF –on Earth and hopefully also in space.

In an experiment to test UFF the observable physical quantity is the differential acceleration Δa of two test masses of different composition, relative to each other, while falling in the gravitational field of a source body with an average acceleration a (also referred to as the “driving acceleration”). A deviation from UFF is therefore quantified by the dimensionless parameter

$$\eta = \frac{\Delta a}{a} \quad . \tag{2.1}$$

The finding of a value $\eta \neq 0$ would disprove the UFF and indicate a violation of the Weak Equivalence Principle on which General Relativity ultimately relies. Instead, $\eta = 0$ –as reported by all experiments so far– confirms the basic assumption of General Relativity. By writing the equations of motion of each individual test mass without assuming a priori the equivalence of their inertial and gravitational mass, the parameter η given by (1) becomes

$$\eta = \frac{2[(m_g / m_i)_A - (m_g / m_i)_B]}{[(m_g / m_i)_A + (m_g / m_i)_B]} \tag{2.2}$$

where subscripts A and B refer to the individual test masses and allow them to be distinguished by their different composition. This parameter η is also known as the Eötvös parameter and it has additional profound significance.

The total mass–energy of a body can be expressed as the sum of many terms, corresponding to the energy of all the conceivable interactions and components: $m = \Sigma_k m_k$. For instance, at the atomic level, the rest mass contributes (as a fraction of the total) for $\cong 1$; the nuclear binding energy for $8 \cdot 10^{-3}$ (for light elements), the mass difference between neutron and proton for $1.4 \cdot 10^{-3} (A - Z)/A$ (A being the number of protons plus neutrons and Z the number of protons in the nucleus), the electrostatic energy of repulsion in the nuclei for $6 \cdot 10^{-4} Z^2 A^{-4/3}$, the mass of electrons for $5 \cdot 10^{-4} Z/A$, the antiparticles for $\cong 10^{-7}$, the weak interactions responsible of β decay for 10^{-9} or less. For an extended spherical body of radius R and (homogeneous) density ρ, the gravitational self–energy contributes by the fraction

$-(4/5)\pi\rho GR^2/c^2$. The conventional Eötvös parameter (2.2*)* can therefore be generalized into:

$$\eta_k = \frac{2[(m_g/m_i)_{A_k} - (m_g/m_i)_{B_k}]}{[(m_g/m_i)_{A_k} + (m_g/m_i)_{B_k}]} \tag{2.3}$$

such that a non-zero value of η_k would define the violation of equivalence between the inertial and gravitational mass-energy of the k^{th} type. From the point of view of conventional field theory, the verification of all these separate "Equivalence Principles" corresponds to a very peculiar coupling of each field to gravity; whether and why it should be so in all cases is a mystery.

It is apparent from (2.1) that –for any given experimental apparatus– the larger the driving acceleration, the more sensitive the UFF test (hence the EP test) that it provides. In a Galileo-type mass dropping experiment the driving acceleration is the gravitational acceleration of the Earth along the local vertical ($9.8\,ms^{-2}$). If the test masses are suspended on a torsion balance the driving acceleration is $0.017\,ms^{-2}$ (at most) in the field of the Earth –directed along the North-South direction of the local horizontal plane– and $0.006\,ms^{-2}$ in the field of the Sun (with components along the North-South and East-West directions of the horizontal plane). Yet, the first experimental apparatus to provide very accurate EP tests (to $10^{-8}-10^{-9}$) was the torsion balance used by Eötvös [10] at the turn of the 20th century, and later on by his students to detect an EP violation in the field of the Earth. This is because torsion balances are extremely sensitive; moreover, they are inherently differential instruments, and although in reality perfect rejection of common mode effects is impossible, the advantages of a differential instrument for detecting differential accelerations are enormous.

The next leap in sensitivity (to $10^{-11}-10^{-12}$) came in the 60s and early 70s using again a torsion balance but also recognizing that by taking the Sun as the source mass rather than the Earth, any differential effect on the test masses of the balance would be modulated by the 24^{hr} rotation of the Earth on which the balance sits [11,12]. Indeed, the modulation frequency should be as high as possible, in order to reduce $1/f$ electronic noise. The best and most reliable results in EP testing (to about 1 part in 10^{13}) have been achieved by the "Eöt-Wash" group at the University of Seattle in a systematic series of experiments using torsion balances placed on a turntable which modulates the signal with a period down to about 20 minutes [13-15].

Despite the much larger driving acceleration, Galileo-type mass dropping tests of UFF have been unable to compete with rotating torsion balances [16]. The success of torsion balances relies on 3 main properties: i) high sensitivity to differential accelerations; ii) long time duration of the experiment; iii) up-conversion of the signal (DC from the Earth and 24-hr from the Sun) to higher frequency. Flying an instrument with these properties in low orbit around the Earth would add the only advantage of mass dropping, namely the very large driving acceleration from Earth. This fact alone would provide –assuming the same sensitivity to differential accelerations as achieved in ground tests– an improvement in η by about 3 orders of magnitude. The difficulties related to running the experiment in space, with no direct access to it, can be compensated by exploiting those peculiarities of the space environment which are relevant for experiments to test UFF, most importantly, absence of weight and isolation of the satellite/experiment once in orbit. Throughout this Report we shall see how absence of weight and system isolation can significantly contribute to improving the current best tests of EP by several orders of magnitude.

