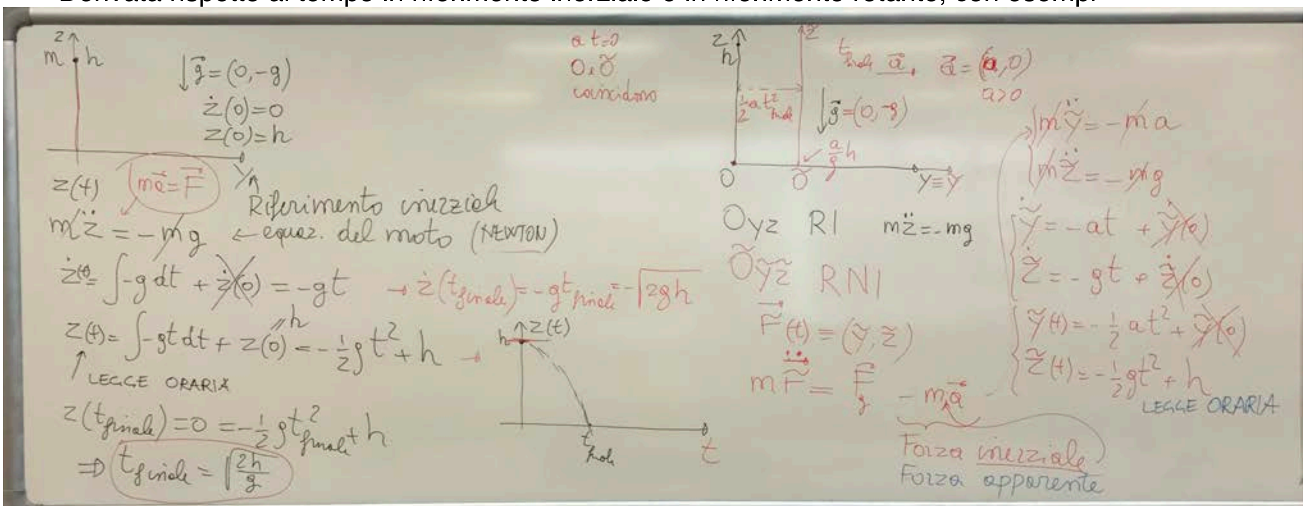
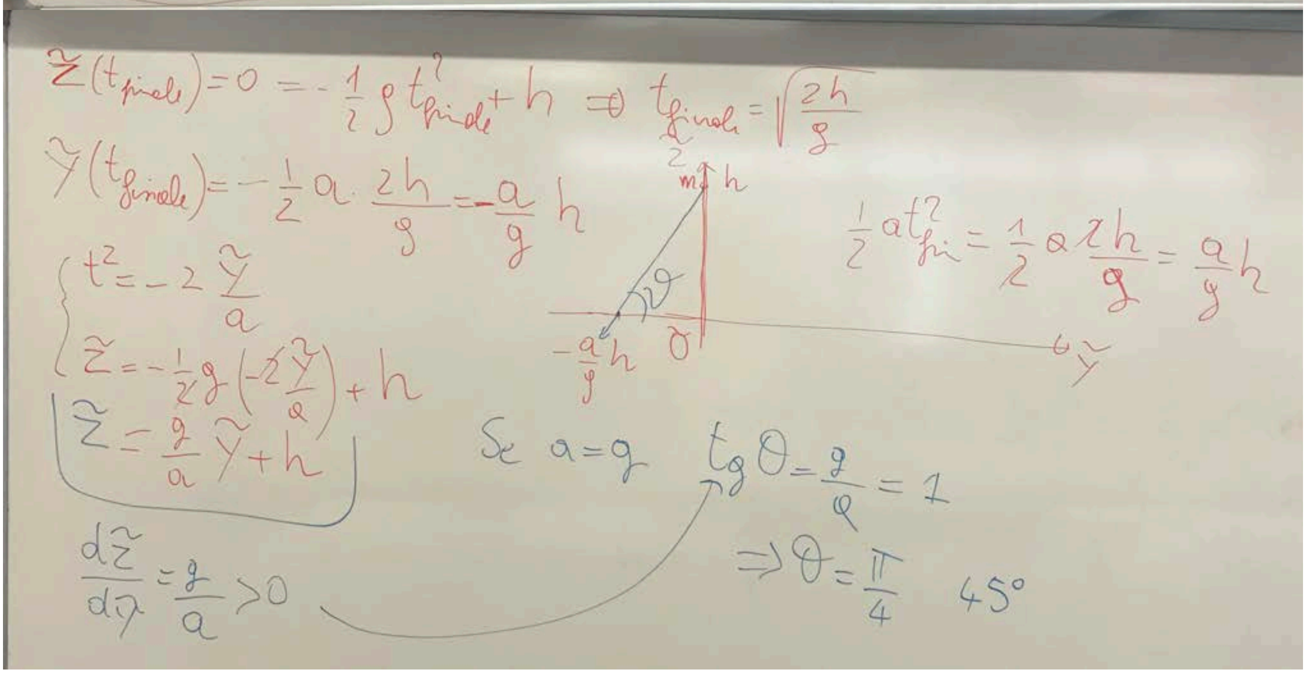
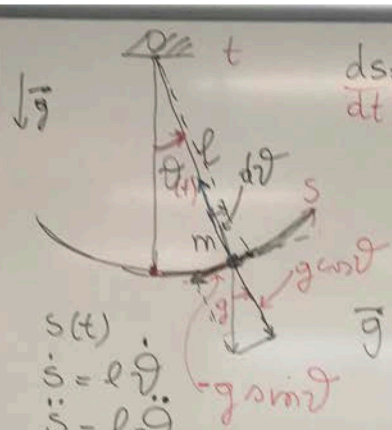
S3 - Terza settimana, lezioni del 28 febbraio e del 2 marzo 2017

