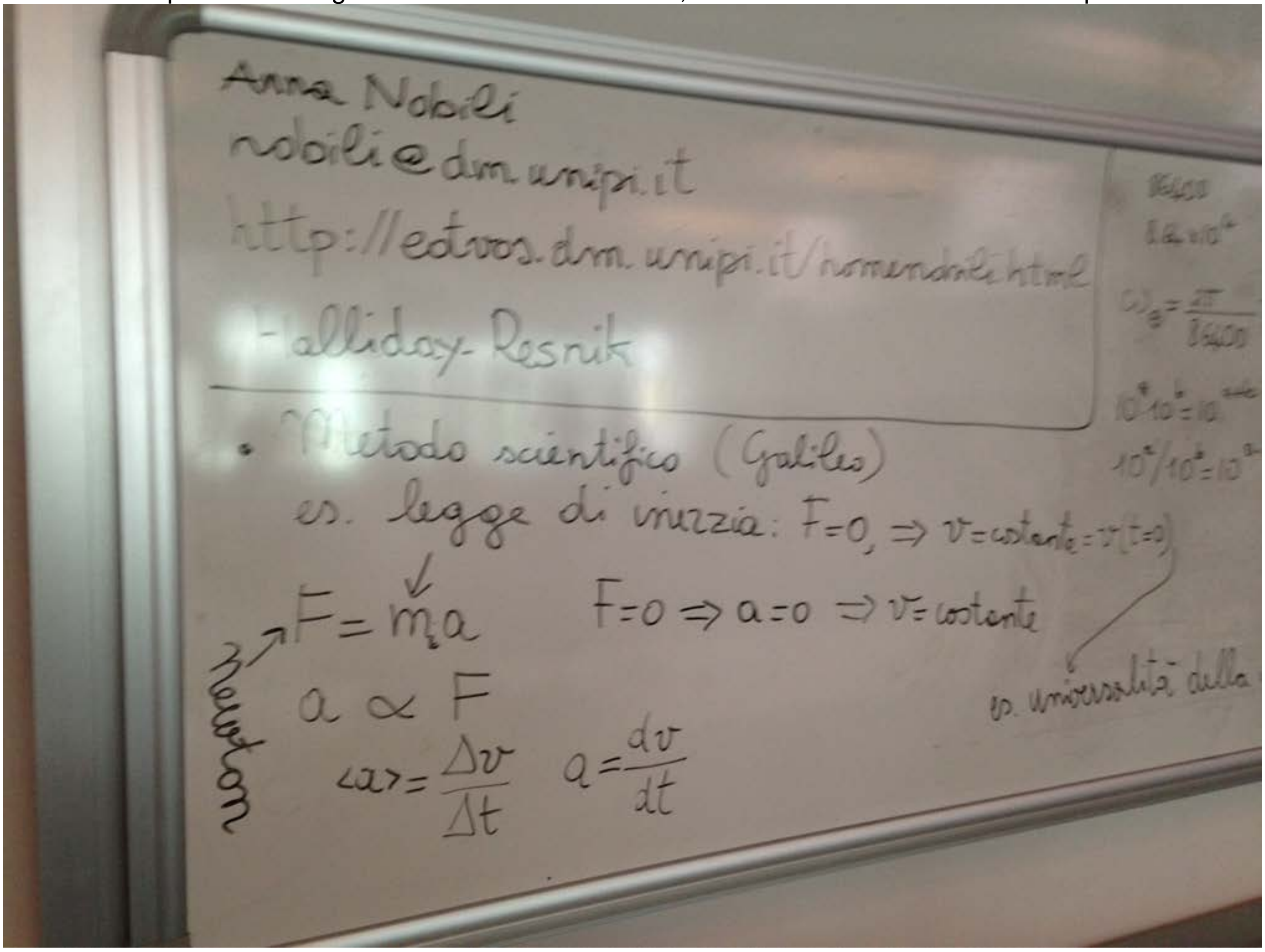
Fisica 1 per studenti di chimica - Anno accademico 2014-2015 - Primo Semestre - Anna Nobili

25 Settembre 2014
-Considerazioni sul metodo scientifico di Galileo su cui si basa la fisica (esempi: legge di inerzia e universalità della caduta libera)
-Importanza di usare le potenze di 10
-Sistema di misura internazionale (SI)
-Dimensioni delle grandezze fisiche: spostamento, velocità lineare, velocità angolare, accelerazione lineare. Esempi: velocità angolare di rotazione della terra, velocità lineare di rotazione all'equatore



Anna Nobili
nobili@dm.unipi.it
http://eotvos.dm.unipi.it/nobilidchimica.html
Halliday-Resnik

- Metodo scientifico (Galileo)
  es. legge di inerzia: $F=0 \Rightarrow v=\text{costante}=v(t=0)$

$F = m_i a$ $F=0 \Rightarrow a=0 \Rightarrow v=\text{costante}$

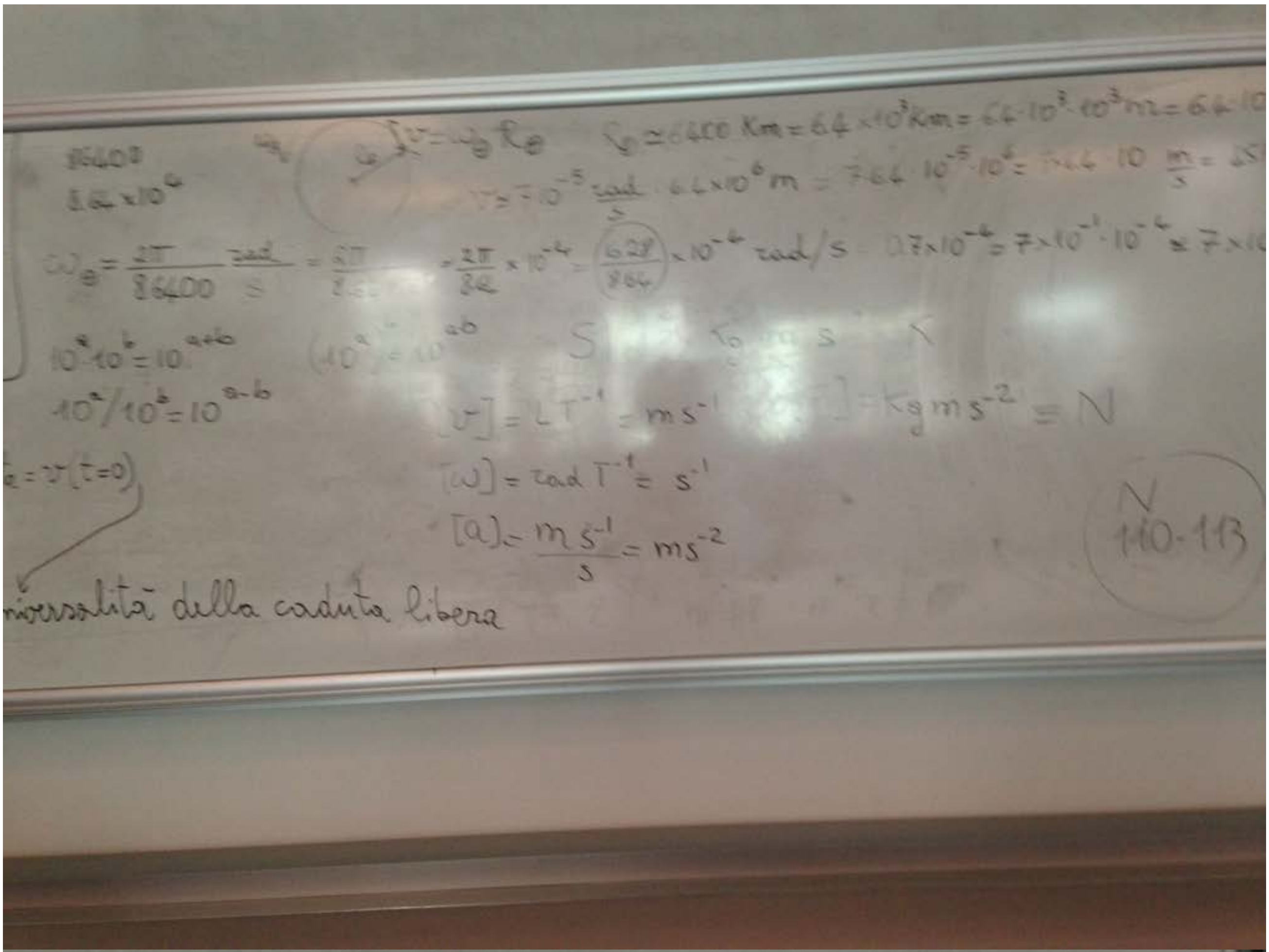
Newton

$a \propto F$

$\langle a \rangle = \frac{\Delta v}{\Delta t}$ $a = \frac{dv}{dt}$

es. universalità della

86400
8.64×10^4
$\omega_\oplus = \frac{2\pi}{86400}$
$10^a 10^b = 10^{a+b}$
$10^a / 10^b = 10^{a-b}$



86400

8.64×10^4

$\omega_\oplus$ $R_\oplus$ $v=\omega_\oplus R_\oplus$ $R_\oplus\simeq6400$ Km $=6.4\times10^3$ Km $=6.4\cdot10^3\cdot10^3$ m $=6.4\cdot10$

$v\simeq7\cdot10^{-5}\frac{rad}{s}\cdot6.4\times10^6\,m=7\cdot6.4\cdot10^{-5}\cdot10^6=7\times6.4\cdot10\,\frac{m}{s}=45$

$\omega_\oplus=\frac{2\pi}{86400}\frac{rad}{s}=\frac{2\pi}{8.64}\ldots=\frac{2\pi}{8.64}\times10^{-4}=\boxed{\frac{6.28}{8.64}}\times10^{-4}\,rad/s\simeq0.7\times10^{-4}=7\times10^{-1}\cdot10^{-4}\simeq7\times1$

$10^a10^b=10^{a+b}$ $(10^a)^b=10^{ab}$ SI Kg m s K

$10^a/10^b=10^{a-b}$

$[v]=LT^{-1}=ms^{-1}$ $[F]=Kg\,m\,s^{-2}=N$

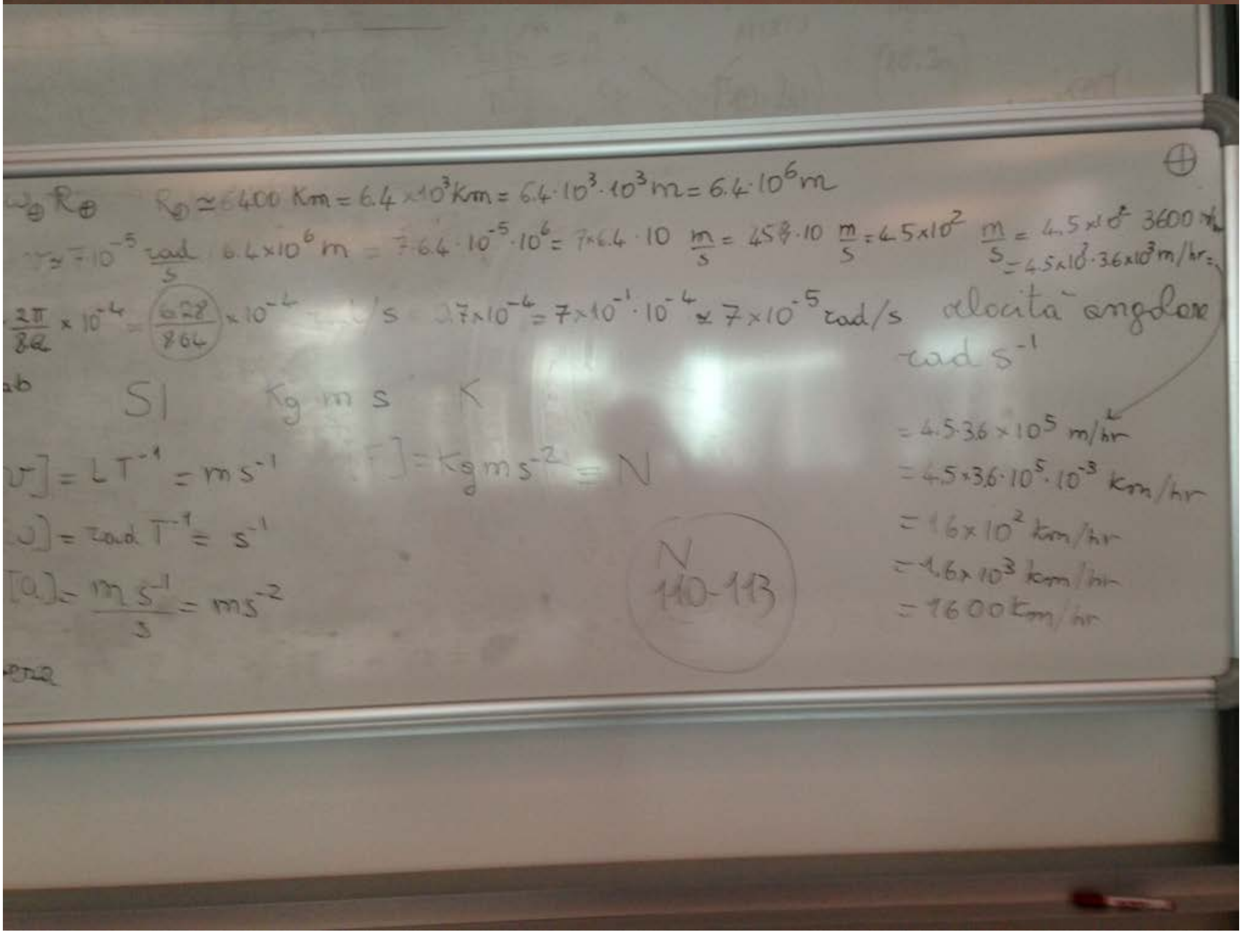
$=v(t=0)$

$[\omega]=rad\,T^{-1}=s^{-1}$

$[a]=\frac{m\,s^{-1}}{s}=ms^{-2}$

N 110-113

niversalità della caduta libera



$\omega_\oplus R_\oplus$ $R_\oplus\simeq6400$ Km $=6.4\times10^3$ Km $=6.4\cdot10^3\cdot10^3$ m $=6.4\cdot10^6$ m

$\oplus$

$v\simeq7\cdot10^{-5}\frac{rad}{s}\,6.4\times10^6\,m=7\cdot6.4\cdot10^{-5}\cdot10^6=7\times6.4\cdot10\,\frac{m}{s}=45\cdot10\,\frac{m}{s}=4.5\times10^2\,\frac{m}{s}=4.5\times10^2\cdot3600\,m/hr=4.5\times10^2\cdot3.6\times10^3\,m/hr=$

$\frac{2\pi}{8.64}\times10^{-4}=\boxed{\frac{6.28}{8.64}}\times10^{-4}\,rad/s\simeq0.7\times10^{-4}=7\times10^{-1}\cdot10^{-4}\simeq7\times10^{-5}\,rad/s$ velocità angolare

rad s^{-1}

ab

SI Kg m s K

$=4.5\cdot3.6\times10^5\,m/hr$

$=4.5\times3.6\cdot10^5\cdot10^{-3}\,Km/hr$

$\simeq16\times10^2\,Km/hr$

$\simeq1.6\times10^3\,Km/hr$

$=1600\,Km/hr$

$[v]=LT^{-1}=ms^{-1}$ $[F]=Kg\,m\,s^{-2}=N$

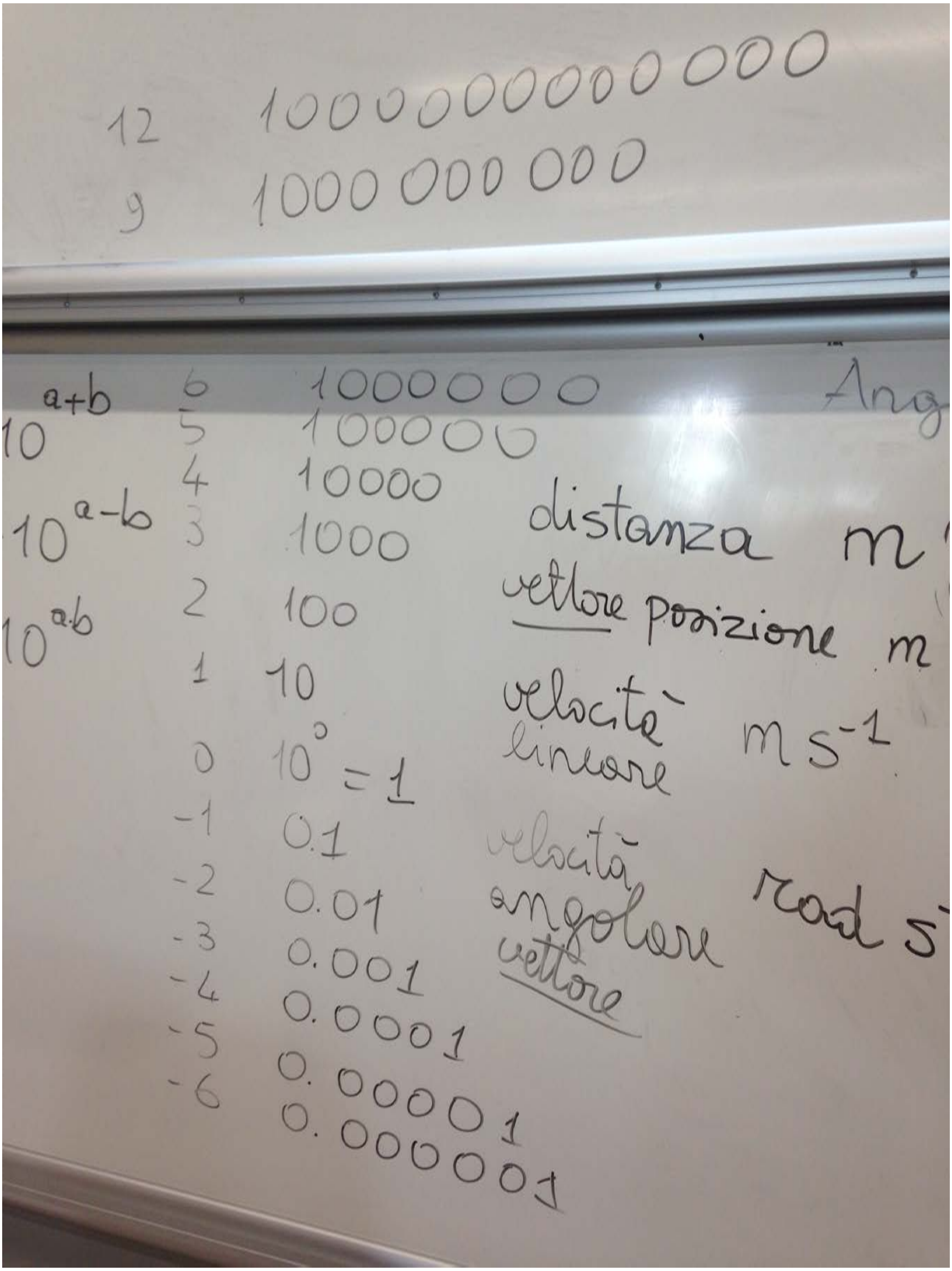
$[\omega]=rad\,T^{-1}=s^{-1}$

$[a]=\frac{m\,s^{-1}}{s}=ms^{-2}$

N 110-113

era

2 ottobre 2014
-Potenze di 10
-Sistema di coordinate cartesiane ortogonali
-Dimensioni delle grandezze fisiche: angoli, distanze/spostamenti, velocità lineare, velocità angolare, accelerazione lineare, accelerazione angolare, quantità di moto lineare, quantità di moto angolare (momento angolare).
-Esempi: misure di angoli in radianti; relazione tra dimensione lineare e dimensione angolare ad una certa distanza (angolo sotteso da un oggetto di dimensioni lineari date visto da un osservatore ad una distanza data); velocità angolare e velocità lineare della terra nella sua orbita intorno al sole



12	1000000000000
9	1000 000 000

10^{a+b}

10^{a-b}

$10^{a \cdot b}$

6	1000000
5	100000
4	10000
3	1000
2	100
1	10
0	$10^0 = \underline{1}$
-1	0.1
-2	0.01
-3	0.001
-4	0.0001
-5	0.00001
-6	0.000001

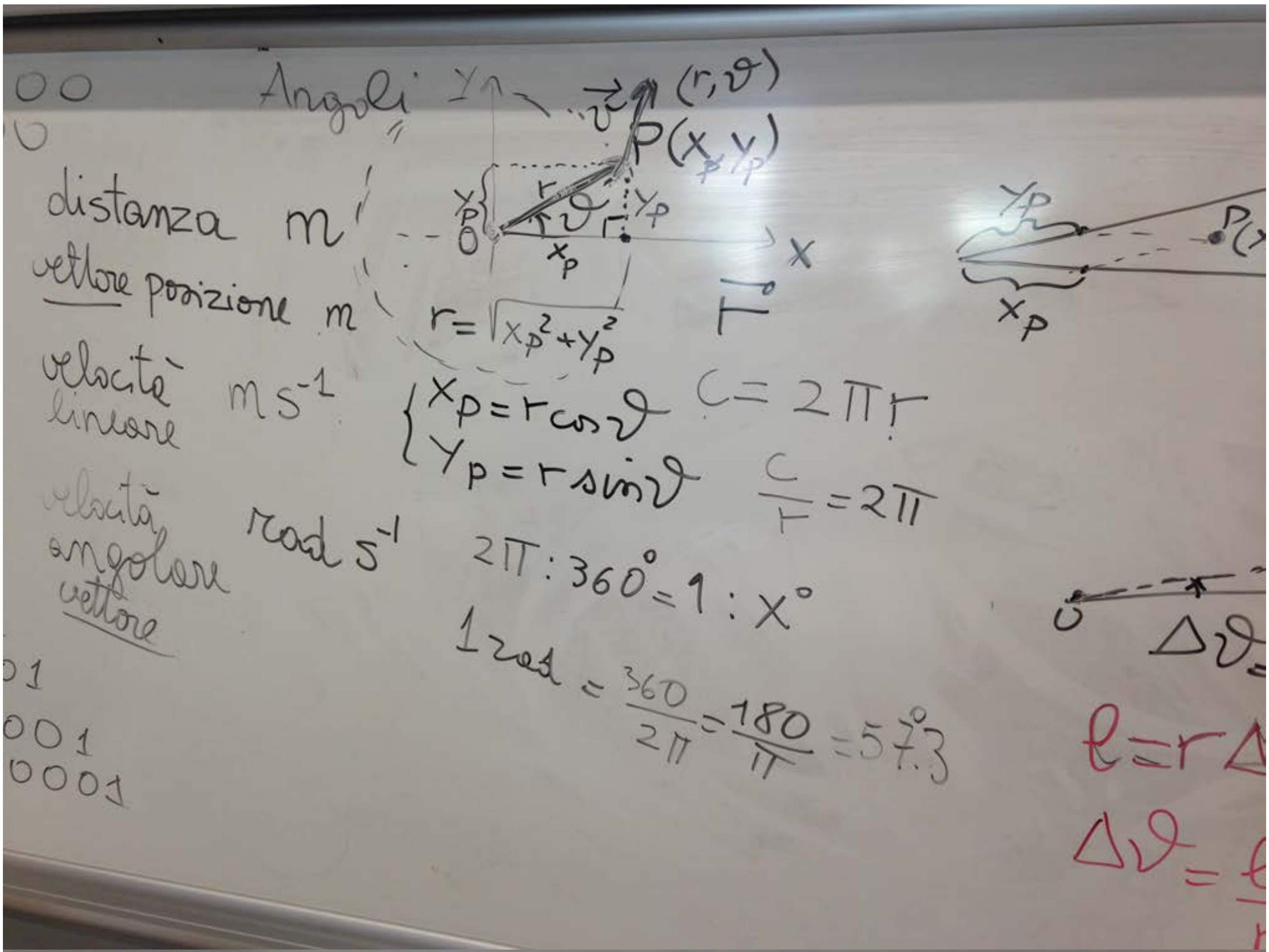
Ang

distanza m

vettore posizione m

velocità lineare $m s^{-1}$

velocità angolare vettore rad s


Y
$\vec{v}$
(r,ϑ)
$P(x_p,y_p)$
r
ϑ
y_p
x_p
O
x
$\vec{r}$
y_p
x_p

Angoli

distanza m

vettore posizione m

velocità lineare m s^{-1}

velocità angolare vettore rad s^{-1}

$$r=\sqrt{x_p^2+y_p^2}$$

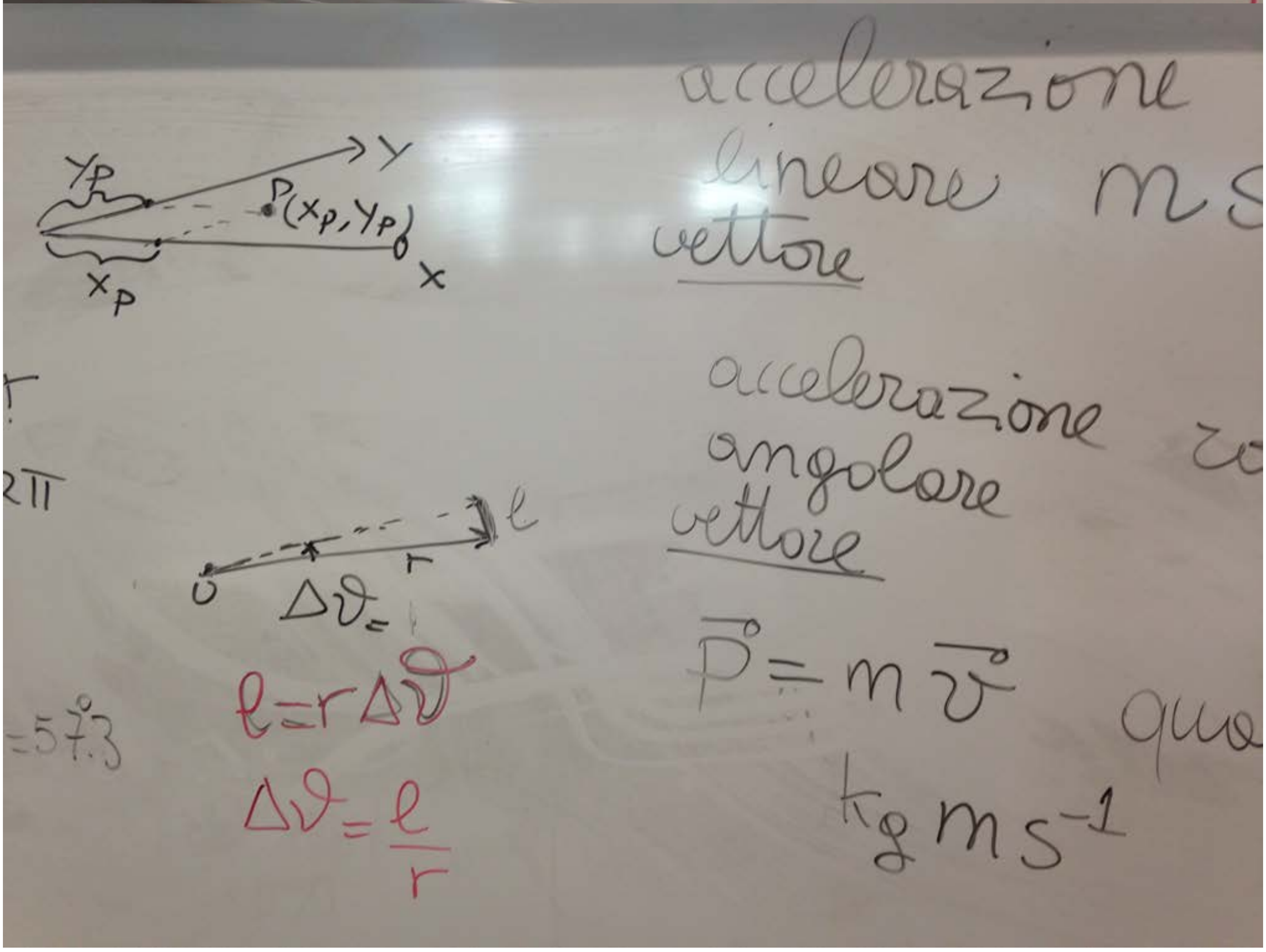
$$\begin{cases} x_p = r\cos\vartheta \\ y_p = r\sin\vartheta \end{cases}$$

$$C = 2\pi r \qquad \frac{C}{r} = 2\pi$$

$$2\pi : 360^\circ = 1 : x^\circ$$

$$1\,\text{rad} = \frac{360}{2\pi} = \frac{180}{\pi} = 57^\circ.3$$

00
0
01
001
0001


y
y_p
$P(x_p,y_p)$
x_p
x
ℓ
r
O
Δϑ =

$$\ell = r\Delta\vartheta$$

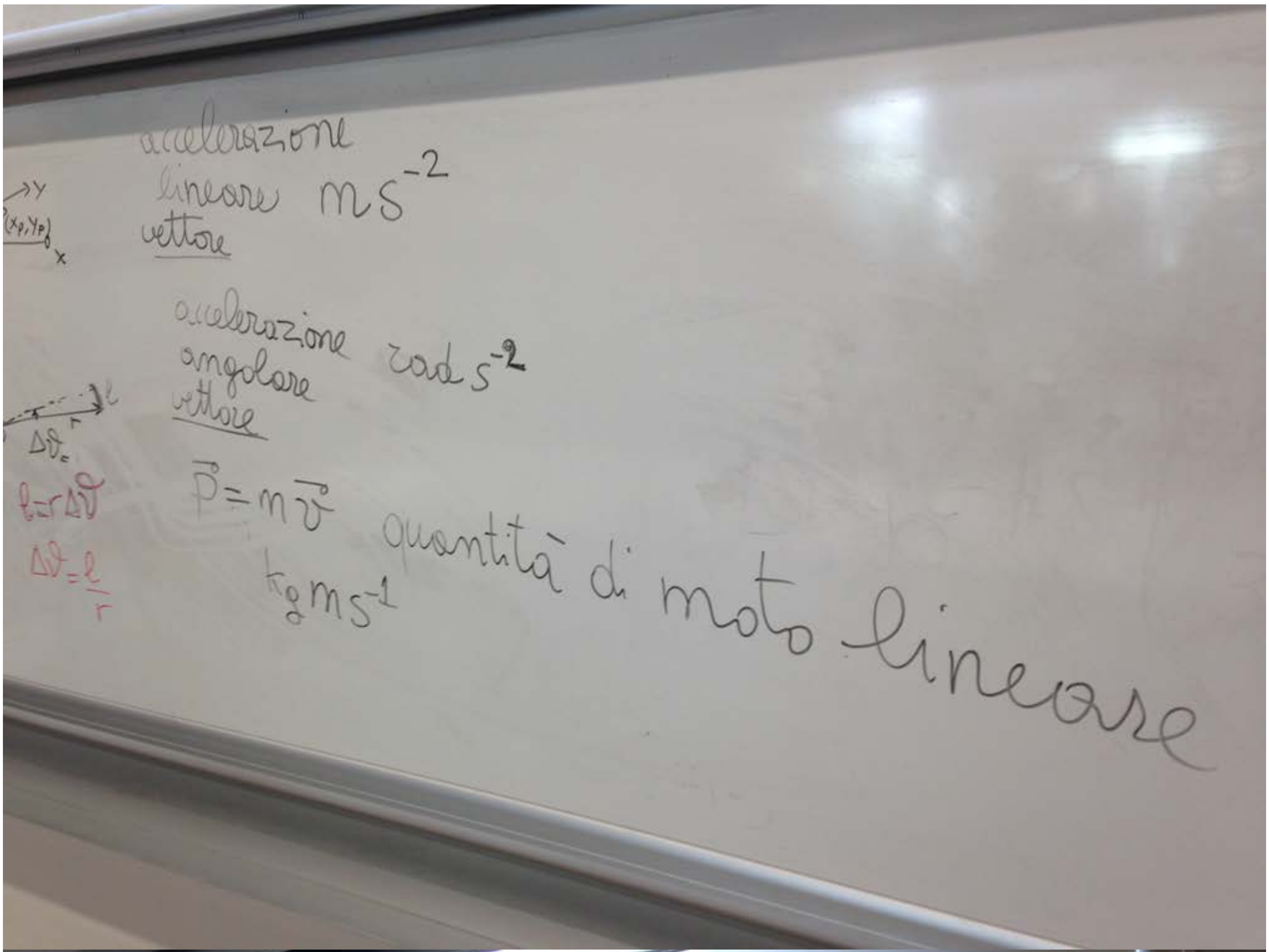
$$\Delta\vartheta = \frac{\ell}{r}$$

accelerazione lineare m s
vettore

accelerazione angolare ra
vettore

$$\vec{p} = m\vec{v}$$ qua

kg m s^{-1}



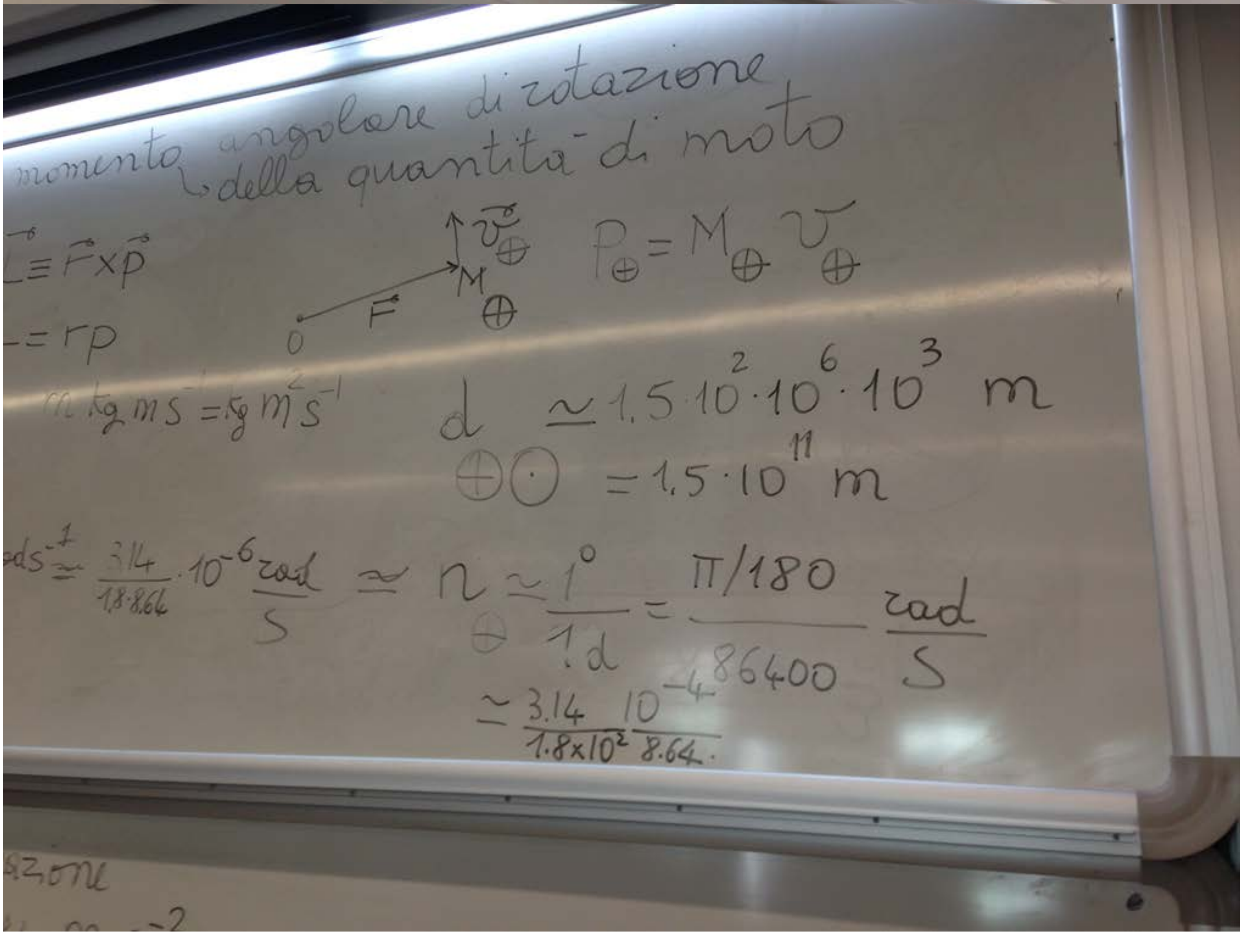
$\ell = r\Delta\vartheta$

$\Delta\vartheta = \frac{\ell}{r}$

accelerazione
lineare $m s^{-2}$
vettore

accelerazione angolare $rad\, s^{-2}$
vettore

$\vec{p} = m\vec{v}$ quantità di moto lineare
$kg\, m s^{-1}$



momento angolare di rotazione
della quantità di moto

$\vec{L} \equiv \vec{r} \times \vec{p}$

$L = rp$

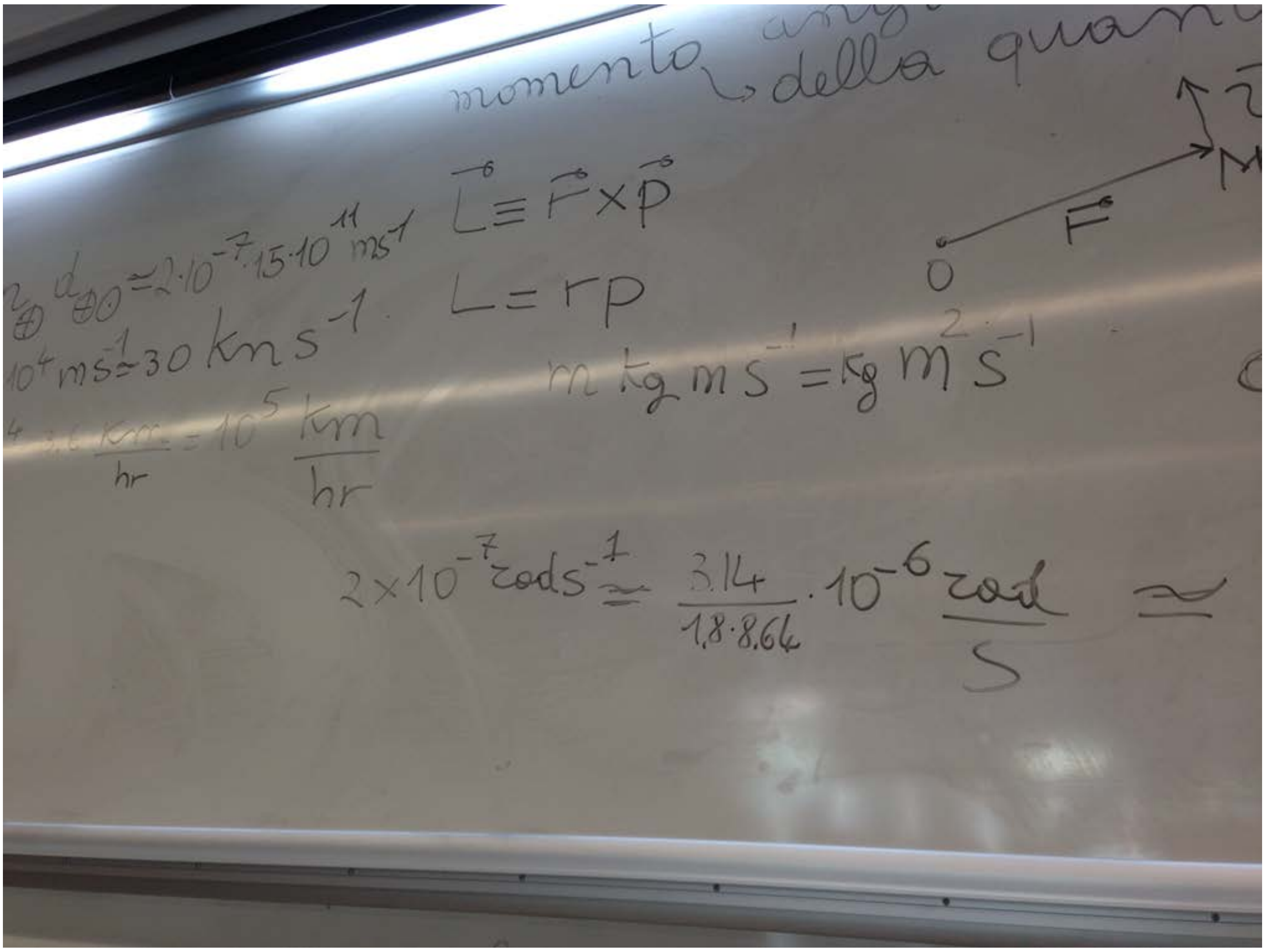
$m\, kg\, m s^{-1} = kg\, m^2 s^{-1}$

$P_\oplus = M_\oplus v_\oplus$

$d_{\oplus\odot} \simeq 1.5\, 10^2 \cdot 10^6 \cdot 10^3\ m = 1.5 \cdot 10^{11}\ m$

$ads^{-1} \simeq \frac{3.14}{1.8 \cdot 8.64} \cdot 10^{-6} \frac{rad}{s} \simeq \omega_\oplus \simeq \frac{1^\circ}{1d} = \frac{\pi/180}{86400} \frac{rad}{s} \simeq \frac{3.14}{1.8\times10^2} \frac{10^{-4}}{8.64}$

azione



momento ang
↳ della quan

$\vec{L} \equiv \vec{r} \times \vec{p}$

$v_{\oplus}\, d_{\oplus\odot} \simeq 2\cdot 10^{-7}\cdot 15\cdot 10^{11}\, m s^{-1}$

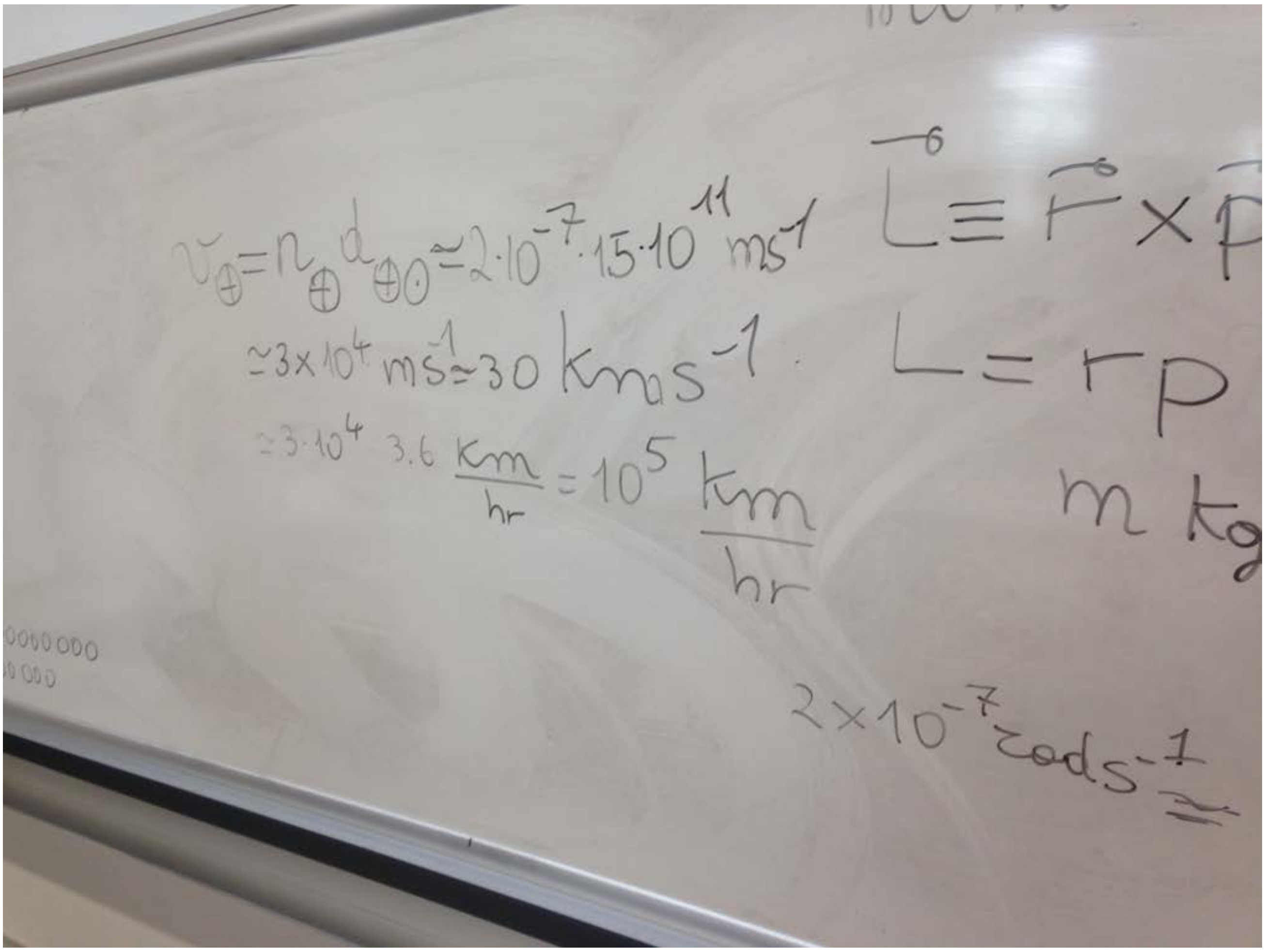
$L = rp$

$10^{4}\, m s^{-1} \simeq 30\, km\, s^{-1}$

$m\, kg\, m\, s^{-1} = kg\, m^{2}\, s^{-1}$

${}^{4}\cdot 3.6\, \frac{km}{hr} = 10^{5}\, \frac{km}{hr}$

$2\times 10^{-7}\, rad\, s^{-1} \simeq \frac{3.14}{1{,}8\cdot 8{,}64}\cdot 10^{-6}\, \frac{rad}{s} \simeq$



