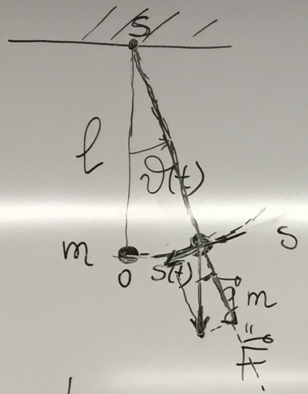
S11 Lezioni 16 e 18 Febbraio 2016:

- Pendolo semplice: grado di di libertà, scelta della coordinata, equazione del moto, approssimazione delle piccole oscillazioni (che porta all'equazione dell'oscillatore armonico) e soluzione (condizioni iniziali e loro uso)
- Riferimento inerziale e riferimento rotante: derivata temporale di un generico vettore nell'uno e nell'altro riferimento e relazione tra di esse. Applicazione di questa relazione in casi noti
- Equazioni del moto di un riferimento rotante in forma generale. Forze inerziali e loro significato fisico
- Esempio di sistema di riferimento rotante: la Terra in rotazione attorno al proprio asse. Forza centrifuga e forza di Coriolis e loro ordini di grandezza

http://eotvos.dm.unipi.it/homendsili.html



1 grado di libertà

$s(t) = l\,\vartheta(t)$

Rif. Inerziale

$\vec{F} = m\vec{a}$

$F_s = m\,a_s$

$\dot{s} \equiv \frac{d}{dt}$

$\dot{s}(t) = l\,\dot{\vartheta}(t)$

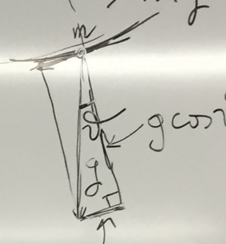
$\ddot{s}(t) = l\,\ddot{\vartheta}(t)$

$m\ddot{s}(t) = m\,l\,\ddot{\vartheta}(t)$

$\cancel{m}\,g\sin\vartheta(t) = -\cancel{m}\,l\,\ddot{\vartheta}(t)$

$T > mg\cos\vartheta$

$> mg$



$g = 9.8\ m s^{-2}$

$\frac{\Delta g}{g} \simeq 10^{-9}$

$l\ddot{\vartheta} = -g\sin\vartheta$ eq. del moto

$m\,\frac{rad}{s^2}$ $\quad m s^{-2}$

$\ddot{\vartheta} = -\frac{g}{l}\,\frac{\sin\vartheta}{?}$

$\vartheta \ll 1$

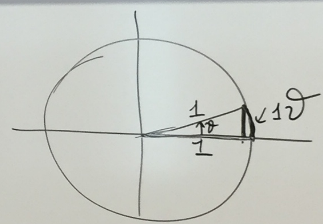
$\sin\vartheta \simeq \vartheta$

$\cos\vartheta \simeq 1$

$\left[\frac{g}{l}\right] = \left[\frac{\cancel{m} s^{-2}}{\cancel{m}}\right]$

ω^2

$\omega = \frac{2\pi}{P}$



piccole oscillazioni $\vartheta \ll 1$

$$\boxed{\ddot{\vartheta}(t) \simeq -\omega^2\,\vartheta(t)}$$

$(\ddot{x} = -\omega^2 x)$

$\cos\varphi$

$\frac{d}{d\varphi}\cos\varphi = -\sin\varphi$

$\frac{d^2}{d\varphi^2}\cos\varphi = -\cos\varphi$

$\varphi = \omega t$

$\frac{d}{dt}\cos(\omega t) = -\sin\omega t \times \frac{d(\omega t)}{dt} = -\omega\sin\omega t$

$\frac{d^2}{dt^2}\cos(\omega t) = $

$= -\omega^2\cos\omega t$

Condizioni iniziali

$t=0 \quad \vartheta(t=0)=\vartheta_0>0 \quad \dot{\vartheta}(t=0)=0$

$\vartheta(t)=A\cos\omega t+B\sin\omega t$

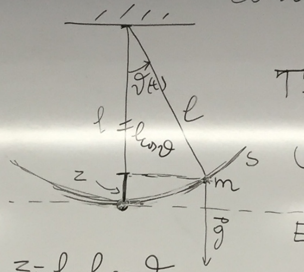
$\vartheta(0)=A\equiv\vartheta_0 \qquad \hookrightarrow \vartheta(t)=C\cos(\omega t+\varphi)$