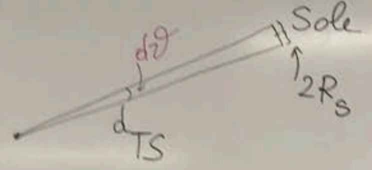
- Esercizio: caduta di una massa in un riferimento fisso e i un riferimento accelerato. Equazioni del moto e traiettorie nei due casi.
- Equazione del moto del pendolo piccole oscillazioni; somiglianza con l'oscillatore armonico e soluzione
- Richiami sulla derivata di funzioni composte
- Richiami sul concetto di fase
- Vettore velocità di rotazione, segno degli angoli. Esempi di prodotto vettore
- Derivata rispetto al tempo in riferimento inerziale e in riferimento rotante, con esempi







$$\frac{ds}{dt} = \ell \frac{d\vartheta}{dt}$$



$$(2R_S) = d_{TS}\, d\vartheta$$

$$s(t) \qquad \dot{s} = \ell \dot{\vartheta} \qquad \ddot{s} = \ell \ddot{\vartheta}$$

$$\vec{g} = (0, -g)$$

$$\sin\vartheta = 0 + \left.\frac{d(\sin\vartheta)}{d\vartheta}\right|_{\vartheta=0} \vartheta + \dots \simeq \vartheta + O(\vartheta^3)$$

$$\cancel{m}\ddot{s} = -\cancel{m}g\sin\vartheta \leftarrow \text{eq. Newton in } R\,I$$

$$\ell\ddot{\vartheta} = -g\sin\vartheta \qquad \left[\frac{g}{\ell}\right] = \left[\frac{m s^{-2}}{m}\right] = s^{-2} \qquad \omega = \sqrt{\frac{g}{\ell}}$$

$$\boxed{\ddot{\vartheta}_{(t)} = -\frac{g}{\ell}\sin\vartheta_{(t)}} \longrightarrow \text{piccole oscillazioni: } \boxed{\ddot{\vartheta}_{(t)} \simeq -\underbrace{\frac{g}{\ell}}_{\omega^2}\vartheta_{(t)}}$$

$$f(t) = \sin(\omega t) \qquad \dot{f} = \frac{d}{d(\omega t)}\sin(\omega t) \cdot \frac{d\omega t}{dt} = \omega\cos\omega t$$