$v_\oplus = n_\oplus d_{\oplus\odot} \simeq 2\cdot10^{-7}\cdot 15\cdot10^{11}\, ms^{-1}$

$\simeq 3\times10^{4}\, ms^{-1} \simeq 30\, kms^{-1}$

$\simeq 3\cdot10^{4}\ 3.6\ \frac{km}{hr} = 10^{5}\ \frac{km}{hr}$

$\vec{L} \equiv \vec{r}\times\vec{p}$

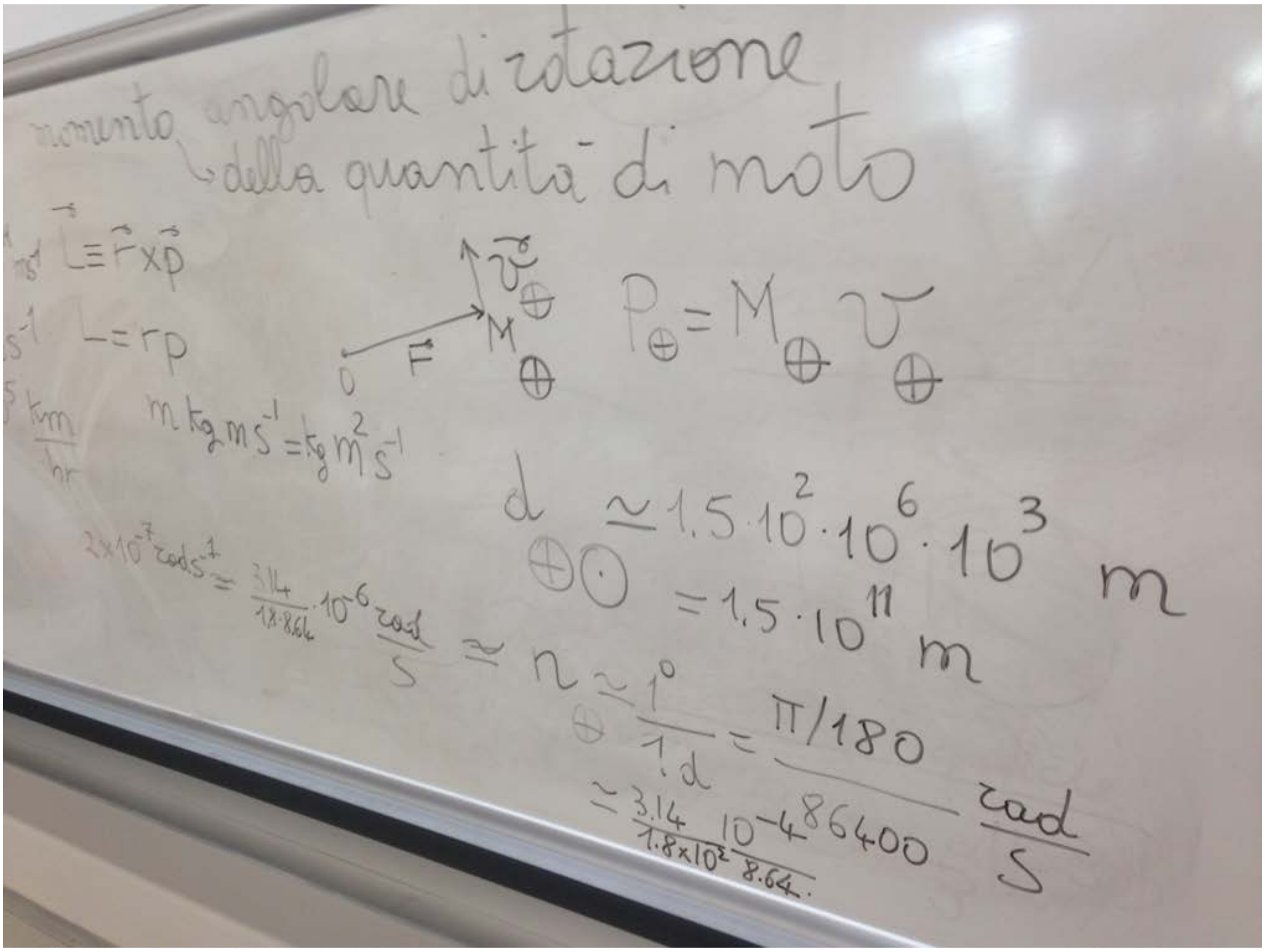
$L = rp$

m kg

0000000 000

0 000

$2\times10^{-7}\, rads^{-1} \simeq$

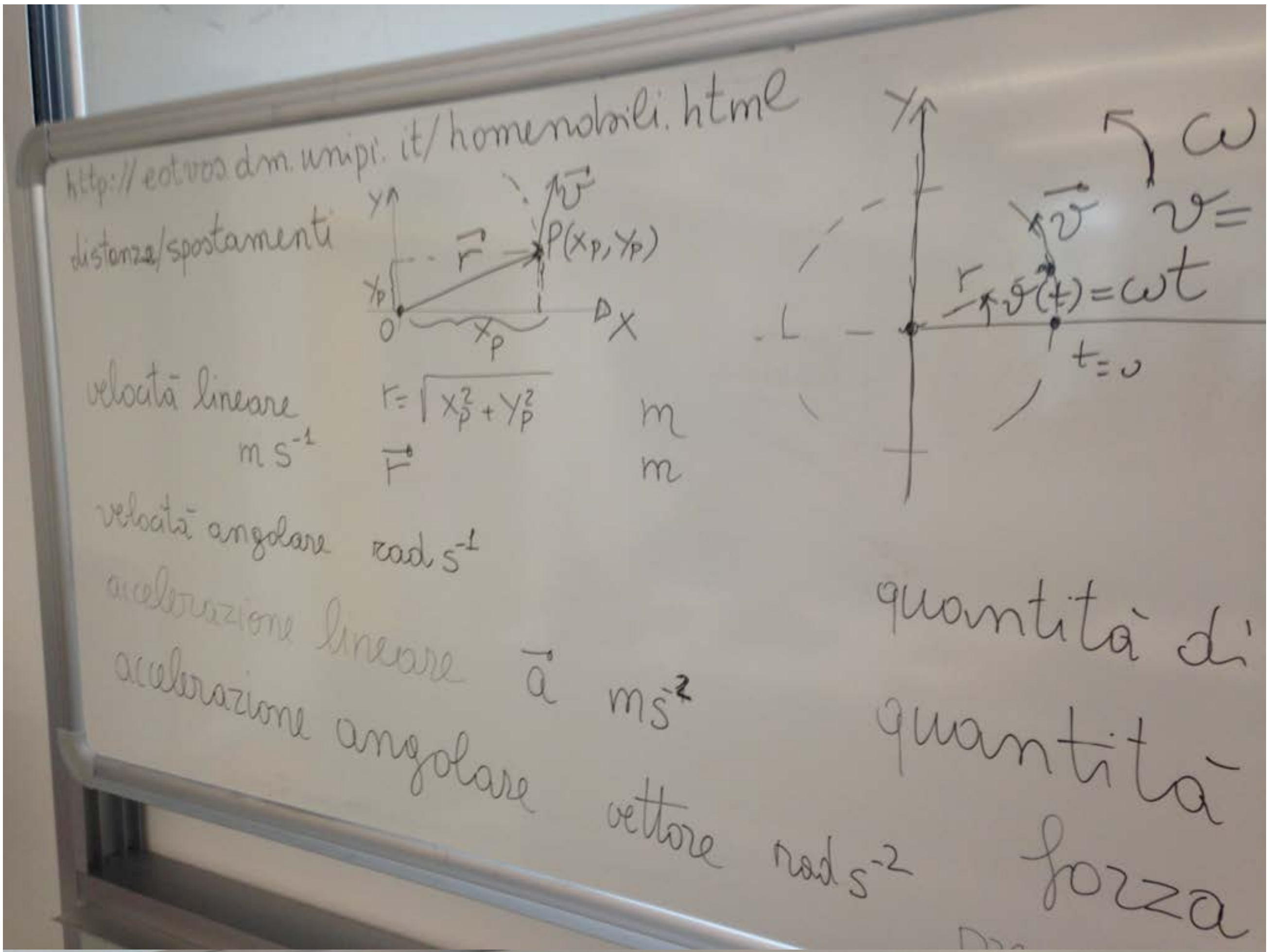

momento angolare di rotazione
della quantità di moto
L ≡ F x p
L = rp
m kg m s^-1 = kg m^2 s^-1
km / hr
v⊕
O F M⊕
P⊕ = M⊕ v⊕
d⊕☉ ≃ 1.5·10^2·10^6·10^3 m
= 1.5·10^11 m
2x10^-7 rads^-1 = 3.14 / 1.8·8.64 · 10^-6 rad/s ≃ ω ≃ 1° / 1 d = π/180 / 86400 rad/s
≃ 3.14 / 1.8x10^2 · 10^-4 / 8.64

9 Ottobre 2014

-Ulteriori grandezze fisiche; distinzione tra grandezze scalari e grandezze vettoriali

-Spostamento; velocità lineare; velocità angolare; accelerazione lineare; accelerazione angolare; quantità di moto lineare; quantità di moto angolare; forza; momento di una forza; energia; potenza; energia per unità di tempo e per unità di superficie; densità volumetrica (massa per unità di volume; es: densità dell'acqua)

-Calcolo della luminosità del sole dalla misura della costante solare


Y
$\vec{v}$
$\vec{r}$
$P(x_P, y_P)$
y_P
O
x_P
X
Y
ω
$\vec{v}$
$v=$
r
$\vartheta(t)=\omega t$
L
$t=0$

http://eotvos.dm.unipi.it/homenobili.html

distanze/spostamenti

velocità lineare $r=\sqrt{x_P^2+y_P^2}$ m

$m\,s^{-1}$ $\vec{r}$ m

velocità angolare $rad\,s^{-1}$

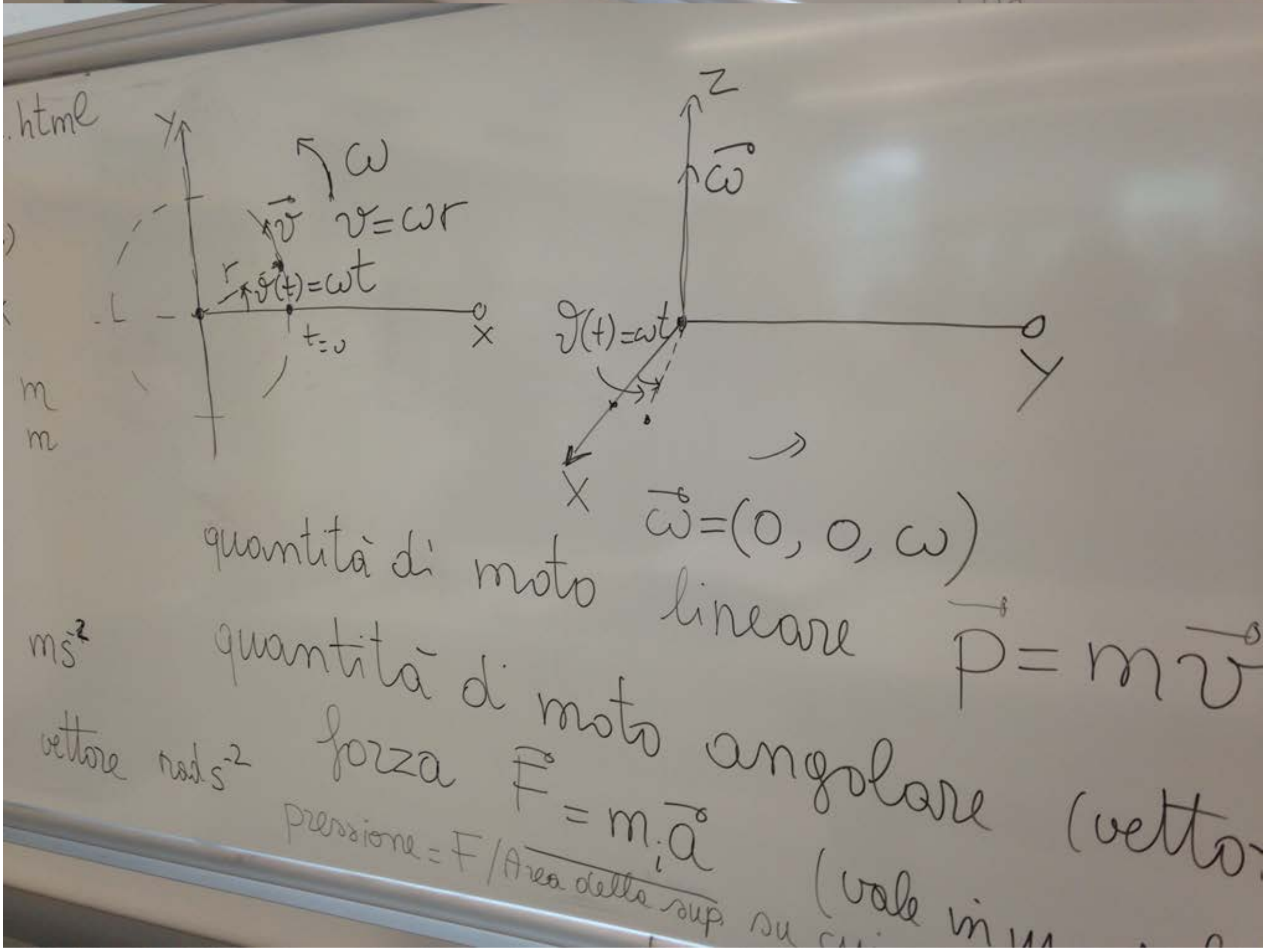
accelerazione lineare $\vec{a}$ $m s^{-2}$

accelerazione angolare vettore $rad\,s^{-2}$

quantità di

quantità

forza


Y
ω
$\vec{v}$
$v=\omega r$
r
$\vartheta(t)=\omega t$
L
$t=0$
X
Z
$\vec{\omega}$
$\vartheta(t)=\omega t$
Y
X

.html

m

m

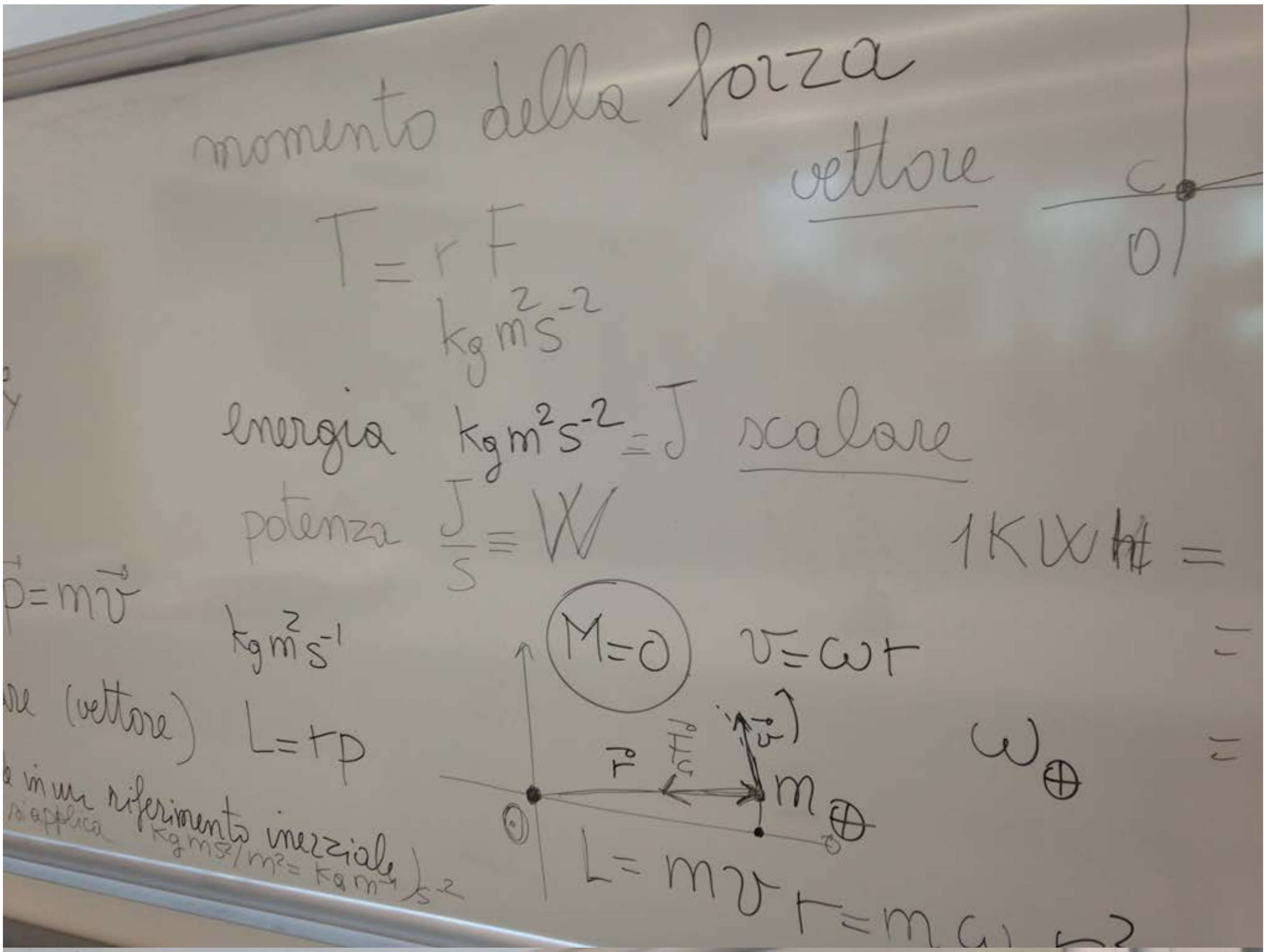
$\vec{\omega}=(0, 0, \omega)$

$m s^{-2}$ quantità di moto lineare $\vec{p}=m\vec{v}$

quantità di moto angolare (vetto

vettore $rad\,s^{-2}$ forza $\vec{F}=m_i\vec{a}$ (vale in

pressione = F/Area della sup. su



momento della forza

vettore

$T = r F$

$kg\,m^2 s^{-2}$

energia $kg\,m^2 s^{-2} = J$ scalare

potenza $\frac{J}{s} = W$

$1KWh =$

$\vec{p} = m\vec{v}$

$kg\,m^2 s^{-1}$

$\boxed{M=0}$ $v = \omega r$

re (vettore) $L = rp$

e in un riferimento inerziale

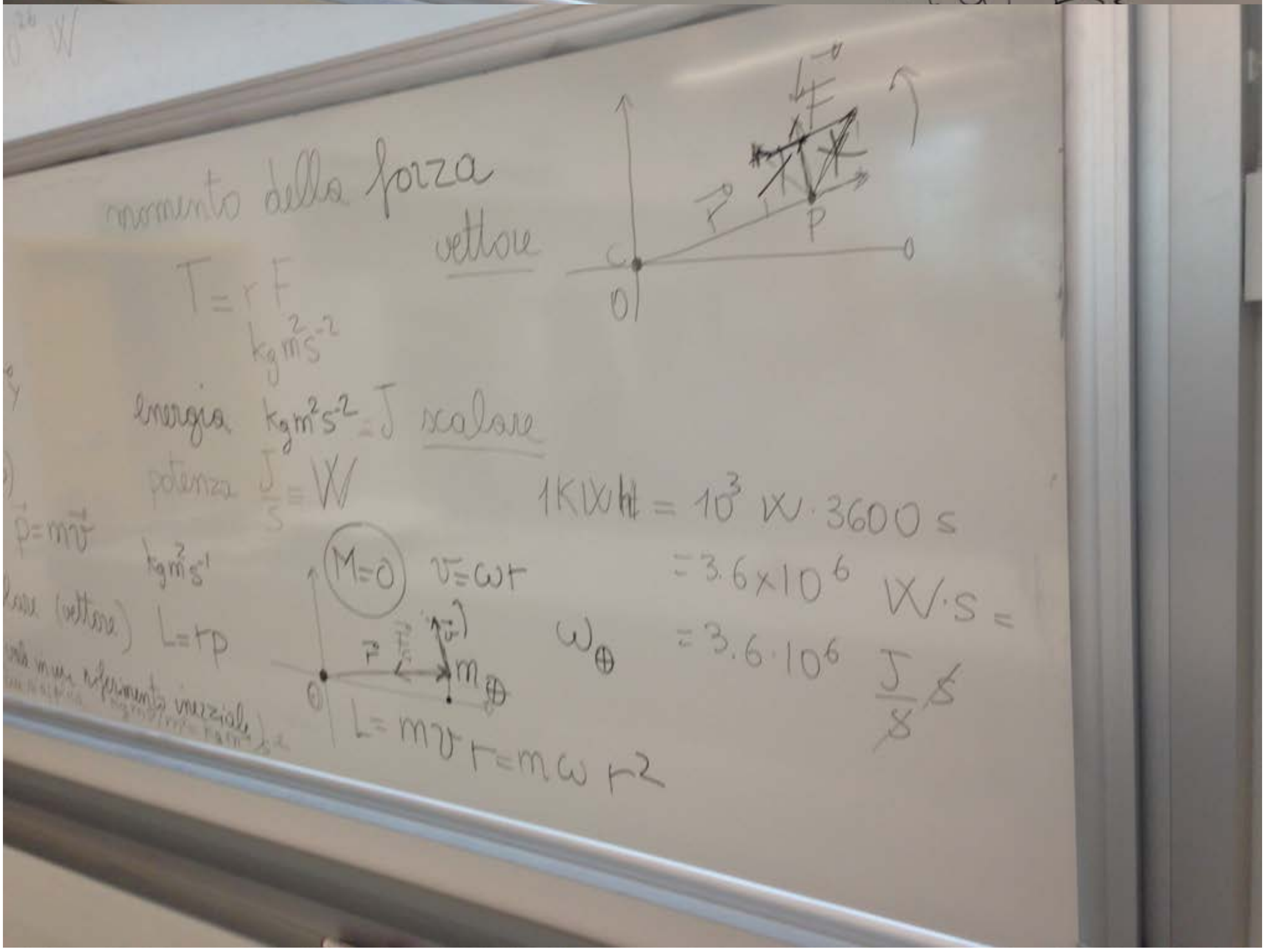
si applica

$kg\,ms^2/m^2 = kg\,m^{-1}) s^{-2}$

$L = m v r = m \omega r^2$

$\omega_\oplus$

C
O
$\vec{r}$
$\vec{F}_c$
$\vec{v}$
$m_\oplus$
O



momento della forza

vettore

$T = r F$

$kg\,m^2 s^{-2}$

energia $kg\,m^2 s^{-2} = J$ scalare

potenza $\frac{J}{s} = W$

$1KWh = 10^3\,W \cdot 3600\,s$

$= 3.6 \times 10^6\,W \cdot s =$

$= 3.6 \cdot 10^6\,\frac{J}{\cancel{s}}\cancel{s}$

$\vec{p} = m\vec{v}$

$kg\,m^2 s^{-1}$

$\boxed{M=0}$ $v = \omega r$

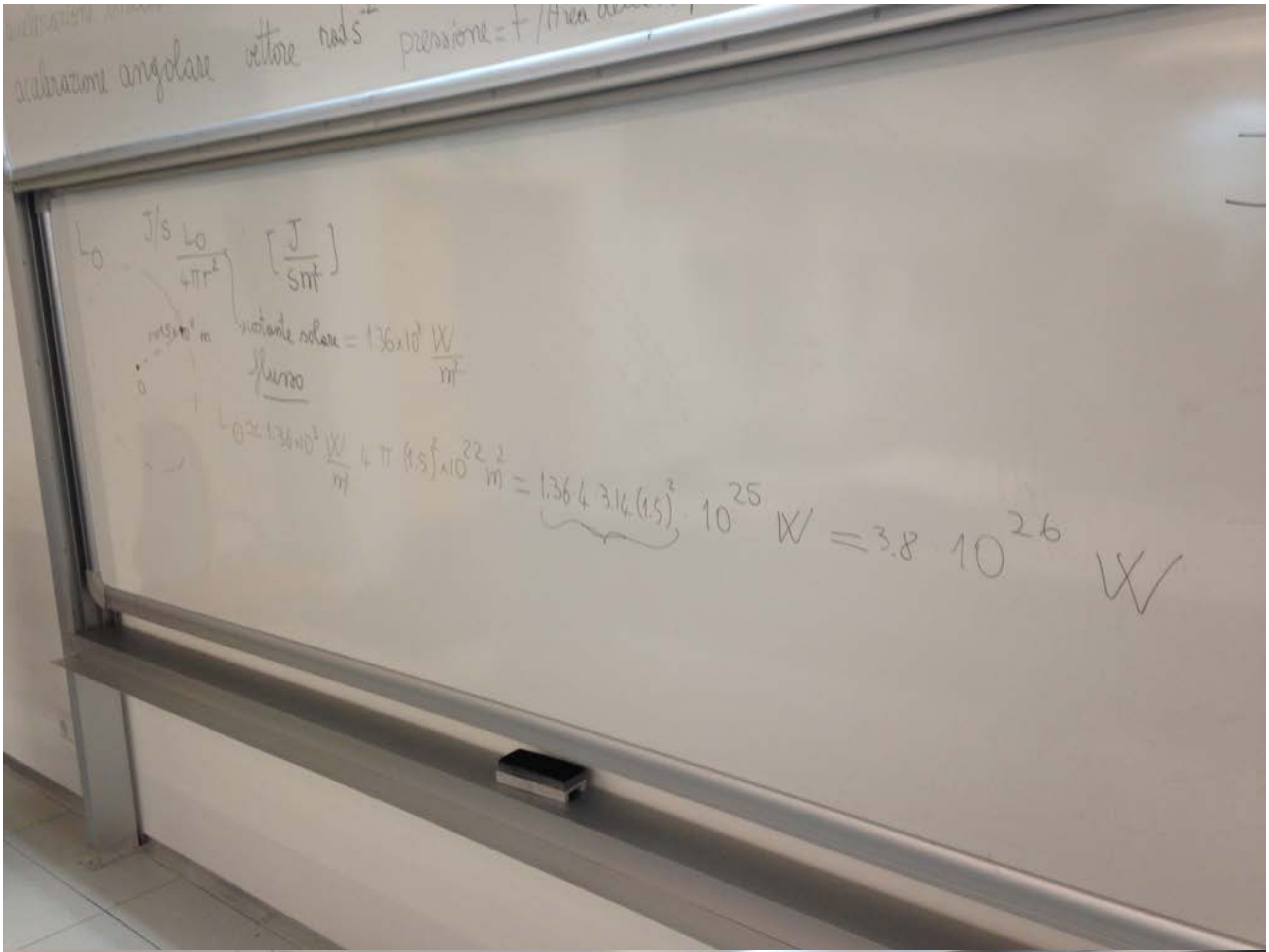
are (vettore) $L = rp$

e in un riferimento inerziale

$L = m v r = m \omega r^2$

$\omega_\oplus$

$\vec{F}$
$\vec{r}$
P
C
O
O
$\vec{r}$
$\vec{F}_c$
$\vec{v}$
$m_\oplus$



accelerazione angolare vettore rad s^{-2} pressione = F/Area

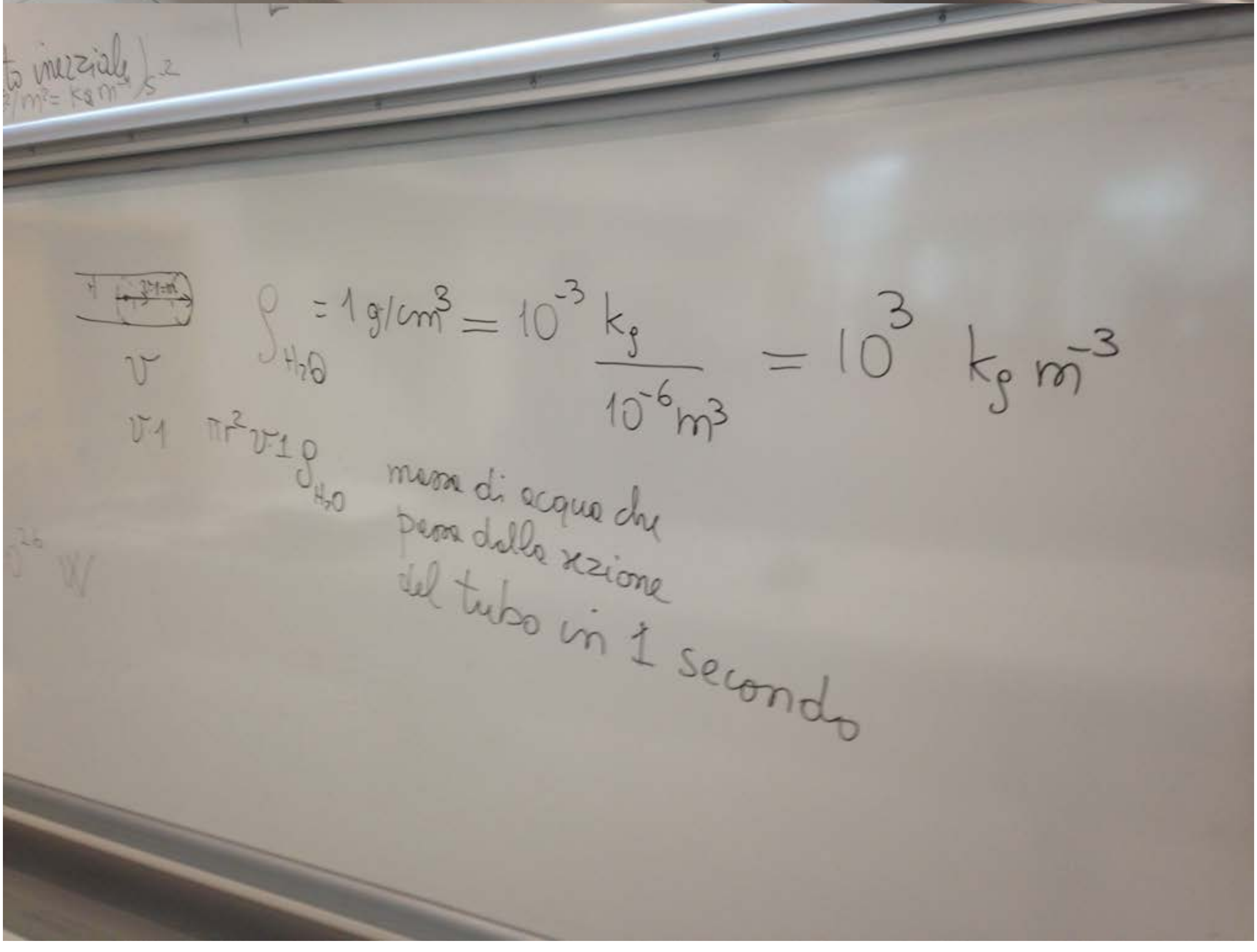
L_0 J/s $\frac{L_0}{4\pi r^2}$ $\left[\frac{J}{s m^2}\right]$

$r = 1.5 \times 10^{11}$ m

costante solare $= 1.36 \times 10^3 \frac{W}{m^2}$

flusso

$L_0 \simeq 1.36 \times 10^3 \frac{W}{m^2} 4\pi (1.5)^2 \times 10^{22} m^2 = \underbrace{1.36 \cdot 4 \cdot 3.14 \cdot (1.5)^2} \cdot 10^{25} W = 3.8 \cdot 10^{26} W$



to inerziale

$/m^2 = kg\, m^{-1} s^{-2}$

$\rho_{H_2O} = 1 g/cm^3 = 10^{-3} \frac{kg}{10^{-6} m^3} = 10^3 \; kg\, m^{-3}$

v

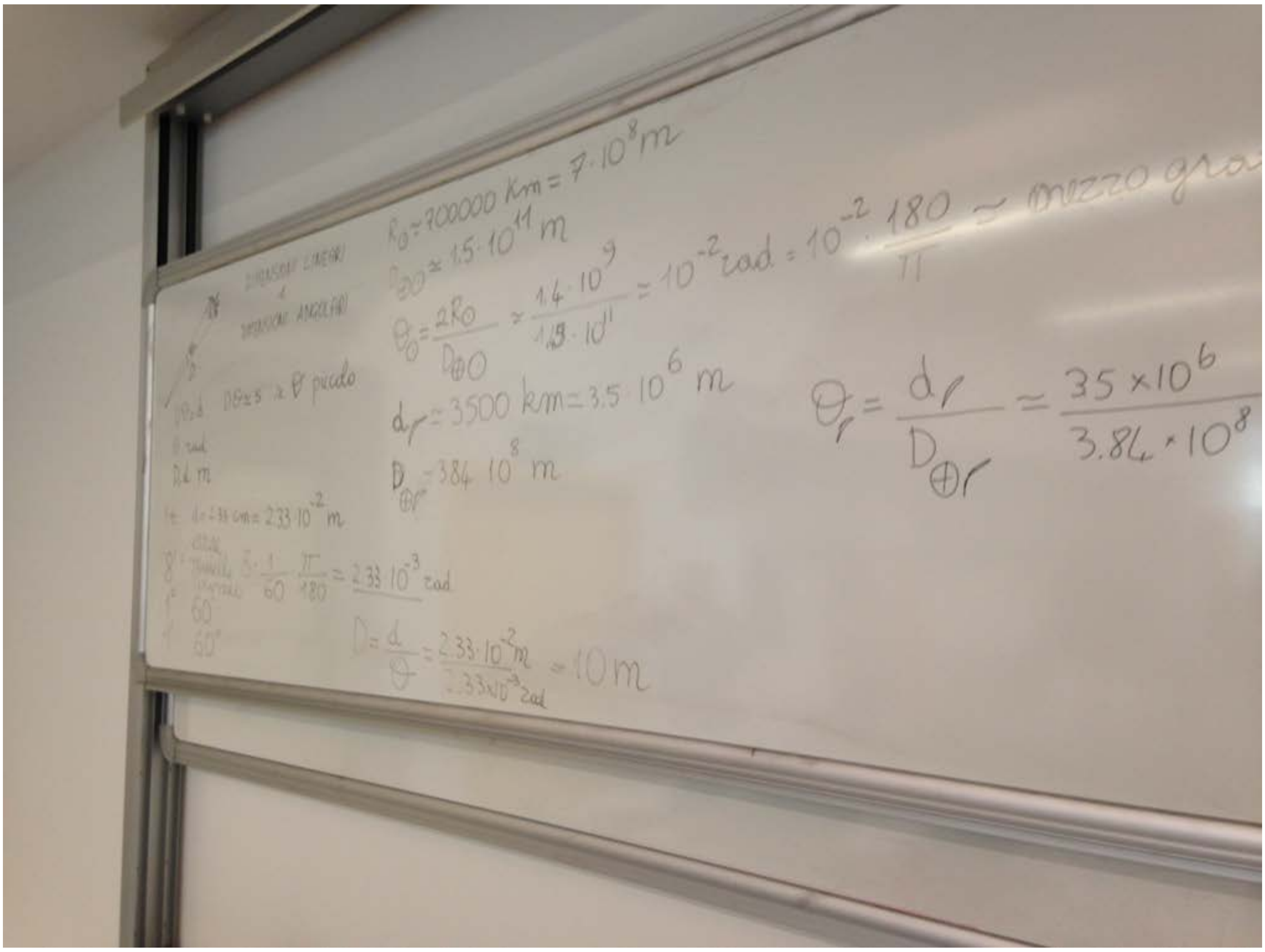
$v \cdot 1$ $\pi r^2 v 1 \rho_{H_2O}$ massa di acqua che passa dalla sezione del tubo in 1 secondo

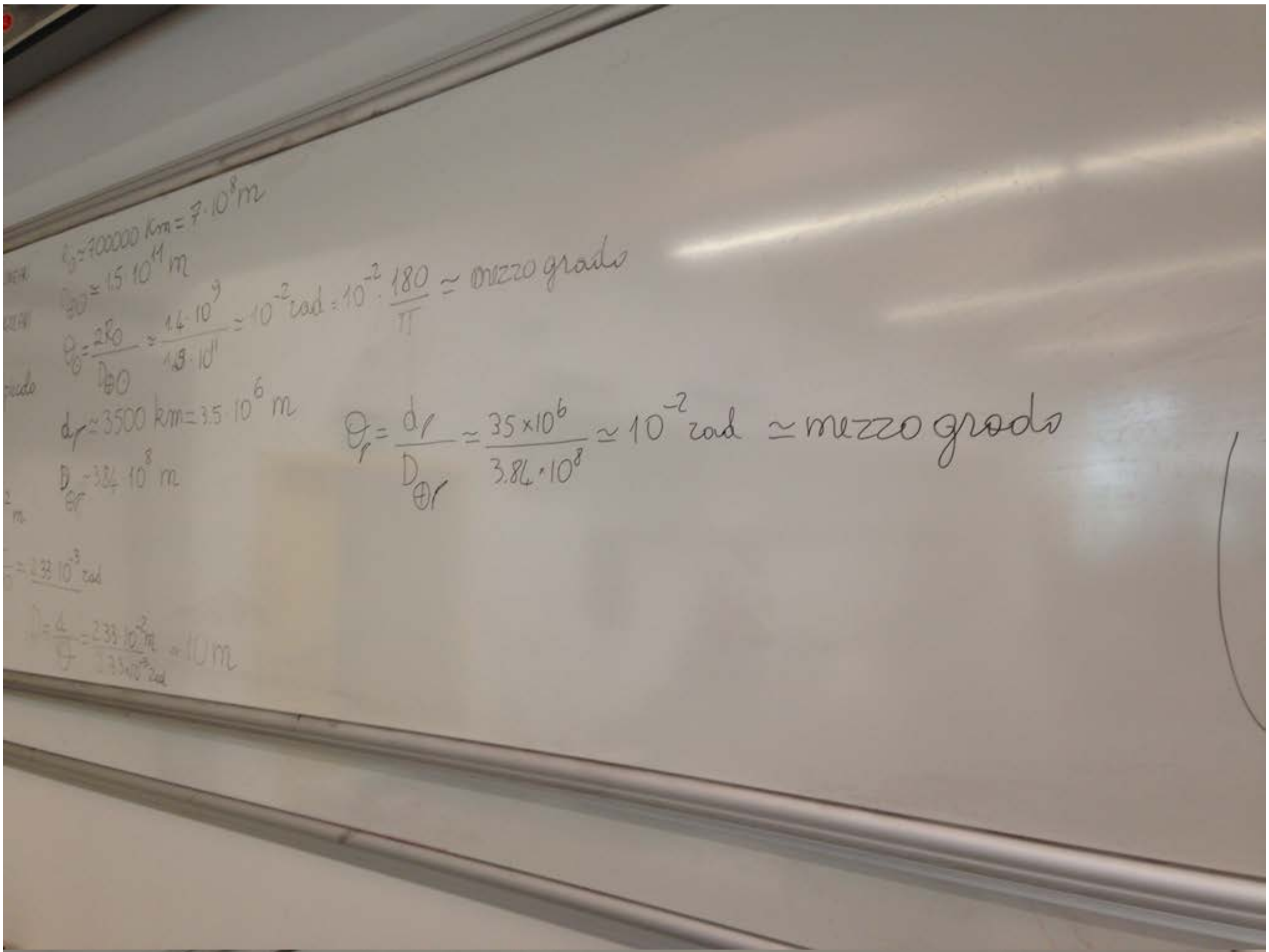
16 Ottobre 2014

-Dimensioni lineari e dimensioni angolari (es: dimensioni angolari del disco del sole e del disco della luna piena conoscendo le rispettive distanze da terra e i rispettivi raggi)

-Coordinate di un vettore in due sistemi di coordinate ruotati aventi la stessa origine e ruotati l'uno rispetto all'altro di un dat angolo; caso piano, calcolo geometrico; matrice di rotazione e relazioni tra le coordinate in forma matriciale (esempi di calcolo di matrici)

-Caso tridimensionale: rotazione attorno ad uno dei 3 assi coordinati e relativa matrice di rotazione


DIMENSIONI LINEARI
e
DIMENSIONI ANGOLARI
θ piccolo
D m
$R_\odot \simeq 700000$ km $= 7 \cdot 10^8$ m
$D_{\oplus\odot} \simeq 1.5 \cdot 10^{11}$ m
$\theta_\odot = \frac{2R_\odot}{D_{\oplus\odot}} \simeq \frac{1.4 \cdot 10^9}{1.5 \cdot 10^{11}} = 10^{-2}$ rad $= 10^{-2} \cdot \frac{180}{\pi} \simeq$ mezzo gra
$d_r \simeq 3500$ km $= 3.5 \cdot 10^6$ m
$D_{\oplus r} = 384 \cdot 10^8$ m
$\theta_r = \frac{d_r}{D_{\oplus r}} \simeq \frac{35 \times 10^6}{3.84 \times 10^8}$
$d = 2.33$ cm $= 2.33 \cdot 10^{-2}$ m
$8' \cdot \frac{1}{60} \cdot \frac{\pi}{180} = 2.33 \cdot 10^{-3}$ rad
1° 60'
1' 60"
$D = \frac{d}{\theta} = \frac{2.33 \cdot 10^{-2} m}{2.33 \times 10^{-3} rad} = 10$ m



$R_\odot \simeq 700000$ Km $= 7\cdot10^8$ m

$D_{\oplus\odot} \simeq 1.5\ 10^{11}$ m

$\theta_\odot = \frac{2R_\odot}{D_{\oplus\odot}} \simeq \frac{1.4\cdot10^9}{1.5\cdot10^{11}} \simeq 10^{-2}$ rad $= 10^{-2}\cdot\frac{180}{\pi} \simeq$ mezzo grado

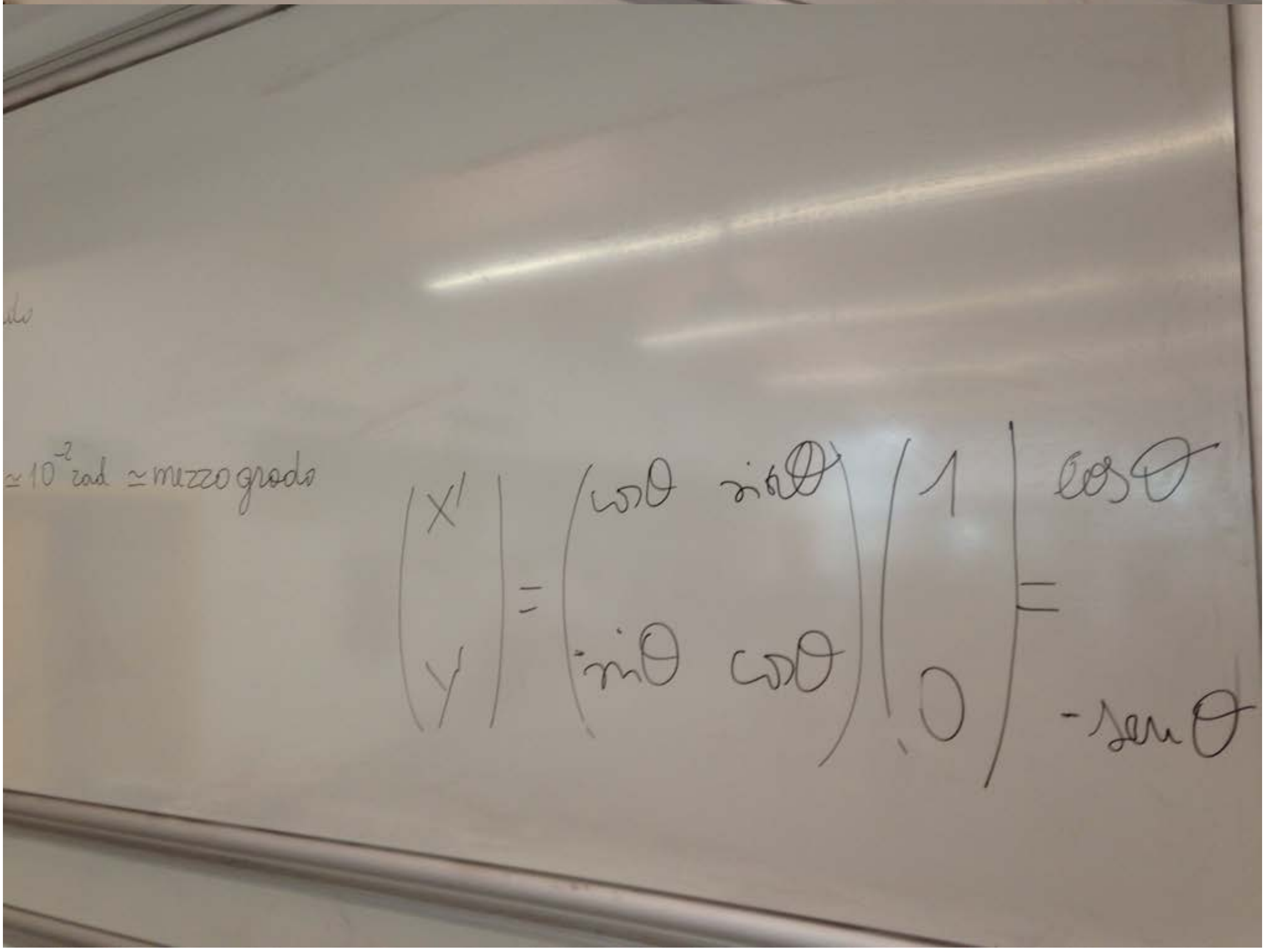
$d_r \simeq 3500$ km $= 3.5\ 10^6$ m

$D_{\oplus r} \simeq 3.84\ 10^8$ m

$\theta_r = \frac{d_r}{D_{\oplus r}} \simeq \frac{35\times10^6}{3.84\cdot10^8} \simeq 10^{-2}$ rad $\simeq$ mezzo grado