$\hookrightarrow \dot{\vartheta}(t)=-A\omega\sin\omega t+B\omega\cos\omega t$

$\dot{\vartheta}(0)=B\omega$
$\quad \underset{0}{\parallel} \quad \Rightarrow B=0$

$\boxed{\vartheta(t)=\vartheta_0\cos\omega t}$ legge oraria

CONSERVAZIONE ENERGIA



$\boxed{U(z)=mgz}$

$T=\frac{1}{2}m\ell^2\dot{\vartheta}^2$

$U=mg\ell(1-\cos\vartheta)$

$E=\frac{1}{2}m\ell^2\dot{\vartheta}^2+mg\ell(1-\cos$

$z=\ell-\ell\cos\vartheta$
$\;=\ell(1-\cos\vartheta)$

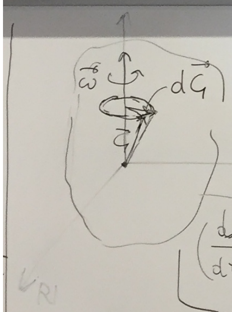
$\dot{E}=0$

$\frac{1}{2}m\ell^2(2\dot{\vartheta}\ddot{\vartheta})-mg\ell(-\dot{\vartheta}\sin\vartheta)$

$m\ell^2\dot{\vartheta}\ddot{\vartheta}+mg\ell\dot{\vartheta}\sin\vartheta=0$

$\cancel{m\ell\dot{\vartheta}}(\ell\ddot{\vartheta}+g\sin\vartheta)=0 \Rightarrow \boxed{\ddot{\vartheta}=-\frac{g}{\ell}\sin\vartheta}$

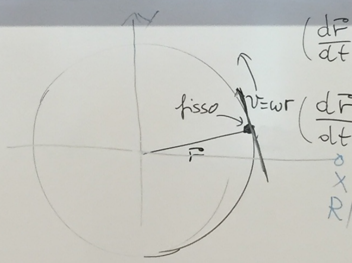
$t>0$



dt

$\boxed{\left(\frac{d\vec{G}}{dt}\right)_{RI}=\vec{\omega}\times\vec{G}}$

$\boxed{\left(\frac{d\vec{G}}{dt}\right)_{RI}=\left(\frac{d\vec{G}}{dt}\right)_{Rot}+\vec{\omega}\times\vec{G}}$

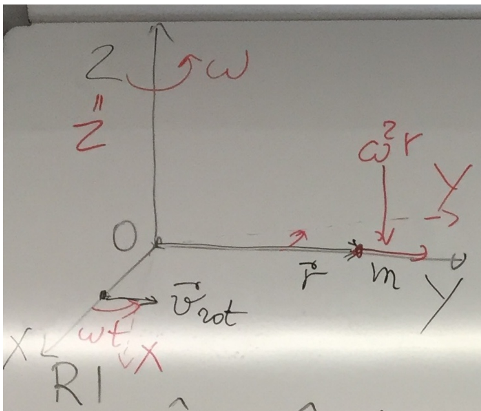


$\left(\frac{d\vec{r}}{dt}\right)_{Rot}=0$

$\left(\frac{d\vec{r}}{dt}\right)_{RI}=\vec{\omega}\times\vec{r}$

$|\vec{\omega}\times\vec{r}|=\omega r=v$

RIFERIMENTI ROTANTI E FORZE INERZIALI

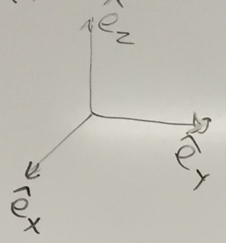


$\vec{G}$

$$\boxed{\left(\frac{d\vec{G}}{dt}\right)_{RI} = \left(\frac{d\vec{G}}{dt}\right)_{Rot} + \vec{\omega}\times\vec{G}}$$

$\vec{r}$

Newton $\boxed{\left(\frac{d^2\vec{r}}{dt^2}\right)_{RI} = \frac{\vec{F}}{m}}$



$$\hat{e}_x\times\hat{e}_y = \hat{e}_z$$
$$\hat{e}_y\times\hat{e}_z = \hat{e}_x$$
$$\hat{e}_z\times\hat{e}_x = \hat{e}_y$$

$$\underbrace{\left(\frac{d\vec{r}}{dt}\right)_{RI}}_{\vec{v}_{RI}} = \underbrace{\left(\frac{d\vec{r}}{dt}\right)_{Rot}}_{\vec{v}_{rot}} + \vec{\omega}\times\vec{r}$$

$$|\vec{\omega}\times\vec{G}| = \omega G \sin(\vec{\omega},\vec{G})$$

$$\left(\frac{d\vec{\omega}}{dt}\right)_{RI} = \left(\frac{d\vec{\omega}}{dt}\right)_{Rot}$$