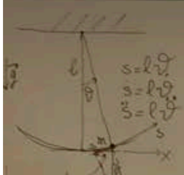
$$\text{rad } s^{-1} \cdot s = \text{rad}$$

$$\ddot{f}(t) = \omega\frac{d}{dt}(\cos\omega t) = \omega(-\sin\omega t)\omega = -\omega^2\sin\omega t = -\omega^2 f(t)$$

$$g(t) = \cos\omega t$$

$$\ddot{g}(t) = -\omega^2\cos\omega t = -\omega^2 g(t)$$

$$T = \frac{2\pi}{\omega} \qquad T = 2\pi\sqrt{\frac{\ell}{g}}$$



$$s = \ell\vartheta \quad \dot{s} = \ell\dot{\vartheta} \quad \ddot{s} = \ell\ddot{\vartheta}$$

$$m\ddot{s} = -mg\sin\vartheta$$

$$\boxed{\ddot{\vartheta} = -\frac{g}{\ell}\sin\vartheta} \qquad \left[\frac{g}{\ell}\right] = s^{-2}$$

piccole oscillazioni $\sin\vartheta \simeq \vartheta$

EQ del moto

$$\ddot{\vartheta}_{(t)} \simeq -\frac{g}{\ell}\vartheta_{(t)}$$

$$(\ell\ddot{\vartheta}) \simeq -\frac{g}{\ell}(\ell\vartheta)$$

$$\ddot{s} \simeq -\frac{g}{\ell}s$$

$$\boxed{\ddot{x}_{(t)} = -\frac{g}{\ell}x_{(t)}}$$

$x(t)$! legge oraria

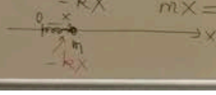
$$\frac{d(\sin\varphi)}{d\varphi} = \cos\varphi \qquad \frac{d^2(\sin\varphi)}{d^2\varphi} = \frac{d}{d\varphi}(\cos\varphi) = -\sin\varphi$$

$$\frac{d}{d\varphi}(\cos\varphi) = -\sin\varphi \qquad \frac{d^2(\cos\varphi)}{d^2\varphi} = \frac{d}{d\varphi}(-\sin\varphi) = -\cos\varphi$$

$$[k] = s^{-2}\,kg\,\frac{m}{m} = N/m$$

$$m\ddot{x} = -\left(\frac{g}{\ell}m\right)x$$

$$-kx \qquad m\ddot{x} = -kx$$



$$\boxed{\ddot{x}_{(t)} = -\frac{k}{m}x_{(t)}}$$

$$\boxed{\varphi(t) = \omega t}$$

rad $\qquad$ rad s^{-1}

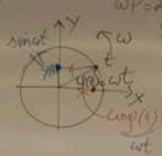
$$\sin\varphi = \sin(\omega t)$$

$$\frac{d(\sin\varphi(t))}{dt} = \frac{d(\sin\varphi)}{d\varphi}\frac{d\varphi}{dt} = \omega\cos\varphi$$

$$\frac{d^2(\sin\varphi)}{dt^2} = \omega\frac{d}{dt}(\cos\varphi) = \omega\frac{d(\cos\varphi)}{d\varphi}\frac{d\varphi}{dt} = -\omega^2\sin\varphi$$

rad s^{-2}

$$\omega t = 2\pi \qquad P = \frac{2\pi}{\omega}$$

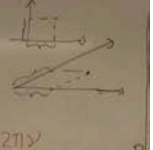


$\sin\omega t$ $\qquad$ ω $\qquad$ t $\qquad$ $\varphi(t) = \omega t$ $\qquad$ $\cos\varphi(t) = x(t)$

