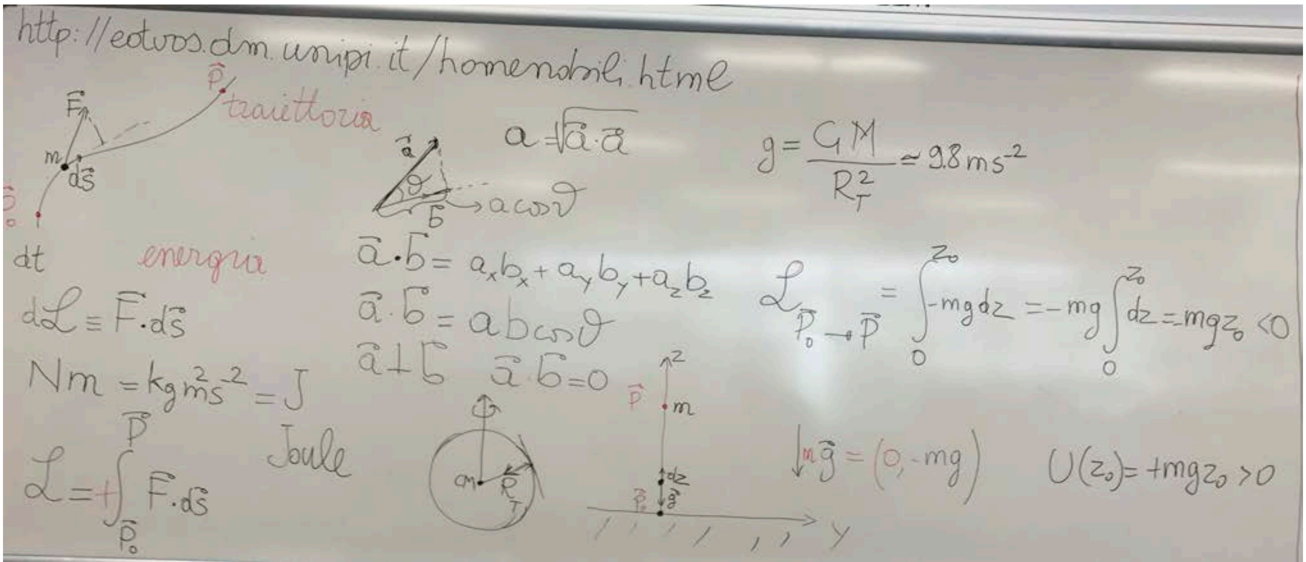
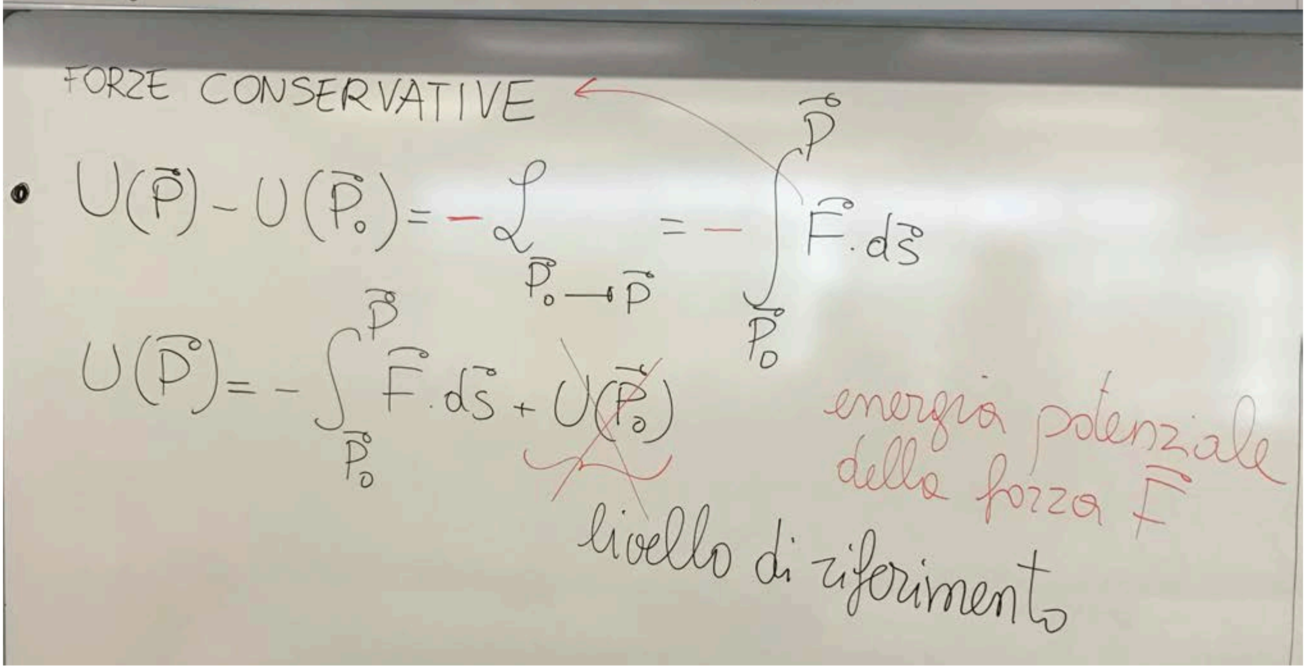
S1 - Prima settimana, lezioni del 14 e 16 febbraio 2017