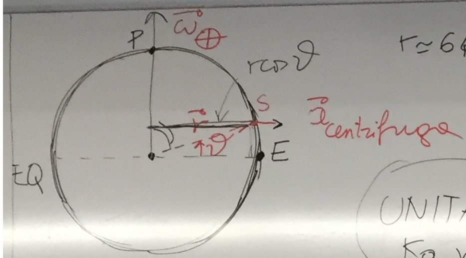
$$\left(\frac{d^2\vec{r}}{dt^2}\right)_{RI} = \underbrace{\left(\frac{d^2\vec{r}}{dt^2}\right)_{Rot}}_{\vec{a}_{Rot}} + \left[\frac{d}{dt}(\vec{\omega}\times\vec{r})\right]_{Rot} + \vec{\omega}\times\left(\vec{v}_{Rot} + \vec{\omega}\times\vec{r}\right) =$$

$$\vec{a}_{RI} = \vec{a}_{Rot} + \underbrace{\left(\frac{d\vec{\omega}}{dt}\right)}_{\dot{\vec{\omega}}}\times\vec{r} + 2\vec{\omega}\times\vec{v}_{Rot} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\boxed{\vec{a}_{Rot} = \underbrace{\vec{a}_{RI}}_{\vec{F}/m} - \vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2\vec{\omega}\times\vec{v}_{Rot} - \dot{\vec{\omega}}\times\vec{r}}$$

$\omega^2 r$

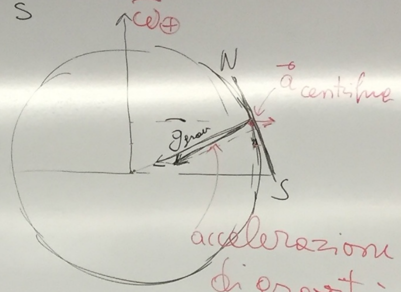
accelerazione centrifuga



$r \simeq 6400\ Km = 6.4\times10^3\ km = 6.4\times10^6\ m$

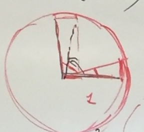
$$\omega_\oplus = \frac{2\pi}{86164\ s} \simeq 7.3\times10^{-5}\ rad\ s^{-1}$$

UNITA' SI
kg m s

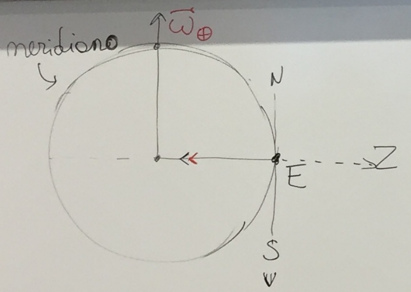


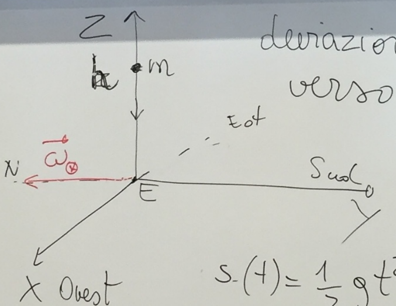
$$\vec{a}_{centrifuga} = -\vec{\omega}_\oplus \times (\vec{\omega}\times\vec{r})$$

$$a_{centrifuga} = \omega^2 \underbrace{r\sin\left(\frac{\pi}{2}-\vartheta\right)}_{=r\cos\vartheta}$$



$$\frac{a_{centrifuga}(\vartheta=0)}{g=9.8\ ms^{-2}} \simeq \frac{\omega^2 r_\oplus}{g} \approx \frac{(7.3)^2\times10^{-10}\times6.4\times10^6}{9.8} \simeq \frac{(7.3)^2\times6.6}{9.8}\times10^{-4} = 3.47\times10^{-3}$$





deviazione dei gravi verso EST

$$\frac{a_{Coriolis}}{g} \simeq \frac{2\cdot7.29\times10^{-5}\times17.15}{9.8} \simeq 2.6\times10^{-4}$$

$h = 15\ m$

$s(t) = \frac{1}{2} g t^2$

$h = \frac{1}{2} g t_{fin}^2$

$1.75\ s = t_{fin} = \sqrt{\frac{2h}{g}}$

$v_{fin} = g t_f = 9.8\cdot1.75 = 17.15\ m\ s^{-1} = 61\ Km/h$