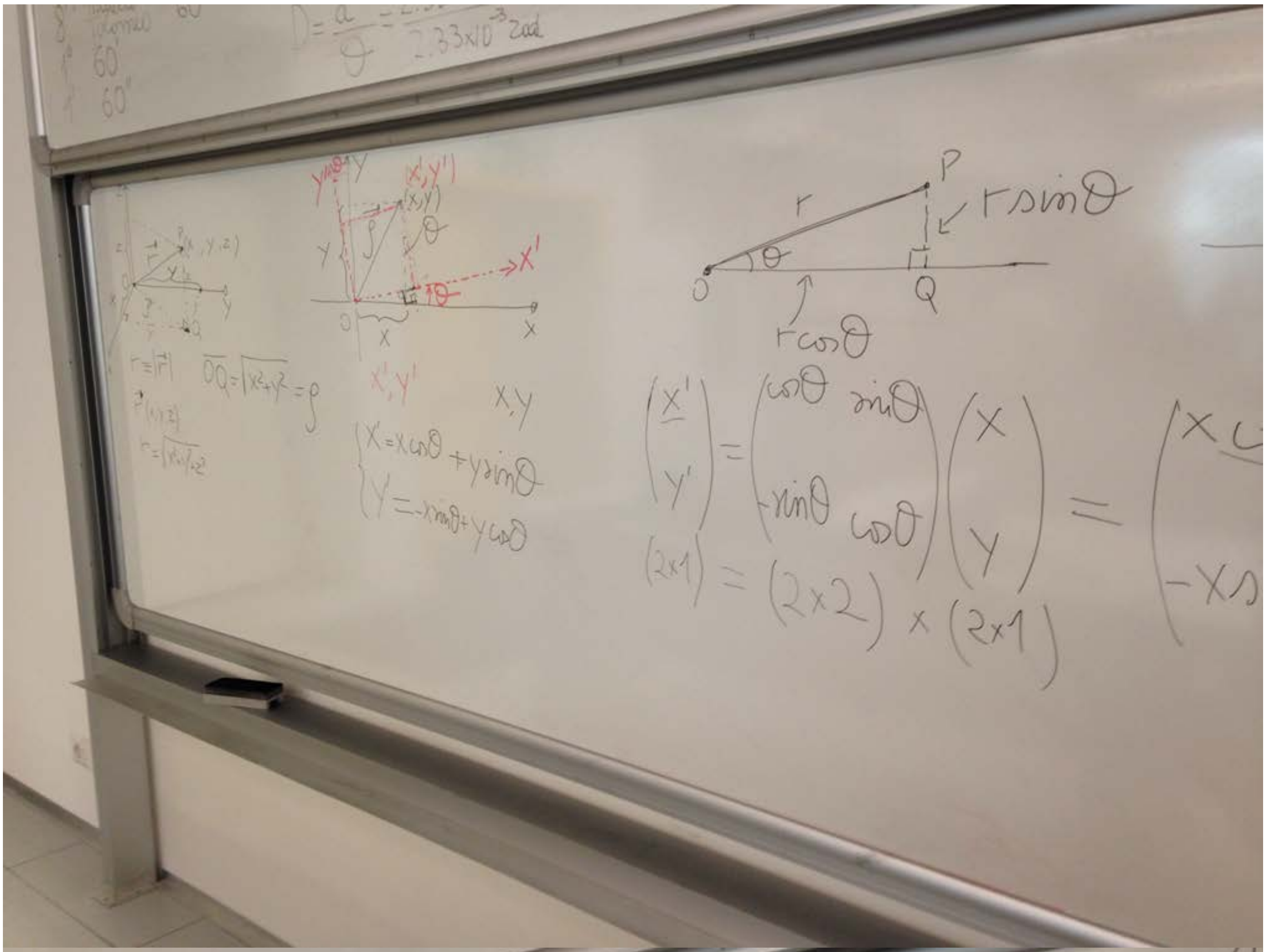
$= 2.33\ 10^{-3}$ rad

$D = \frac{d}{\theta} = \frac{2.33\ 10^{-2}\ m}{2.33\cdot10^{-3}\ rad} = 10$ m



$\simeq 10^{-2}$ rad $\simeq$ mezzo grado

$$\begin{pmatrix} X' \\ Y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ -\mathrm{sen}\,\theta \end{pmatrix}$$



$D = \frac{a}{\theta} = \frac{2.3\ldots}{2.33\times10^{-3}\,\text{rad}}$

60'

60'

$r = |\vec{r}|$

$\overline{OQ} = \sqrt{x^2+y^2} = \rho$

$\vec{r}(x,y,z)$

$r = \sqrt{x^2+y^2+z^2}$

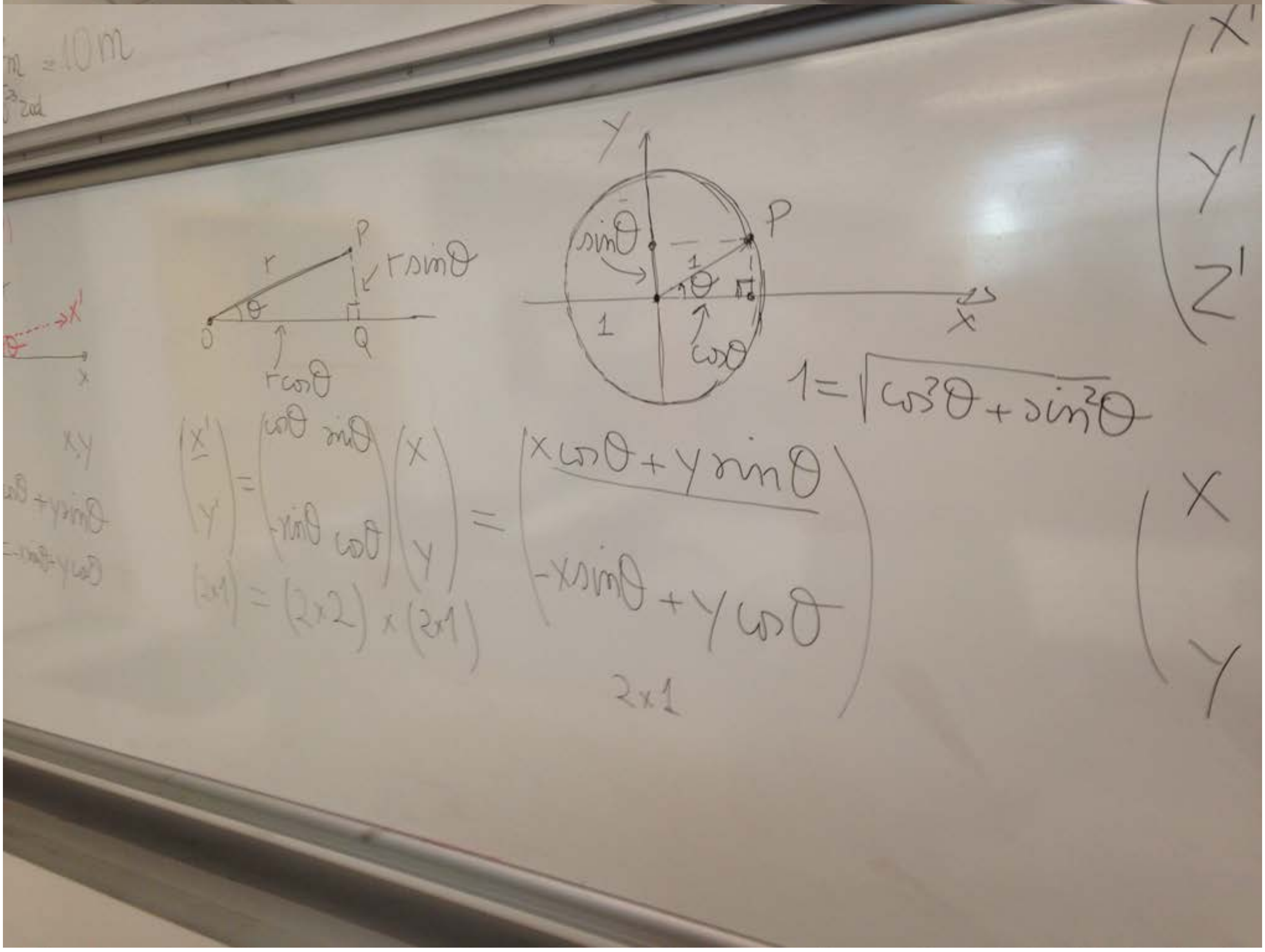
x', y' $\quad x, y$

$$\begin{cases} X = x\cos\theta + y\sin\theta \\ Y = -x\sin\theta + y\cos\theta \end{cases}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} =$$

$(2\times1) = (2\times2)\times(2\times1)$

P(x, y, z)
z
y
x
O
y
(x', y')
(x,y)
ρ
θ
x'
O
x
x
P
r
r sinθ
θ
O
Q
r cosθ



$= 10\,m$

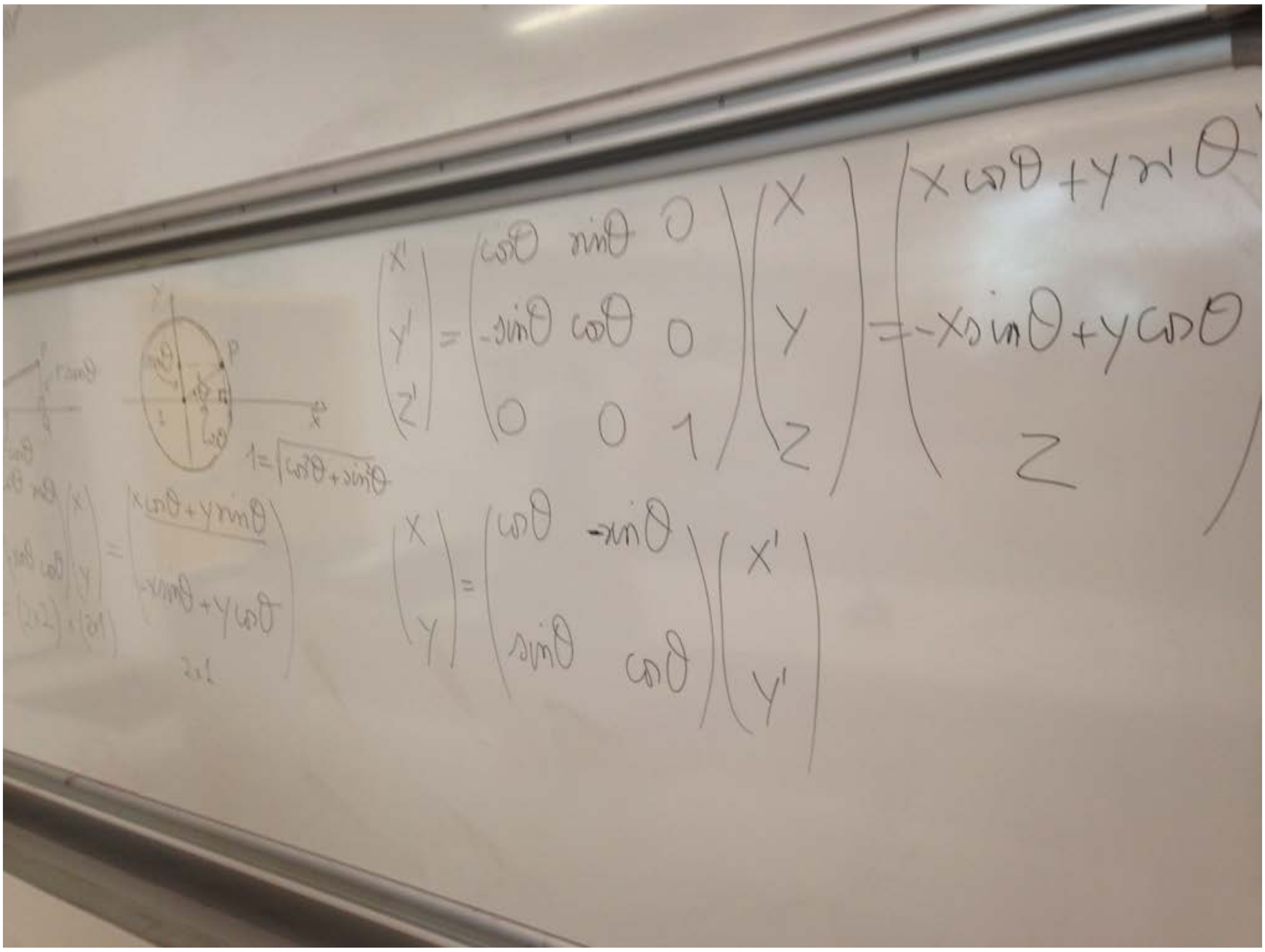
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x\cos\theta + y\sin\theta \\ -x\sin\theta + y\cos\theta \end{pmatrix}$$

$(2\times1) = (2\times2)\times(2\times1) \qquad 2\times1$

$1 = \sqrt{\cos^2\theta + \sin^2\theta}$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \qquad \begin{pmatrix} x \\ y \end{pmatrix}$$

P
r
r sinθ
θ
O
Q
r cosθ
y
P
sinθ
1
θ
1
cosθ
x

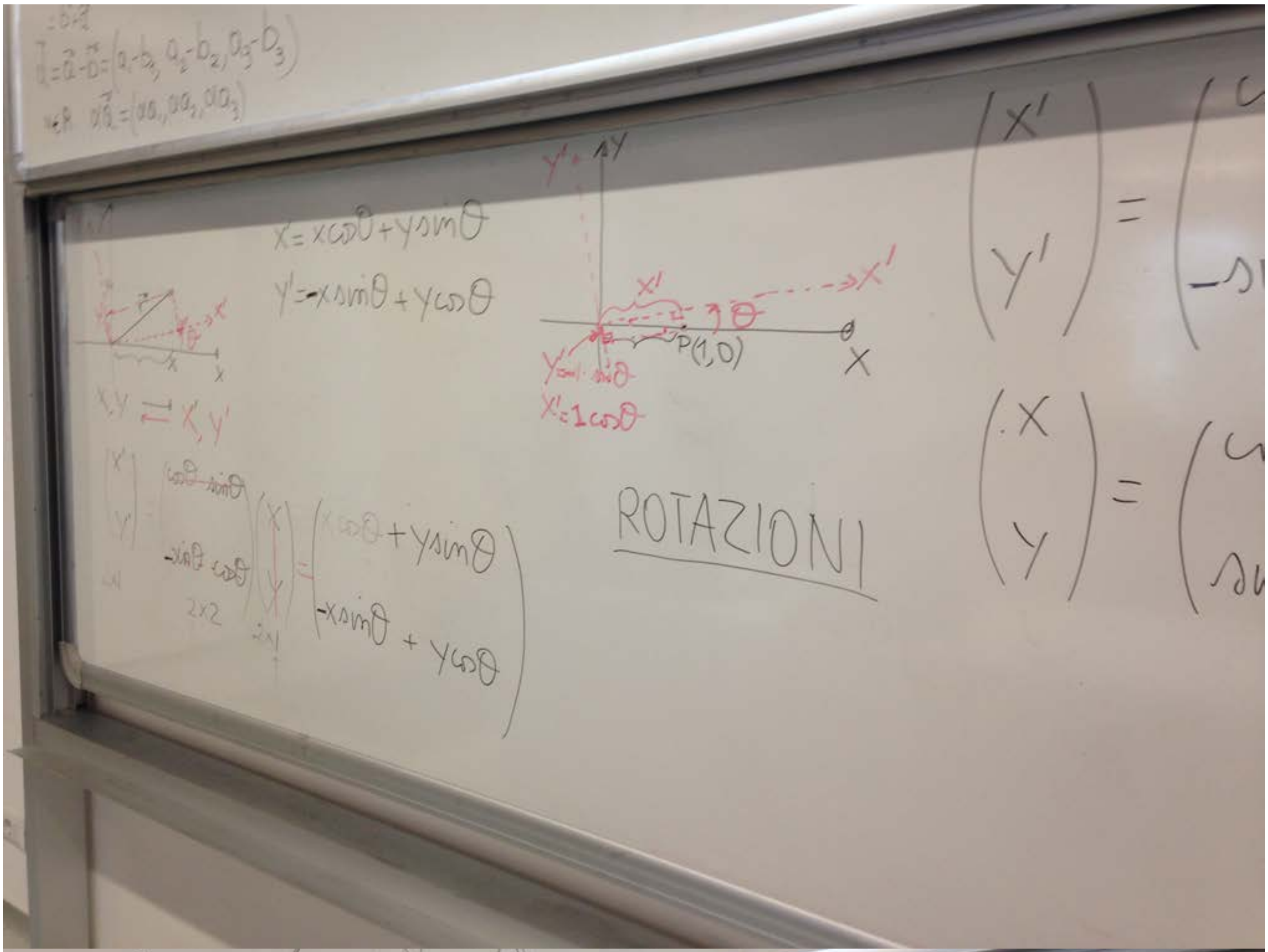

x' y' z' = cosθ sinθ 0 / -sinθ cosθ 0 / 0 0 1 · x y z = x cosθ + y sinθ / -x sinθ + y cosθ / z
x y = cosθ -sinθ / sinθ cosθ · x' y'
1 = √(cos²θ + sin²θ)
x cosθ + y sinθ / -x sinθ + y cosθ
2x1
P
x
y

23 Ottobre 2014

-Sommario su rotazioni e matrici di rotazione in due e in tre dimensioni; un metodo semplice per stabilire il segno

-Vettori: somma e differenza di vettori con la regola del parallelogrammo. Come sommare e sottrarre correttamente i vettori. Esempi di moto curvilineo e in particolare di moto circolare uniforme (vettore velocità e vettore accelerazione nel moto circolare uniforme)

-Prodotto scalare e prodotto vettore (modulo) e loro significato geometrico



$\vec{d}=\vec{a}-\vec{b}=(a_1-b_1, a_2-b_2, a_3-b_3)$

$\alpha\in R \quad \alpha\vec{a}=(\alpha a_1, \alpha a_2, \alpha a_3)$

$x'=x\cos\theta+y\sin\theta$

$y'=-x\sin\theta+y\cos\theta$

$y'=-1\sin\theta$

$x'=1\cos\theta$

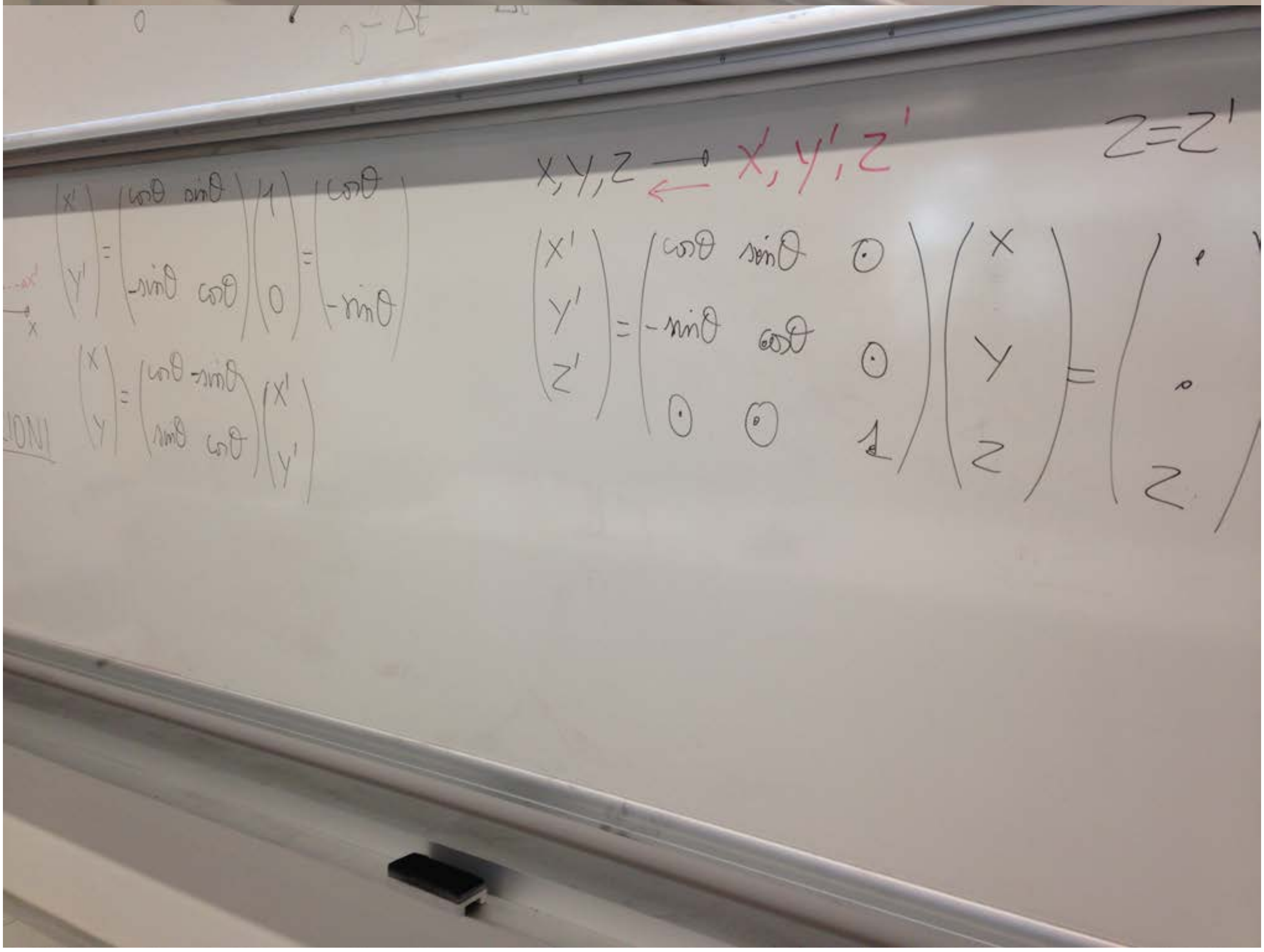
$x,y \rightleftarrows x',y'$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \underset{2\times2}{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}} \underset{2\times1}{\begin{pmatrix} x \\ y \end{pmatrix}} = \begin{pmatrix} x\cos\theta+y\sin\theta \\ -x\sin\theta+y\cos\theta \end{pmatrix}$$

ROTAZIONI

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} c \\ -s \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c \\ s \end{pmatrix}$$



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$$

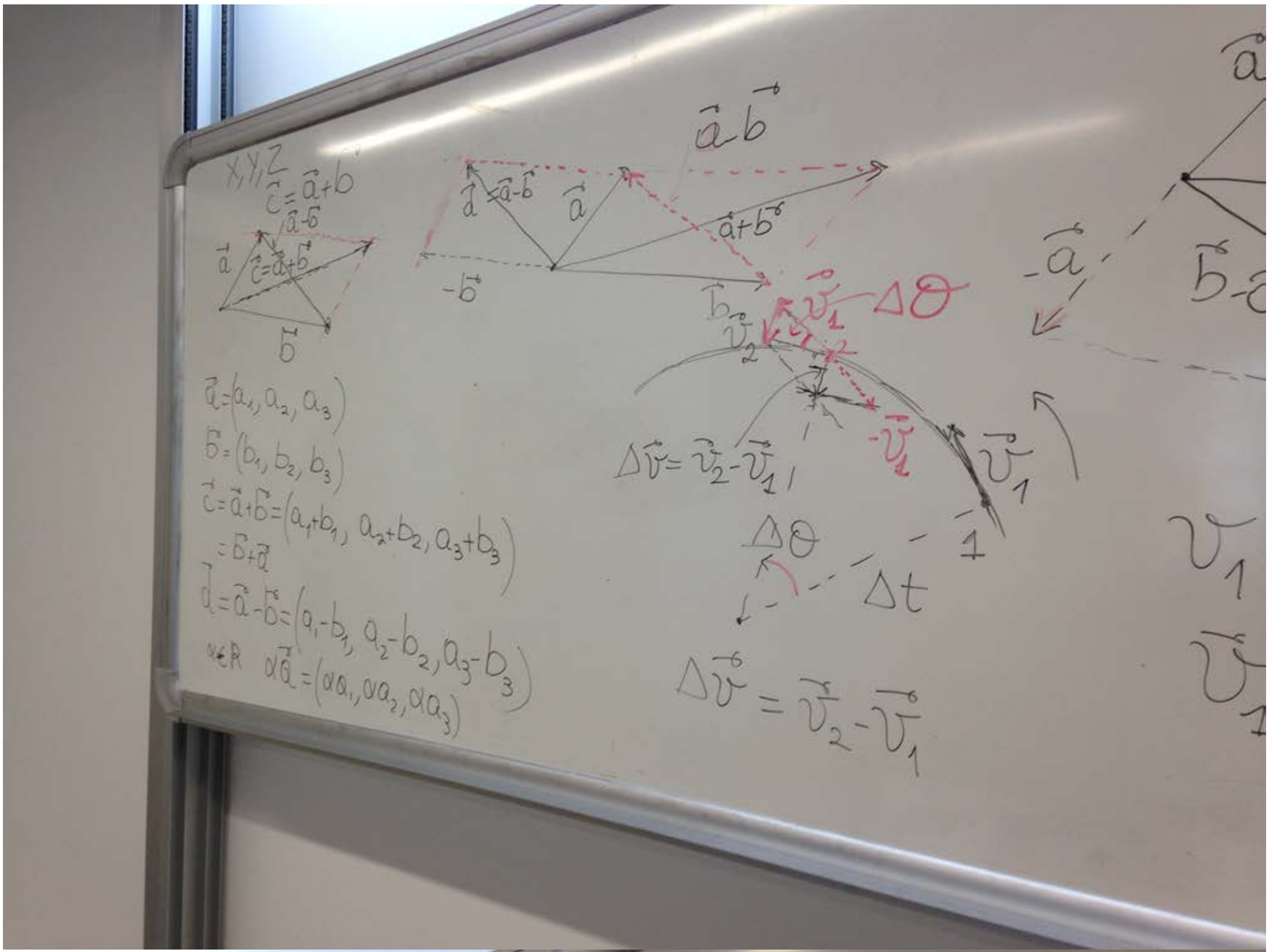
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

ZIONI

$x,y,z \rightleftarrows x',y',z'$

$z=z'$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cdot \\ \cdot \\ z \end{pmatrix}$$


x, y, z
$\vec{c} = \vec{a} + \vec{b}$
$\vec{a} - \vec{b}$
$\vec{a}$
$\vec{c} = \vec{a} + \vec{b}$
$\vec{b}$
$\vec{a} \cdot \vec{b}$
$\vec{d}$
$\vec{a} - \vec{b}$
$\vec{a}$
$-\vec{b}$
$\vec{a} + \vec{b}$
$\vec{b}$
$\vec{v}_1$
$\Delta\theta$
$\vec{v}_2$
2
$-\vec{v}_1$
$\vec{v}_1$
1
$\Delta\theta$
Δt
$\vec{a}$
$-\vec{a}$
$\vec{b} - \vec{a}$
v_1
$\vec{v}_1$

$\vec{a} = (a_1, a_2, a_3)$

$\vec{b} = (b_1, b_2, b_3)$

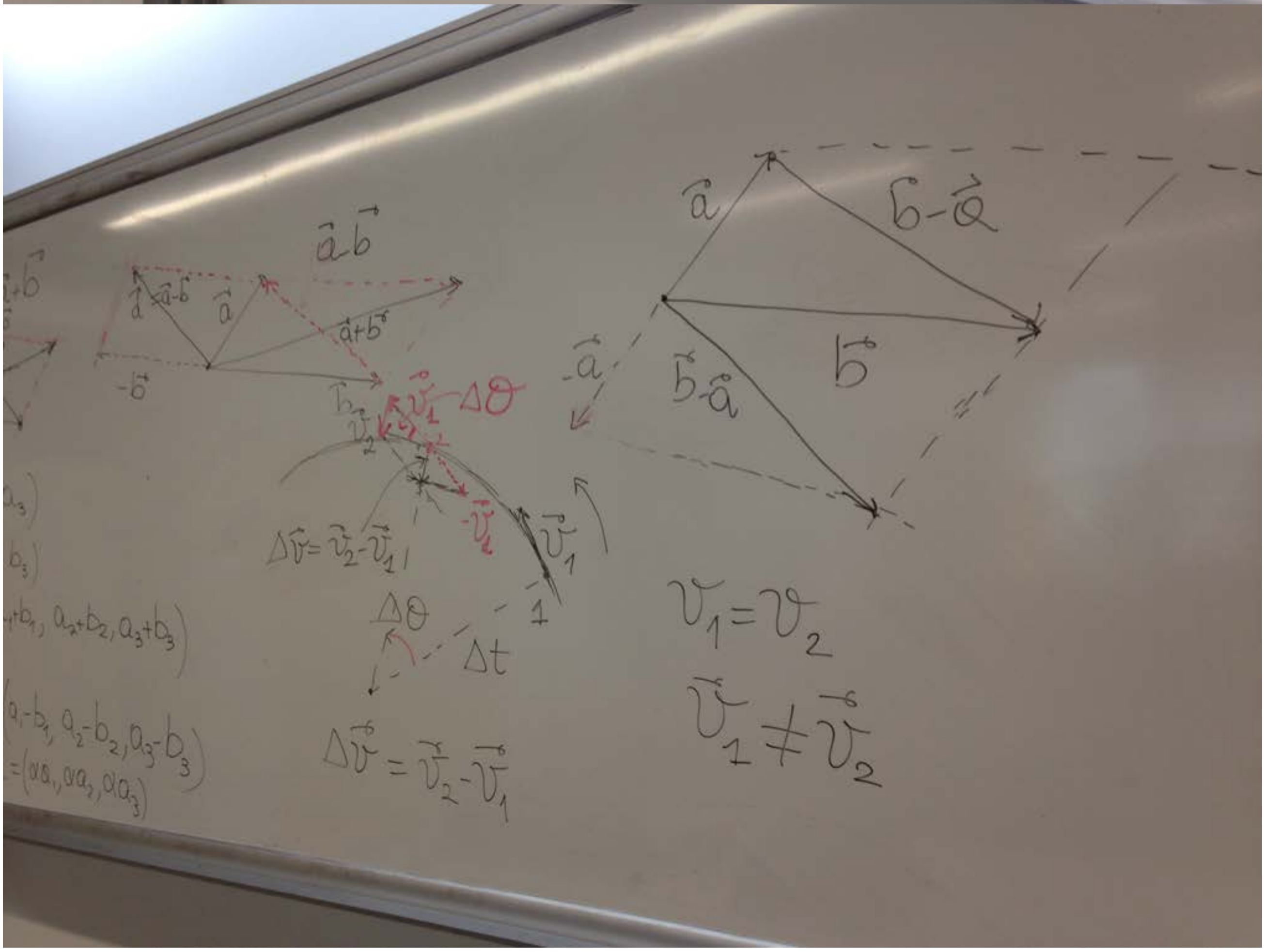
$\vec{c} = \vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
$= \vec{b} + \vec{a}$

$\vec{d} = \vec{a} - \vec{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

$\alpha \in R \quad \alpha\vec{a} = (\alpha a_1, \alpha a_2, \alpha a_3)$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

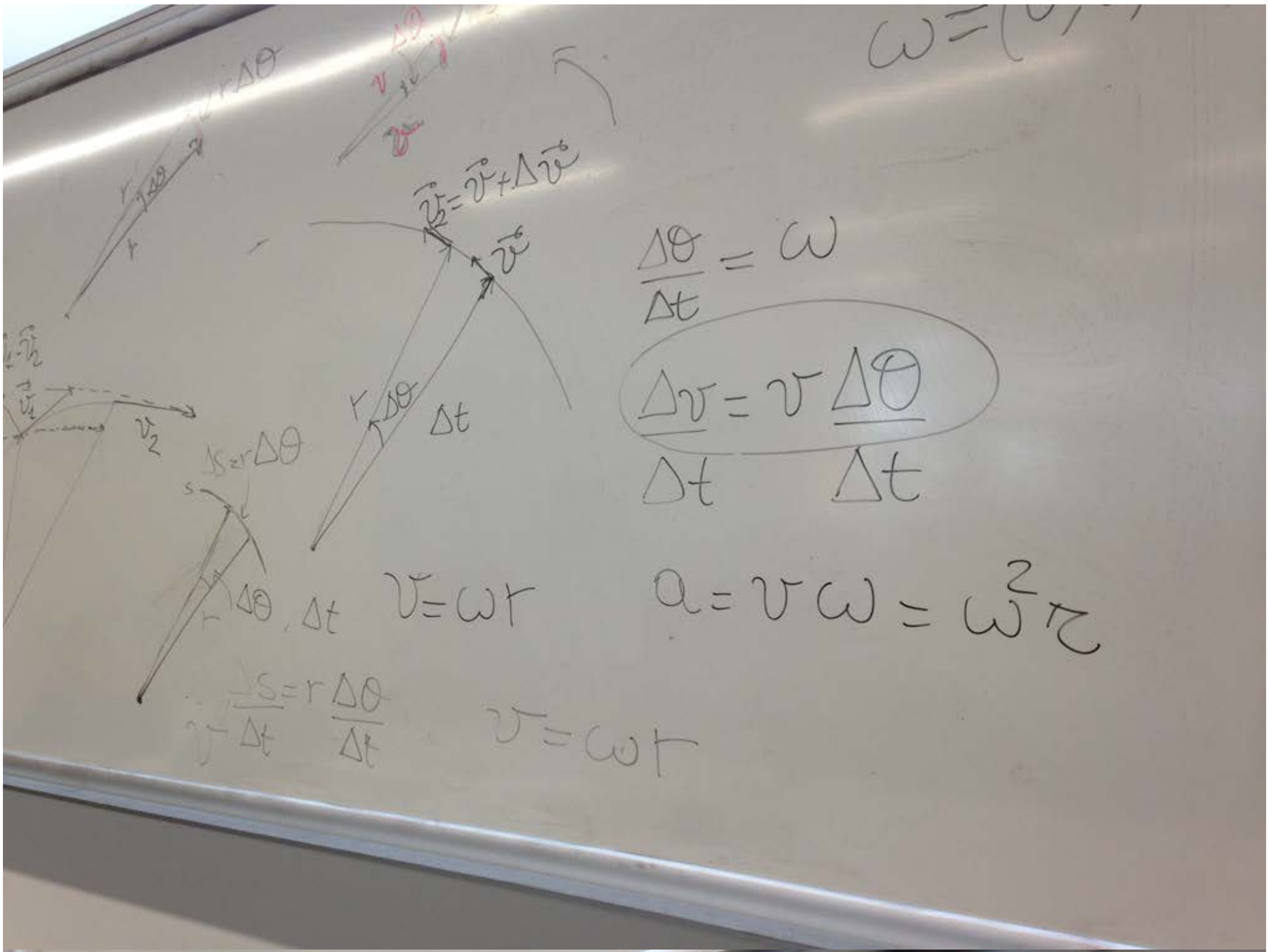

$\vec{a} \cdot \vec{b}$
$\vec{d}$
$\vec{a} - \vec{b}$
$\vec{a}$
$\vec{a} + \vec{b}$
$-\vec{b}$
$\vec{b}$
$\vec{v}_1$
$\Delta\theta$
$\vec{v}_2$
$-\vec{v}_1$
$\vec{v}_1$
1
$\Delta\theta$
Δt
$\vec{a}$
$\vec{b} - \vec{a}$
$-\vec{a}$
$\vec{b}$
$\vec{b} - \vec{a}$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

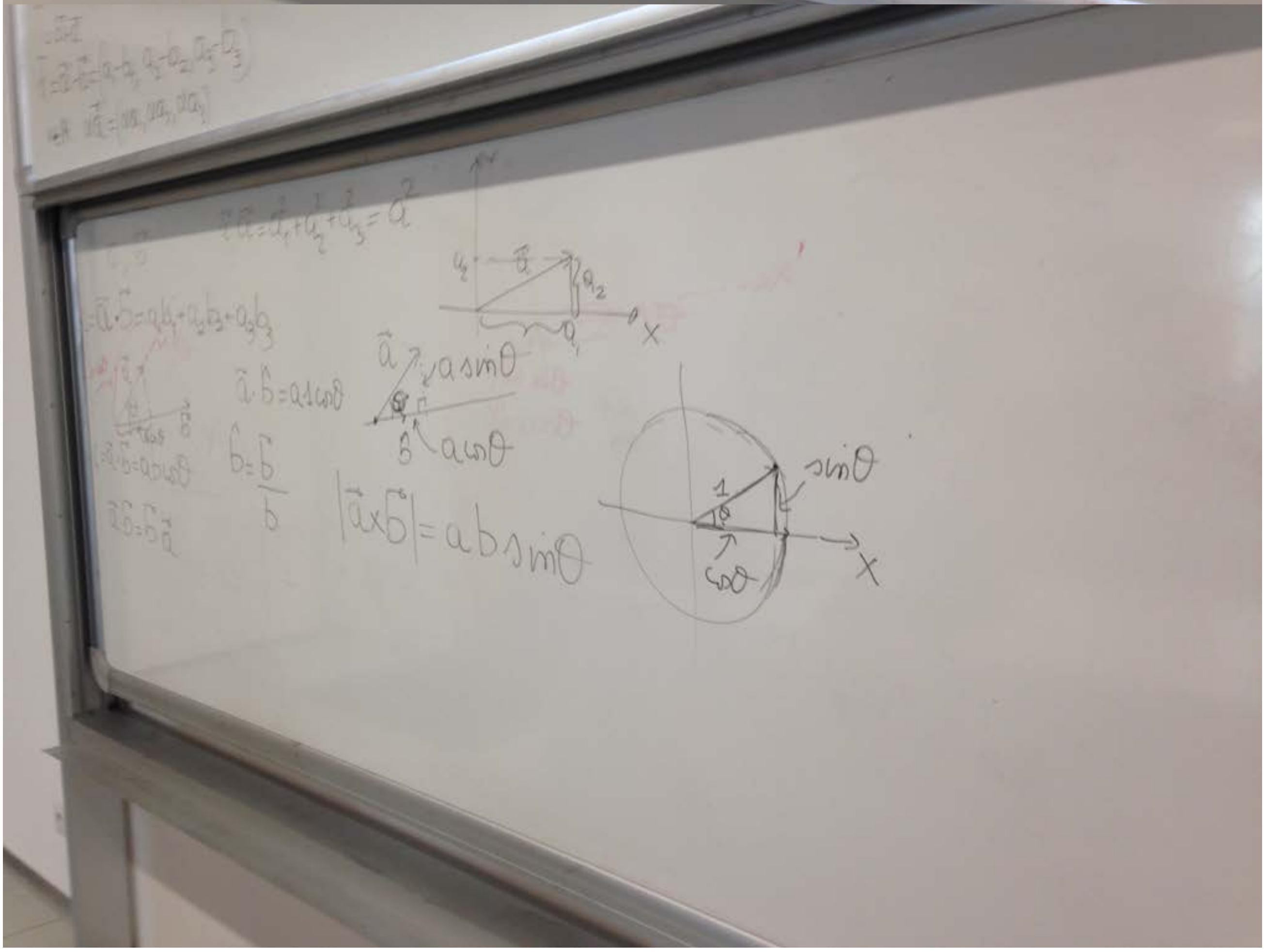
$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$v_1 = v_2$