$$\omega P = 2\pi \qquad P = \frac{2\pi}{\omega}$$

$$\omega = 2\pi\nu$$

$$\nu = \frac{1}{P} \quad Hz \quad (s^{-1})$$



clockrate $3\,GHz = 3 \cdot 10^9\,Hz$

$$P = \frac{1}{3\times 10^9}s = \frac{1}{3}10^{-9}s$$

$$\ddot{x} \simeq -\frac{g}{\ell}\,x \qquad \frac{g}{\ell} = \omega_0^2 \qquad \omega_0 = \sqrt{\frac{g}{\ell}} \;\Rightarrow\; \omega_0 = \sqrt{\frac{k}{m}}$$

FREQ. NATURALE DI OSCILLAZIONE

Es $\ell = 1m$

$$P_0 = 2\pi\sqrt{\frac{1}{9.8}} \simeq \;= \frac{2\pi}{\sqrt{9.8}}\,s \simeq 2s$$

$$\begin{cases} x(0) = x_0 \\ \dot{x}(0) = v_0 = 0 \end{cases}$$

Condizioni iniziali

$$\begin{cases} x(t) = A\cos\omega_0 t + B\sin\omega_0 t \\ \dot{x}(t) = -A\,\omega_0 \sin\omega_0 t + B\,\omega_0\cos\omega_0 t \end{cases}$$

$$x(0) = A\cdot 1 + 0 = A = x_0$$

$$\dot{x}(0) = -A\cancel{\omega_0}\cdot 0 + B\omega_0 = 0 \qquad \omega_0 \neq 0 \Rightarrow B = 0$$

$$\boxed{x(t) = x_0\cos\omega_0 t}$$

costante

$$x(t) = A\cos(\omega_0 t + \psi)$$

$$\dot{x}(t) = -A\sin(\omega_0 t + \psi)\cdot\frac{d(\omega_0 t + \psi)}{dt} = -A\omega_0\sin(\omega_0 t + \psi)$$