- Lavoro, forze conservative e definizione di energia potenziale (con particolare attenzione al suo segno)
- Energia potenziale, energia cinetica e conservazione dell'energia totale
- Energia potenziale gravitazionale; definizione per un corpo sulla superficie della Terra e per un corpo a distanza generica. Risoluzione di un apparente paradosso.

Richiami:

- significato geometrico del prodotto scalare
- parametri dimensionali piccoli rispetto ad 1, sviluppi in serie di potenze e loro importanza in fisica
- ordini di grandezza e uso delle potenze di 10





CONSERVAZIONE DELL'ENERGIA

$$U(\vec P) - U(\vec P_0) = -\int_{\vec P_0}^{\vec P} \vec F \cdot d\vec s =$$

$$\frac{d\vec s}{dt} = \vec v \qquad d\vec s = \vec v\, dt \qquad \vec v \cdot d\vec v$$

$$\boxed{\vec F = m\vec a} = m\frac{d\vec v}{dt} \qquad x\,dx = d\left(\frac{x^2}{2}\right) \qquad \cancel{2}x\,dx$$

$$= -\int_{\vec P_0}^{\vec P} m\frac{d\vec v}{\cancel{dt}} \cdot \vec v\, \cancel{dt} = -m\int_{\vec v(\vec P_0)}^{\vec v(\vec P)} \vec v \cdot d\vec v = \text{R1}$$

$$v^2 = \vec v \cdot \vec v$$

$$d(v^2) = d(\vec v \cdot \vec v) = \vec v \cdot d\vec v + d\vec v \cdot \vec v = 2\vec v \cdot d\vec v$$

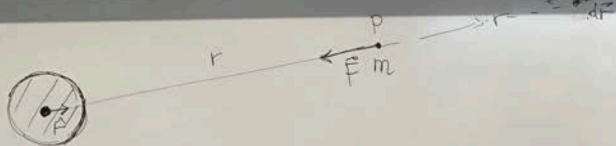
$$\vec v(\vec P_0) = \vec v_0$$

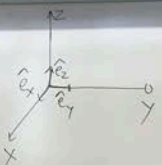
$$= m\int_{\vec v(\vec P)}^{\vec v(\vec P_0)} \vec v \cdot d\vec v = \frac{1}{2}mv_0^2 - \frac{1}{2}mv^2$$

$$\vec v(\vec P) = \vec v \qquad d\left(\frac{1}{2}\vec v \cdot \vec v\right) = d\left(\frac{1}{2}v^2\right)$$

$$\Downarrow$$

$$U(\vec P) + \frac{1}{2}mv^2 = U(\vec P_0) + \frac{1}{2}mv_0^2$$





$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

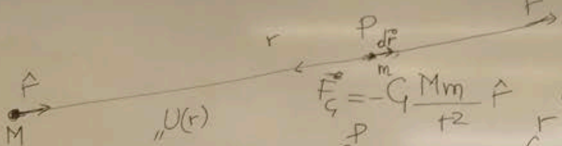
$$\hat r = \frac{\vec r}{r} \qquad \vec F = -G\frac{Mm}{r^2}\frac{\vec r}{r} = -\frac{GMm}{r^3}\vec r$$

$$|F| = \frac{GMm}{r^2}$$

$$U(\vec P) = -\int_\infty^r \left(-\frac{GMm}{r^3}\vec r \cdot d\vec r\right) = +GMm\int_\infty^r \frac{dr}{r^2} = -\frac{GMm}{r}$$