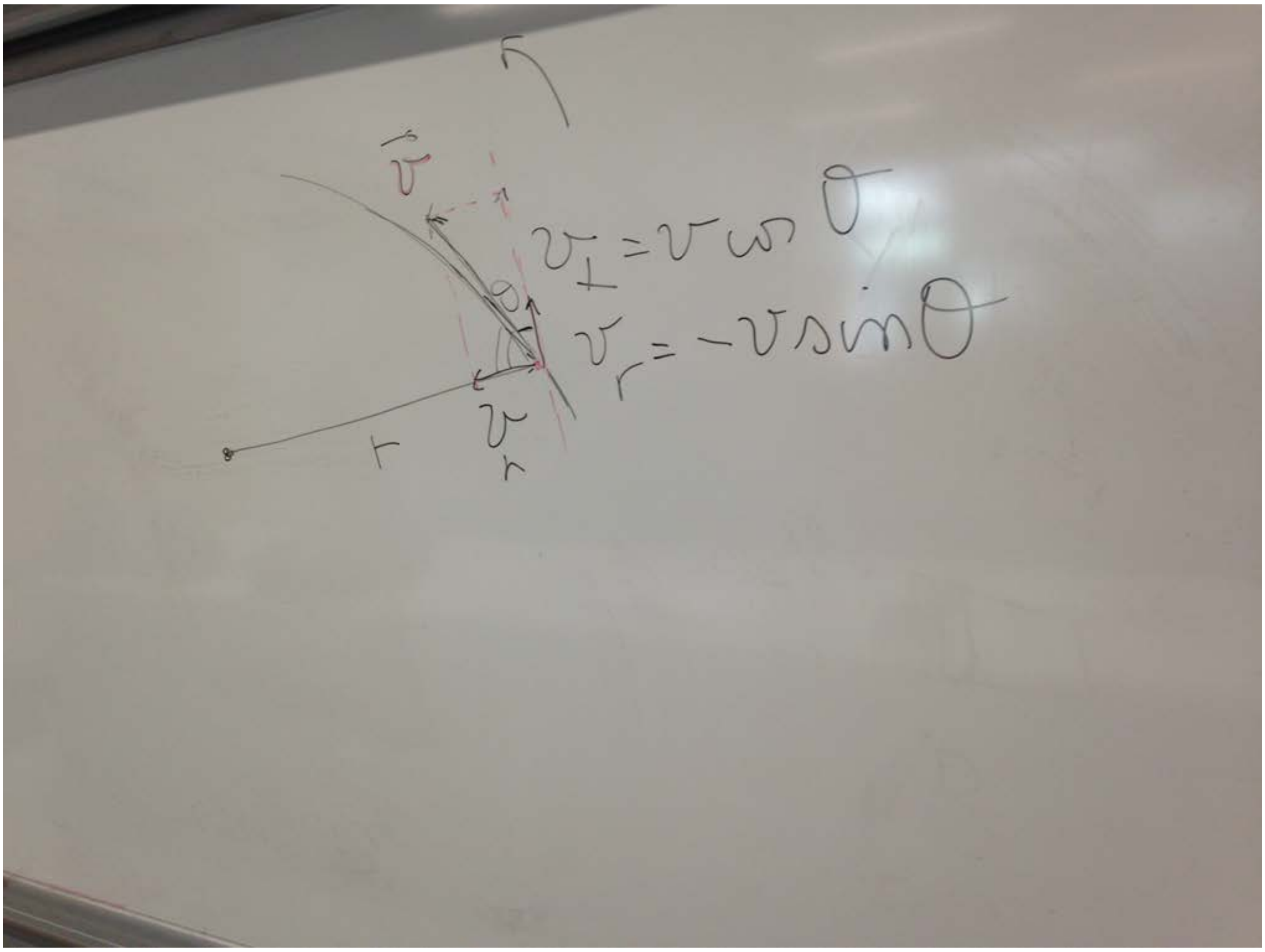
$\vec{v}_1 \neq \vec{v}_2$



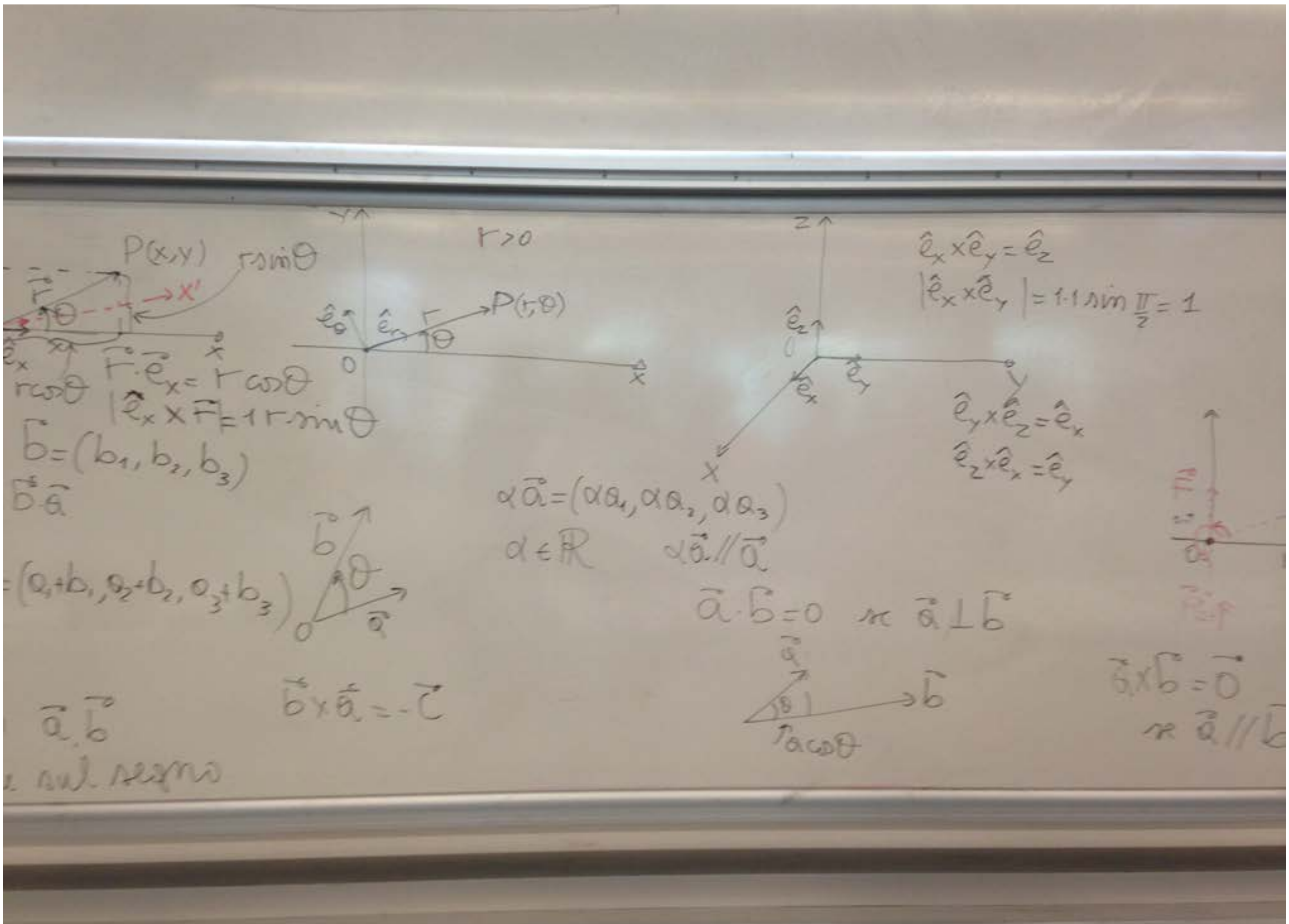
$\omega = ($
$r\Delta\theta$
$\Delta\theta$
$\vec{v}_2 = \vec{v} + \Delta\vec{v}$
$\vec{v}$
$\frac{\Delta\theta}{\Delta t} = \omega$
$\frac{\Delta v}{\Delta t} = v\frac{\Delta\theta}{\Delta t}$
$\vec{v}_1$
v_2
$\Delta s = r\Delta\theta$
r
$\Delta\theta$
Δt
$\Delta\theta,\ \Delta t$
$v = \omega r$
$a = v\omega = \omega^2 r$
$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$
$v = \omega r$



$\vec{a} = \vec{a}_1 + \vec{a}_2 + \vec{a}_3 = \vec{a}$
a_2
$\vec{a}$
a_1
x
$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
$\vec{a}$
$a\sin\theta$
$a\cos\theta$
$\vec{b}$
$\vec{a}\cdot\vec{b} = a\,1\cos\theta$
$\hat{b} = \frac{\vec{b}}{b}$
$\vec{a}\cdot\vec{b} = ab\cos\theta$
$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
$|\vec{a}\times\vec{b}| = ab\sin\theta$
1
θ
$\sin\theta$
$\cos\theta$
x


$\vec{v}$
$v_t = v\cos\theta$
$v_r = -v\sin\theta$
θ
v_r
r

30 Ottobre 2014
-Vettori: prodotto scalare, prodotto vettore, significato geometrico e proprietà. Esempi in fisica. Scomposizione di un vettore in componenti. Versori. Versori degli assi coordinati; versori nel caso di coordinate polari


P(x,y)
r sinθ
x'
θ
$\hat{e}_x$
x
r cosθ
y
O
$\hat{e}_\theta$
$\hat{e}_r$
r
P(r,θ)
z
$\hat{e}_z$
$\hat{e}_y$
$\vec{b}$
$\vec{a}$
a cosθ

$\vec{r}\cdot\hat{e}_x = r\cos\theta$

$|\hat{e}_x \times \vec{r}| = 1\, r \sin\theta$

$\vec{b} = (b_1, b_2, b_3)$

$\vec{b}\cdot\vec{a}$

$= (a_1+b_1, a_2+b_2, a_3+b_3)$

$\vec{a}, \vec{b}$

sul segno

$\vec{b}\times\vec{a} = -\vec{c}$

$r>0$

$\alpha\vec{a} = (\alpha a_1, \alpha a_2, \alpha a_3)$

$\alpha \in \mathbb{R}$ $\quad \alpha\vec{a} \,//\, \vec{a}$

$\vec{a}\cdot\vec{b} = 0$ se $\vec{a} \perp \vec{b}$

$\hat{e}_x \times \hat{e}_y = \hat{e}_z$

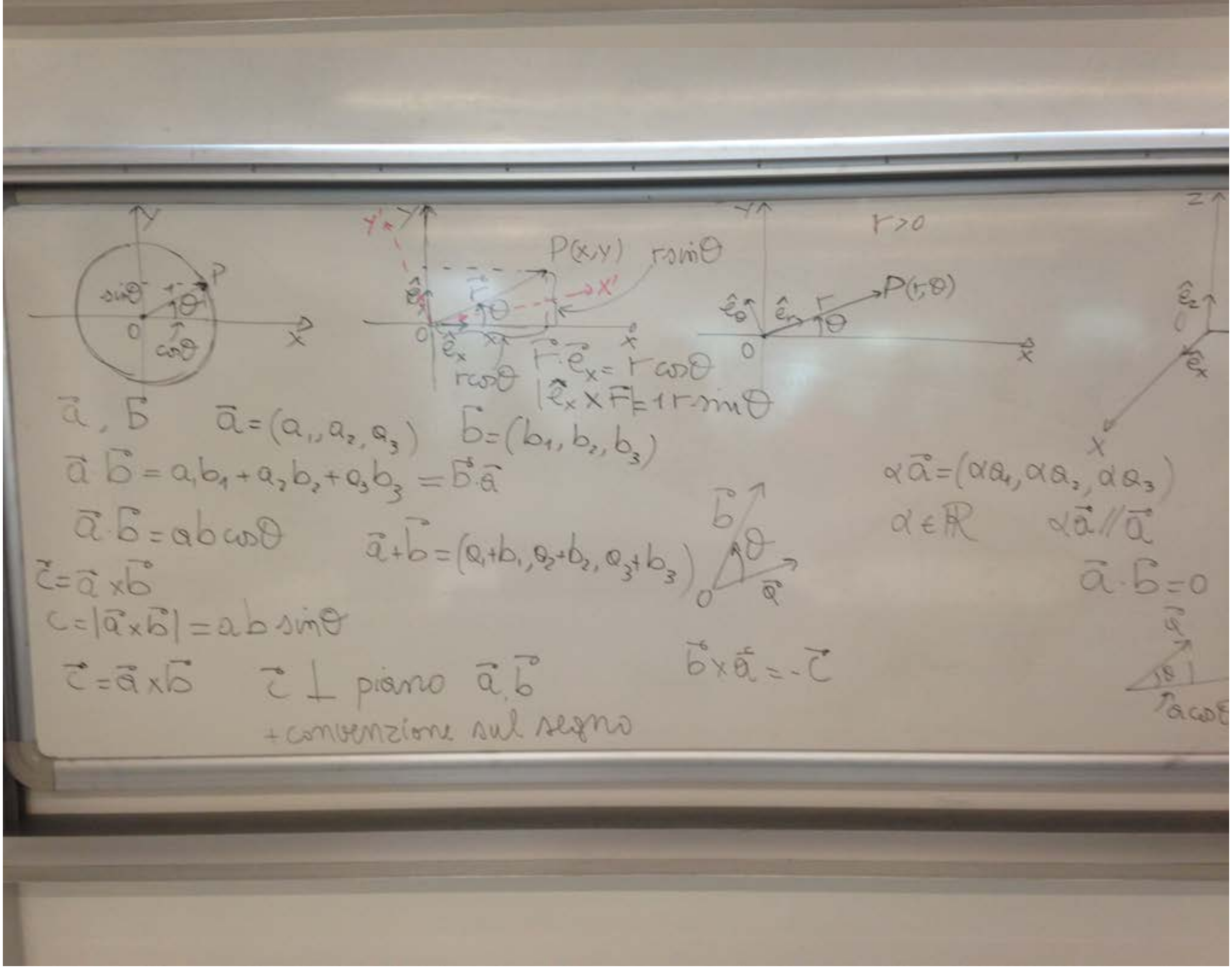
$|\hat{e}_x \times \hat{e}_y| = 1\cdot 1 \sin\frac{\pi}{2} = 1$

$\hat{e}_y \times \hat{e}_z = \hat{e}_x$

$\hat{e}_z \times \hat{e}_x = \hat{e}_y$

$\vec{a}\times\vec{b} = \vec{0}$

se $\vec{a} \,//\, \vec{b}$


y
P
sinθ
cosθ
θ
O
x
y'
P(x,y)
r sinθ
x'
$\hat{e}_y$
$\hat{e}_x$
r cosθ
$\hat{e}_\theta$
$\hat{e}_r$
r
P(r,θ)
z
$\hat{e}_z$
$\vec{b}$
$\vec{a}$
a cosθ

$\vec{r}\cdot\hat{e}_x = r\cos\theta$

$|\hat{e}_x \times \vec{r}| = 1\, r \sin\theta$

$r>0$

$\vec{a}, \vec{b} \quad \vec{a} = (a_1, a_2, a_3) \quad \vec{b} = (b_1, b_2, b_3)$

$\vec{a}\,\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \vec{b}\cdot\vec{a}$

$\vec{a}\cdot\vec{b} = ab\cos\theta \qquad \vec{a}+\vec{b} = (a_1+b_1, a_2+b_2, a_3+b_3)$

$\vec{c} = \vec{a}\times\vec{b}$

$c = |\vec{a}\times\vec{b}| = ab\sin\theta$

$\vec{c} = \vec{a}\times\vec{b} \quad \vec{c} \perp$ piano $\vec{a}, \vec{b}$

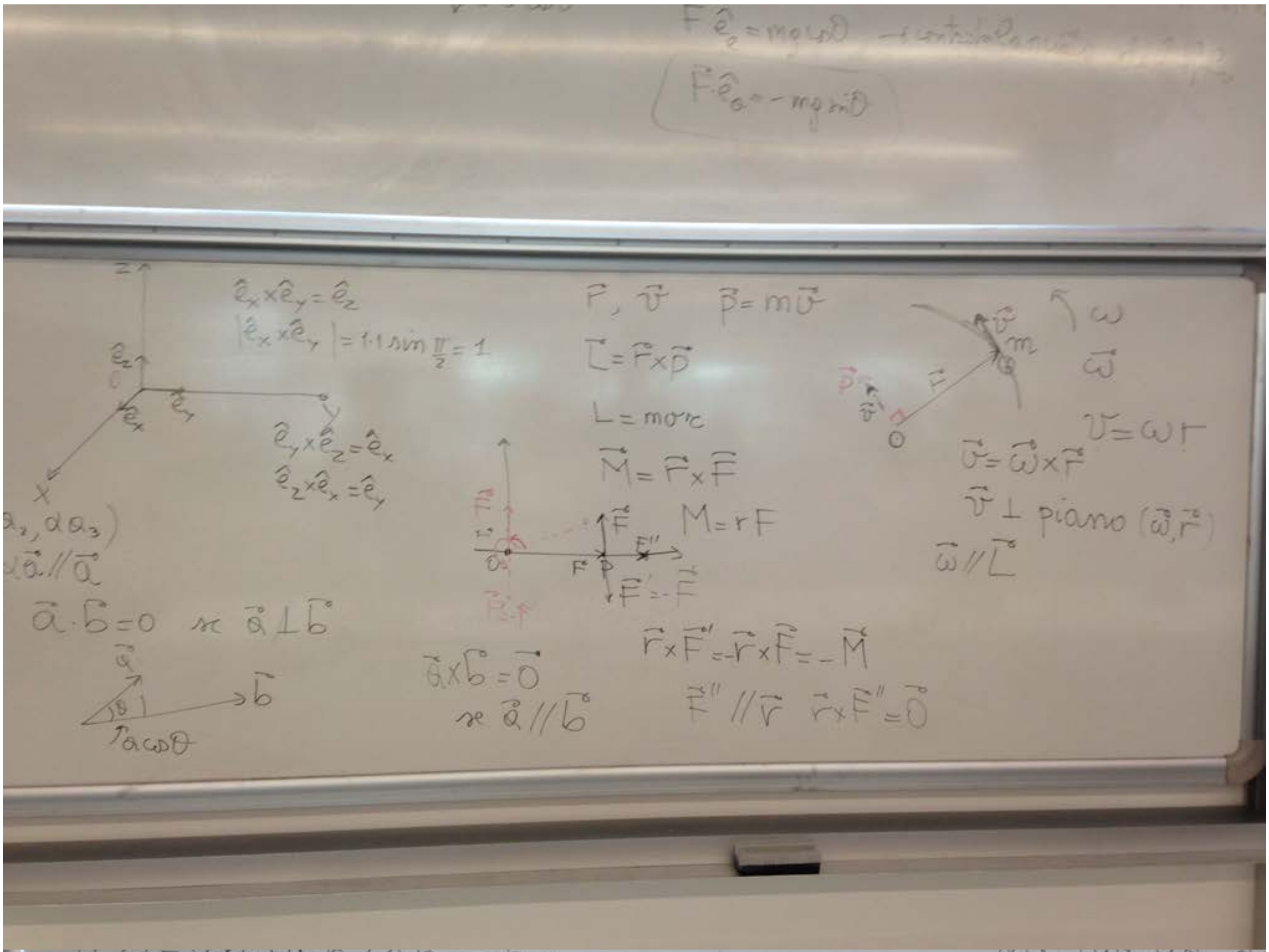
+ convenzione sul segno

$\vec{b}\times\vec{a} = -\vec{c}$

$\alpha\vec{a} = (\alpha a_1, \alpha a_2, \alpha a_3)$

$\alpha \in \mathbb{R} \quad \alpha\vec{a} \,//\, \vec{a}$

$\vec{a}\cdot\vec{b} = 0$


z
x
y
$\hat e_z$
$\hat e_x$
$\hat e_y$
O
$\vec a$
$\vec b$
θ
$a\cos\theta$
$\vec F$
$\vec F'$
$\vec F''$
P
O
$\vec v$
m
$\vec r$
$\vec p$

$F\cdot\hat e_z = mg\cos\theta$ → controllare

$$\boxed{F\cdot\hat e_\theta = -mg\sin\theta}$$

$\hat e_x\times\hat e_y=\hat e_z$

$|\hat e_x\times\hat e_y| = 1\cdot 1\sin\frac{\pi}{2} = 1$

$\hat e_y\times\hat e_z=\hat e_x$

$\hat e_z\times\hat e_x=\hat e_y$

$a_2, \alpha a_3)$

$\alpha\vec a // \vec a$

$\vec a\cdot\vec b = 0$ se $\vec a\perp\vec b$

$\vec r,\ \vec v \quad \vec p = m\vec v$

$\vec L = \vec r\times\vec p$

$L = mvr$

$\vec M = \vec r\times\vec F$

$M = rF$

$\vec F' = -\vec F$

$\vec r\times\vec F' = -\vec r\times\vec F = -\vec M$

$\vec a\times\vec b = \vec 0$ se $\vec a // \vec b$

$\vec F'' // \vec r \quad \vec r\times\vec F'' = \vec 0$

ω

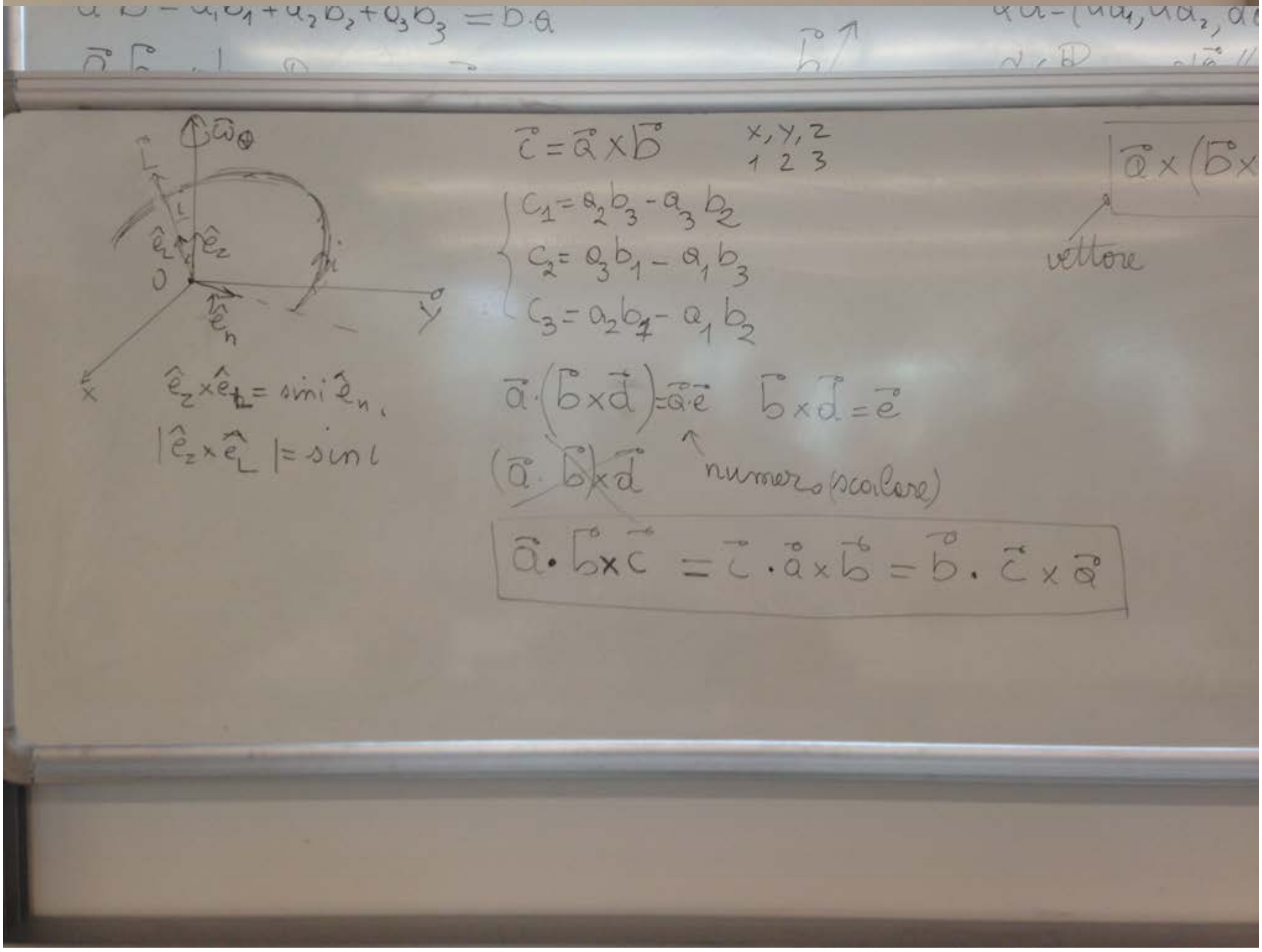
$\vec\omega$

$v = \omega r$

$\vec v = \vec\omega\times\vec r$

$\vec v\perp$ piano $(\vec\omega,\vec r)$

$\vec\omega // \vec L$


$\vec\omega_\oplus$
$\vec L$
i
$\hat e_L$
$\hat e_z$
O
$\hat e_n$
x
y

$a_1b_1 + a_2b_2 + a_3b_3 = b\cdot a$

$\vec c = \vec a\times\vec b$ x, y, z
1 2 3

$$\begin{cases} c_1 = a_2b_3 - a_3b_2 \\ c_2 = a_3b_1 - a_1b_3 \\ c_3 = a_2b_1 - a_1b_2 \end{cases}$$

$\hat e_z\times\hat e_L = \sin i\,\hat e_n$

$|\hat e_z\times\hat e_L| = \sin i$

$\vec a\cdot(\vec b\times\vec d) = \vec a\cdot\vec e \quad \vec b\times\vec d = \vec e$

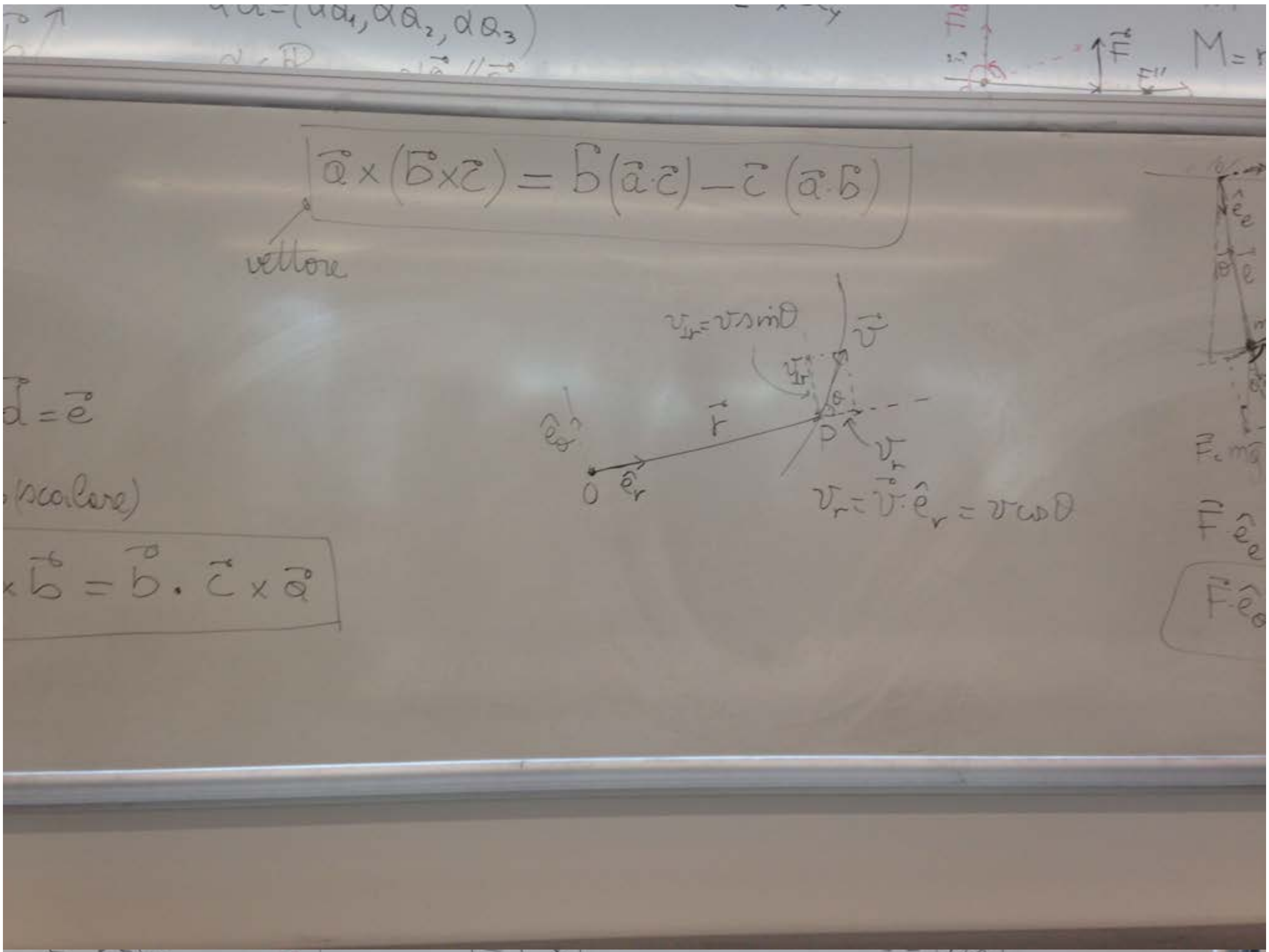
numero (scalare)

$\cancel{(\vec a\cdot\vec b)\times\vec d}$

$$\boxed{\vec a\cdot\vec b\times\vec c = \vec c\cdot\vec a\times\vec b = \vec b\cdot\vec c\times\vec a}$$

$\vec a\times(\vec b\times$

vettore



(dQ_1, dQ_2, dQ_3)

$M =$

$$\boxed{\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})}$$

vettore

$\vec{d} = \vec{e}$

(scalare)

$$\boxed{\times\vec{b} = \vec{b}\cdot\vec{c}\times\vec{a}}$$

$v_{\perp r} = v\sin\theta$

$\vec{v}$

$v_{\perp r}$

θ

$\hat{e}_\theta$

$\vec{r}$

P

v_r

O $\hat{e}_r$

$v_r = \vec{v}\cdot\hat{e}_r = v\cos\theta$

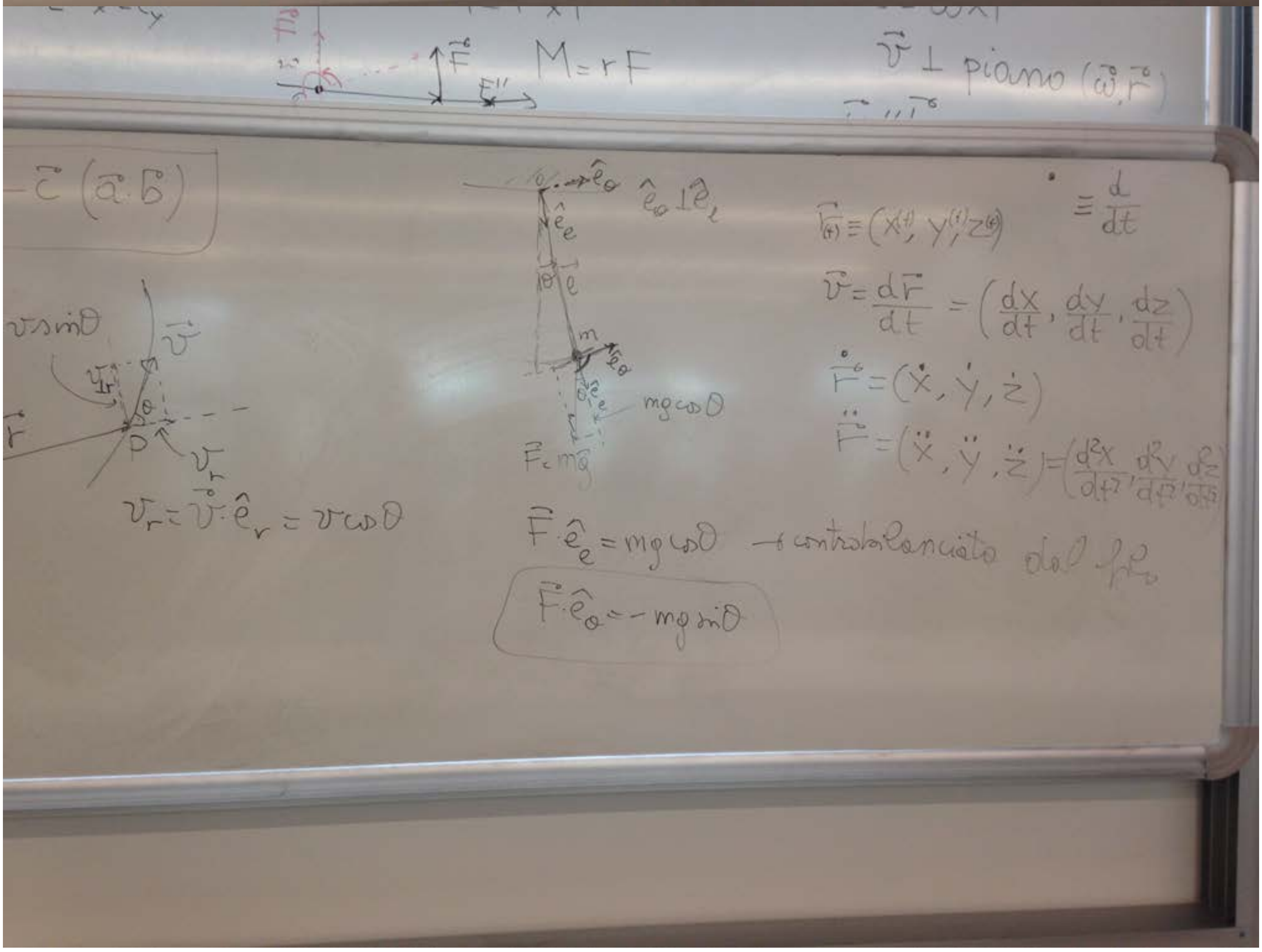
$\hat{e}_\theta$

$\vec{e}$

$\vec{F} = m\vec{g}$

$\vec{F}\hat{e}_e$

$\vec{F}\hat{e}_\theta$



$\vec{F}$ $\vec{F}''$ $M = rF$

$\vec{v} \perp$ piano $(\vec{\omega}, \vec{r})$

$-\vec{c}(\vec{a}\cdot\vec{b})$

$v\sin\theta$

$\vec{v}$

$v_{\perp r}$

θ

$\vec{r}$

P

v_r

$v_r = \vec{v}\cdot\hat{e}_r = v\cos\theta$

$\hat{e}_\theta$ $\hat{e}_\theta \perp \hat{e}_l$

$\hat{e}_e$

θ $\vec{e}$

m

$\hat{e}_\theta$

θ $\hat{e}_e$

$mg\cos\theta$

$\vec{F} = m\vec{g}$

$\vec{F}\hat{e}_e = mg\cos\theta$ → controbilanciato dal filo

$$\boxed{\vec{F}\cdot\hat{e}_\theta = -mg\sin\theta}$$

$\vec{r}_{(t)} \equiv (x^{(t)}, y^{(t)}, z^{(t)})$

$\dot{} \equiv \frac{d}{dt}$

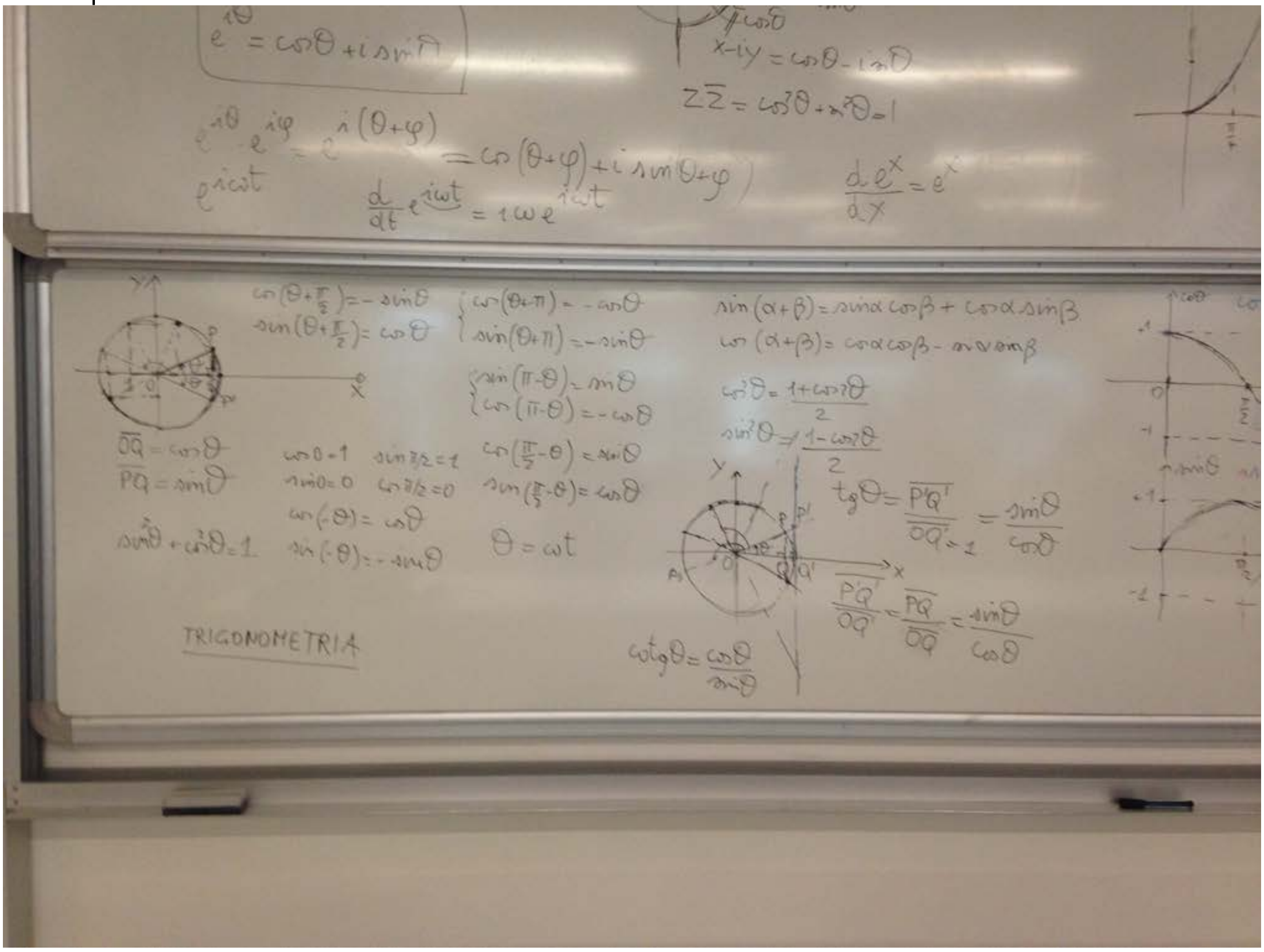
$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$

$\dot{\vec{r}} = (\dot{x}, \dot{y}, \dot{z})$

$\ddot{\vec{r}} = (\ddot{x}, \ddot{y}, \ddot{z}) = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\right)$

6 Novembre 2014

-Richiami di trigonometria; concetto di fase ed esempi di sfasamenti; caso di angoli variabili linearmente nel tempo



$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$x - iy = \cos\theta - i\sin\theta$

$z\bar{z} = \cos^2\theta + \sin^2\theta = 1$

$e^{i\theta} e^{i\varphi} = e^{i(\theta+\varphi)} = \cos(\theta+\varphi) + i\sin(\theta+\varphi)$

$e^{i\omega t}$

$\frac{d}{dt} e^{i\omega t} = i\omega e^{i\omega t}$

$\frac{de^x}{dx} = e^x$

TRIGONOMETRIA

$\overline{OQ} = \cos\theta$

$\overline{PQ} = \sin\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\cos(\theta + \frac{\pi}{2}) = -\sin\theta$

$\sin(\theta + \frac{\pi}{2}) = \cos\theta$

$\cos 0 = 1$ $\sin\pi/2 = 1$

$\sin 0 = 0$ $\cos\pi/2 = 0$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\begin{cases} \cos(\theta+\pi) = -\cos\theta \\ \sin(\theta+\pi) = -\sin\theta \end{cases}$

$\begin{cases} \sin(\pi-\theta) = \sin\theta \\ \cos(\pi-\theta) = -\cos\theta \end{cases}$

$\cos(\frac{\pi}{2} - \theta) = \sin\theta$

$\sin(\frac{\pi}{2} - \theta) = \cos\theta$

$\theta = \omega t$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

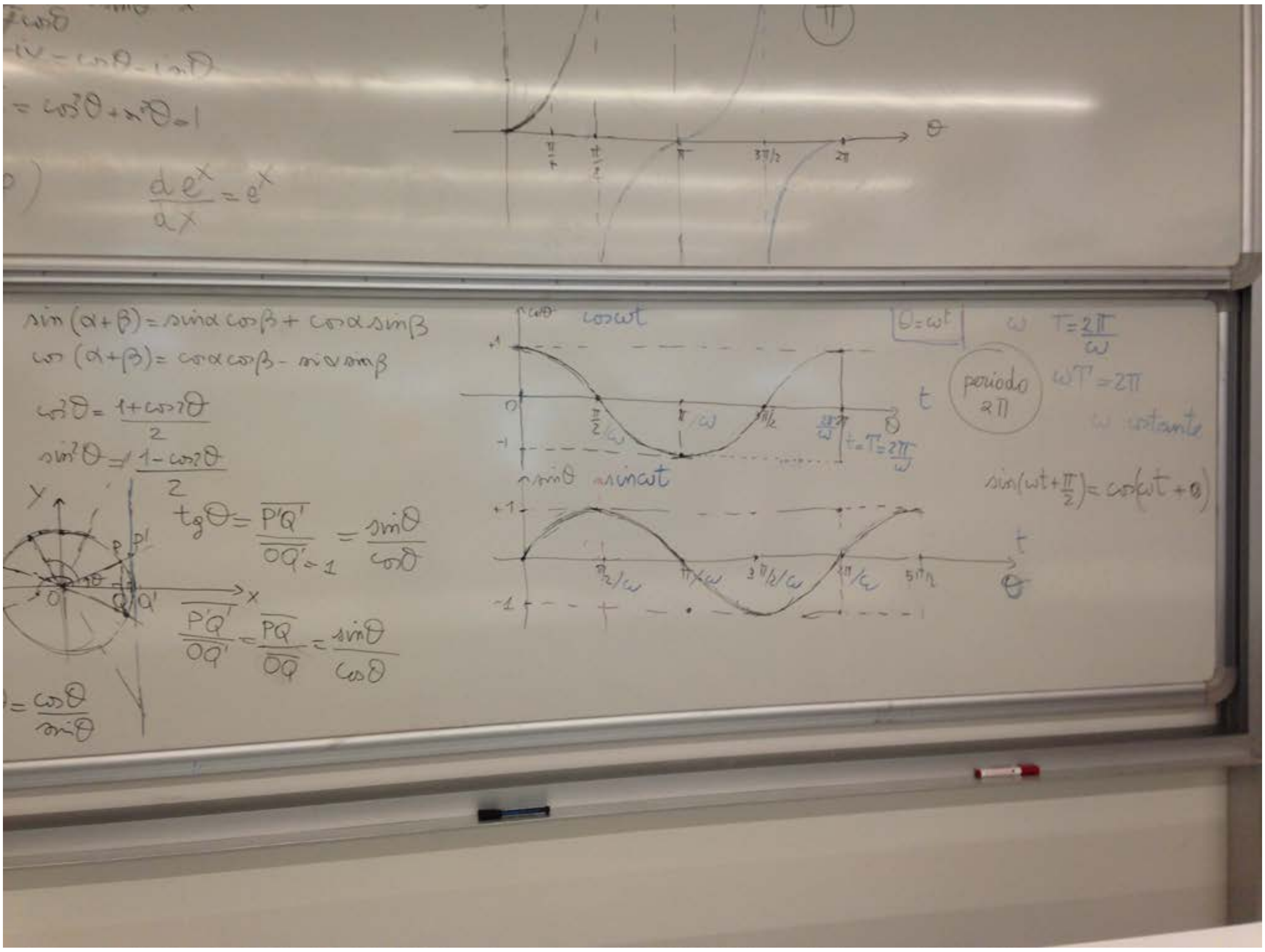
$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$tg\theta = \frac{\overline{P'Q'}}{\overline{OQ'} = 1} = \frac{\sin\theta}{\cos\theta}$

$\frac{\overline{P'Q'}}{\overline{OQ'}} = \frac{\overline{PQ}}{\overline{OQ}} = \frac{\sin\theta}{\cos\theta}$

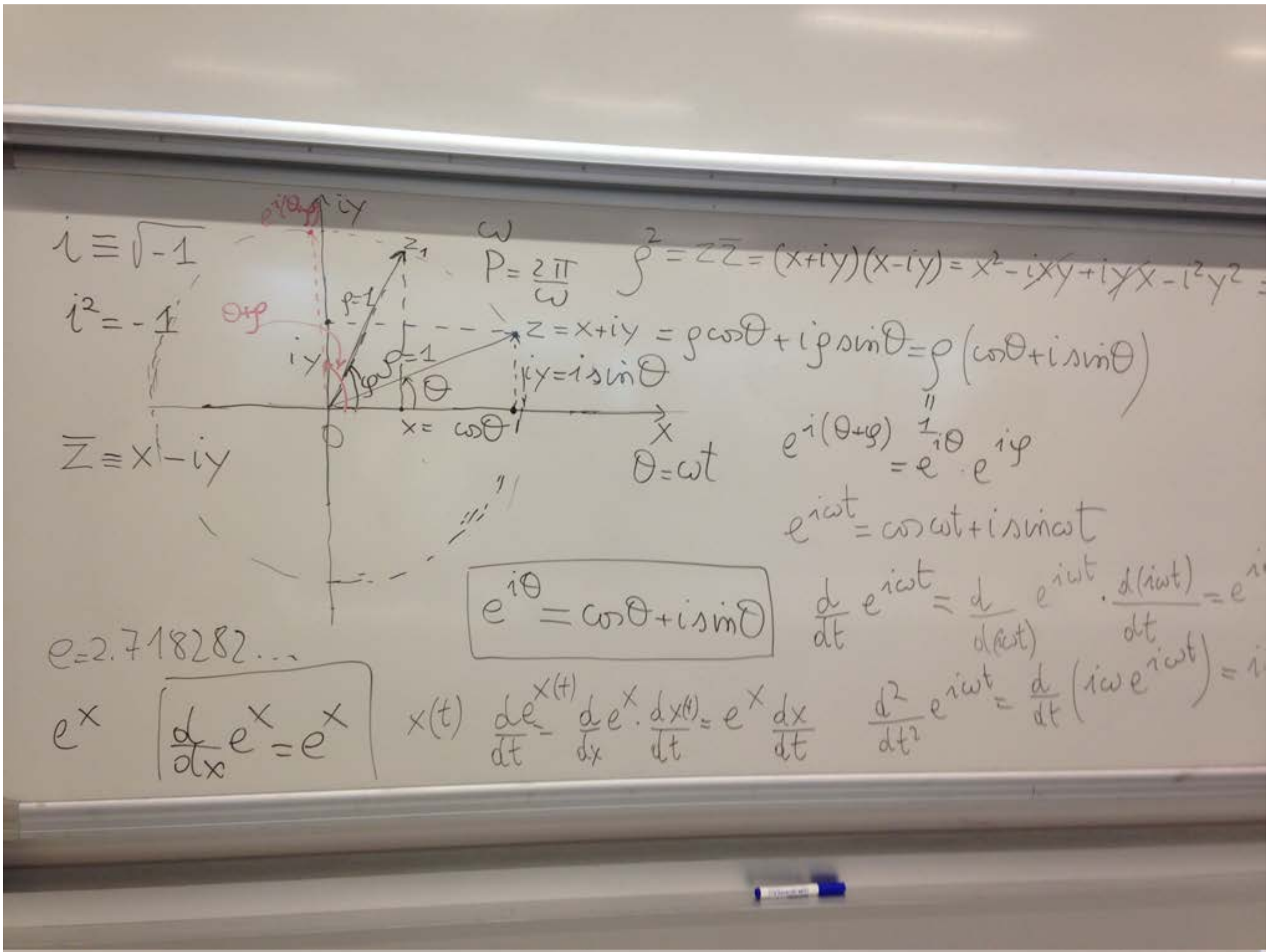
$cotg\theta = \frac{\cos\theta}{\sin\theta}$


= cos²θ + sin²θ = 1
d eˣ / dx = eˣ
θ
π/4
π/2
π
3π/2
2π
sin(α+β) = sinα cosβ + cosα sinβ
cos(α+β) = cosα cosβ − sinα sinβ
cos²θ = (1+cos2θ)/2
sin²θ = (1−cos2θ)/2
tgθ = P'Q'/OQ' = 1 = sinθ/cosθ
P'Q'/OQ' = PQ/OQ = sinθ/cosθ
= cosθ/sinθ
cosθ
cosωt
θ=ωt
ω
T=2π/ω
ωT=2π
periodo 2π
ω costante
t
π/2/ω
π/ω
t=T=2π/ω
sinθ
sinωt
sin(ωt+π/2) = cos(ωt+0)
+1
−1
0
3π/2/ω
2π/ω
5π/2
x
y
P
P'
Q
Q'
O

13 Novembre 2014

-Richiami sui numeri complessi; piano complesso, interpretazione geometrica e proprietà. Esponenziali complessi e loro importanza

-Richiami sulle derivate. Derivate rispetto al tempo. Concetto di traiettoria e concetto di legge oraria. Significato geometrico della derivata (caso della velocità e della accelerazione). Calcolo di alcune derivate. Proprietà.


