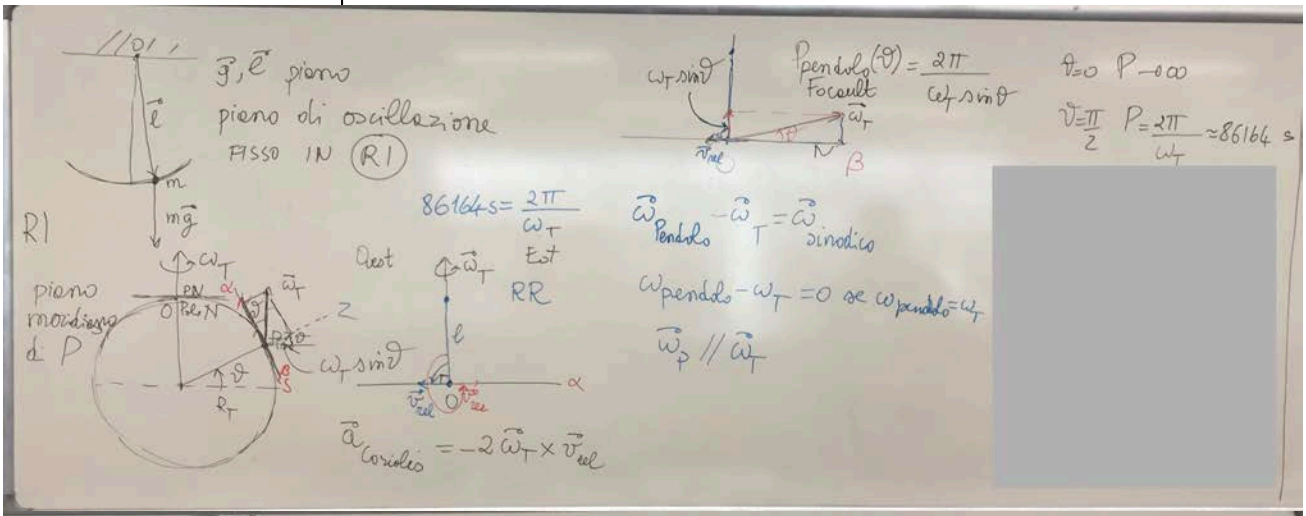
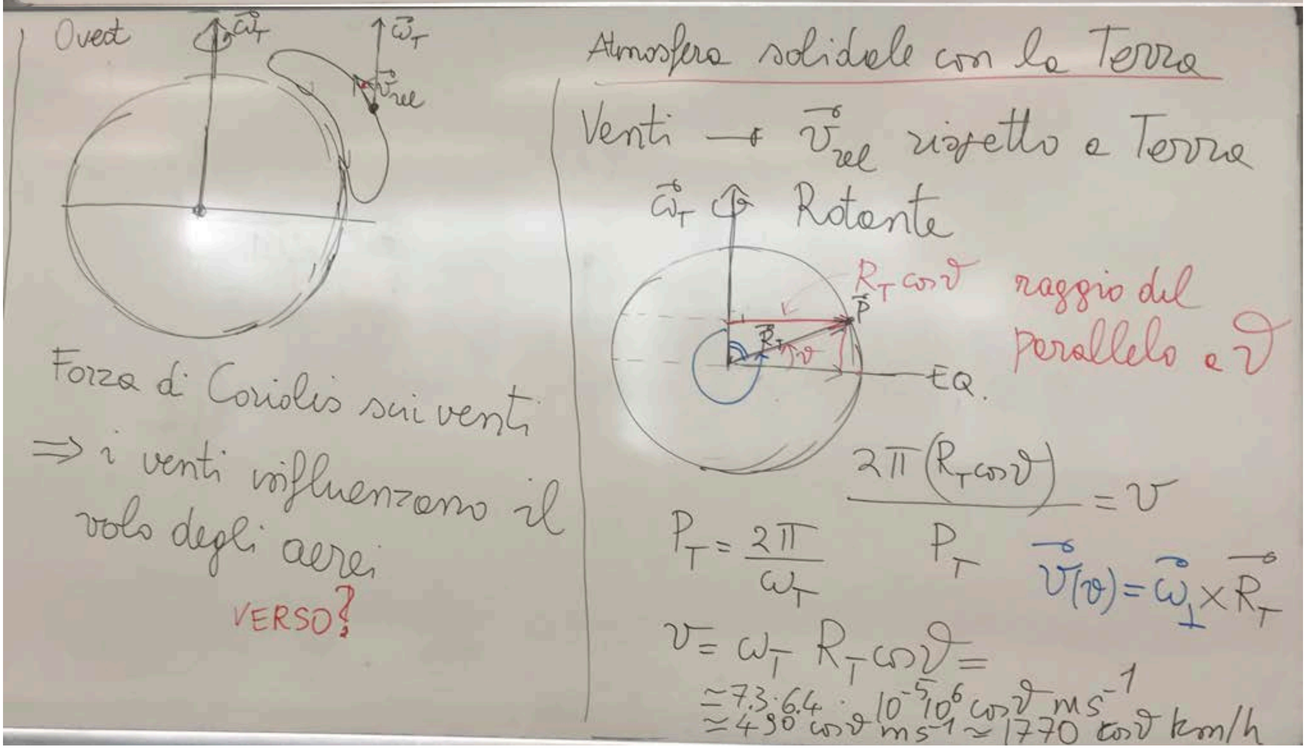
S5 - Quinta settimana, lezioni del 14 e 16 marzo 2017