ENERGIA POTENZIALE GRAVITAZIONALE

$$\boxed{G = 6.67\times10^{-11}\, m^3 s^{-2} kg^{-1}}$$

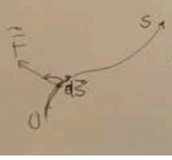


$$\vec F_G = -G\frac{Mm}{r^2}\hat r \qquad N = kg\, m s^{-2} \qquad [G] = m^3 s^{-2} kg^{-1}$$

$$\hat r = \frac{\vec r}{r}$$

$$\underbrace{U(P) - U(\infty)}_{U(r)} \equiv -\int_\infty^P d\mathcal{L} = +GMm\int_\infty^r \frac{1}{r'^2}dr' = GMm\int_0^r r'^{-2}dr' = -GMm\left[r'^{-1}\right]_\infty^r = -\frac{GMm}{r}$$

$$d\mathcal{L} = \vec F \cdot d\vec s \qquad d\mathcal{L} = \vec F_G \cdot d\vec r = -\frac{GMm}{r^2}\hat r \cdot d\vec r = -\frac{GMm}{r^2}dr$$

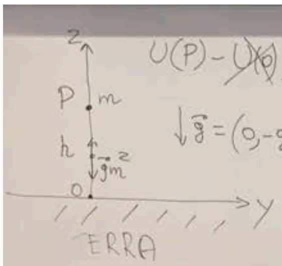


$$d\mathcal{L} = \vec F \cdot d\vec s$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

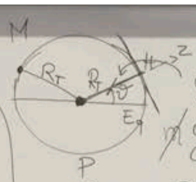
$$U(r) = -\frac{GMm}{r} < 0$$

$$\frac{m^3 s^{-2} kg^{-1} kg^2}{m} = kg\, m^2 s^{-2} = J$$



$U(P) - \cancel{U(0)}$

$\downarrow \vec{g} = (0, -g)$



$h \ll R_T$

$\vec{g} = (0, -g)$

$\cancel{m} g = \frac{GM\cancel{m}}{R_T^2} \simeq 9.8\, ms^{-2}$

$$= -\int_0^P d\mathcal{L} = -\int_0^P \vec{F} \cdot d\vec{z} = -\int_0^h (-mg)\, dz = +mg \int_0^h dz = mg[z]_0^h = mgh > 0$$

R_T

$$U(P) - U(0) = -\frac{GMm}{R_T + h} + \frac{GMm}{R_T}$$

$$\frac{h}{R_T} = \frac{1.5\, m}{6.4 \cdot 10^6} = \frac{1.5}{6.4} 10^{-6} = \frac{15}{64} \cdot 10^{-1} 10^{-6} = \frac{15}{64} \cdot 10^{-7} = 2.3 \times 10^{-7}$$

CM Terra $h \ll R_T$ $\quad \frac{h}{R_T} \ll 1$ $\quad 10^a 10^b = 10^{a+b}$ $\quad (10^a)^b = 10^{ab}$ $\quad = 0.00000023$

$\boxed{\varepsilon \ll 1}$

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \dots$$

$2! = 2 \qquad 3! = 3 \cdot 2 \cdot 1 = 6$

$x \ll 1$

$\frac{df}{dx} \equiv f'(x)$

SVILUPPO IN SERIE DI POTENZE (DI TAYLOR)

$\frac{d^2 f}{dx^2} \equiv f''(x)$

$$f(x) = \frac{1}{1+x} = (1+x)^{-1} = 1 - x + \cancel{2}\frac{x^2}{\cancel{2}} + O(x^3)$$

$x = \varepsilon \ll 1$

$f'(x) = -1(1+x)^{-2} \qquad f'(0) = -1$

$f''(x) = +2(1+x)^{-3} \qquad f''(0) = +2$

$$U(P) - U(0) = -GMm\left(\frac{1}{R_T + h} + \frac{1}{R_T}\right) = -GMm\left(\frac{1}{R_T\left(1 + \frac{h}{R_T}\right)} - \frac{1}{R_T}\right) =$$

$\frac{h}{R_T} = \varepsilon \ll 1$

$$= -\frac{GMm}{R_T}\left(\frac{1}{1+\varepsilon} - 1\right) \qquad \varepsilon = h/R_T$$

$$\simeq -\frac{GMm}{R_T}\left(\cancel{1} - \frac{h}{R_T} + \frac{h^2}{R_T^2} + \dots - \cancel{1}\right) \simeq -\frac{GMm}{R_T^2}\left(-h + \frac{h^2}{R_T} + \dots\right)$$

$$\simeq \frac{GMm}{R_T^2} h - \frac{GMm}{R_T^2} h \left(\frac{h}{R_T}\right) + \dots \simeq mgh - mgh\left(\frac{h}{R_T}\right)$$

$$\simeq mgh \underbrace{\left(1 - \frac{h}{R_T}\right)}_{>0} > 0$$