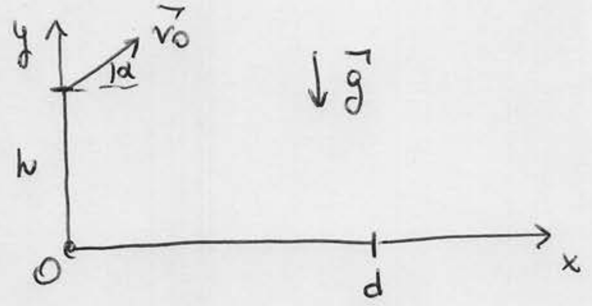
Compitino del 13/12/2016

Problema 1



1) $$\begin{cases} x(t) = v_0 \cos\alpha \, t \\ y(t) = h + v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \end{cases} \qquad (1)$$

Per trovare v_0 impongo che esista $\bar{t}$ t.c. $\begin{cases} x(\bar{t}) = d \\ y(\bar{t}) = 0 \end{cases}$ $\Rightarrow$

$$\begin{cases} d = v_0 \cos\alpha \, \bar{t} \\ 0 = h + v_0 \sin\alpha \, \bar{t} - \frac{1}{2} g \bar{t}^2 \end{cases} \Rightarrow \quad \bar{t} = \frac{d}{v_0 \cos\alpha}$$

$$\Downarrow$$

$$0 = h + \cancel{v_0} \sin\alpha \frac{d}{\cancel{v_0} \cos\alpha} - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2\alpha}$$

$$\frac{g d^2}{2 v_0^2 \cos^2\alpha} = h + d \frac{\sin\alpha}{\cos\alpha} \Rightarrow \quad v_0 = \sqrt{\frac{g d^2}{2\cos^2\alpha} \; \frac{1}{h + d \frac{\sin\alpha}{\cos\alpha}}}$$

2) Ora conosco v_0. Eq. (1) è ancora valida. In più

$$\begin{cases} v_x(t) = v_0 \cos\alpha \\ v_y(t) = v_0 \sin\alpha - g t \end{cases}$$

Nel punto più alto della traiettoria $v_y(t) = 0 \Rightarrow$

$$|\vec{v}| = v_x = v_0 \cos\alpha$$

3) Nel punto più alto $v_0 \sin\alpha - g t_{max} = 0 \Rightarrow t_{max} = \frac{v_0 \sin\alpha}{g}$

$$\Rightarrow \quad x_{max} \equiv x(t_{max}) = \frac{v_0^2 \cos\alpha \sin\alpha}{g}$$

4) $\vec{v}(t) = v_x(t)\,\hat{x} + v_y(t)\,\hat{y} =$

$= v_0 \cos\alpha\,\hat{x} + (v_0 \sin\alpha - gt)\,\hat{y}$

$$|\vec{v}| = \sqrt{(v_0\cos\alpha)^2 + (v_0\sin\alpha - gt)^2} = \sqrt{v_0^2 + g^2t^2 - 2v_0\sin\alpha\, gt}$$

Per y_{max} $|\vec{v}| = v_0\cos\alpha$ come dal punto 2) o dalle formule prec. con $t_{max} = \frac{v_0 \sin\alpha}{g}$

$$\Rightarrow \quad \vec{v} = v\,\hat{T} \Rightarrow \quad \hat{T} = \hat{x} + \cancel{\frac{v_0\sin\alpha - gt_{max}}{v_0\cos\alpha}}\,\hat{y} = \hat{x}$$

$$\Rightarrow \hat{N} = -\hat{y}$$

$$\vec{a} = \dot{v}\,\hat{T} + \frac{v^2}{R}\hat{N} = -g\,\hat{y}$$

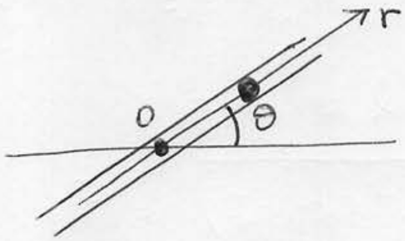
$$\dot{v} = \frac{1}{\cancel{2}\sqrt{v_0^2 + g^2t^2 - 2v_0\sin\alpha\, gt}} \cdot \left[\cancel{2}g^2t - \cancel{2}v_0\sin\alpha\, g\right] =$$