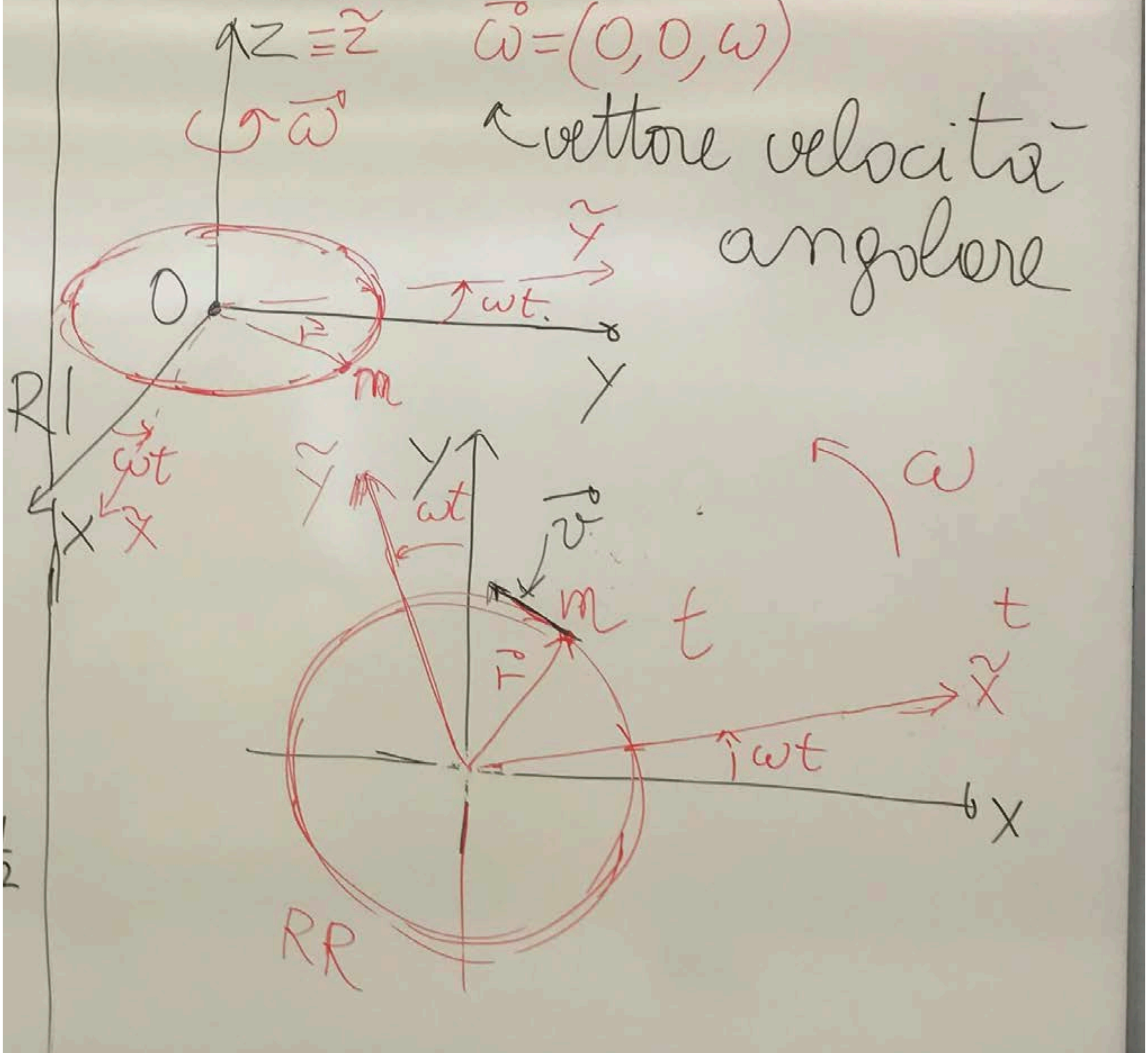
$$x(0) = A\underbrace{\cos\psi}_{=1} = x_0$$

$$\dot{x}(0) = A\omega_0\sin\psi = 0 \Rightarrow \sin\psi = 0$$

$$\Rightarrow \psi = 0 \Rightarrow x(t) = x_0\cos\omega_0 t$$

EQUAZIONI DEL MOTO IN RIFERIMENTI ROTANTI

$\vec{\omega} = (0, 0, \omega)$ ← vettore velocità angolare



$\vec{r}(t)$ vettore posizione istantaneo

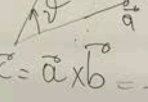
$$\boxed{\left(\frac{d\vec{r}}{dt}\right)_{RI} = \left(\frac{d\vec{r}}{dt}\right)_{RR} + \vec{\omega} \times \vec{r}}$$

$\vec{r}$ fisso in RR $\Rightarrow \left(\frac{d\vec{r}}{dt}\right)_{RR} = 0$

$$\left(\frac{d\vec{r}}{dt}\right)_{RI} = \vec{\omega} \times \vec{r}$$

$$|\vec{\omega} \times \vec{r}| = \omega r \sin(\text{angolo tra } \vec{\omega} \text{ ed } \vec{r}) = \omega r \sin\frac{\pi}{2} = \omega r$$

$\left(\frac{d\vec{r}}{dt}\right)_{RR} = \vec{v}_{rel} \neq 0$ in generale



$\vec{c} = \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$$\vec{v}_{RI} = \vec{v}_{rel} + \vec{\omega} \times \vec{r}$$

$\downarrow$

$\vec{a}_{RI}$

$$m\,\vec{a}_{RI} = \vec{F}$$