$e^{i(\theta+\varphi)}$
iy
z_1
$\rho=1$
$\theta+\varphi$
iy
φ
$\rho=1$
θ
$iy = i\sin\theta$
O
$x = \cos\theta$
X

$i \equiv \sqrt{-1}$

$i^2 = -1$

$\bar{Z} \equiv x - iy$

ω

$P = \frac{2\pi}{\omega}$

$\rho^2 = Z\bar{Z} = (x+iy)(x-iy) = x^2 - \cancel{ixy} + \cancel{iyx} - i^2y^2 =$

$z = x + iy = \rho\cos\theta + i\rho\sin\theta = \underset{\overset{\parallel}{1}}{\rho}(\cos\theta + i\sin\theta)$

$\theta = \omega t$

$e^{i(\theta+\varphi)} = e^{i\theta} \cdot e^{i\varphi}$

$e^{i\omega t} = \cos\omega t + i\sin\omega t$

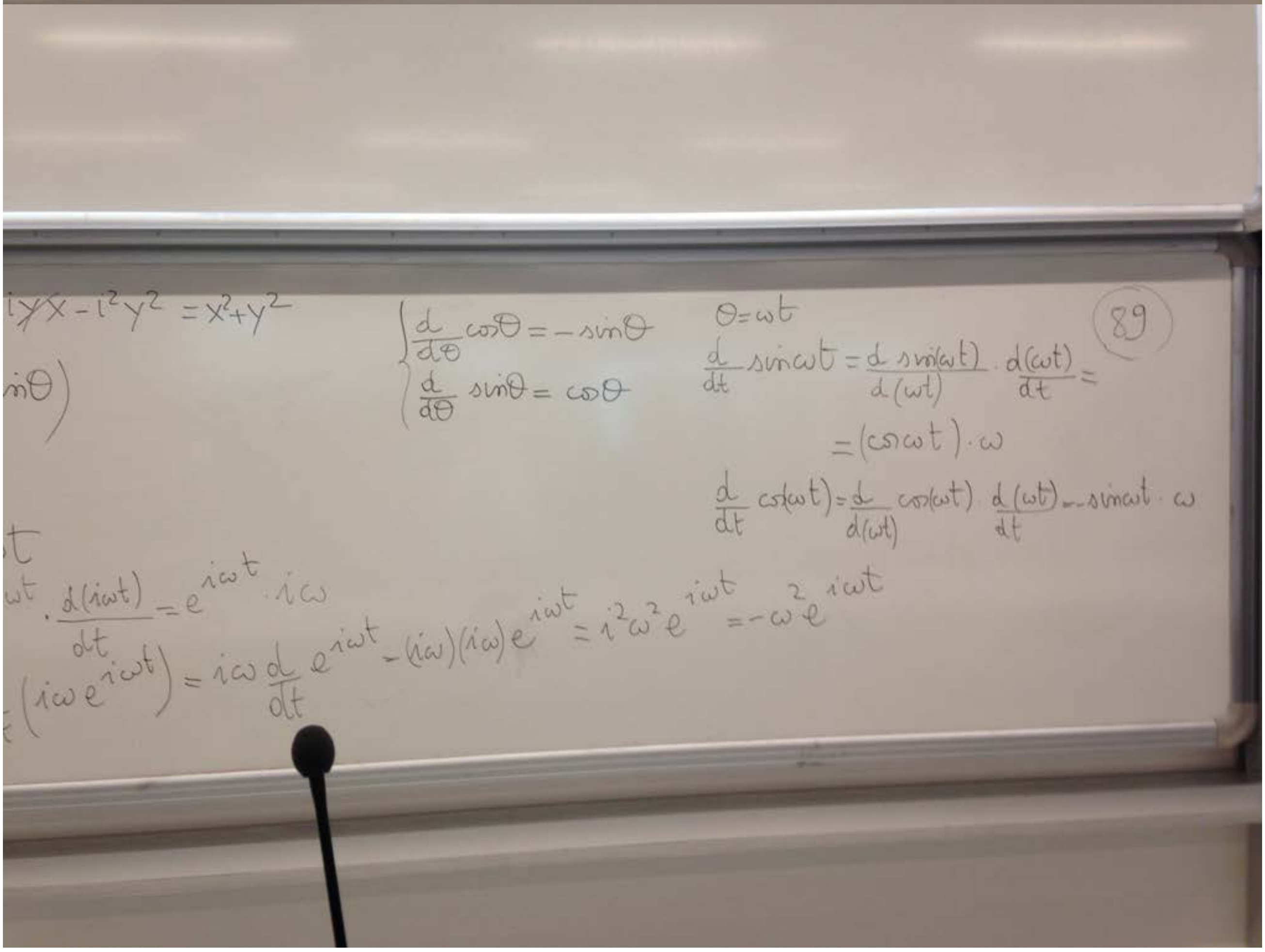
$e = 2.718282\ldots$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$\frac{d}{dt} e^{i\omega t} = \frac{d}{d(i\omega t)} e^{i\omega t} \cdot \frac{d(i\omega t)}{dt} = e^{i}$

e^x $\boxed{\frac{d}{dx} e^x = e^x}$ $x(t)$ $\frac{de^{x(t)}}{dt} = \frac{d}{dx} e^x \cdot \frac{dx(t)}{dt} = e^x \frac{dx}{dt}$

$\frac{d^2}{dt^2} e^{i\omega t} = \frac{d}{dt}\left(i\omega e^{i\omega t}\right) = i$



$iyx - i^2y^2 = x^2 + y^2$

$\sin\theta)$

t

$\omega t \cdot \frac{d(i\omega t)}{dt} = e^{i\omega t} \cdot i\omega$

$\left(i\omega e^{i\omega t}\right) = i\omega \frac{d}{dt} e^{i\omega t} = (i\omega)(i\omega) e^{i\omega t} = i^2\omega^2 e^{i\omega t} = -\omega^2 e^{i\omega t}$

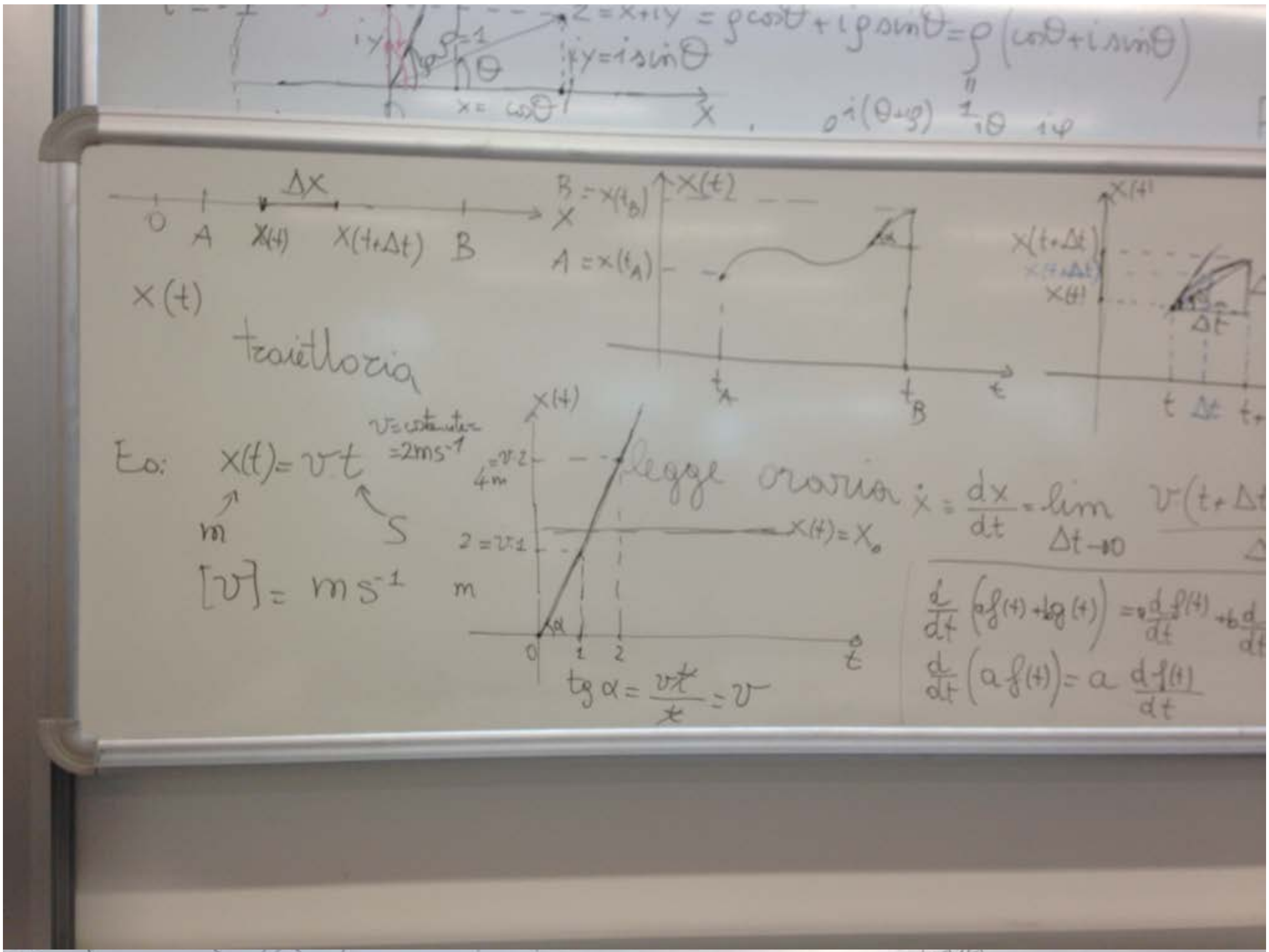
$$\begin{cases} \frac{d}{d\theta}\cos\theta = -\sin\theta \\ \frac{d}{d\theta}\sin\theta = \cos\theta \end{cases}$$

$\theta = \omega t$

$\frac{d}{dt}\sin\omega t = \frac{d\sin(\omega t)}{d(\omega t)} \cdot \frac{d(\omega t)}{dt} =$

$= (\cos\omega t) \cdot \omega$

$\frac{d}{dt}\cos(\omega t) = \frac{d}{d(\omega t)}\cos(\omega t) \cdot \frac{d(\omega t)}{dt} = -\sin\omega t \cdot \omega$


$z = x+iy = \rho\cos\theta + i\rho\sin\theta = \rho(\cos\theta + i\sin\theta)$

$iy = i\sin\theta$

$x = \cos\theta$

$e^{i(\theta+\varphi)}$ $\overset{\shortparallel}{1}$ $i\theta$ $i\varphi$

0 A X(t) X(t+Δt) B x

x(t)

traiettoria

$B = x(t_B)$

$A = x(t_A)$

Es: $x(t) = vt$ v=costante =2ms⁻¹

m s

$[v] = ms^{-1}$

legge oraria

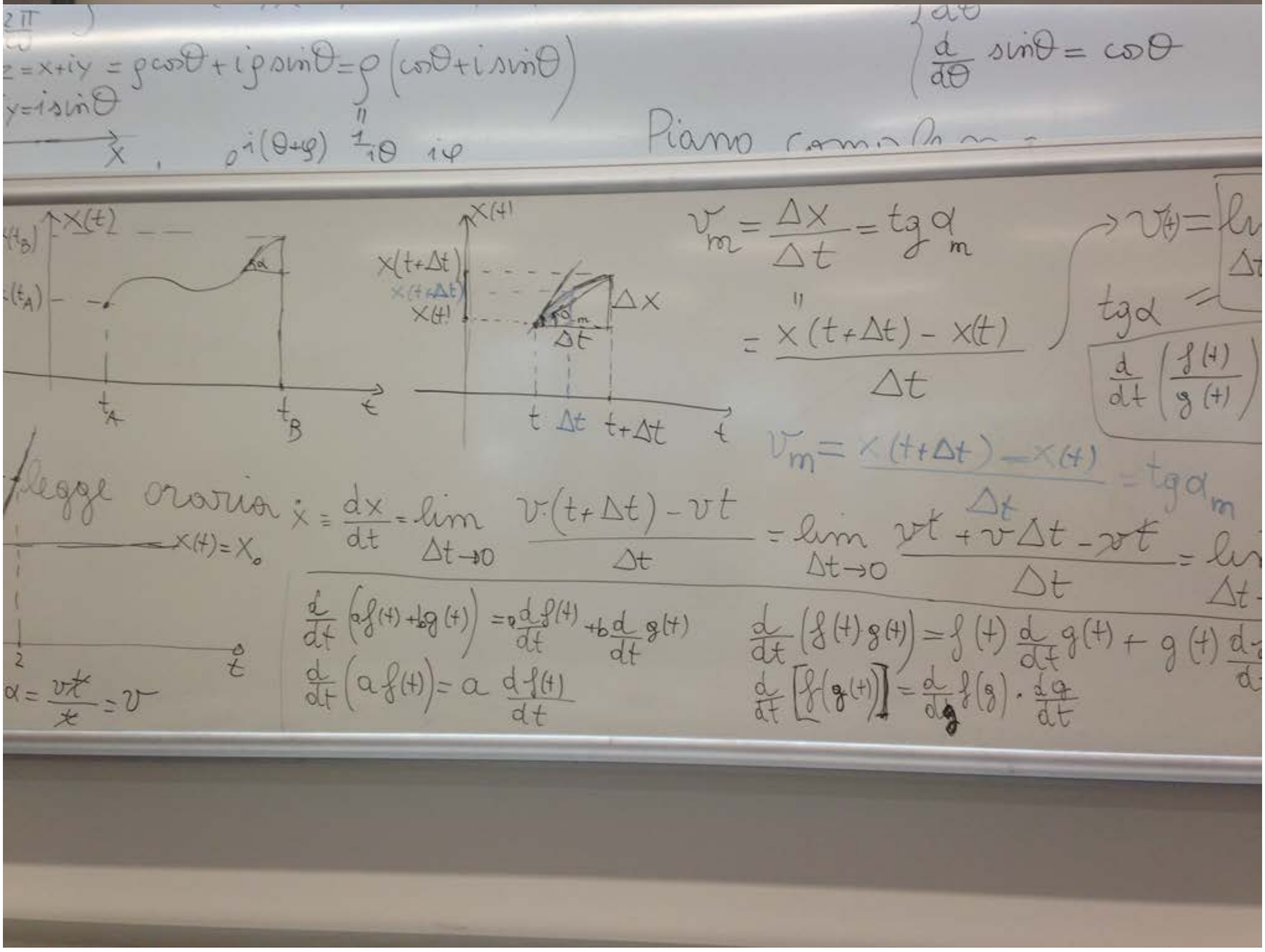
$x(t) = x_0$

$\operatorname{tg}\alpha = \frac{v\cancel{t}}{\cancel{t}} = v$

$\dot{x} = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{v(t+\Delta t}{}$

$\frac{d}{dt}\left(af(t) + bg(t)\right) = a\frac{df(t)}{dt} + b\frac{d}{dt}$

$\frac{d}{dt}\left(af(t)\right) = a\frac{df(t)}{dt}$


$z = x+iy = \rho\cos\theta + i\rho\sin\theta = \rho(\cos\theta + i\sin\theta)$

$y = i\sin\theta$

$e^{i(\theta+\varphi)}$ $\overset{\shortparallel}{1}$ $i\theta$ $i\varphi$

$\frac{d}{d\theta}\sin\theta = \cos\theta$

Piano compl

$v_m = \frac{\Delta x}{\Delta t} = \operatorname{tg}\alpha_m$

$= \frac{x(t+\Delta t) - x(t)}{\Delta t}$

$v(t) = \lim$

$\operatorname{tg}\alpha =$

$\frac{d}{dt}\left(\frac{f(t)}{g(t)}\right)$

$v_m = \frac{x(t+\Delta t) - x(t)}{\Delta t} = \operatorname{tg}\alpha_m$

legge oraria

$x(t) = x_0$

$\dot{x} = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - vt}{\Delta t} = \lim_{\Delta t \to 0} \frac{vt + v\Delta t - vt}{\Delta t} = \lim$

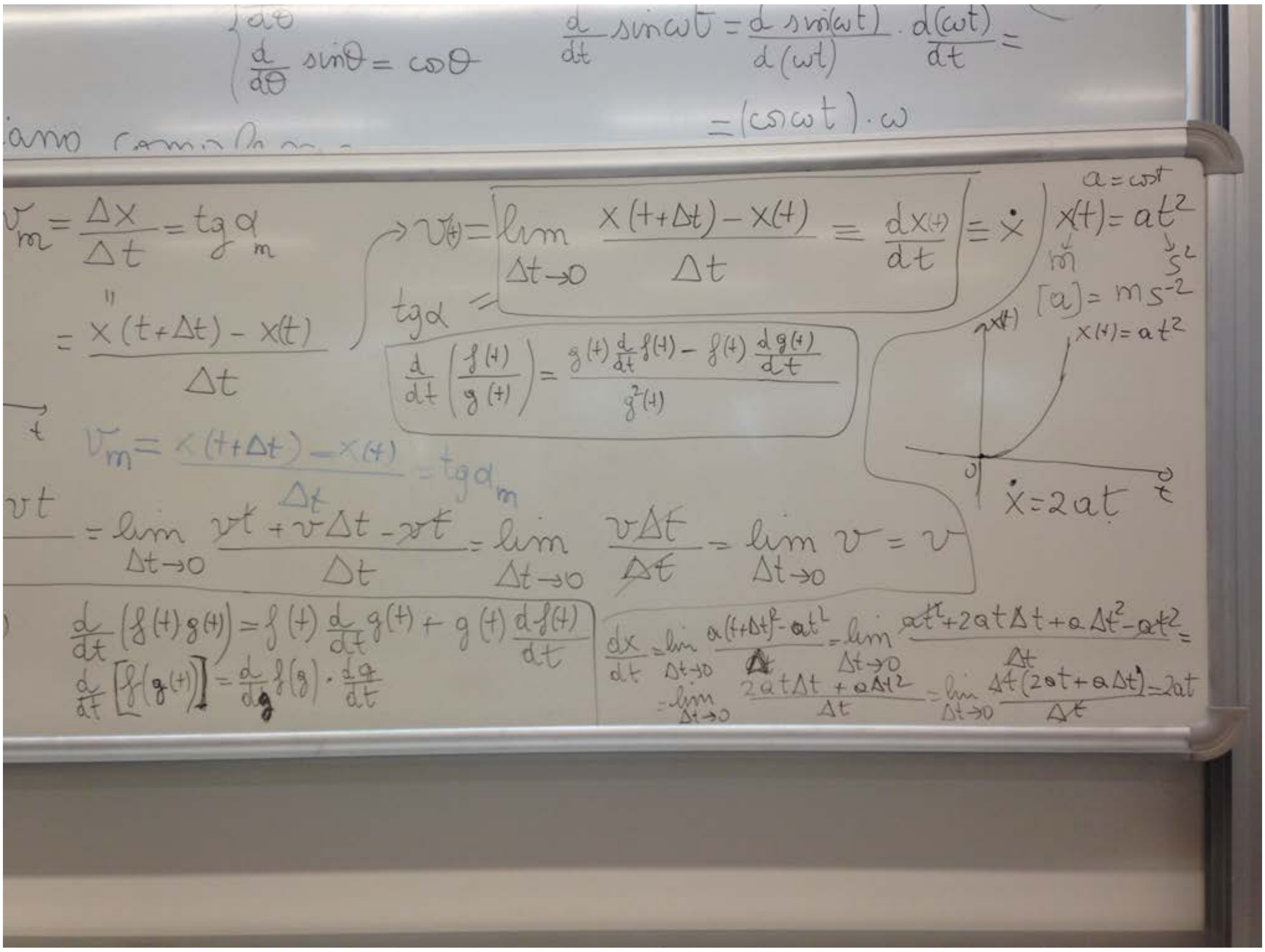
$\frac{d}{dt}\left(af(t) + bg(t)\right) = a\frac{df(t)}{dt} + b\frac{d}{dt}g(t)$

$\frac{d}{dt}\left(f(t)g(t)\right) = f(t)\frac{d}{dt}g(t) + g(t)\frac{d}{d}$

$\frac{d}{dt}\left(af(t)\right) = a\frac{df(t)}{dt}$

$\frac{d}{dt}\left[f(g(t))\right] = \frac{d}{dg}f(g)\cdot\frac{dg}{dt}$

$\alpha = \frac{v\cancel{t}}{\cancel{t}} = v$



$$\frac{d}{d\theta}\sin\theta = \cos\theta$$

$$\frac{d}{dt}\sin\omega t = \frac{d\sin(\omega t)}{d(\omega t)}\cdot\frac{d(\omega t)}{dt} = (\cos\omega t)\cdot\omega$$

$$v_m = \frac{\Delta x}{\Delta t} = \operatorname{tg}\alpha_m = \frac{x(t+\Delta t)-x(t)}{\Delta t}$$

$$v(t) = \boxed{\lim_{\Delta t\to 0}\frac{x(t+\Delta t)-x(t)}{\Delta t} \equiv \frac{dx(t)}{dt}} \equiv \dot{x} = \operatorname{tg}\alpha$$

$$\boxed{\frac{d}{dt}\left(\frac{f(t)}{g(t)}\right) = \frac{g(t)\frac{d}{dt}f(t) - f(t)\frac{dg(t)}{dt}}{g^2(t)}}$$

$$v_m = \frac{x(t+\Delta t)-x(t)}{\Delta t} = \operatorname{tg}\alpha_m$$

$$= \lim_{\Delta t\to 0}\frac{\cancel{vt}+v\Delta t-\cancel{vt}}{\Delta t} = \lim_{\Delta t\to 0}\frac{v\cancel{\Delta t}}{\cancel{\Delta t}} = \lim_{\Delta t\to 0} v = v$$

$$\frac{d}{dt}\big(f(t)g(t)\big) = f(t)\frac{d}{dt}g(t) + g(t)\frac{df(t)}{dt}$$

$$\frac{d}{dt}\big[f(g(t))\big] = \frac{d}{dg}f(g)\cdot\frac{dg}{dt}$$

$a = \text{cost}$

$x(t) = at^2$ (m, s^2)

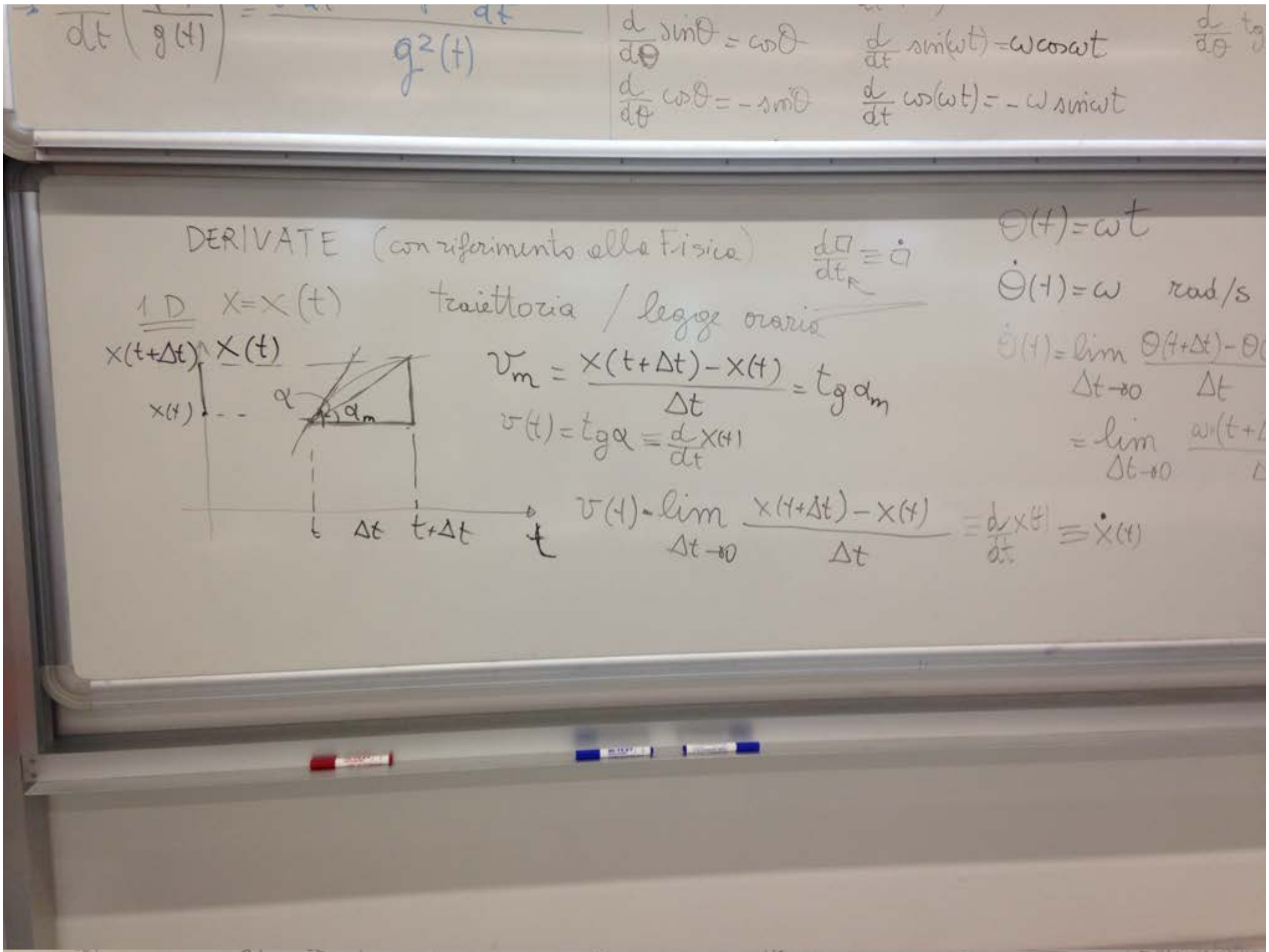
$[a] = m s^{-2}$

$\dot{x} = 2at$

$$\frac{dx}{dt} = \lim_{\Delta t\to 0}\frac{a(t+\Delta t)^2 - at^2}{\Delta t} = \lim_{\Delta t\to 0}\frac{\cancel{at^2}+2at\Delta t+a\Delta t^2-\cancel{at^2}}{\Delta t} = \lim_{\Delta t\to 0}\frac{2at\Delta t + a\Delta t^2}{\Delta t} = \lim_{\Delta t\to 0}\frac{\cancel{\Delta t}(2at+a\Delta t)}{\cancel{\Delta t}} = 2at$$

20 Novembre 2014

- Ancora su derivate ed integrali son riferimento al loro significato geometrico, al loro uso in Fisica e alle regole principali che occorre conoscere



$$\frac{d}{dt}\left(\frac{\quad}{g(t)}\right) = \frac{\quad dt}{g^2(t)}$$

$\frac{d}{d\theta}\sin\theta = \cos\theta$ $\quad \frac{d}{dt}\sin(\omega t) = \omega\cos\omega t$ $\quad \frac{d}{d\theta}\,tg$

$\frac{d}{d\theta}\cos\theta = -\sin\theta$ $\quad \frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

DERIVATE (con riferimento alla Fisica) $\quad \frac{d\square}{dt} \equiv \dot{\square}$

1 D $\quad X = X(t) \quad$ traiettoria / legge oraria

x(t+Δt)
x(t)
x(t)
α
α_m
t
Δt
t+Δt
t

$$v_m = \frac{x(t+\Delta t)-x(t)}{\Delta t} = tg\,\alpha_m$$

$$v(t) = tg\,\alpha = \frac{d}{dt}x(t)$$

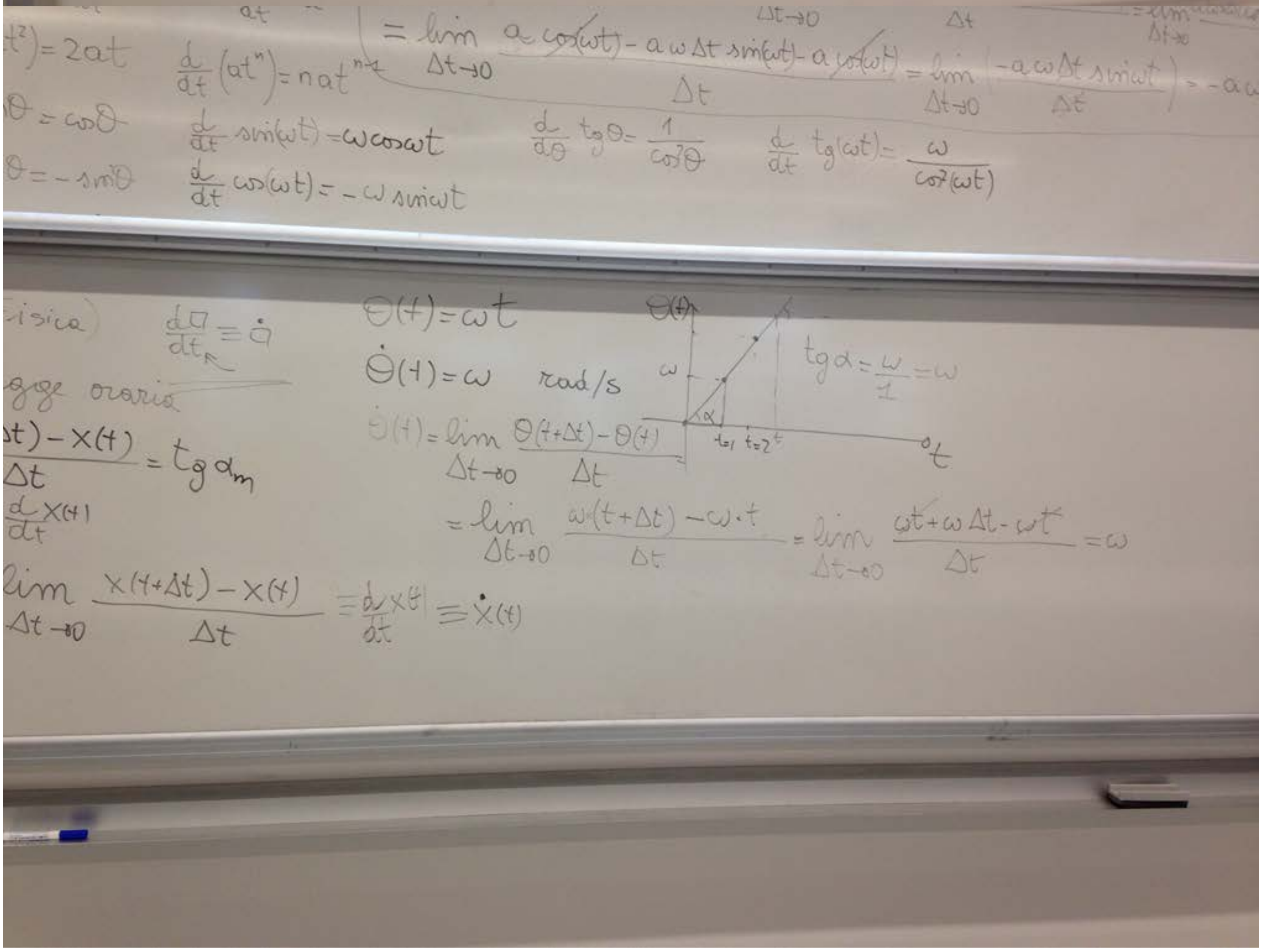
$$v(t) = \lim_{\Delta t\to 0}\frac{x(t+\Delta t)-x(t)}{\Delta t} \equiv \frac{d}{dt}x(t) \equiv \dot{x}(t)$$

$\theta(t) = \omega t$

$\dot{\theta}(t) = \omega$ rad/s

$$\dot{\theta}(t) = \lim_{\Delta t\to 0}\frac{\theta(t+\Delta t)-\theta(}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\omega(t+}{}$$



$t^2) = 2at \quad \frac{d}{dt}(at^n) = nat^{n-1}$

$$= \lim_{\Delta t\to 0}\frac{a\cancel{\cos(\omega t)} - a\omega\Delta t\sin(\omega t) - a\cancel{\cos(\omega t)}}{\Delta t} = \lim_{\Delta t\to 0}\left(\frac{-a\omega\Delta t\sin\omega t}{\Delta t}\right) = -a\omega$$

$\frac{d}{d\theta}\,tg\,\theta = \frac{1}{\cos^2\theta}$ $\quad \frac{d}{dt}\,tg(\omega t) = \frac{\omega}{\cos^2(\omega t)}$

$\theta = \cos\theta \quad \frac{d}{dt}\sin(\omega t) = \omega\cos\omega t$

$\theta = -\sin\theta \quad \frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

isica) $\quad \frac{d\square}{dt} \equiv \dot{\square}$

gge oraria

$\frac{t)-x(t)}{\Delta t} = tg\,\alpha_m$

$\frac{d}{dt}x(t)$

$\lim_{\Delta t\to 0}\frac{x(t+\Delta t)-x(t)}{\Delta t} \equiv \frac{d}{dt}x(t) \equiv \dot{x}(t)$

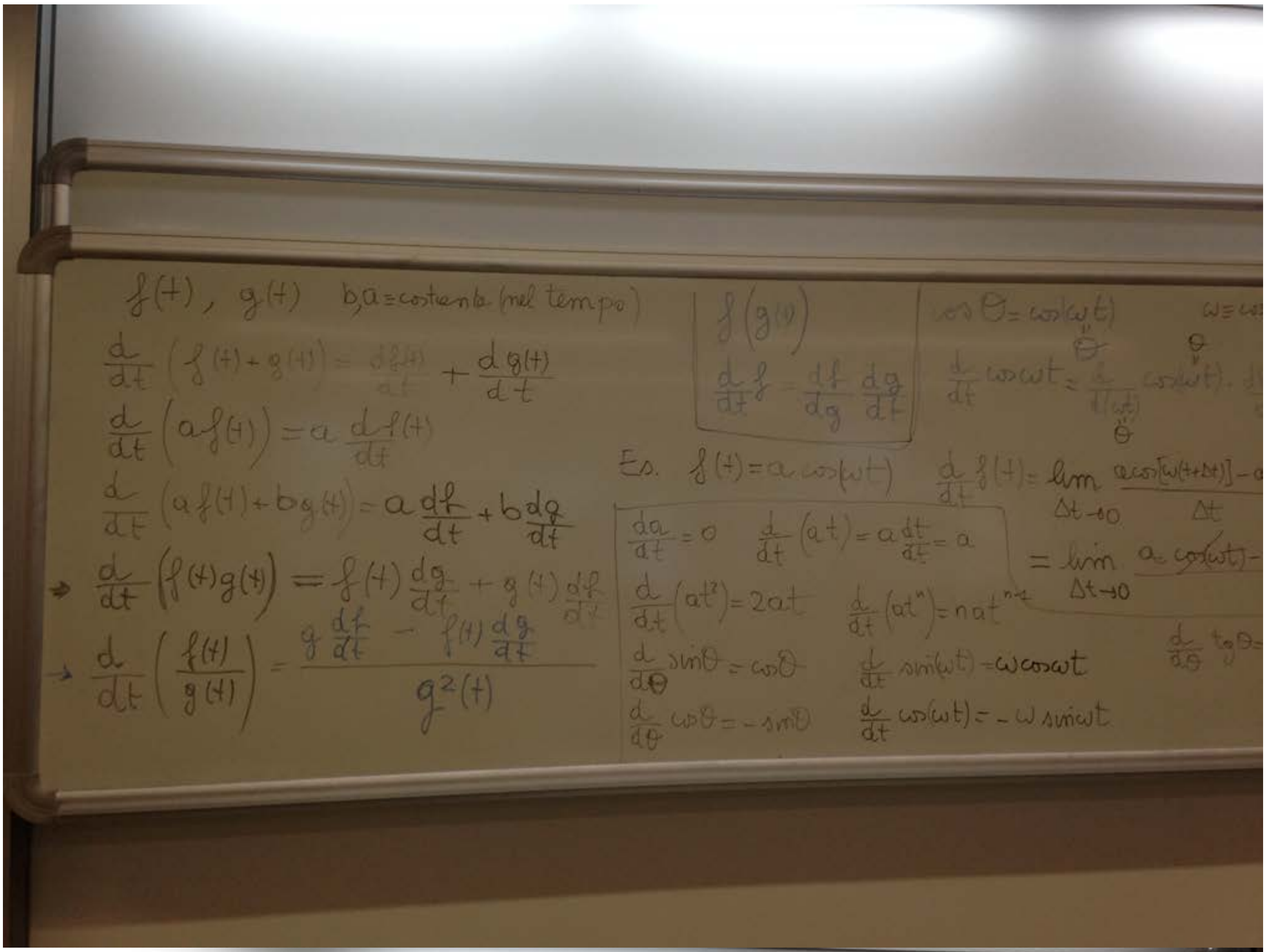
$\theta(t) = \omega t$

$\dot{\theta}(t) = \omega$ rad/s

$$\dot{\theta}(t) = \lim_{\Delta t\to 0}\frac{\theta(t+\Delta t)-\theta(t)}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\omega(t+\Delta t)-\omega\cdot t}{\Delta t} = \lim_{\Delta t\to 0}\frac{\cancel{\omega t}+\omega\Delta t-\cancel{\omega t}}{\Delta t} = \omega$$

$tg\,\alpha = \frac{\omega}{1} = \omega$



$f(t)$, $g(t)$ b, a ≡ costante (nel tempo)

$$\frac{d}{dt}\left(f(t)+g(t)\right)=\frac{df(t)}{dt}+\frac{dg(t)}{dt}$$

$$\frac{d}{dt}\left(af(t)\right)=a\frac{df(t)}{dt}$$

$$\frac{d}{dt}\left(af(t)+bg(t)\right)=a\frac{df}{dt}+b\frac{dg}{dt}$$

$$\Rightarrow \frac{d}{dt}\left(f(t)g(t)\right)=f(t)\frac{dg}{dt}+g(t)\frac{df}{dt}$$

$$\rightarrow \frac{d}{dt}\left(\frac{f(t)}{g(t)}\right)=\frac{g\frac{df}{dt}-f(t)\frac{dg}{dt}}{g^2(t)}$$

$$\boxed{\begin{gathered} f(g(t)) \\ \frac{d}{dt}f=\frac{df}{dg}\frac{dg}{dt}\end{gathered}}$$

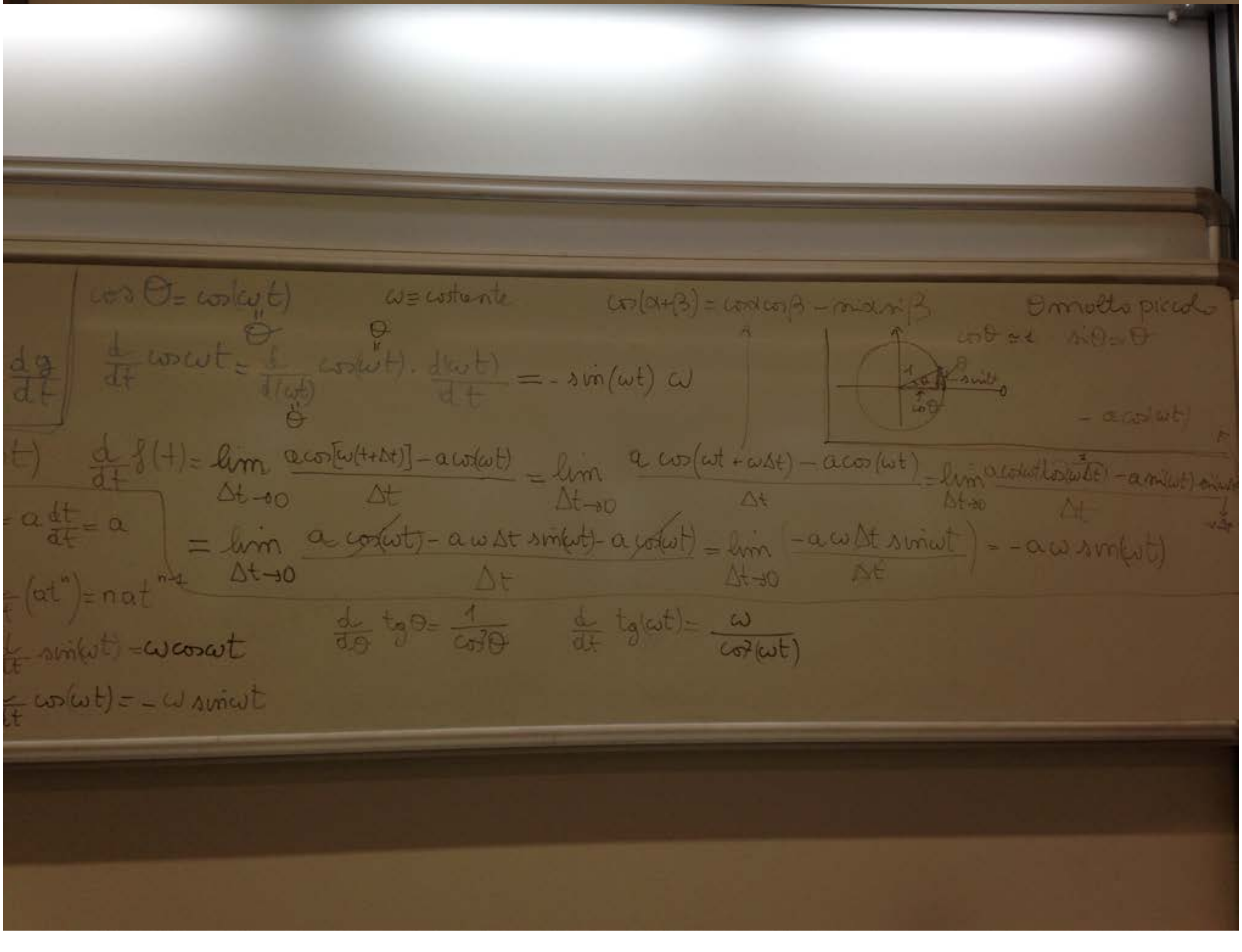
Es. $f(t)=a\cos(\omega t)$

$$\frac{da}{dt}=0 \qquad \frac{d}{dt}(at)=a\frac{dt}{dt}=a$$

$$\frac{d}{dt}(at^2)=2at \qquad \frac{d}{dt}(at^n)=nat^{n-1}$$

$$\frac{d}{d\theta}\sin\theta=\cos\theta \qquad \frac{d}{dt}\sin(\omega t)=\omega\cos\omega t$$

$$\frac{d}{d\theta}\cos\theta=-\sin\theta \qquad \frac{d}{dt}\cos(\omega t)=-\omega\sin\omega t$$



$$\cos\theta=\cos(\underset{\overset{\shortparallel}{\theta}}{\omega t}) \qquad \omega \equiv \text{costante}$$

$$\frac{d}{dt}\cos\omega t=\frac{d}{d(\underset{\overset{\shortparallel}{\theta}}{\omega t})}\cos(\overset{\overset{\theta}{\shortparallel}}{\omega t})\cdot\frac{d(\omega t)}{dt}=-\sin(\omega t)\,\omega$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