per $t = t_{max}$

$$= \frac{g\left[gt - v_0\sin\alpha\right]}{\sqrt{v_0^2 + g^2t^2 - 2v_0\sin\alpha\, gt}} = 0$$

$$\Rightarrow \quad \frac{v^2}{R}(-\hat{y}) = -g\,\hat{y} \Rightarrow \frac{v_0^2\cos^2\alpha}{R} = g \Rightarrow \quad R = \frac{v_0^2\cos^2\alpha}{g}$$

Problema 2



$$\begin{cases} r(t) = v_0 t \\ \theta(t) = \omega_0 t \end{cases} \qquad \begin{cases} \dot r = v_0 \\ \dot\theta = \omega_0 \end{cases} \qquad \begin{cases} \ddot r = 0 \\ \ddot\theta = 0 \end{cases}$$

1) $$\vec{v} = \dot r \, \hat e_r + r\dot\theta \, \hat e_\theta = v_0 \, \hat e_r + (v_0 \omega_0 t)\, \hat e_\theta$$

$$|\vec v| = \sqrt{v_0^2 + (v_0 t \omega_0)^2} = v_0 \sqrt{1 + \omega^2 t^2}$$

2) $$\vec a = (\ddot r - r\dot\theta^2)\,\hat e_r + (r\ddot\theta + 2\dot r\dot\theta)\,\hat e_\theta = -v_0\omega_0^2 t\,\hat e_r + 2 v_0 \omega_0 \hat e_\theta$$

$$|\vec a| = \sqrt{4 v_0^2 \omega_0^2 + v_0^2 \omega_0^4 t^2} = v_0 \omega_0 \sqrt{4 + \omega_0^2 t^2}$$

3) SR assoluto vs. SR scanalatura

$$\vec v = \vec v_{tr} + \vec v\,'$$

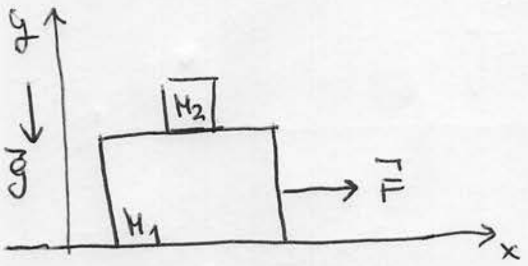
$$\vec v\,' = v_0 \hat e_r$$
$$\vec v = v_0 \hat e_r + (v_0 \omega_0 t)\,\hat e_\theta$$
$$\Rightarrow \vec v_{tr} = \vec v - \vec v\,' = (v_0\omega_0 t)\,\hat e_\theta$$
$$\Rightarrow |\vec v_{tr}| = v_0 \omega_0 t$$

4) $$\vec a = \underset{0}{\underset{\parallel}{\vec a\,'}} + \vec\omega \times (\vec\omega \times \vec r\,') + \underset{0}{\underset{\parallel}{\dot{\vec\omega}}} \times \vec r\,' + 2\vec\omega \times \vec v\,'$$

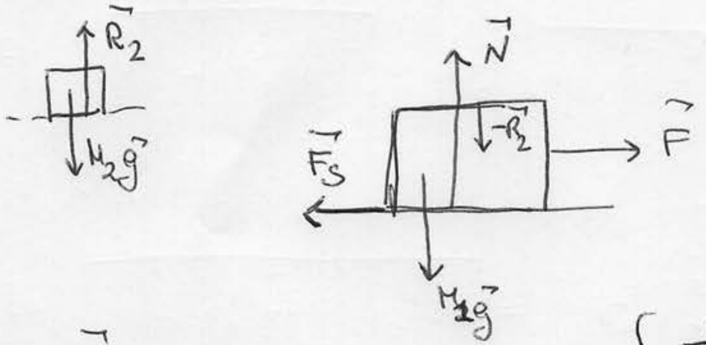
$$= -\omega_0^2 \vec r\,' + 2\vec\omega \times \vec v\,' = -\omega_0^2 r \,\hat e_r + \underbrace{2\vec\omega \times \vec v\,'}_{\vec a_{co}}$$

$$\Rightarrow |\vec a_{co}| = |2\vec\omega \times \vec v\,'| = 2\omega_0 v_0 |\hat z \times \hat e_r| = 2\omega_0 v_0$$