- Accelerazione di Coriolis e calcolo del periodo di rotazione del piano di un pendolo ad una qualunque latitudine
- Accelerazione di Coriolis e effetto dei venti sul volo degli aerei quando viaggiano da ovest verso est o viceversa (velocità della atmosfera rispetto alla terra e dei venti rispetto all'atmosfera)
- Richiami sul prodotto vettore e sulla somma e differenza di vettori con la regola del parallelogramma
- Richiami sui numeri complessi; interpretazione geometrica relazione con i vettori nel piano reale
- Introduzione del numero di Eulero. Potenze del numero di Eulero con esponente immaginario e relazioni con la trigonometria.
- Richiami sull'uso delle potenze





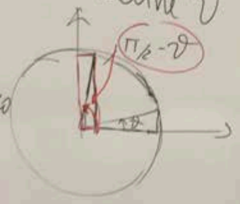
$v\left(\frac{\pi}{4}\right) = 1770 \times \frac{\sqrt{2}}{2}$ km/h

$\simeq 1250$ km/h

$$\vec{v}(\vartheta) = \left(\frac{d\vec{R}_T}{dt}\right)_{RI} = \underbrace{\cancel{\left(\frac{d\vec{R}_T}{dt}\right)_{RR}}}_{0 \,=\, \vec{v}_{rel}(P)} + \vec{\omega}_T \times \vec{R}_T$$

velocità di P (fermo) sul parallelo a latitudine ϑ

cerchio trigonometrico $r=1$

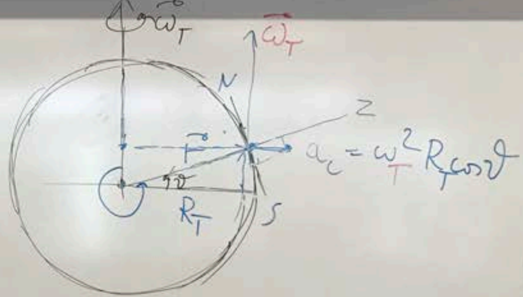


$$v(\vartheta) = \omega_T R_T \sin(\vec{\omega}_T, \vec{R}_T) = \omega_T R_T \sin\left(\frac{\pi}{2} - \vartheta\right) = \omega_T R_T \cos\vartheta$$

$\omega_T = \frac{2\pi}{86164}$ rad $s^{-1} \simeq 7.3 \times 10^{-5}$ rad s^{-1}

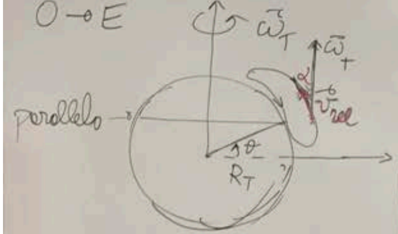
$R_T \simeq 6378$ km $\simeq 6.4 \times 10^6$ m

$$\vec{a}_{rel} = \frac{\vec{F}}{m} - 2\vec{\omega} \times \vec{v}_{rel} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

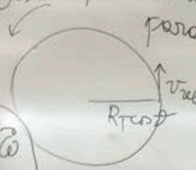


RR $\quad \vec{a}_{coriolis} = -2\vec{\omega}_T \times \vec{v}_{rel}$

O → E



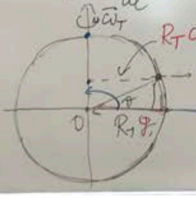
O → E $\quad \vec{\omega}_T \times \vec{v}_{rel}$ verso tangente parallelo $\Rightarrow \vec{a}_c$ è radiale lungo il raggio del parallelo



$a_{coriolis}$: O → E venti

RR $\vec{\omega}$

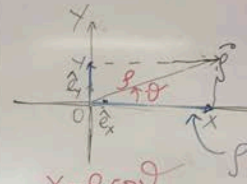
$$\vec{a}_{rel} = \underbrace{\vec{a}_i}_{= \frac{\vec{F}}{m}} - 2\vec{\omega}_T \times \vec{v}_{rel} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$