Torsion balance tests indicate that considerable progress beyond the current level is extremely hard to achieve. A new type of experiments based on interferometry of free falling cooled atoms is in preparation [17] with the very ambitious goal of performing in the lab an EP to 10^{-15} and even to 10^{-17} sometime in the future. So far a measurement of the local acceleration of falling atoms has been performed achieving $\Delta g / g \simeq 3 \cdot 10^{-9}$ [18]. The proposed EP tests with cold atoms interferometry will measure the differential acceleration between isotopes ^{85}Rb and ^{87}Rb, whose difference in composition is unfortunately limited to two neutrons only.

All other experiment proposals aiming at a considerable improvement over the results achieved by rotating torsion balances are to be performed inside a capsule dropped during a balloon flight [19], in a suborbital flight with a sounding rocket [20] or inside a spacecraft in low Earth orbit [21-23].

Theoretical predictions have been made as to what level an EP violation is likely to occur [24, 25, 9]. In [9] it is shown with a rigorous calculation, within a classical framework which does not postulate any new interaction, that if gravity couples anomalously to the energy of neutrino-antineutrino exchange, its contribution to the mass-energy of the nucleus would lead to an Equivalence Principle violation to the level of about 10^{-17}. The most recent work [9] indicates that, for test masses made of Be-Cu and Pt-Ti, a violation might occur at a level which should be observed with the rotating torsion balances of the “Eöt-Wash” group. However, it is apparent from the speculative nature of these analyses that only a very high accuracy EP test will provide a major breakthrough or –if not– severely constrain the theoretical framework. In this context, tests of composition dependent effects and of post-Newtonian ones are quantitatively compared –as mentioned above [9]– to conclude that UFF tests put much more stringent limits than solar system or binary-pulsar test, by several orders of magnitude.

Equivalence principle experiments involving man made test masses do not allow to test it for gravitational self-energy itself because the contribution from gravitational binding energy –mentioned above in relation to the generalized Eötvös parameter

(2.3)– is negligible for artificial bodies. This form of equivalence is often referred to as the Strong Equivalence Principle (SEP) and can be tested only with experiments in which the test masses are celestial bodies –as in the case of the Earth and the Moon falling in the gravitational field of the Sun, the lunar orbit being determined by laser ranging to the Moon. In addition to differing in composition, Earth and Moon have a significant component of gravitational binding energy ($4.6 \cdot 10^{-10}$ for the Earth and $1.6 \cdot 10^{-11}$ for the Moon) whose equivalence can be tested with the current sensitivity of Lunar Laser Ranging (LLR) tests which have reached the level of 10^{-13} as tests of the weak equivalence principle [26]. In this sense LLR tests are unique, though they need to be combined with composition dependent tests of laboratory size bodies of Earth-like and Moon-like composition in order to remove their ambiguity as tests of the SEP [14].

The new APOLLO (Apache Point Observatory Lunar Laser-ranging Operation) facility in southern new Mexico [27] will provide –together with an improved physical model of all perturbations involved– a better determination of the lunar orbit and a more accurate test of the equivalence principle, confirming LLR once more as the most important scientific legacy of the Apollo project to the Moon.

As a test of the equivalence principle, lunar laser ranging is ultimately limited by the non uniformity of the gravity field of the Sun, a limitation expressed by the dimensionless quantity $3\Delta a_{sma}/d$ (Δa_{sma} being the measurement error in the semimajor axis of the orbit of the Moon around the Earth and d the distance of the Earth-Moon system from the Sun) [28]. A $1cm$ error in semimajor axis (due to $1cm$ accuracy of lunar laser ranging) is consistent with the current level of LLR tests to 10^{-13}, 1 order of magnitude improvement is expected with the capability of the APOLLO facility to perform laser ranging to the Moon at $1mm$ 1 level.

The effect of non uniformity of the gravitational field is a real limitation to EP tests with laser ranging because they rely on absolute distance measurements from Earth. However, it is apparent from (2.1) that EP tests require to measure only differential accelerations –and the displacements they give rise to– of the test masses relative to one another. If artificial test masses are placed inside a spacecraft and differential measurements are performed in situ, displacement sensors are available to measure their relative displacements many orders of magnitude more accurately than by laser ranging from Earth. For testing the equivalence principle there is no need for a very accurate measurement of the absolute distance of the spacecraft from Earth. This is why differential accelerometers to fly inside a spacecraft in low Earth orbit can aim at a far more accurate EP test than lunar laser ranging so as to put General Relativity to the most stringent test ever.