Problema 3



1)

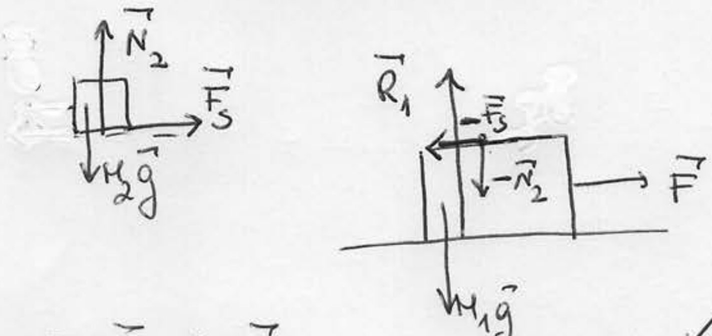


$$\begin{cases} M_2\vec{g} + \vec{R}_2 = 0 \\ M_1\vec{g} + \vec{N} - \vec{R}_2 + \vec{F} + \vec{F}_S = 0 \end{cases} \qquad \begin{cases} -M_2 g + R_2 = 0 \\ -M_1 g + N - R_2 = 0 \\ F - F_S = 0 \end{cases} \qquad \begin{aligned} R_2 &= M_2 g \\ N &= (M_1+M_2)g \\ F &= F_S \end{aligned}$$

Legge dell' attrito statico

$$F = F_S \le \mu_S N = \mu_S (M_1+M_2)\, g \Rightarrow F_{Max} = \mu_S (M_1+M_2) g$$

2) $\vec{F}$ nota



$$\begin{cases} \vec{N}_2 + M_2\vec{g} + \vec{F}_S = M_2\vec{a}_2 \\ \vec{F} - \vec{F}_S + \vec{R}_1 + M_1\vec{g} - \vec{N}_2 = M_1\vec{a}_1 \end{cases}$$

acc. di trascin.

$$\vec{a}_2 = \vec{a}_1 + \vec{a}_2'$$

acc. relativa $= 0 \Rightarrow$ $\vec{a}_1 = \vec{a}_2$

$$\begin{cases} +F_S = M_2\, a_{1x} \\ N_2 - M_2 g = 0 \end{cases} \qquad \begin{aligned} F - F_S &= M_1 a_{1x} \\ -M_1 g + R_1 - N_2 &= 0 \end{aligned}$$

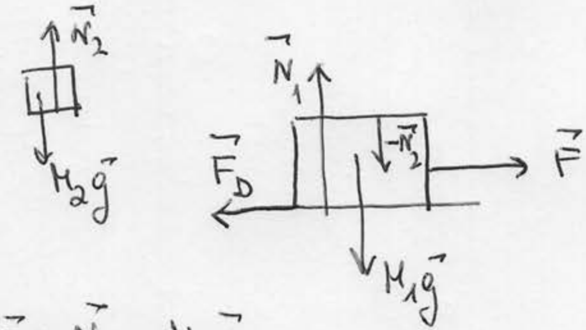
$N_2 = M_2 g \qquad a_{1x} = \frac{1}{M_1}(F - F_S)$

$$F_S = M_2 a_{1x} = \frac{M_2}{M_1}(F - F_S) \Rightarrow F_S\left(1 + \frac{M_2}{M_1}\right) = \frac{M_2}{M_1}F$$

$$F_S = \frac{M_2}{M_1+M_2}F \leq \mu_S N_2$$

$$\Rightarrow \mu_S \geq \frac{M_2 F}{M_1+M_2}\frac{1}{N_2} = \frac{\cancel{M_2}}{M_1+M_2}\frac{F}{\cancel{M_2}g} = \frac{F}{g(M_1+M_2)}$$