θ molto piccolo

$\cos\theta \simeq 1$ $\sin\theta \simeq \theta$

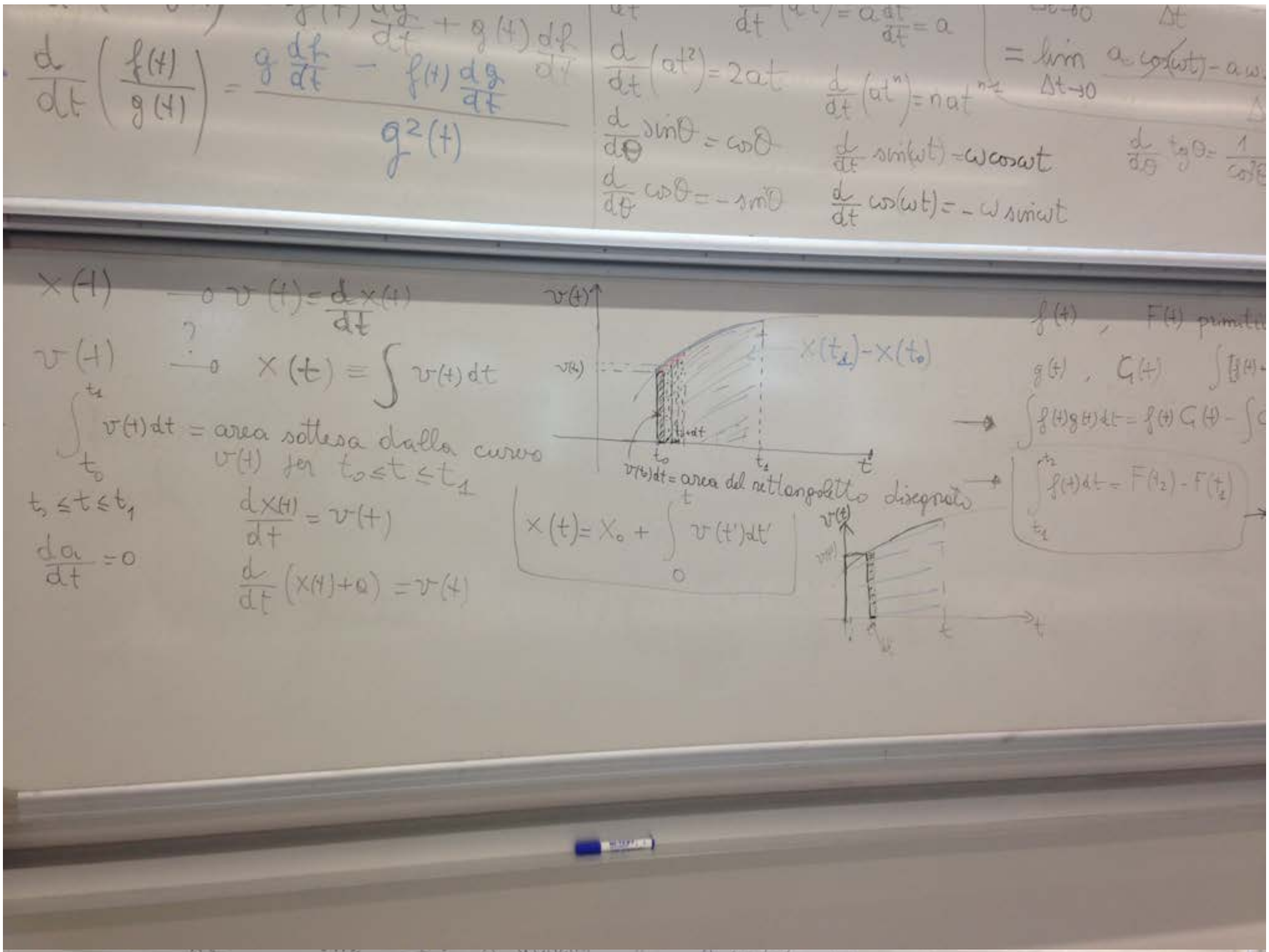
1
α
β
sin θ
cos θ
θ

$-a\cos(\omega t)$

$$\frac{d}{dt}f(t)=\lim_{\Delta t\to 0}\frac{a\cos[\omega(t+\Delta t)]-a\cos(\omega t)}{\Delta t}=\lim_{\Delta t\to 0}\frac{a\cos(\omega t+\omega\Delta t)-a\cos(\omega t)}{\Delta t}=\lim_{\Delta t\to 0}\frac{a\cos(\omega t)\overset{1}{\cos(\omega\Delta t)}-a\sin(\omega t)\underset{\omega\Delta t}{\underset{\downarrow}{\sin(\omega\Delta t)}}}{\Delta t}$$

$$=\lim_{\Delta t\to 0}\frac{\cancel{a\cos(\omega t)}-a\omega\Delta t\sin(\omega t)-\cancel{a\cos(\omega t)}}{\Delta t}=\lim_{\Delta t\to 0}\left(\frac{-a\omega\Delta t\sin\omega t}{\Delta t}\right)=-a\omega\sin(\omega t)$$

$$\frac{d}{d\theta}\tan\theta=\frac{1}{\cos^2\theta} \qquad \frac{d}{dt}\tan(\omega t)=\frac{\omega}{\cos^2(\omega t)}$$



$$\frac{d}{dt}\left(\frac{f(t)}{g(t)}\right) = \frac{g\frac{df}{dt} - f(t)\frac{dg}{dt}}{g^2(t)}$$

$\frac{d}{dt}(at^2) = 2at$ $\frac{d}{dt}(at^n) = nat^{n-1}$

$\frac{d}{d\theta}\sin\theta = \cos\theta$ $\frac{d}{dt}\sin(\omega t) = \omega\cos\omega t$ $\frac{d}{d\theta}tg\theta = \frac{1}{\cos^2\theta}$

$\frac{d}{d\theta}\cos\theta = -\sin\theta$ $\frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

$x(t) \longrightarrow v(t) = \frac{d}{dt}x(t)$

$v(t) \overset{?}{\longrightarrow} x(t) = \int v(t)\,dt$

$\int_{t_0}^{t_1} v(t)\,dt$ = area sottesa dalla curva $v(t)$ per $t_0 \le t \le t_1$

$t_0 \le t \le t_1$

$\frac{da}{dt} = 0$ $\frac{dx(t)}{dt} = v(t)$

$\frac{d}{dt}(x(t)+a) = v(t)$

$v(t_0)dt$ = area del rettangolotto disegnato

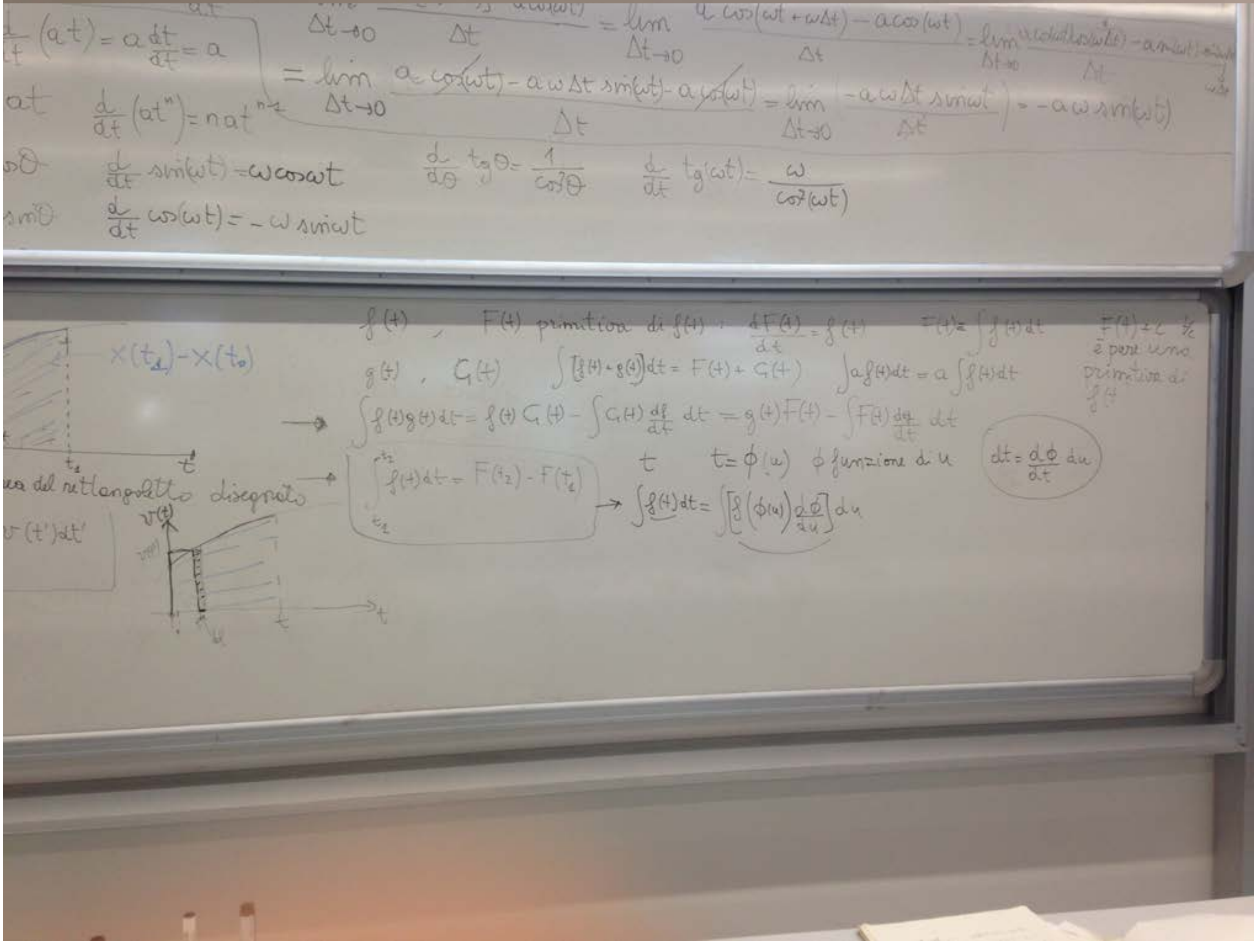
$$\boxed{x(t) = x_0 + \int_0^t v(t')\,dt'}$$

$f(t)$, $F(t)$ primitiva

$g(t)$, $G(t)$

$\longrightarrow \int f(t)g(t)\,dt = f(t)G(t) - \int$

$\longrightarrow \int_{t_1}^{t_2} f(t)\,dt = F(t_2) - F(t_1)$



$\frac{d}{dt}(at) = a\frac{dt}{dt} = a$

$= \lim_{\Delta t\to 0} \frac{a\cos(\omega t+\omega\Delta t) - a\cos(\omega t)}{\Delta t}$

$= \lim_{\Delta t\to 0} \frac{a\cancel{\cos(\omega t)} - a\omega\Delta t\sin(\omega t) - a\cancel{\cos(\omega t)}}{\Delta t} = \lim_{\Delta t\to 0}\left(\frac{-a\omega\Delta t\sin\omega t}{\Delta t}\right) = -a\omega\sin(\omega t)$

$\frac{d}{dt}(at^n) = nat^{n-1}$

$\frac{d}{dt}\sin(\omega t) = \omega\cos\omega t$ $\frac{d}{d\theta}tg\theta = \frac{1}{\cos^2\theta}$ $\frac{d}{dt}tg(\omega t) = \frac{\omega}{\cos^2(\omega t)}$

$\frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

$f(t)$, $F(t)$ primitiva di $f(t)$: $\frac{dF(t)}{dt} = f(t)$ $F(t) = \int f(t)\,dt$ $F(t)+c$ è pure una primitiva di $f(t)$

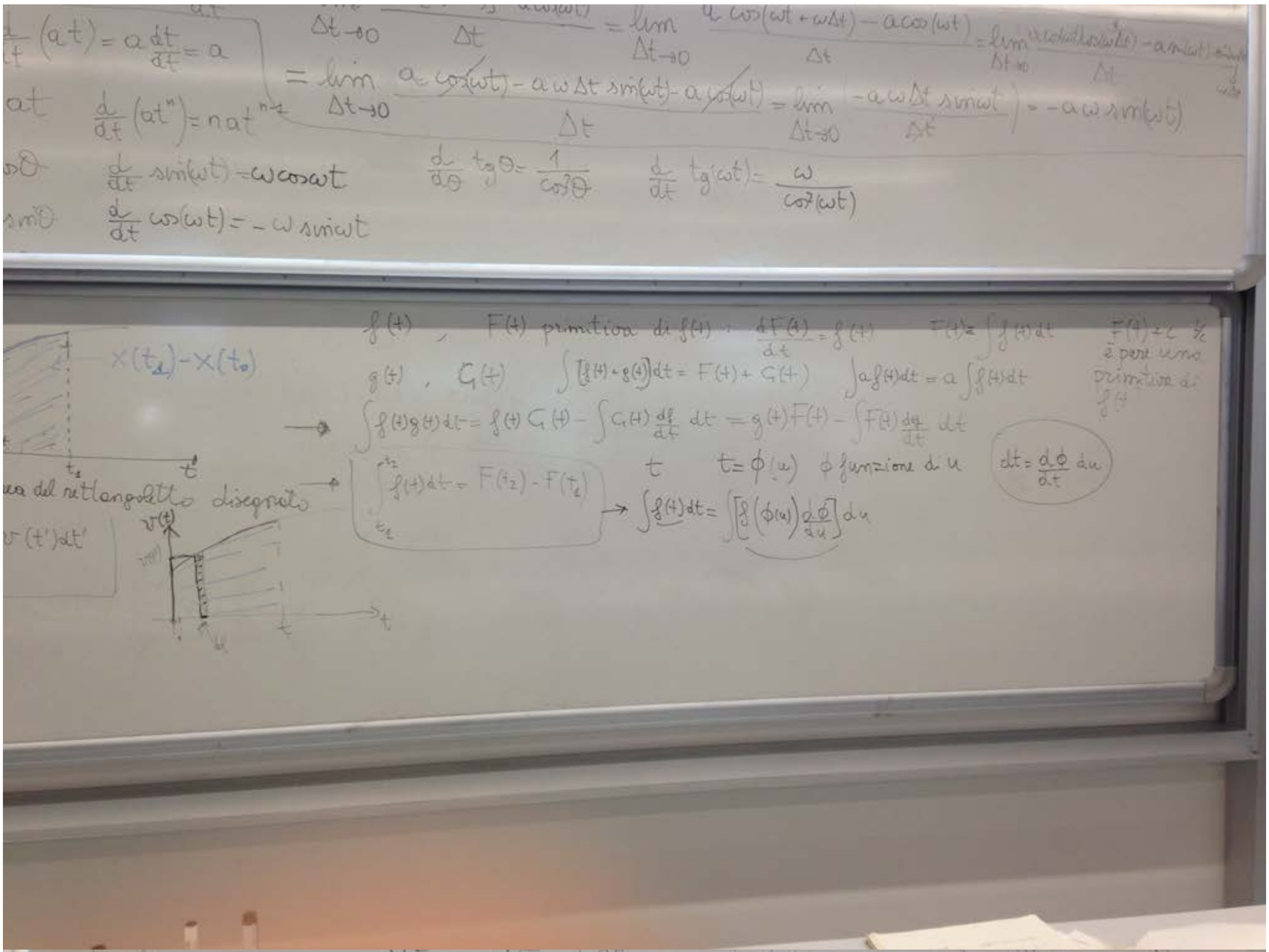
$g(t)$, $G(t)$ $\int[f(t)+g(t)]dt = F(t)+G(t)$ $\int af(t)dt = a\int f(t)dt$

$\longrightarrow \int f(t)g(t)\,dt = f(t)G(t) - \int G(t)\frac{df}{dt}dt = g(t)F(t) - \int F(t)\frac{dg}{dt}dt$

$\longrightarrow \boxed{\int_{t_1}^{t_2} f(t)\,dt = F(t_2) - F(t_1)}$ t $t = \phi(u)$ ϕ funzione di u $dt = \frac{d\phi}{dt}du$

$\rightarrow \int f(t)\,dt = \int\left[f(\phi(u))\frac{d\phi}{du}\right]du$

rea del rettangolotto disegnato



$\frac{d}{dt}(at) = a\frac{dt}{dt} = a$

$\frac{d}{dt}(at^n) = nat^{n-1}$

$\frac{d}{dt}\sin(\omega t) = \omega\cos\omega t$

$\frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

$$= \lim_{\Delta t\to 0}\frac{a\cos(\omega t+\omega\Delta t) - a\cos(\omega t)}{\Delta t} = \lim_{\Delta t\to 0}\frac{\cancel{a\cos(\omega t)} - a\omega\Delta t\sin(\omega t) - \cancel{a\cos(\omega t)}}{\Delta t} = \lim_{\Delta t\to 0}\left(\frac{-a\omega\Delta t\sin\omega t}{\Delta t}\right) = -a\omega\sin(\omega t)$$

$\frac{d}{d\theta}\mathrm{tg}\,\theta = \frac{1}{\cos^2\theta}$ $\frac{d}{dt}\mathrm{tg}(\omega t) = \frac{\omega}{\cos^2(\omega t)}$

$f(t)$, $F(t)$ primitiva di $f(t)$: $\frac{dF(t)}{dt} = f(t)$ $F(t) = \int f(t)\,dt$

$F(t)+c$ è pure una primitiva di $f(t)$

$g(t)$, $G(t)$ $\int[f(t)+g(t)]\,dt = F(t)+G(t)$ $\int af(t)\,dt = a\int f(t)\,dt$

$\rightarrow \int f(t)g(t)\,dt = f(t)G(t) - \int G(t)\frac{df}{dt}\,dt = g(t)F(t) - \int F(t)\frac{dg}{dt}\,dt$

$\rightarrow \int_{t_1}^{t_2} f(t)\,dt = F(t_2) - F(t_1)$

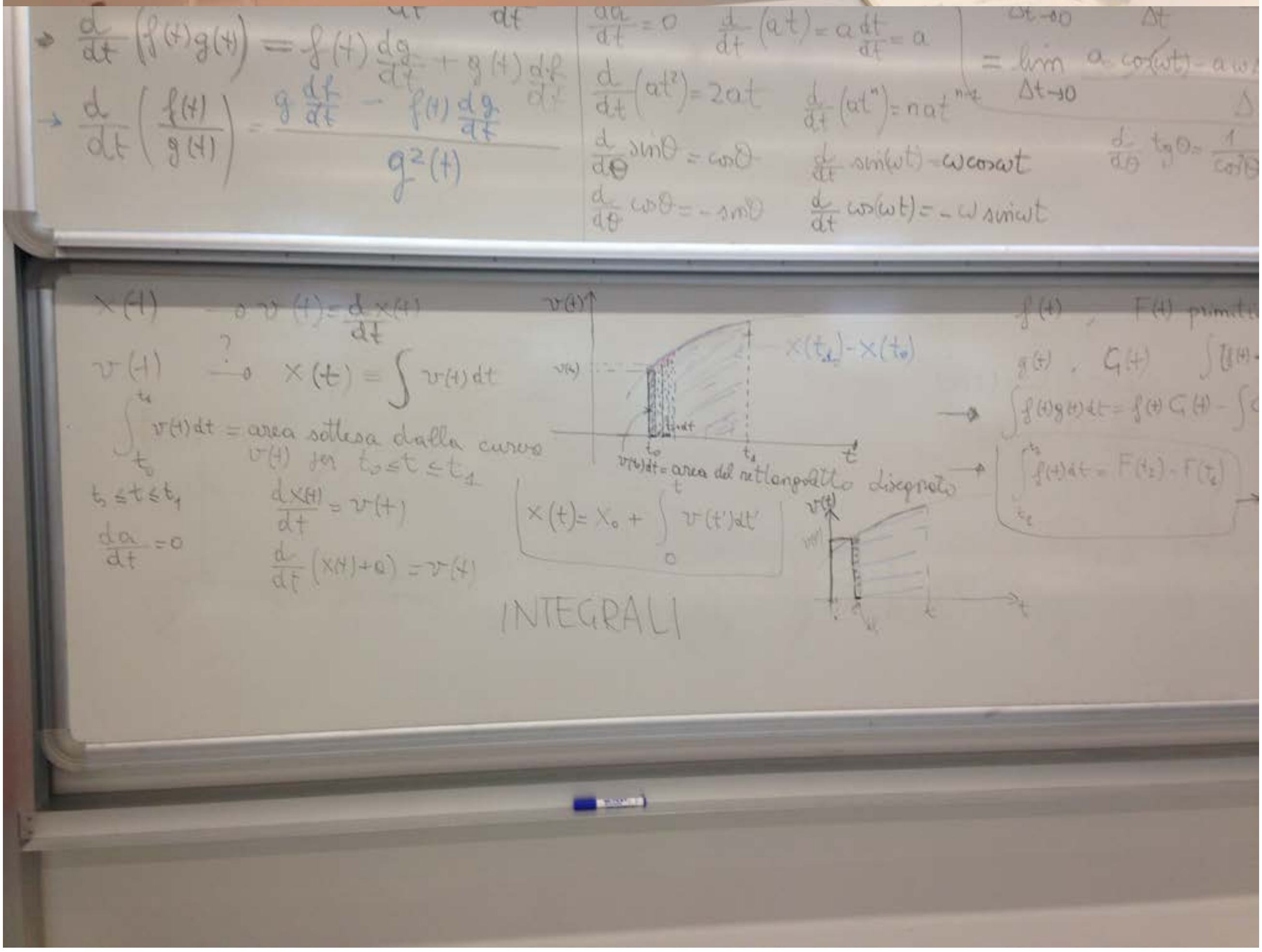
t $t = \phi(u)$ ϕ funzione di u $dt = \frac{d\phi}{dt}du$

$\rightarrow \int f(t)\,dt = \int\left[f(\phi(u))\frac{d\phi}{du}\right]du$

$x(t_1) - x(t_0)$

area del rettangolotto disegnato

$v(t')dt'$



$\rightarrow \frac{d}{dt}(f(t)g(t)) = f(t)\frac{dg}{dt} + g(t)\frac{df}{dt}$

$\rightarrow \frac{d}{dt}\left(\frac{f(t)}{g(t)}\right) = \frac{g\frac{df}{dt} - f(t)\frac{dg}{dt}}{g^2(t)}$

$\frac{da}{dt} = 0$ $\frac{d}{dt}(at) = a\frac{dt}{dt} = a$

$\frac{d}{dt}(at^2) = 2at$ $\frac{d}{dt}(at^n) = nat^{n-1}$

$\frac{d}{d\theta}\sin\theta = \cos\theta$ $\frac{d}{dt}\sin(\omega t) = \omega\cos\omega t$

$\frac{d}{d\theta}\cos\theta = -\sin\theta$ $\frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

$x(t) \rightarrow v(t) = \frac{d\,x(t)}{dt}$

$v(t) \overset{?}{\rightarrow} x(t) = \int v(t)\,dt$

$\int_{t_0}^{t_1} v(t)\,dt$ = area sottesa dalla curva $v(t)$ per $t_0 \le t \le t_1$

$t_0 \le t \le t_1$

$\frac{da}{dt} = 0$

$\frac{dx(t)}{dt} = v(t)$

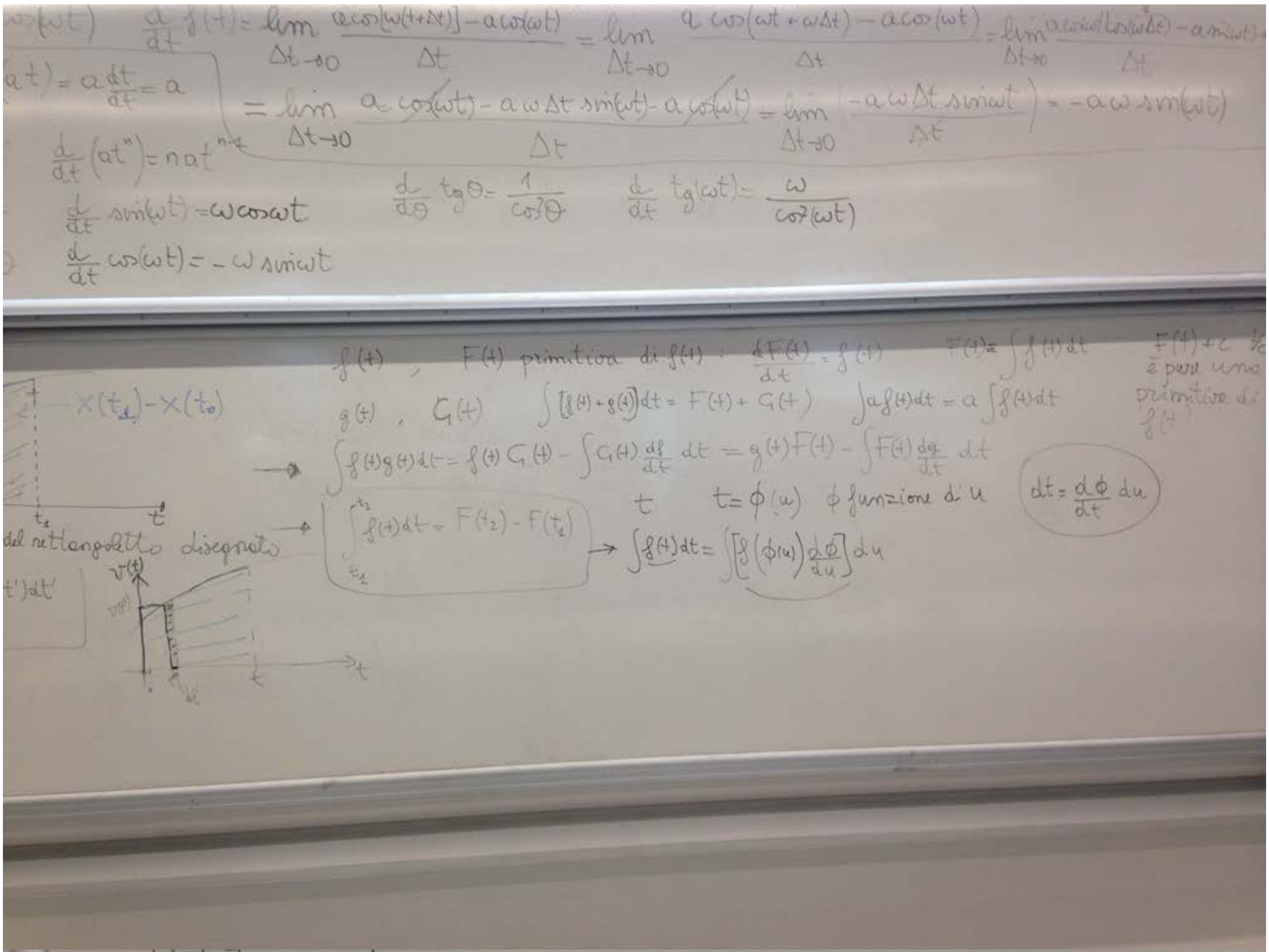
$\frac{d}{dt}(x(t)+a) = v(t)$

$$\boxed{x(t) = x_0 + \int_0^t v(t')\,dt'}$$

$v(t_0)dt$ = area del rettangolotto disegnato

$x(t_1) - x(t_0)$

INTEGRALI



$\cos(\omega t)$ $\frac{d}{dt} f(t) = \lim_{\Delta t \to 0} \frac{a\cos[\omega(t+\Delta t)] - a\cos(\omega t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{a\cos(\omega t + \omega\Delta t) - a\cos(\omega t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{a\cos(\omega t)\cos(\omega\Delta t) - a\sin(\omega t)}{\Delta t}$

$(at) = a\frac{dt}{dt} = a$

$= \lim_{\Delta t \to 0} \frac{a\cancel{\cos(\omega t)} - a\omega\Delta t\sin(\omega t) - a\cancel{\cos(\omega t)}}{\Delta t} = \lim_{\Delta t \to 0}\left(\frac{-a\omega\Delta t\sin\omega t}{\Delta t}\right) = -a\omega\sin(\omega t)$

$\frac{d}{dt}(at^n) = nat^{n-1}$

$\frac{d}{dt}\sin(\omega t) = \omega\cos\omega t$ $\frac{d}{d\theta}\mathrm{tg}\,\theta = \frac{1}{\cos^2\theta}$ $\frac{d}{dt}\mathrm{tg}(\omega t) = \frac{\omega}{\cos^2(\omega t)}$

$\frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

$x(t_1) - x(t_0)$

del rettangoletto disegnato

$t')dt'$

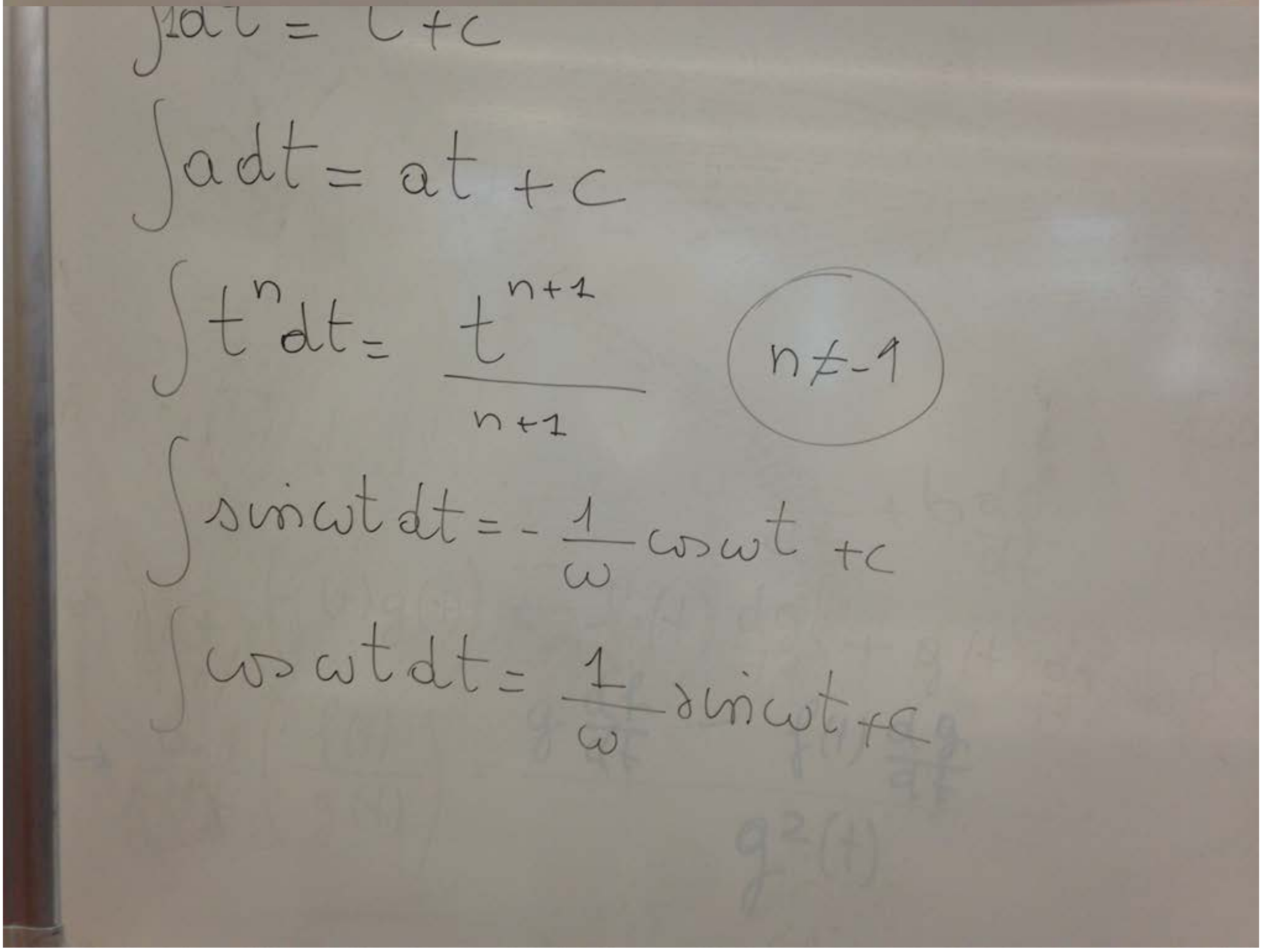
$f(t)$, $F(t)$ primitiva di $f(t)$: $\frac{dF(t)}{dt} = f(t)$ $F(t) = \int f(t)\,dt$ $F(t) + c$ $\forall c$ è pure una primitiva di $f(t)$

$g(t)$, $G(t)$ $\int [f(t) + g(t)]\,dt = F(t) + G(t)$ $\int a f(t)\,dt = a\int f(t)\,dt$

$\longrightarrow \int f(t)g(t)\,dt = f(t)G(t) - \int G(t)\frac{df}{dt}\,dt = g(t)F(t) - \int F(t)\frac{dg}{dt}\,dt$

t $t = \phi(u)$ ϕ funzione di u $\boxed{dt = \frac{d\phi}{dt}\,du}$

$\longrightarrow \boxed{\int_{t_1}^{t_2} f(t)\,dt = F(t_2) - F(t_1)} \longrightarrow \int f(t)\,dt = \int \underbrace{\left[f(\phi(u))\frac{d\phi}{du}\right]} du$



$\int 1\,dt = t + c$

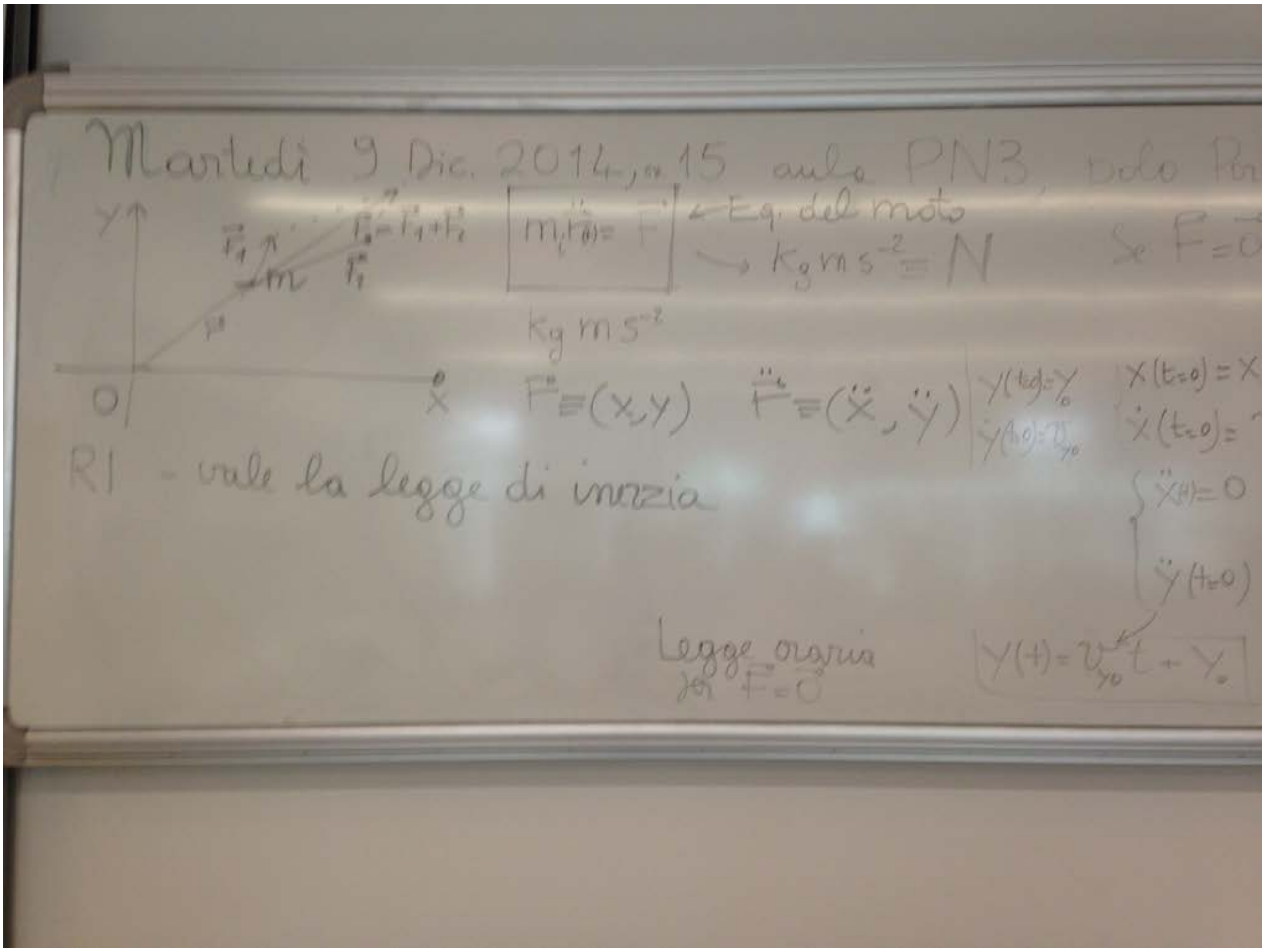
$\int a\,dt = at + c$

$\int t^n\,dt = \frac{t^{n+1}}{n+1}$ $\boxed{n \neq -1}$

$\int \sin\omega t\,dt = -\frac{1}{\omega}\cos\omega t + c$

$\int \cos\omega t\,dt = \frac{1}{\omega}\sin\omega t + c$

4 Dicembre 2014
-Equazione del moto in un riferimento inerziale e sua soluzione
-Legge oraria
-Traiettoria


y
x
O
m
$\vec F_1$
$\vec F_2$
$\vec F = \vec F_1 + \vec F_2$
$\vec r$

Martedì 9 Dic. 2014, ore 15 aula PN3, polo Por

$\boxed{m\ddot{\vec r}(t) = \vec F}$ ← Eq. del moto

→ $kg\,m\,s^{-2} \equiv N$

$kg\,m\,s^{-2}$

$\vec r \equiv (x, y)$ $\ddot{\vec r} \equiv (\ddot x, \ddot y)$

RI - vale la legge di inerzia

Se $\vec F = \vec 0$

$y(t=0) = y_0$
$\dot y(t=0) = v_{y0}$

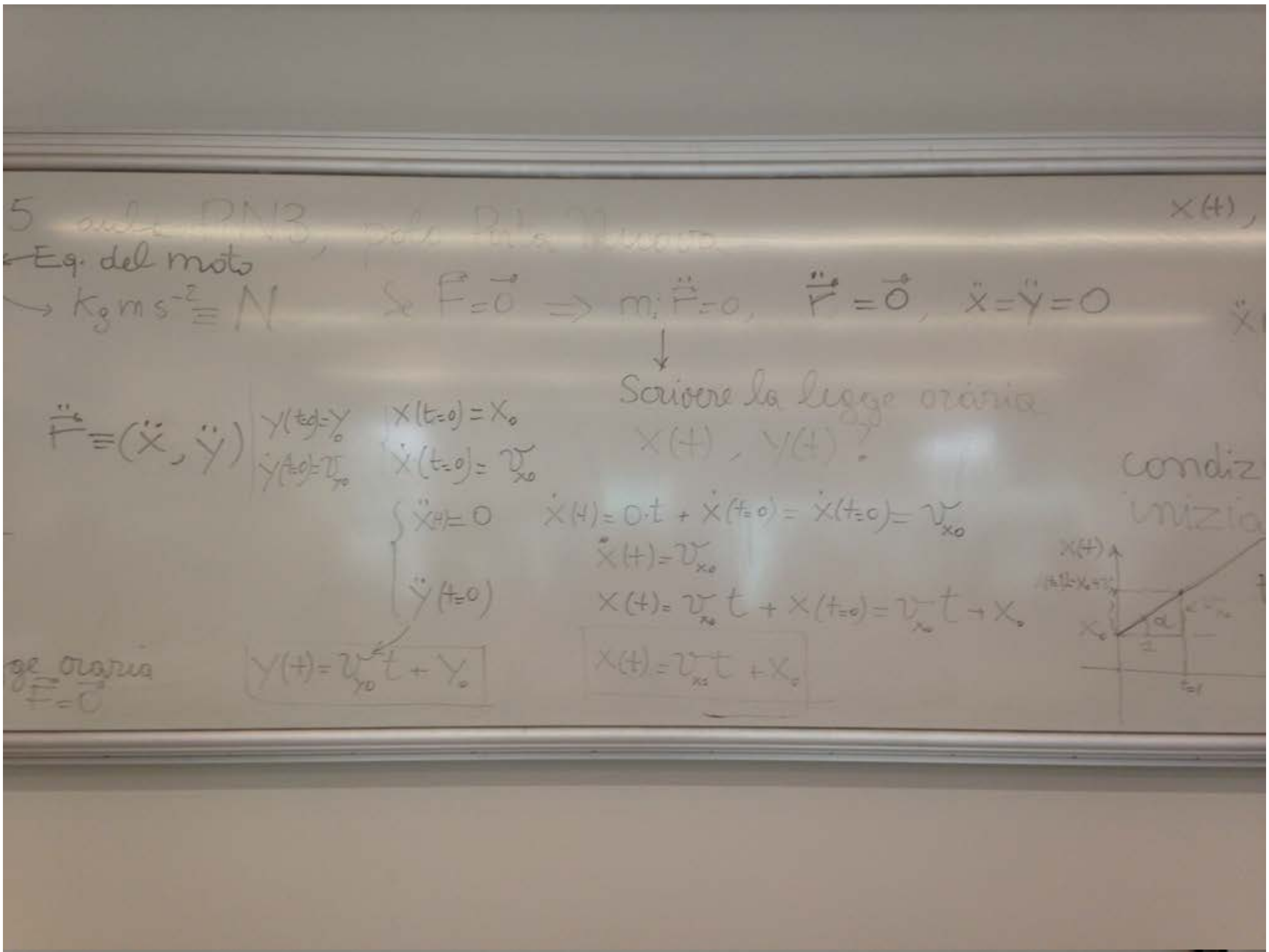
$x(t=0) = x$
$\dot x(t=0) =$

$\ddot x(t) = 0$

$\ddot y(t=0)$

Legge oraria per $\vec F = \vec 0$

$\boxed{y(t) = v_{y0}t + y_0}$



← Eq. del moto

→ $Kg\,m\,s^{-2} \equiv N$

Se $\vec F = \vec 0 \Rightarrow m_i \ddot{\vec r} = 0$, $\ddot{\vec r} = \vec 0$, $\ddot x = \ddot y = 0$

↓

Scrivere la legge oraria

$x(t)$, $y(t)$?

$\ddot{\vec r} \equiv (\ddot x, \ddot y)$

$y(t=0) = y_0$

$\dot y(t=0) = v_{y0}$

$x(t=0) = x_0$

$\dot x(t=0) = v_{x0}$

$$\begin{cases} \ddot x(t) = 0 \\ \ddot y(t=0) \end{cases}$$

$\dot x(t) = 0 \cdot t + \dot x(t=0) = \dot x(t=0) = v_{x0}$

$\dot x(t) = v_{x0}$

$x(t) = v_{x0} t + x(t=0) = v_{x0} t + x_0$

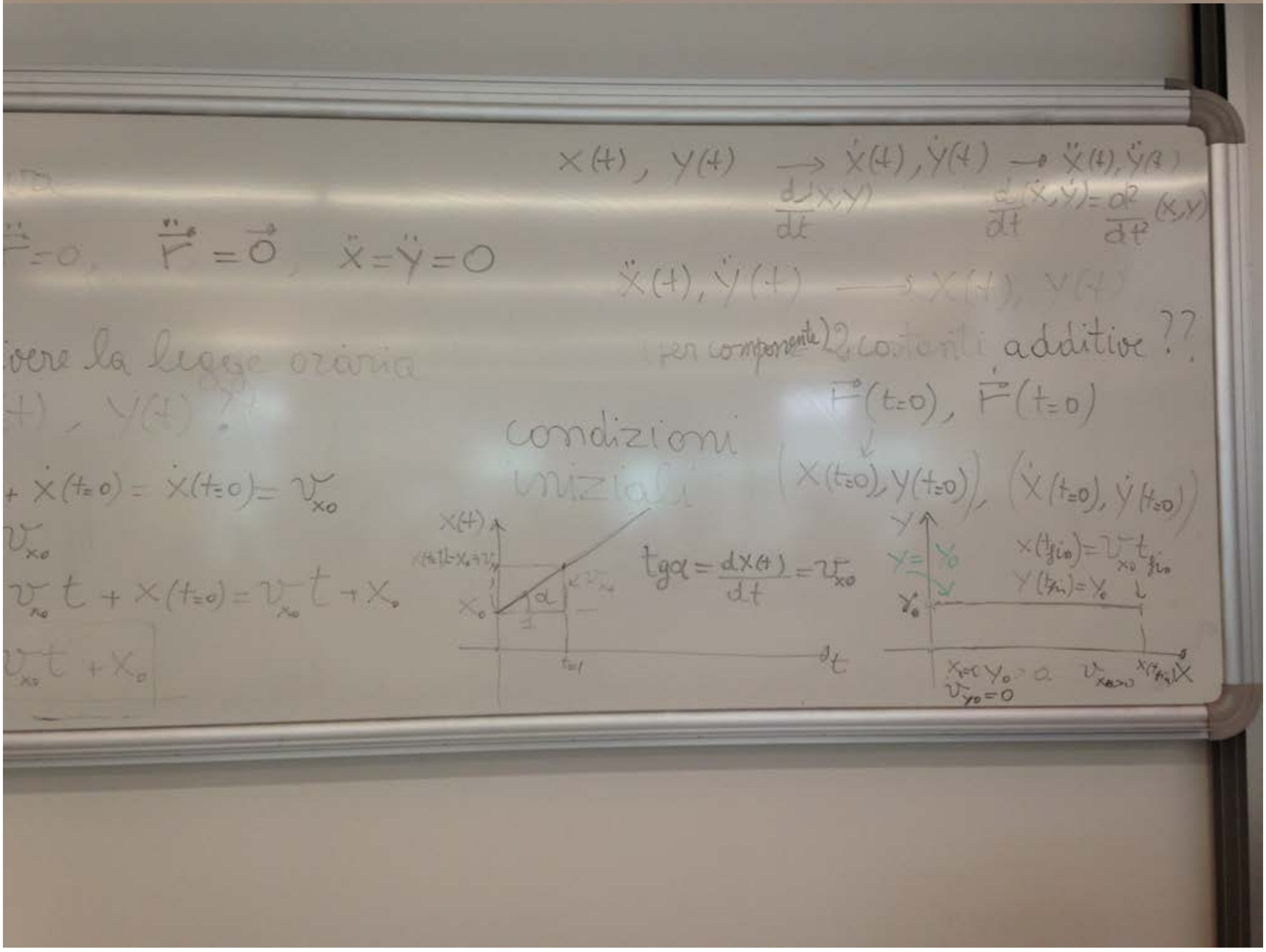
$\boxed{y(t) = v_{y0} t + y_0}$

$\boxed{x(t) = v_{x0} t + x_0}$

ge oraria $\vec F = \vec 0$

condiz

inizia



$x(t)$, $y(t)$ $\xrightarrow[\frac{d}{dt}(x,y)]{}$ $\dot x(t), \dot y(t)$ $\rightarrow$ $\ddot x(t), \ddot y(t)$

$\frac{d}{dt}(\dot x, \dot y) = \frac{d^2}{dt^2}(x,y)$

$\ddot x(t), \ddot y(t) \longrightarrow x(t), y(t)$

(per componente) 2 costanti additive ??