3)



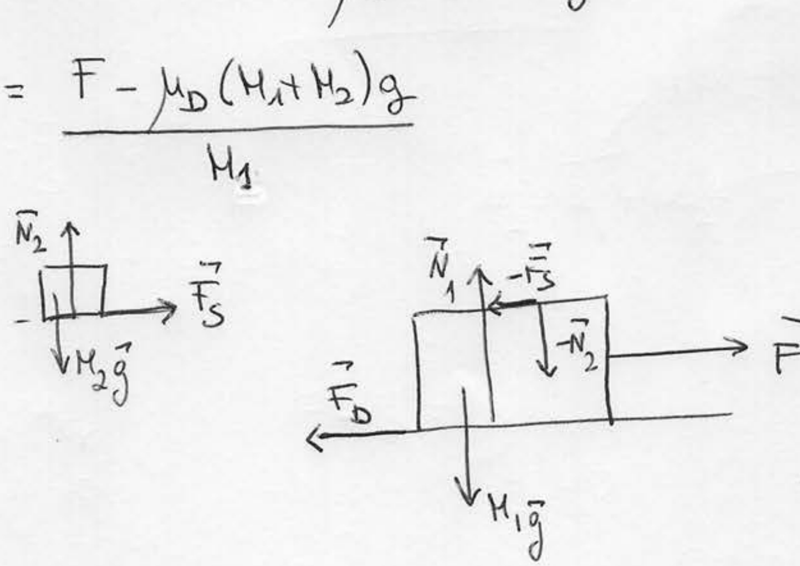
$$\begin{cases} M_2\vec{g} + \vec{N_2} = M_2\vec{a}_2 \\ \vec{F_D} + \vec{N_1} - \vec{N_2} + M_1\vec{g} + \vec{F} = M_1\vec{a}_1 \end{cases} \qquad \begin{cases} -M_2 g + N_2 = 0 \qquad N_2 = M_2 g \\ a_{2x} = 0 \\ -F_D + F = M_1 a_{1x} \\ N_1 - M_1 g - N_2 = 0 \end{cases}$$

$$\Rightarrow N_1 = (M_1 + M_2)g$$

$$M_1 a_{1x} = F - F_D = F - \mu_D (M_1 + M_2) g$$

$$a_{1x} = \frac{F - \mu_D (M_1 + M_2) g}{M_1}$$

4)



$$\begin{cases} \vec{N}_2 + M_2\vec{g} + \vec{F}_S = M_2\vec{a}_2 \\ \vec{F} + \vec{F}_D - \vec{F}_S + \vec{N}_1 - \vec{N}_2 + M_1\vec{g} = M_1\vec{a}_1 \end{cases}$$

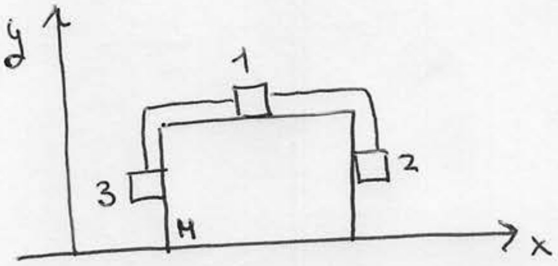
$$\begin{cases} N_2 - M_2 g = M_2 a_{2y} = 0 \\ F_S = M_2 a_{2x} = M_2 a_{1x} \\ F - F_D - F_S = M_1 a_{1x} \\ N_1 - N_2 - M_1 g = 0 \end{cases}$$

$$\Rightarrow \begin{cases} N_2 = M_2 g \\ F - F_D - M_2 a_{1x} = M_1 a_{1x} \\ N_1 = N_2 + M_1 g \end{cases} \Rightarrow \quad \begin{matrix} (M_1 + M_2)\, a_{1x} = F - F_D = F - \mu_D N_1 \\ N_1 = (M_1 + M_2) g \end{matrix}$$

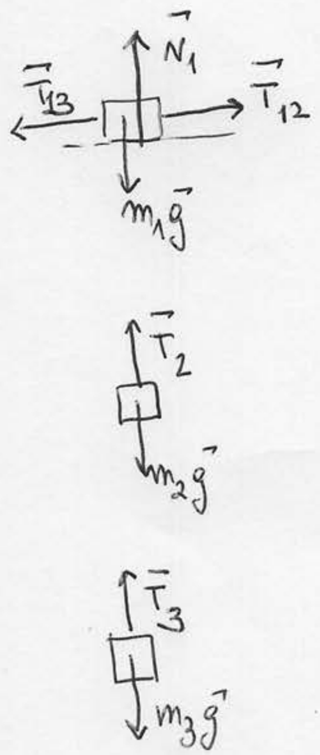
$$\Rightarrow \quad a_{1x} = \frac{F - \mu_D (M_1 + M_2) g}{M_1 + M_2}$$

$$= \frac{F}{M_1 + M_2} - \mu_D g$$

Problema 4



1)



$$\begin{cases} m_1\vec{a}_1 = \vec{N}_1 + m_1\vec{g} + \vec{T}_{12} + \vec{T}_{13} \\ m_2\vec{a}_2 = m_2\vec{g} + \vec{T}_2 \\ m_3\vec{a}_3 = m_3\vec{g} + \vec{T}_3 \end{cases}$$

$$\begin{cases} m_1 a_{1x} = T_{12} - T_{13} \\ 0 = N_1 - m_1 g \\ m_2 a_{2y} = T_2 - m_2 g \\ m_3 a_{3y} = T_3 - m_3 g \end{cases} \quad (1)$$

Richiedo $a_{1x} = a_{2y} = a_{3y} = 0$

So $T_{12} = T_2$, $T_{13} = T_3$ $\Rightarrow$

$$\begin{cases} T_{12} - T_{13} = 0 \\ N_1 - m_1 g = 0 \\ T_2 - m_2 g = 0 \\ T_3 - m_3 g = 0 \end{cases}$$

$T_2 = T_3$

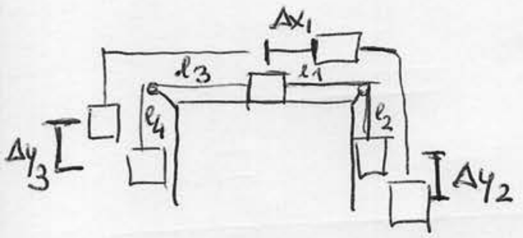
$\downarrow$

$m_2 g = T_2 = T_3 = m_3 g$

$\Rightarrow m_2 = m_3$

era intuitivo!

2) Ritorno al sistema (1) e impongo il fatto che i fili sono inestensibili ⇒



$$\ell_1 + \ell_2 = \ell_1 - \Delta x_1 + \ell_2 + \Delta y_2$$

$$\Rightarrow \Delta x_1 = \Delta y_2 \Rightarrow |v_{1x}| = |v_{2y}|$$

$$|a_{1x}| = |a_{2y}|$$

$$\ell_3 + \ell_4 = \ell_3 + \Delta x_1 + \ell_4 - \Delta y_3$$

$$\Rightarrow \Delta x_1 = \Delta y_3 \Rightarrow |v_{1x}| = |v_{3y}|$$

$$|a_{1x}| = |a_{3y}|$$

Per i segni, con la scelta degli assi, trovo

$$a_{1x} = a_{3y} = -a_{2y} \Rightarrow$$

$$\begin{cases} m_1 a_{1x} = T_2 - T_3 \\ -m_2 a_{1x} = T_2 - m_2 g \\ m_3 a_{1x} = T_3 - m_3 g \end{cases} \Rightarrow \quad (m_1 + m_2 + m_3) a_{1x} = \cancel{T_2} - \cancel{T_3} - \cancel{T_2} + m_2 g + \cancel{T_3} - m_3 g$$