$\vec r(t=0)$, $\dot{\vec r}(t=0)$

↓

$(x(t=0), y(t=0))$, $(\dot x(t=0), \dot y(t=0))$

condizioni iniziali

$tg\alpha = \frac{dx(t)}{dt} = v_{x0}$

$\ddot{\vec r} = 0$, $\ddot{\vec r} = \vec 0$, $\ddot x = \ddot y = 0$

vere la legge oraria

$t)$, $y(t)$??

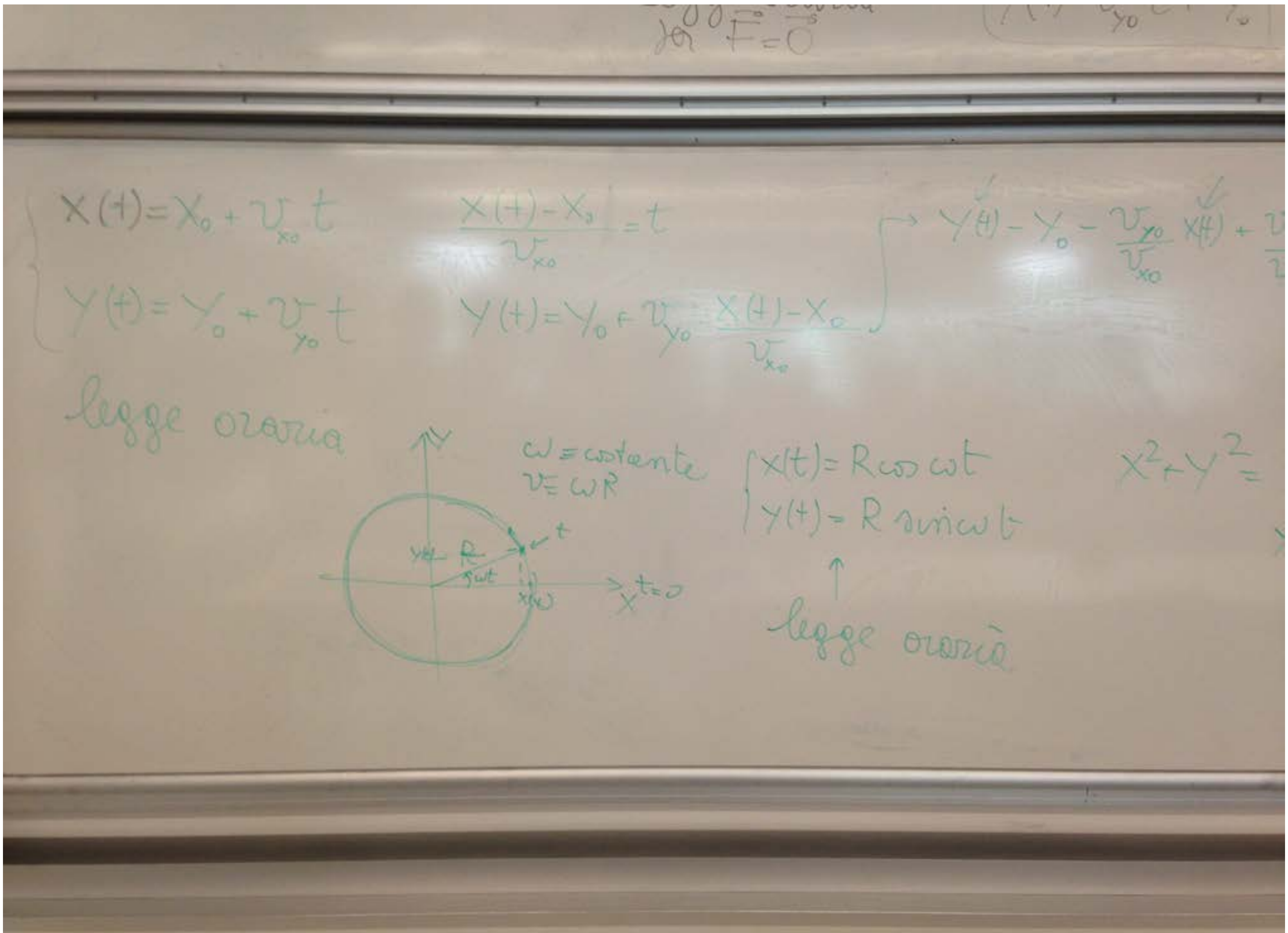
$+ \dot x(t=0) = \dot x(t=0) = v_{x0}$

v_{x0}

$v_{x0} t + x(t=0) = v_{x0} t + x_0$

$v_{x0} t + x_0$

x(t)
x(t=1)-x_0=v_{x0}
x_0
α
1
t=1
t
y
y = y_0
y_0
x(t_fin) = v_{x0} t_fin
y(t_fin) = y_0
x_0=0 y_0=a
v_{y0}=0
v_{x0}>0
x(t_fin)
X


y
R
ωt
y(t)
x(t)
t
x
t=0

se $\vec{F}=\vec{0}$

$$\begin{cases} x(t)=x_0+v_{x0}\,t \\ y(t)=y_0+v_{y0}\,t \end{cases}$$

legge oraria

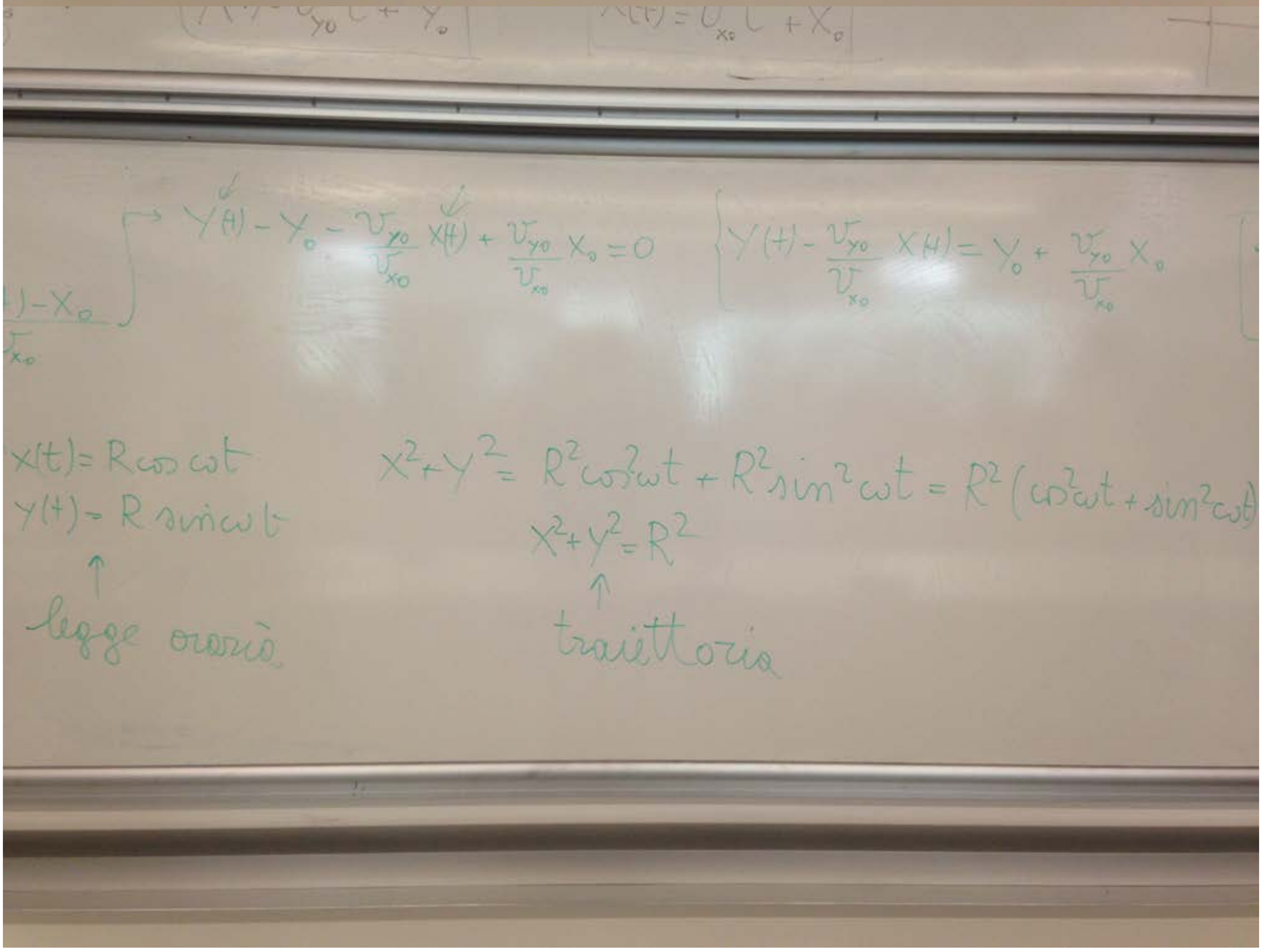
$$\frac{x(t)-x_0}{v_{x0}}=t$$

$$y(t)=y_0+v_{y0}\,\frac{x(t)-x_0}{v_{x0}}$$

ω = costante
v = ωR

$$\begin{cases} x(t)=R\cos\omega t \\ y(t)=R\sin\omega t \end{cases}$$

↑
legge oraria



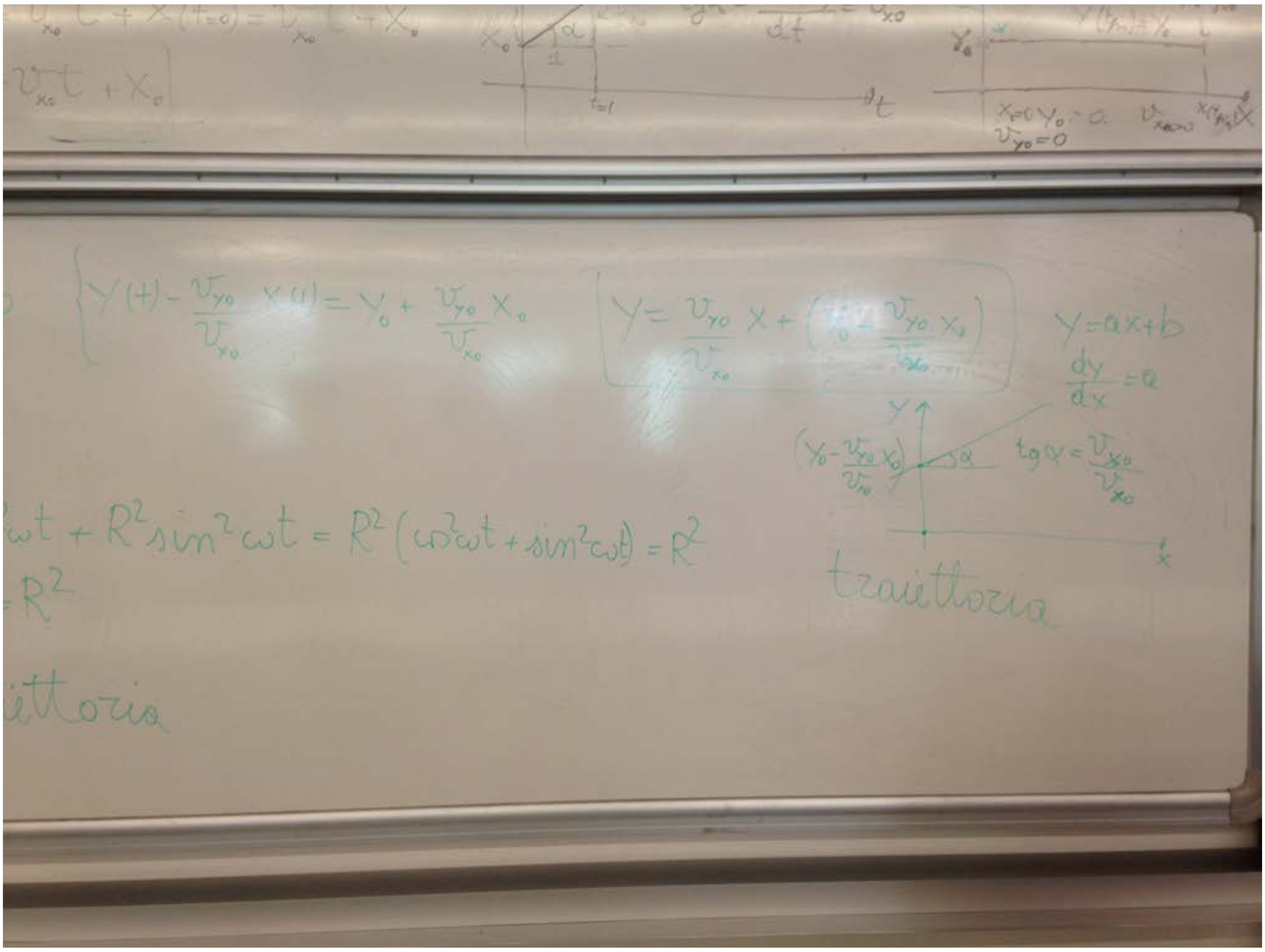
$$\rightarrow y(t)-y_0-\frac{v_{y0}}{v_{x0}}\,x(t)+\frac{v_{y0}}{v_{x0}}\,x_0=0$$

$$\left\{ y(t)-\frac{v_{y0}}{v_{x0}}\,x(t)=y_0+\frac{v_{y0}}{v_{x0}}\,x_0 \right.$$

$$x^2+y^2=R^2\cos^2\omega t+R^2\sin^2\omega t=R^2(\cos^2\omega t+\sin^2\omega t)$$

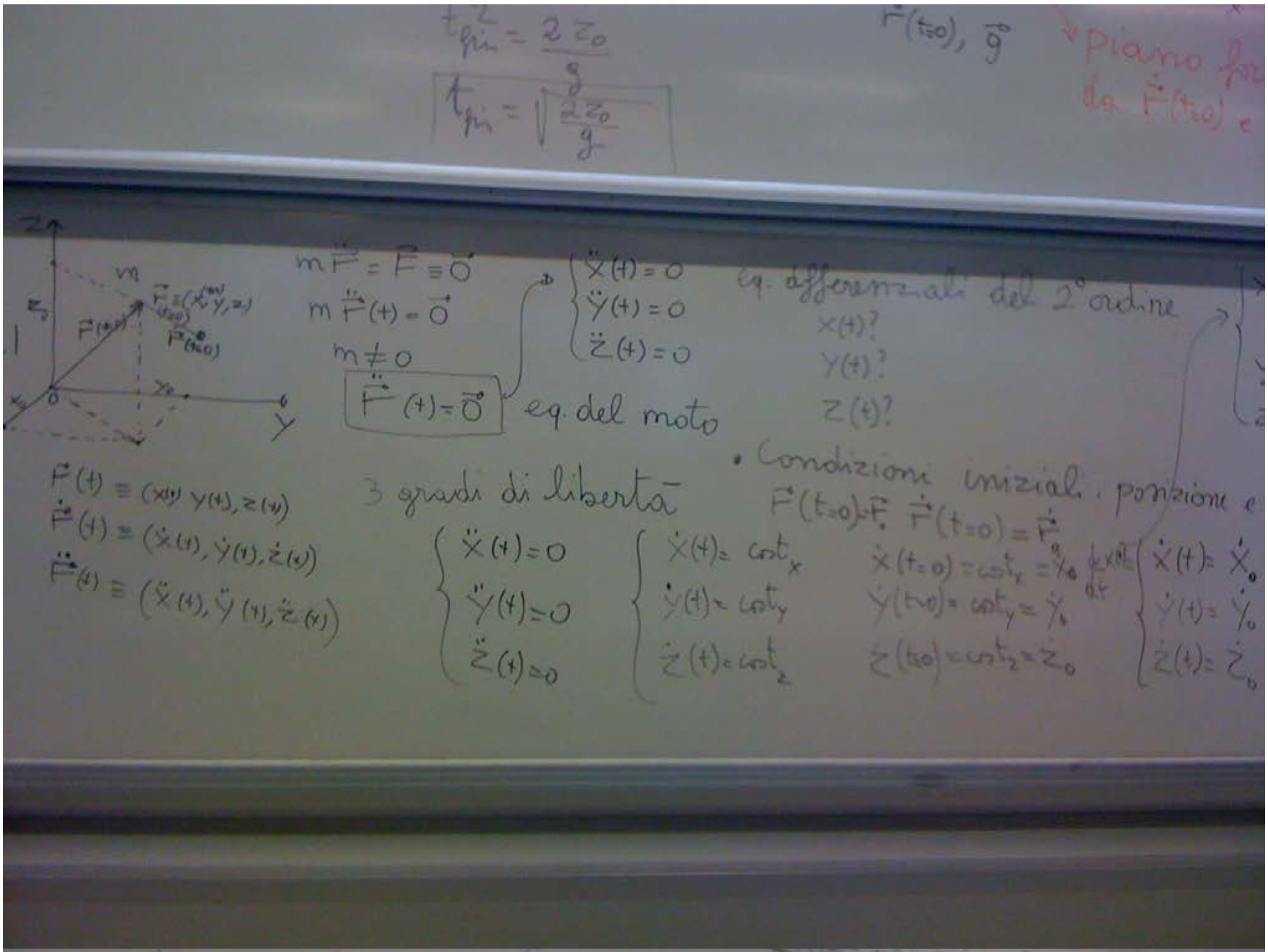
$$x^2+y^2=R^2$$

↑
traiettoria


$y(t) - \frac{v_{y0}}{v_{x0}} x(t) = y_0 + \frac{v_{y0}}{v_{x0}} x_0$
$y = \frac{v_{y0}}{v_{x0}} x + \left(y_0 - \frac{v_{y0}}{v_{x0}} x_0\right)$
$y = ax + b$
$\frac{dy}{dx} = a$
$\left(y_0 - \frac{v_{y0}}{v_{x0}} x_0\right)$
α
$\mathrm{tg}\,\alpha = \frac{v_{y0}}{v_{x0}}$
y
x
traiettoria
$\omega t + R^2 \sin^2\omega t = R^2(\cos^2\omega t + \sin^2\omega t) = R^2$
$= R^2$
ettoria
$v_{y0} = 0$
t=1
t

11 Dicembre 2014

- Eq. del moto e loro integrazione; moto in assenza di forza
- Moto in presenza della forza gravitazionale (sulla superficie della Terra)
- Moto del proiettile



$t_{fin}^2 = \frac{2 z_0}{g}$

$\boxed{t_{fin} = \sqrt{\frac{2 z_0}{g}}}$

$\vec r(t=0), \vec g$ → piano fo

da $\dot{\vec r}(t=0)$ e

$m\ddot{\vec r} = \vec F \equiv \vec 0$

$m\ddot{\vec r}(t) = \vec 0$

$m \neq 0$

$\boxed{\ddot{\vec r}(t) = \vec 0}$ eq. del moto

$\begin{cases} \ddot x(t) = 0 \\ \ddot y(t) = 0 \\ \ddot z(t) = 0 \end{cases}$ eq. differenziali del 2° ordine

x(t)?
y(t)?
z(t)?

3 gradi di libertà

$\vec r(t) \equiv (x(t), y(t), z(t))$

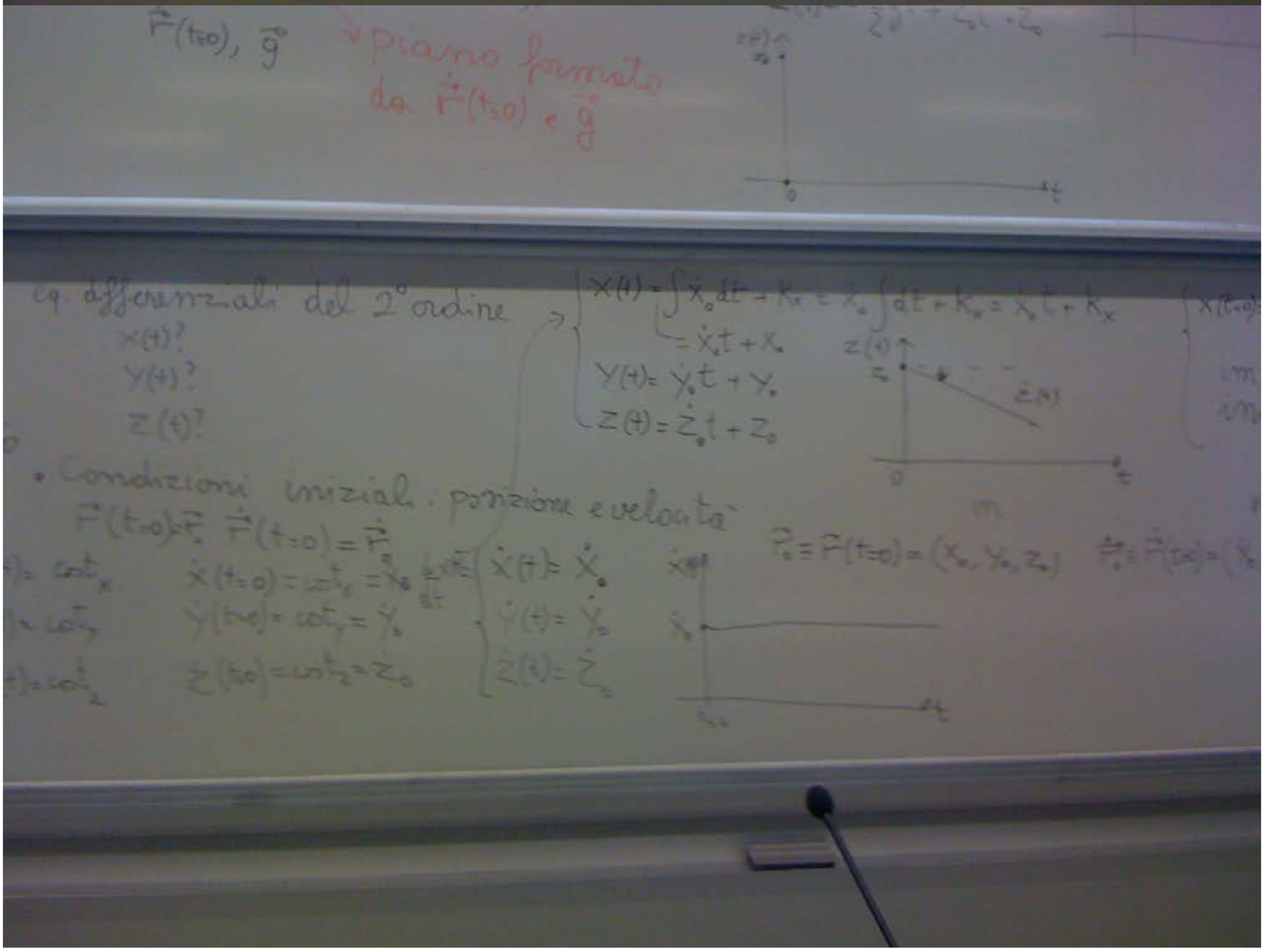
$\dot{\vec r}(t) \equiv (\dot x(t), \dot y(t), \dot z(t))$

$\ddot{\vec r}(t) \equiv (\ddot x(t), \ddot y(t), \ddot z(t))$

• Condizioni iniziali: posizione e

$\vec r(t=0) = \vec r_0$ $\dot{\vec r}(t=0) = \dot{\vec r}_0$

$\begin{cases} \ddot x(t) = 0 \\ \ddot y(t) = 0 \\ \ddot z(t) = 0 \end{cases}$ $\begin{cases} \dot x(t) = cost_x \\ \dot y(t) = cost_y \\ \dot z(t) = cost_z \end{cases}$ $\begin{matrix} \dot x(t=0) = cost_x = \dot x_0 \\ \dot y(t=0) = cost_y = \dot y_0 \\ \dot z(t=0) = cost_z = \dot z_0 \end{matrix}$ $\frac{dx(t)}{dt}$ $\begin{cases} \dot x(t) = \dot x_0 \\ \dot y(t) = \dot y_0 \\ \dot z(t) = \dot z_0 \end{cases}$



$\vec r(t=0), \vec g$ → piano formato da $\dot{\vec r}(t=0)$ e $\vec g$

$\begin{cases} x(t) = \int \dot x_0 \, dt + k_x = \dot x_0 \int dt + k_x = \dot x_0 t + k_x \\ \quad = \dot x_0 t + x_0 \\ y(t) = \dot y_0 t + y_0 \\ z(t) = \dot z_0 t + z_0 \end{cases}$

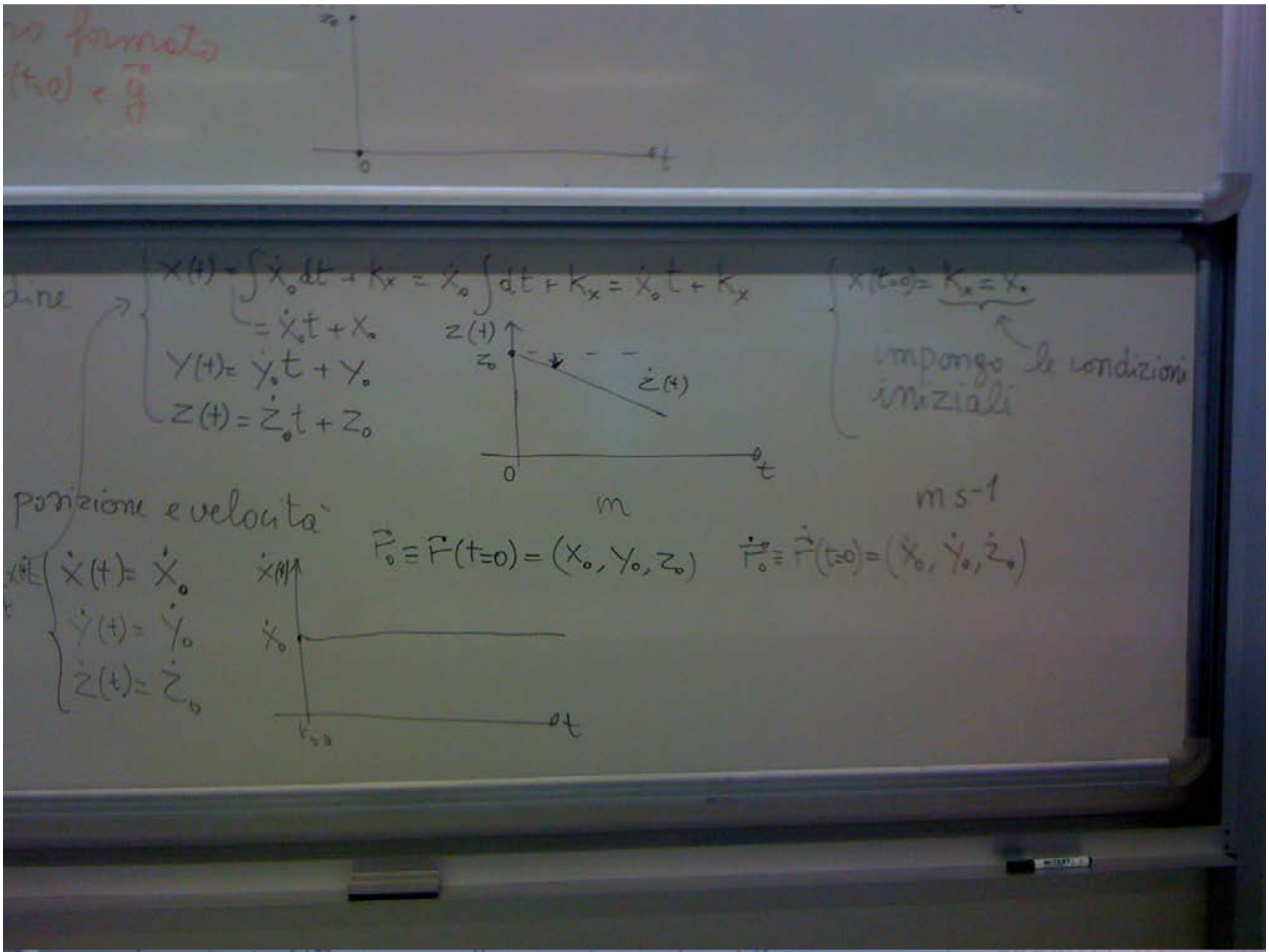
• Condizioni iniziali: posizione e velocità

$\vec r_0 = \vec r(t=0) = (x_0, y_0, z_0)$ $\dot{\vec r}_0 = \dot{\vec r}(t=0) = (\dot x_0$

eq. differenziali del 2° ordine
x(t)?
y(t)?
z(t)?

$\vec r(t=0) = \vec r_0$ $\dot{\vec r}(t=0) = \dot{\vec r}_0$

$\begin{matrix} \dot x(t=0) = cost_x = \dot x_0 \\ \dot y(t=0) = cost_y = \dot y_0 \\ \dot z(t=0) = cost_z = \dot z_0 \end{matrix}$ $\frac{dx(t)}{dt}$ $\begin{cases} \dot x(t) = \dot x_0 \\ \dot y(t) = \dot y_0 \\ \dot z(t) = \dot z_0 \end{cases}$



x_0
0
t
z(t)
z_0
$\dot z(t)$
0
t
$\dot x(t)$
$\dot x_0$
t

…o famoso

$(t=0)$ e $\vec g$

…dine

$$\begin{cases} x(t) = \int \dot x_0 \, dt + k_x = \dot x_0 \int dt + k_x = \dot x_0 t + k_x \\ \quad = \dot x_0 t + x_0 \\ y(t) = \dot y_0 t + y_0 \\ z(t) = \dot z_0 t + z_0 \end{cases}$$

$$\Big\{ x(t=0) = \underbrace{k_x = x_0}$$

impongo le condizioni iniziali

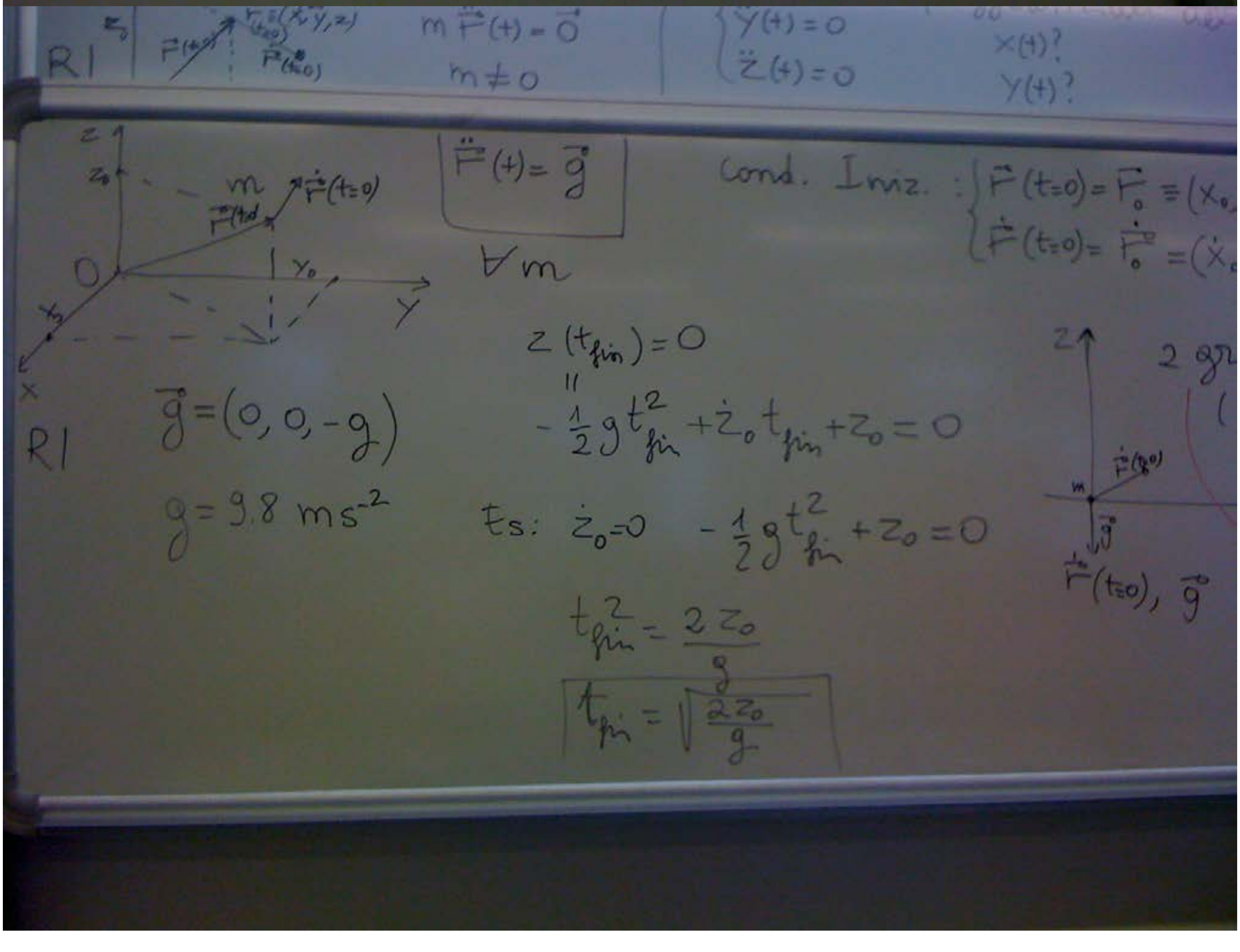
posizione e velocità

m

m s-1

$$\vec r_0 \equiv \vec r(t=0) = (x_0, y_0, z_0) \qquad \dot{\vec r}_0 \equiv \dot{\vec r}(t=0) = (\dot x_0, \dot y_0, \dot z_0)$$

$$\begin{cases} \dot x(t) = \dot x_0 \\ \dot y(t) = \dot y_0 \\ \dot z(t) = \dot z_0 \end{cases}$$



R1
$\vec r(t)$
$\vec r \equiv (x, y, z)$
$\dot{\vec r}(t=0)$
z
z_0
m
$\dot{\vec r}(t=0)$
$\vec r(t=0)$
O
y_0
Y
x_0
X
z
m
$\dot{\vec r}(t=0)$
$\vec g$
2 gr

$$m \ddot{\vec r}(t) = \vec 0$$

$$m \neq 0$$

$$\begin{cases} \ddot y(t) = 0 \\ \ddot z(t) = 0 \end{cases}$$

x(t)?

y(t)?

$$\boxed{\ddot{\vec r}(t) = \vec g}$$

$\forall m$

Cond. Iniz.: $\begin{cases} \vec r(t=0) = \vec r_0 \equiv (x_0, \\ \dot{\vec r}(t=0) = \dot{\vec r}_0 = (\dot x_0 \end{cases}$

R1

$$\vec g = (0, 0, -g)$$

$$g = 9.8 \text{ m s}^{-2}$$

$$z(t_{fin}) = 0$$

$$\shortparallel$$

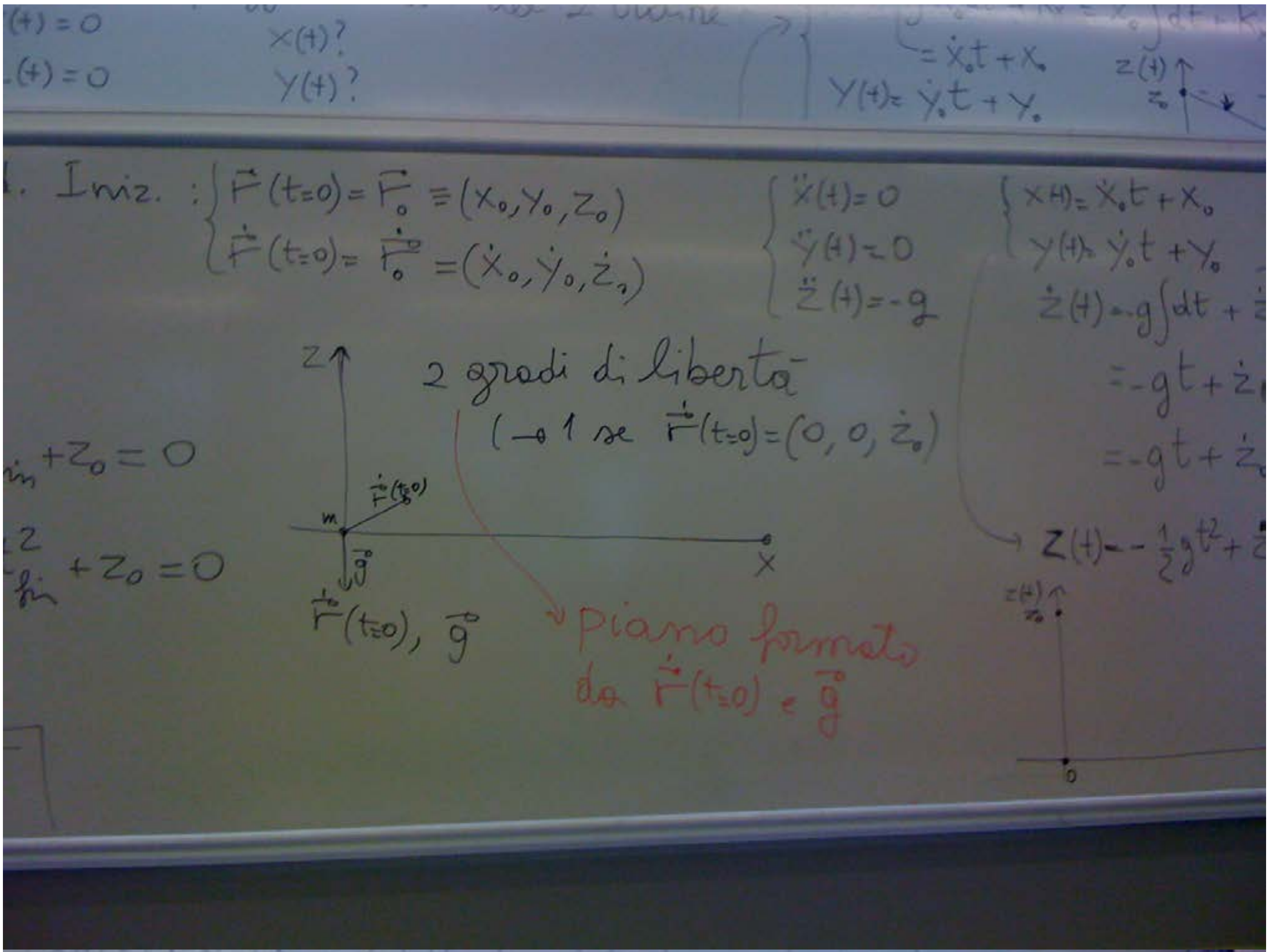
$$-\frac{1}{2} g t_{fin}^2 + \dot z_0 t_{fin} + z_0 = 0$$

Es: $\dot z_0 = 0 \qquad -\frac{1}{2} g t_{fin}^2 + z_0 = 0$

$$t_{fin}^2 = \frac{2 z_0}{g}$$

$$\boxed{t_{fin} = \sqrt{\frac{2 z_0}{g}}}$$

$\ddot{\vec r}(t=0)$, $\vec g$



$(t) = 0$

$(t) = 0$

$x(t)$?

$y(t)$?