$$\Rightarrow a_{1x} = \frac{(m_2 - m_3)\, g}{m_1 + m_2 + m_3}$$

$$\Rightarrow |a_{1x}| = \frac{|m_2 - m_3|\, g}{m_1 + m_2 + m_3}$$

3) $T_2 = m_2 g - m_2 a_{1x} =$

$$= m_2 g - m_2 \frac{m_2 - m_3}{m_1 + m_2 + m_3} g = \frac{m_1 m_2 + m_2 m_3 + m_2 m_3}{m_1 + m_2 + m_3} g$$

$$= \frac{(m_1 + 2m_3) m_2}{m_1 + m_2 + m_3} g$$

$T_3 = m_3 g + m_3 a_{1x} =$

$$= m_3 g + m_3 \frac{m_2 - m_3}{m_1 + m_2 + m_3} g = \frac{m_1 m_3 + m_2 m_3 + m_2 m_3}{m_1 + m_2 + m_3} g = \frac{(m_1 + 2m_2) m_3}{m_1 + m_2 + m_3} g$$

4)

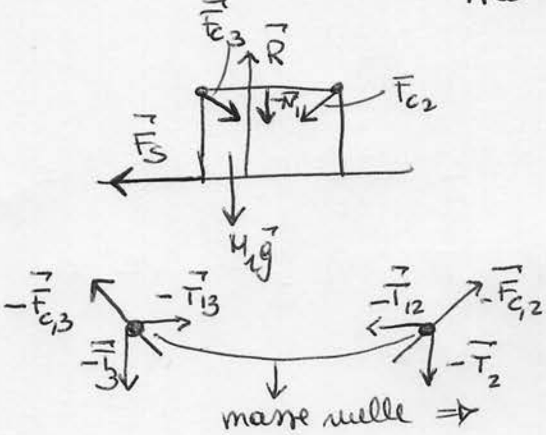
$$m_1 a_{1x} = T_2 - T_3$$

$$0 = N_1 - m_1 g$$

$$m_2 a_{2y} = T_2 - m_2 g$$

$$m_3 a_{3y} = T_3 - m_3 g$$

$$M\vec{a} = M\vec{g} + \vec{R} + \vec{F}_S + \vec{F}_{c2} + \vec{F}_{c3}$$



$$\vec{F}_{c3} = -\vec{T}_{13} - \vec{T}_3$$

$$\vec{F}_{c2} = -\vec{T}_{12} - \vec{T}_2$$

$$M a_x = -F_S + T_3 - T_2 = 0$$

$$0 = -Mg + R - T_3 - T_2 - N_1$$

$$F_S = T_3 - T_2 = \frac{(m_1 + 2m_2)m_3 g}{m_1+m_2+m_3} - \frac{(m_1+2m_3)m_2 g}{m_1+m_2+m_3}$$

$$F_S = \frac{m_1 m_3 + \cancel{2m_2 m_3} - m_1 m_2 - \cancel{2m_2 m_3}}{m_1+m_2+m_3} g = \frac{m_1(m_3-m_2)}{m_1+m_2+m_3} g$$

$$R = Mg + T_3 + T_2 + N_1 = Mg + \frac{(m_1+2m_2)m_3 g + (m_1+2m_3)m_2 g}{m_1+m_2+m_3} + N_1$$

$$= Mg + \frac{m_1 m_3 + \overset{4}{\cancel{2}} m_2 m_3 + m_1 m_2 + \cancel{2m_2 m_3}}{m_1+m_2+m_3} g + m_1 g$$

$$= Mg + \frac{m_1 m_3 + m_1 m_2 + 4m_2 m_3}{m_1+m_2+m_3} g + m_1 g$$

$$= (M+m_1) + \frac{m_1 m_3 + m_1 m_2 + 4m_2 m_3}{m_1+m_2+m_3} g$$

$F_S \leq \mu_S R \Rightarrow$

$$\mu_S \geq \frac{\dfrac{m_1(m_3-m_2)}{m_1+m_2+m_3}}{(M+m_1) + \dfrac{m_1 m_3 + m_1 m_2 + 4m_2 m_3}{m_1+m_2+m_3}} = \frac{m_1(m_3-m_2)}{(M+m_1)(m_1+m_2+m_3) + m_1 m_3 + m_1 m_2 + 4m_2 m_3}$$