$= \dot{x}_0 t + x_0$

$y(t) = \dot{y}_0 t + y_0$

1. Iniz. : $\begin{cases} \vec{r}(t=0) = \vec{r}_0 \equiv (x_0, y_0, z_0) \\ \dot{\vec{r}}(t=0) = \dot{\vec{r}}_0 = (\dot{x}_0, \dot{y}_0, \dot{z}_0) \end{cases}$

$\begin{cases} \ddot{x}(t) = 0 \\ \ddot{y}(t) = 0 \\ \ddot{z}(t) = -g \end{cases}$

$\begin{cases} x(t) = \dot{x}_0 t + x_0 \\ y(t) = \dot{y}_0 t + y_0 \end{cases}$

$\dot{z}(t) = -g\int dt +$

$= -gt + \dot{z}$

$= -gt + \dot{z}$

$Z(t) = -\frac{1}{2}gt^2 +$

2 gradi di libertà

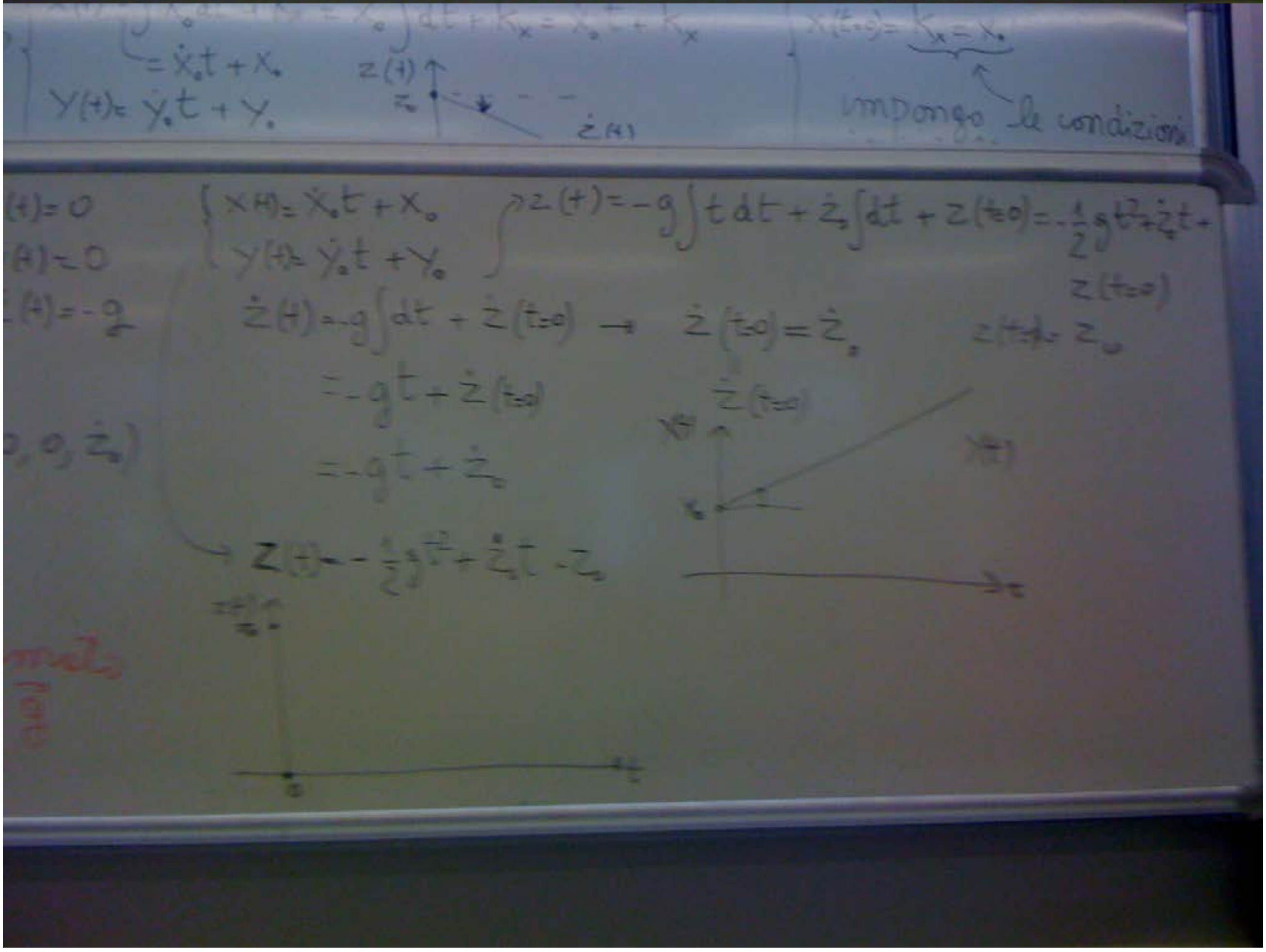
$(\rightarrow 1$ se $\dot{\vec{r}}(t=0) = (0, 0, \dot{z}_0))$

$+ z_0 = 0$

$+ z_0 = 0$

$\dot{\vec{r}}(t=0)$, $\vec{g}$

piano formato da $\dot{\vec{r}}(t=0)$ e $\vec{g}$



$x(t=0) = \underbrace{k_x = x_0}$

impongo le condizioni

$z(t) = -g\int t\,dt + \dot{z}_0\int dt + z(t=0) = -\frac{1}{2}gt^2 + \dot{z}_0 t + z(t=0)$

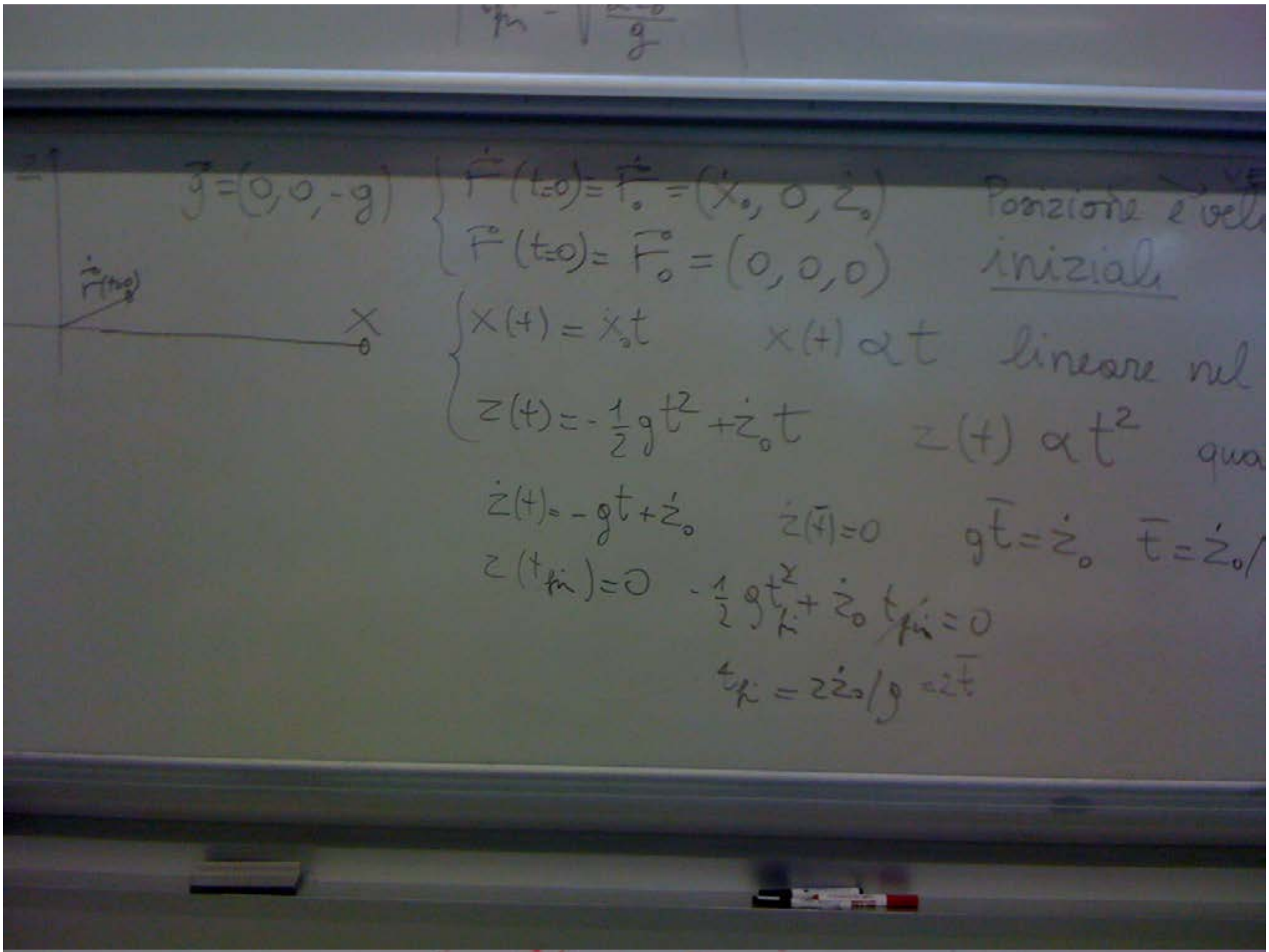
$\dot{z}(t) = -g\int dt + \dot{z}(t=0) \rightarrow \dot{z}(t=0) = \dot{z}_0$

$z(t=0) = z_0$

$= -gt + \dot{z}(t=0)$

$= -gt + \dot{z}_0$

$Z(t) = -\frac{1}{2}gt^2 + \dot{z}_0 t - z_0$



$\vec g = (0,0,-g)$ $\begin{cases}\dot{\vec r}(t=0) = \dot{\vec r}_0 = (\dot x_0, 0, \dot z_0)\\ \vec r(t=0) = \vec r_0 = (0,0,0)\end{cases}$ Posizione e vel iniziali

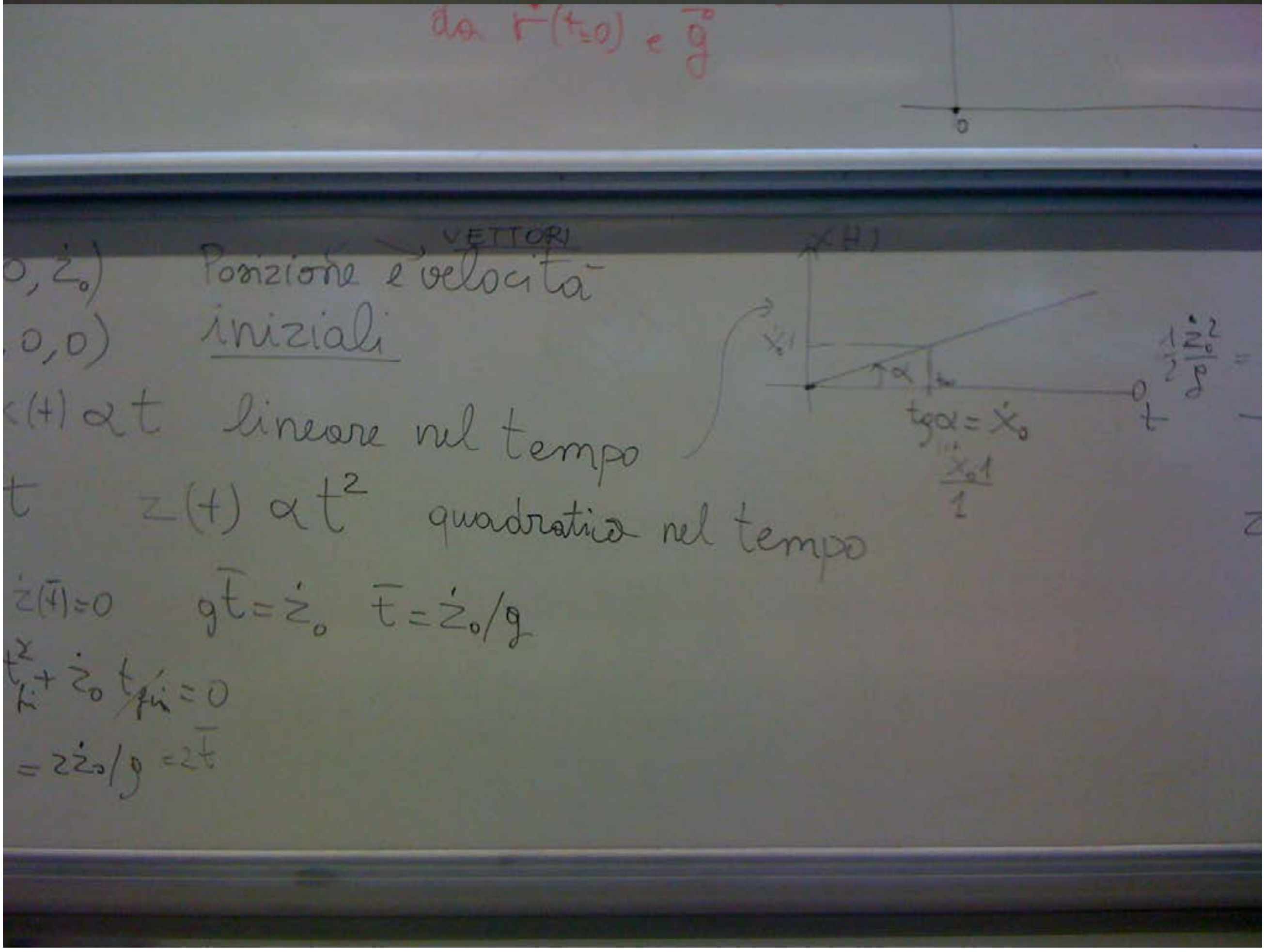
$\begin{cases} x(t) = \dot x_0 t \\ z(t) = -\frac{1}{2} g t^2 + \dot z_0 t \end{cases}$ $x(t) \propto t$ lineare nel

$z(t) \propto t^2$ qua

$\dot z(t) = -gt + \dot z_0$ $\dot z(\bar t) = 0$ $g\bar t = \dot z_0$ $\bar t = \dot z_0/$

$z(t_{fin}) = 0$ $-\frac{1}{2} g t_{fin}^{\cancel{2}} + \dot z_0 \cancel{t_{fin}} = 0$

$t_{fin} = 2\dot z_0/g = 2\bar t$



da $\dot{\vec r}(t=0)$ e $\vec g$

VETTORI

$0, \dot z_0)$ Posizione e velocità

$0,0)$ iniziali

$x(t) \propto t$ lineare nel tempo

$tg\alpha = \dot x_0 = \frac{\dot x_0 1}{1}$

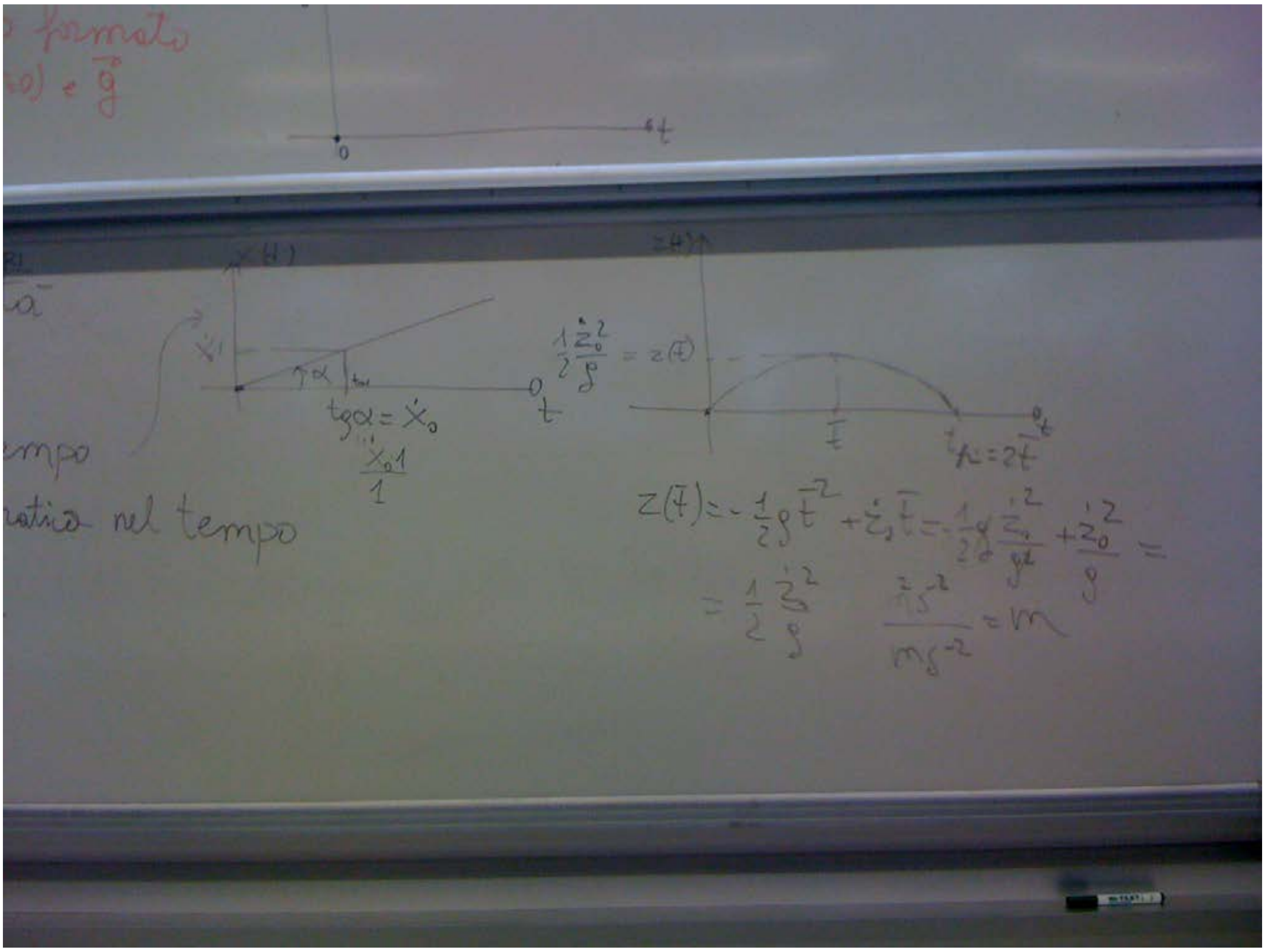
$\frac{1}{2}\frac{\dot z_0^2}{g} =$

t $z(t) \propto t^2$ quadratica nel tempo

$\dot z(\bar t) = 0$ $g\bar t = \dot z_0$ $\bar t = \dot z_0/g$

$t_{fin}^{\cancel{2}} + \dot z_0 \cancel{t_{fin}} = 0$

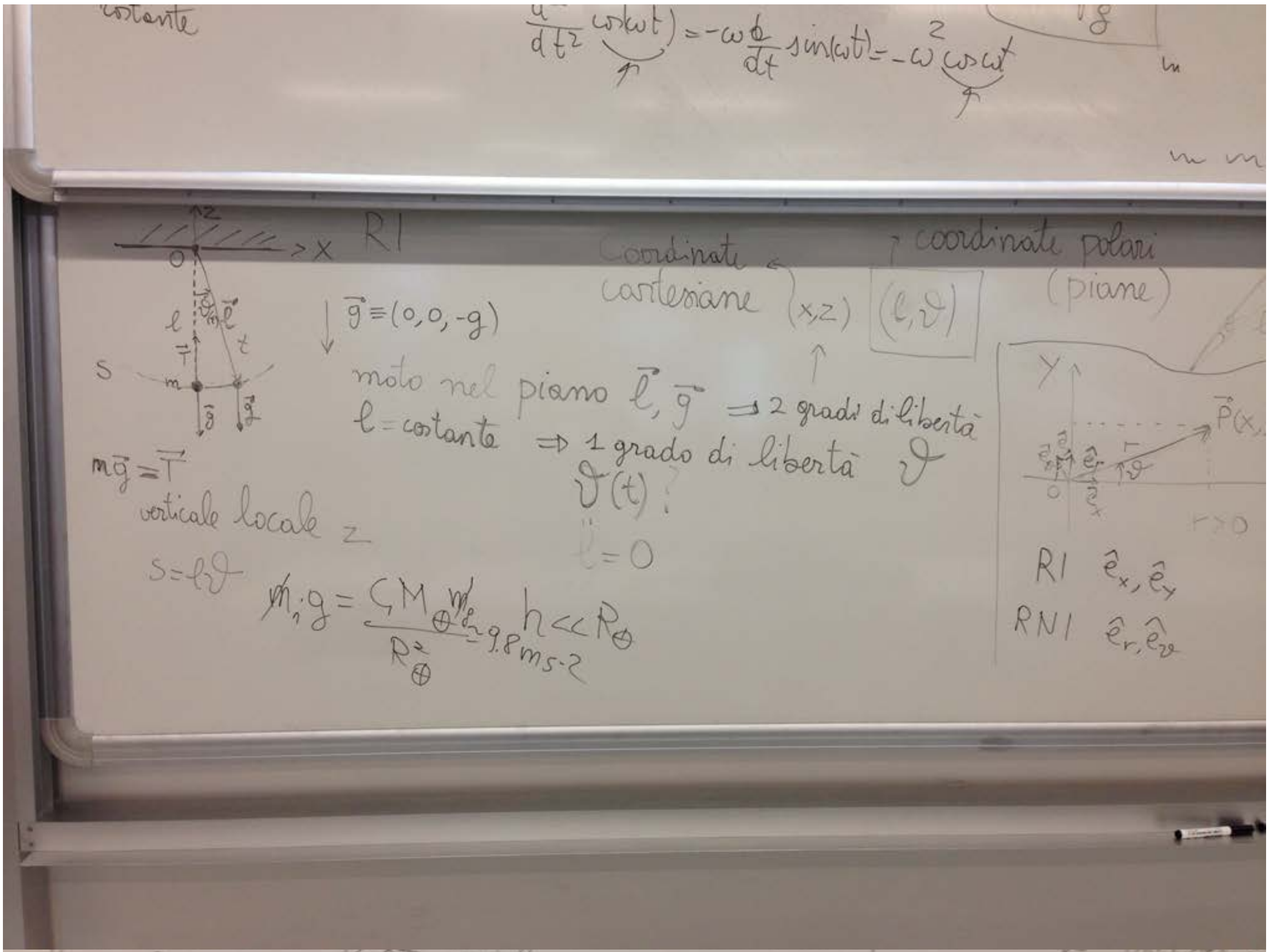
$= 2\dot z_0/g = 2\bar t$



formato
$\vec{g}$
t
0
$x(t)$
$\dot{x}_0$
α
$\operatorname{tg}\alpha = \dot{x}_0 = \frac{\dot{x}_0 1}{1}$
t
mpo
atico nel tempo
$z(t)$
$\frac{1}{2}\frac{\dot{z}_0^2}{g} = z(\bar{t})$
$\bar{t}$
t
$t_A = 2\bar{t}$
$z(\bar{t}) = -\frac{1}{2} g \bar{t}^2 + \dot{z}_0 \bar{t} = -\frac{1}{2} g \frac{\dot{z}_0^2}{g^2} + \frac{\dot{z}_0^2}{g} = \frac{1}{2}\frac{\dot{z}_0^2}{g}$
$\frac{m^2 s^{-2}}{m s^{-2}} = m$

18 Dicembre 2014

- Studio dettagliato del moto del pendolo (in approssimazione di piccole oscillazioni)
- Riferimento inerziale e riferimento non inerziale



costante

$\frac{d^2}{dt^2}\cos(\omega t) = -\omega \frac{d}{dt}\sin(\omega t) = -\omega^2 \cos\omega t$

RI

z
O
x
ℓ
$\vec T$
$\vartheta(t)$
$\vec\ell$
t
s
m
$\vec g$

$\vec g \equiv (0,0,-g)$

moto nel piano $\vec\ell, \vec g$ ⇒ 2 gradi di libertà

ℓ = costante ⇒ 1 grado di libertà ϑ

$\vartheta(t)$?

$\ddot\ell = 0$

$m\vec g = \vec T$

verticale locale z

$s = \ell\vartheta$

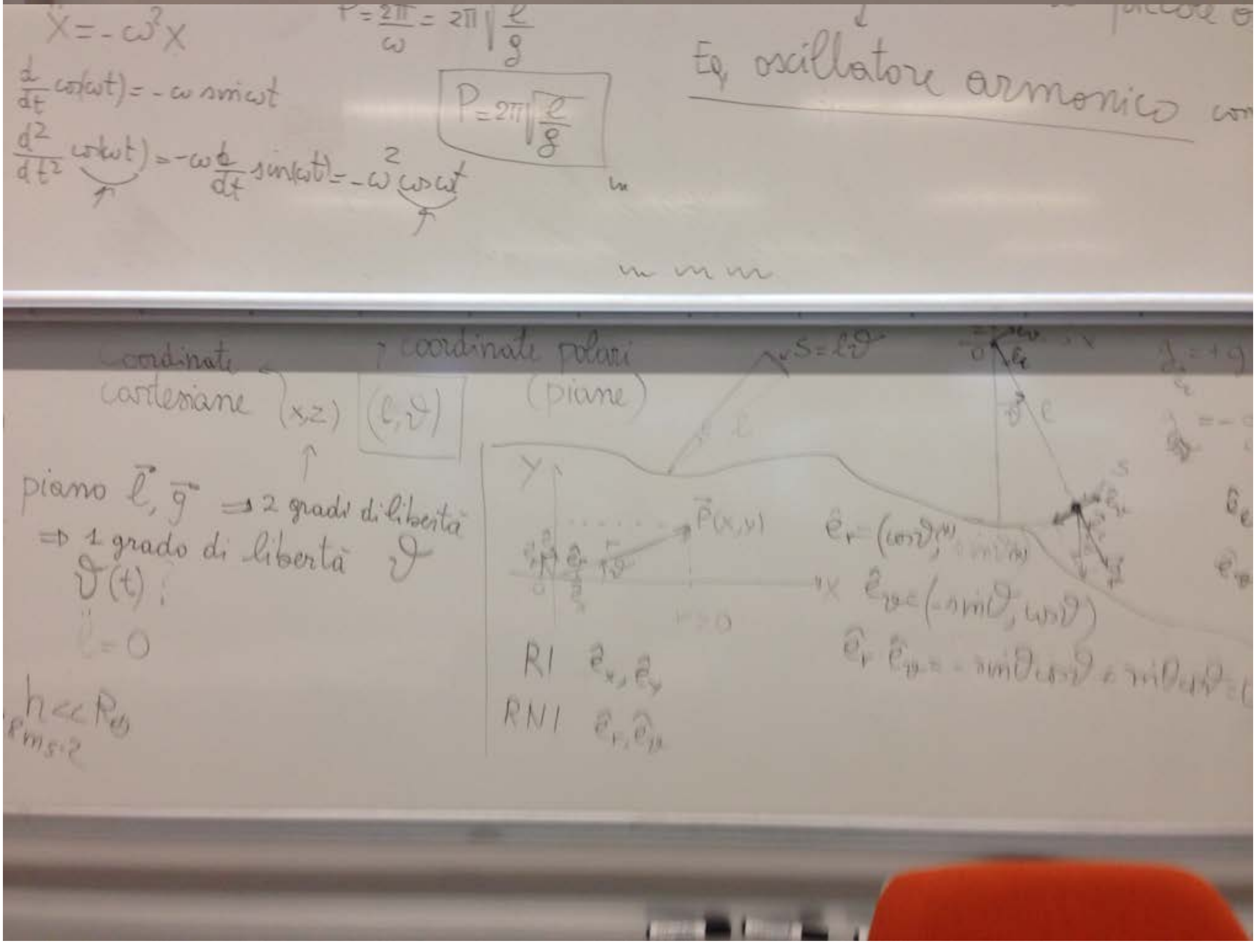
$\cancel{m_i} g = \frac{G M_\oplus \cancel{m_g}}{R_\oplus^2} = 9.8\ ms^{-2}$ $h \ll R_\oplus$

Coordinate cartesiane (x,z) $\boxed{(\ell,\vartheta)}$ coordinate polari (piane)

Y
$\vec P(x,$
$\hat e_\vartheta$
$\hat e_r$
r
ϑ
O
$\hat e_x$
$r>0$

RI $\hat e_x, \hat e_y$

RNI $\hat e_r, \hat e_\vartheta$



$\ddot X = -\omega^2 X$

$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$

$\frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

$\frac{d^2}{dt^2}\cos(\omega t) = -\omega \frac{d}{dt}\sin(\omega t) = -\omega^2 \cos\omega t$

$\boxed{P = 2\pi\sqrt{\frac{\ell}{g}}}$

Eq. oscillatore armonico co

piccole ϑ

Coordinate cartesiane (x,z) (ℓ,ϑ) coordinate polari (piane)

piano $\vec\ell, \vec g$ ⇒ 2 gradi di libertà

⇒ 1 grado di libertà ϑ

$\vartheta(t)$?

$\ddot\ell = 0$

$h \ll R_\oplus$

ms^{-2}

Y
$\vec P(x,y)$
r
ϑ
O
x
$r>0$
$s = \ell\vartheta$
O
x
$\hat e_r$
ϑ
ℓ
s
$\hat e_\vartheta$

$\hat e_r = (\cos\vartheta, \sin\vartheta)$

$\hat e_\vartheta = (-\sin\vartheta, \cos\vartheta)$

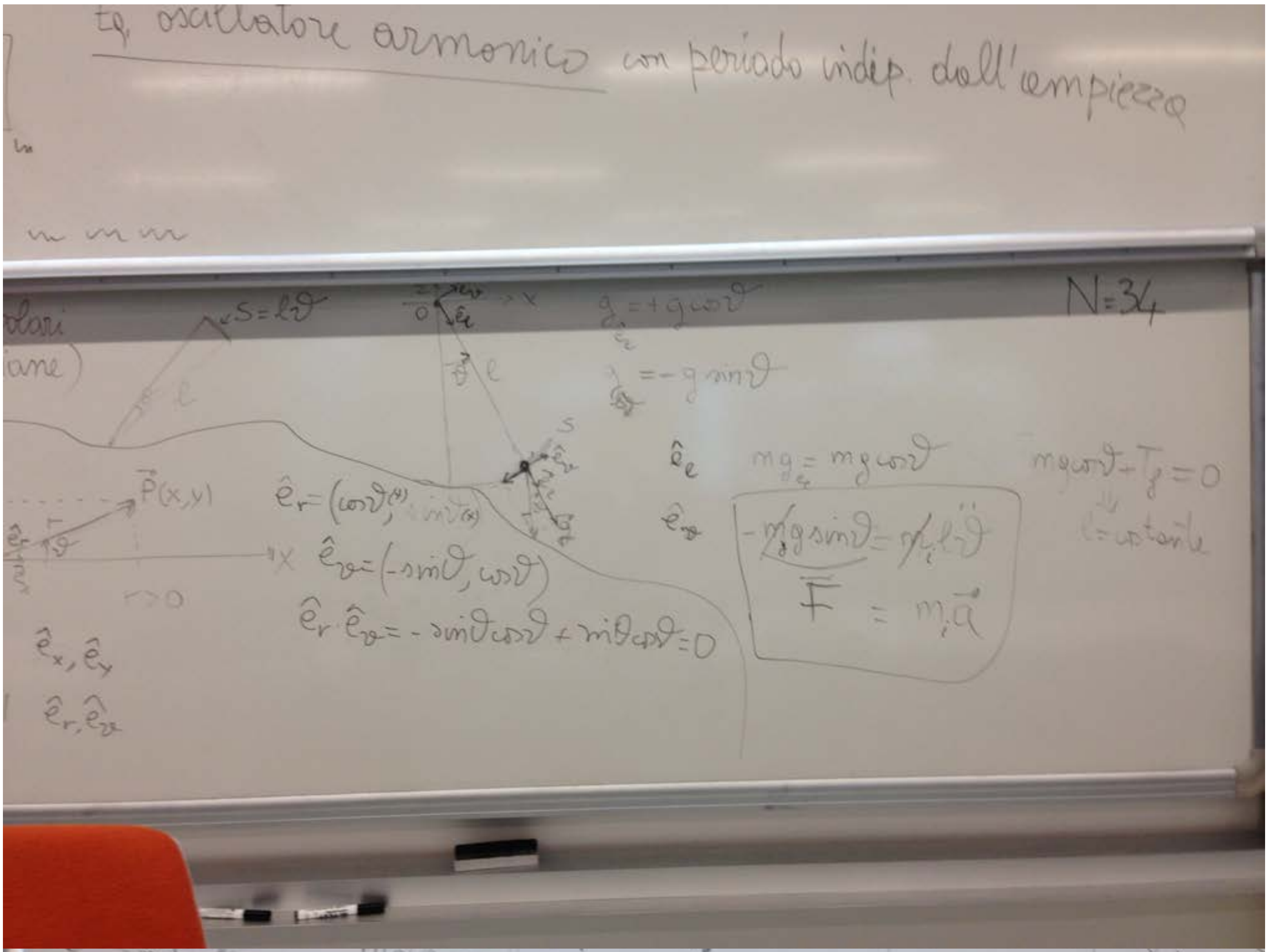
$\hat e_r \cdot \hat e_\vartheta = -\sin\vartheta\cos\vartheta + \sin\vartheta\cos\vartheta = ($

RI $\hat e_x, \hat e_y$

RNI $\hat e_r, \hat e_\vartheta$

$\hat e_r$

$\hat e_\vartheta$



eq. oscillatore armonico con periodo indip. dall'ampiezza

N=34

olari

iane)

$s = \ell\vartheta$

$g_{\hat e_\ell} = +g\cos\vartheta$

$g_{\hat e_\vartheta} = -g\sin\vartheta$

$\hat e_\ell$

$\hat e_\vartheta$

$mg_{\hat e_\ell} = mg\cos\vartheta$

$mg\cos\vartheta + T_\ell = 0$

$\Downarrow$

$\ell = $ costante

$$\boxed{\begin{array}{c} -\cancel{m}g\sin\vartheta = \cancel{m_i}\ell\ddot\vartheta \\ \vec F = m_i\vec a \end{array}}$$

$\vec P(x,y)$

$r>0$

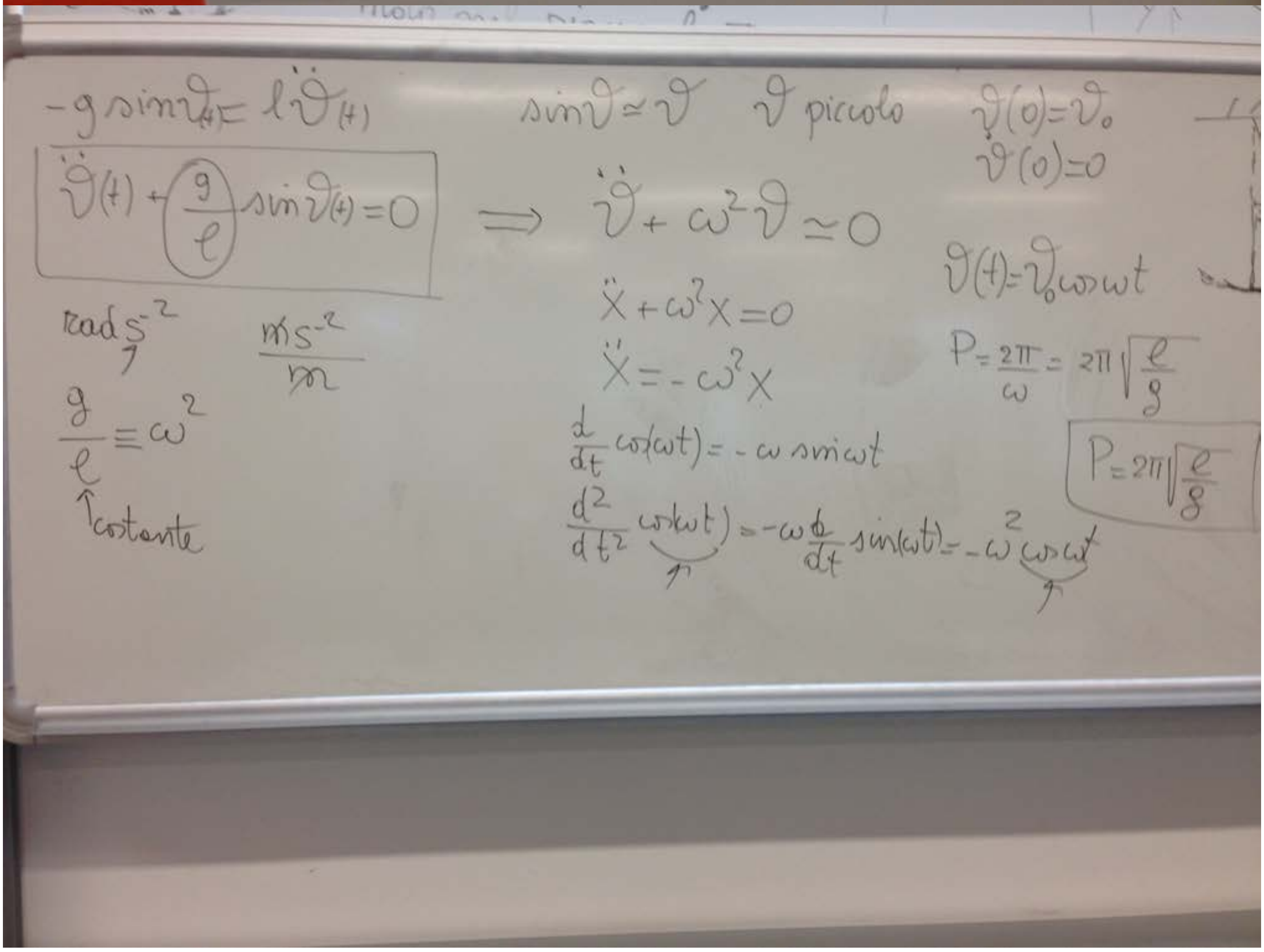
$\hat e_r = (\cos\vartheta(t), \sin\vartheta(t))$

$\hat e_\vartheta = (-\sin\vartheta, \cos\vartheta)$

$\hat e_r \cdot \hat e_\vartheta = -\sin\vartheta\cos\vartheta + \sin\vartheta\cos\vartheta = 0$

$\hat e_x, \hat e_y$

$\hat e_r, \hat e_\vartheta$



$-g\sin\vartheta(t) = \ell\ddot\vartheta(t)$

$$\boxed{\ddot\vartheta(t) + \frac{g}{\ell}\sin\vartheta(t) = 0}$$

rad s^{-2}

$\frac{m s^{-2}}{m}$

$\frac{g}{\ell} \equiv \omega^2$

↑costante

$\sin\vartheta \simeq \vartheta$ ϑ piccolo

$\Longrightarrow \ddot\vartheta + \omega^2\vartheta \simeq 0$

$\ddot x + \omega^2 x = 0$

$\ddot x = -\omega^2 x$

$\frac{d}{dt}\cos(\omega t) = -\omega\sin\omega t$

$\frac{d^2}{dt^2}\underbrace{\cos(\omega t)}_{\uparrow} = -\omega\frac{d}{dt}\sin(\omega t) = -\omega^2\underbrace{\cos\omega t}_{\uparrow}$

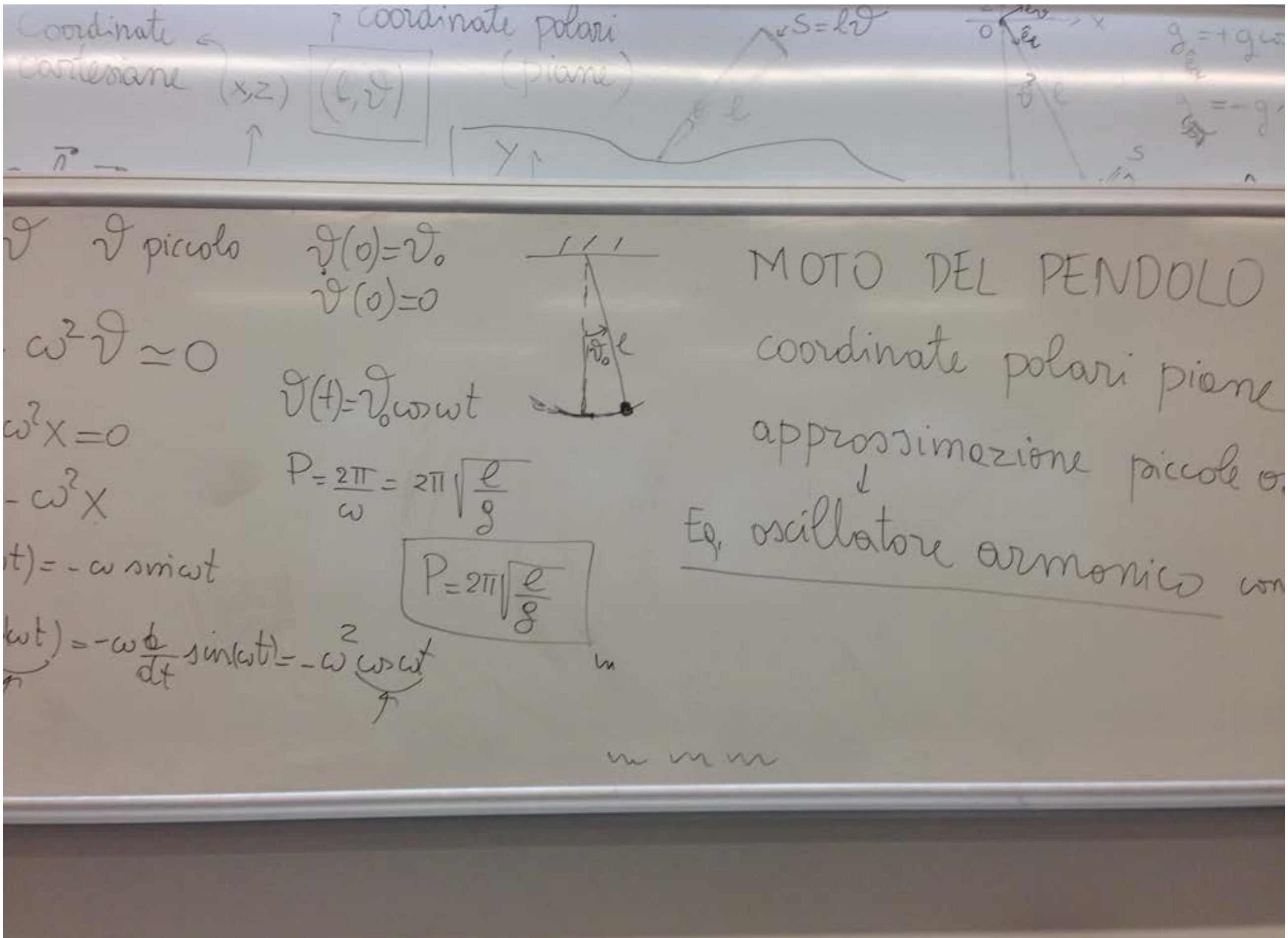
$\vartheta(0) = \vartheta_0$

$\dot\vartheta(0) = 0$

$\vartheta(t) = \vartheta_0\cos\omega t$

$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$

$$\boxed{P = 2\pi\sqrt{\frac{\ell}{g}}}$$



Coordinate cartesiane (x,z)

coordinate polari $\boxed{(l,\vartheta)}$ (piane)

$s = l\vartheta$

$g_{\hat{e}_r} = +g\cos$

$g_{\hat{e}_\vartheta} = -g$

ϑ ϑ piccolo

$\vartheta(0)=\vartheta_0$

$\dot{\vartheta}(0)=0$

$\omega^2\vartheta \simeq 0$

$\omega^2 x = 0$

$-\omega^2 x$

$t) = -\omega\sin\omega t$

$\vartheta(t)=\vartheta_0\cos\omega t$

$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$

$\boxed{P = 2\pi\sqrt{\frac{l}{g}}}$

$\omega t) = -\omega\frac{d}{dt}\sin(\omega t) = -\omega^2\cos\omega t$

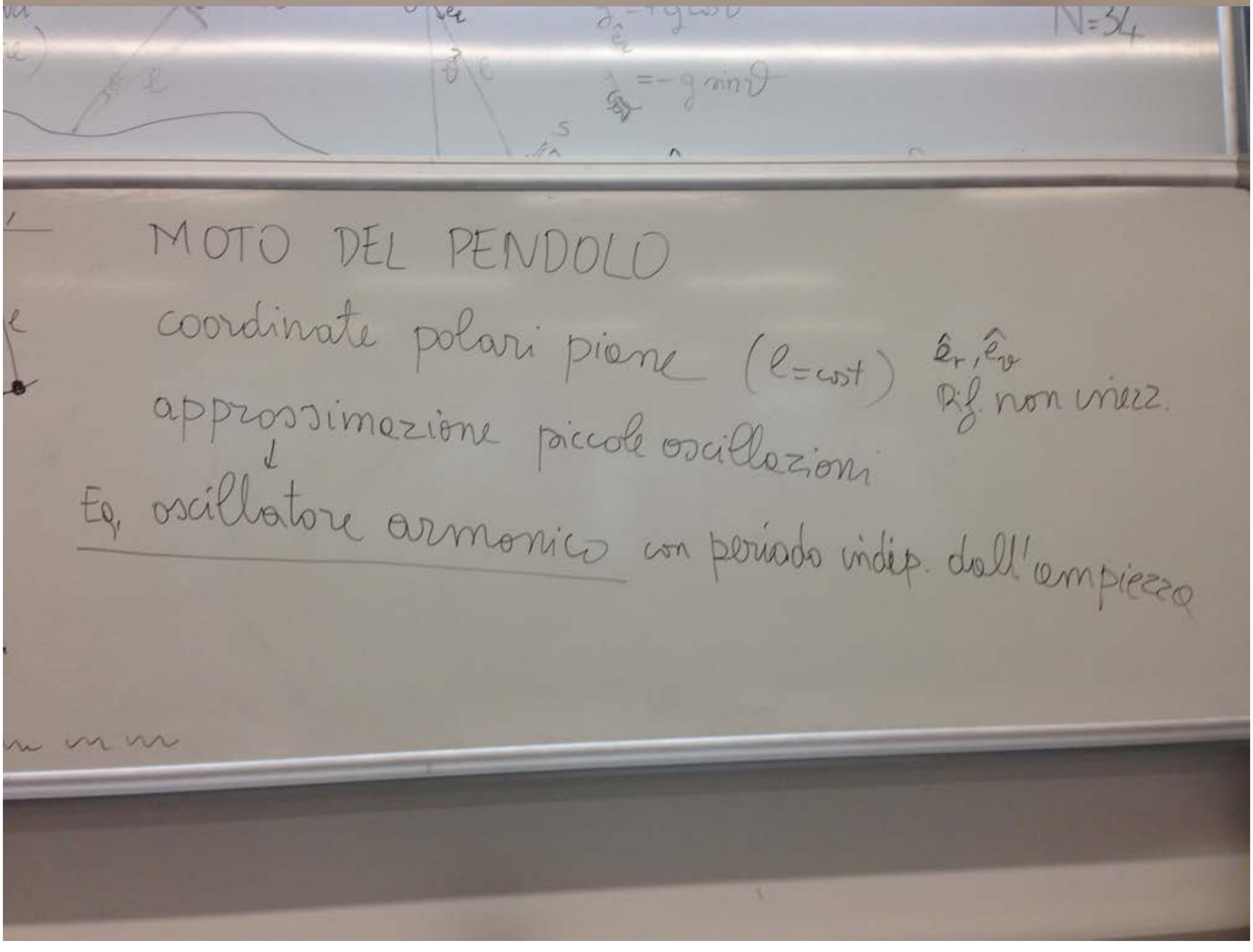
MOTO DEL PENDOLO

coordinate polari piane

approssimazione piccole o

↓

Eq. oscillatore armonico con



$g_{\hat{e}_\vartheta} = -g\sin\vartheta$

N=34

MOTO DEL PENDOLO

coordinate polari piane $(l = \text{cost})$ $\hat{e}_r, \hat{e}_\vartheta$ Rif. non inerz.

approssimazione piccole oscillazioni

↓

Eq. oscillatore armonico con periodo indip. dall'ampiezza