$a_{centr.}(\vartheta) = \omega_T^2 R_T \cos\vartheta$

$$a_{centr}(\vartheta = 0) = a_{centr\,max} = (7.3)^2 \times 10^{-10} \cdot 6.378 \cdot 10^6 \text{ m s}^{-2} \simeq 3.3 \times 10^{-2} \text{ m s}^{-2}$$

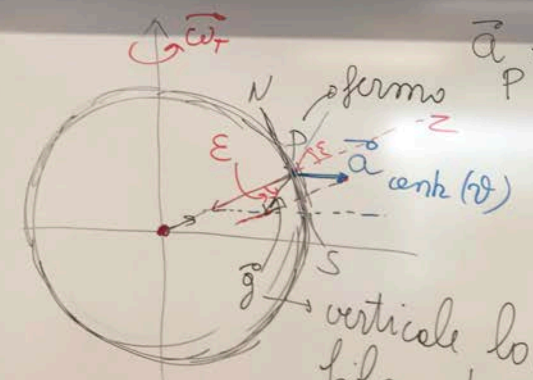
$$\frac{a_{centr.\,max}}{g} \simeq 3.3 \times 10^{-3}$$



$x = \rho\cos\vartheta$

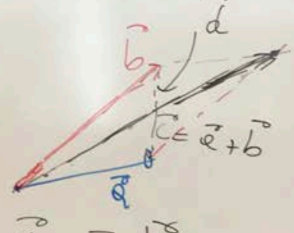
$y = \rho\sin\vartheta$

$x^2 + y^2 = \rho^2\cos^2\vartheta + \rho$

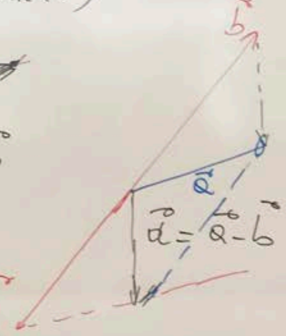


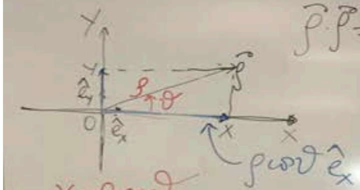
$\vec{g}$ → verticale locale
filo a piombo

$$\vec{a}_P = -\frac{G M_T}{R_T^2}\hat{R}_T - \underbrace{\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\vec{a}_{centr}(\vartheta)}$$



$\vec{c} = \vec{a}+\vec{b}$





$\vec{\rho}\cdot\vec{\rho} = x^2+y^2 = \rho^2$

$x = \rho\cos\vartheta$

$y = \rho\sin\vartheta$

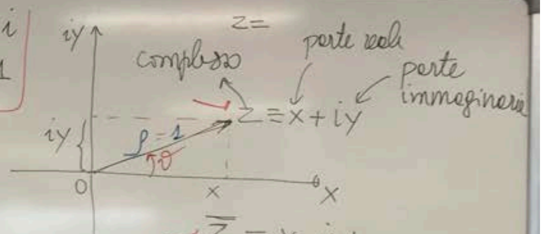
$\vec{\rho} = x\hat{e}_x + y\hat{e}_y \qquad \vec{\rho} = (x,y)$

$$x^2+y^2 = \rho^2\cos^2\vartheta + \rho^2\sin^2\vartheta = \rho^2\underbrace{(\cos^2\vartheta+\sin^2\vartheta)}_{=1} = \rho^2$$

$10^{-10}\cdot 6.378\cdot 10^{6}\ \mathrm{m\,s^{-2}}$
$3.3\times 10^{-2}\ \mathrm{m s^{-2}}$

NUMERO DI EULERO → $e = 2.71828182845\ldots$

$\sqrt{-1} = i$
$i^2 = -1$



$\bar{z} \equiv x - iy$

$$z\bar{z} = (x+iy)(x-iy) = x^2 - \cancel{ixy} + \cancel{ixy} - i^2y^2$$
$$= x^2+y^2 = \rho^2$$

$x = \rho\cos\vartheta$

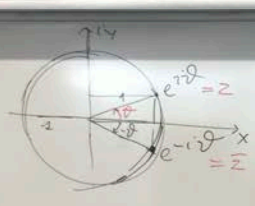
$iy = i\rho\sin\vartheta$

$$z = x+iy = \rho\cos\vartheta + i\rho\sin\vartheta =$$
$$= \rho(\cos\vartheta + i\sin\vartheta)$$

$$\boxed{e^{i\vartheta} = \cos\vartheta + i\sin\vartheta}$$

$e^{-i\vartheta} = \cos\vartheta - i\sin\vartheta$

$e^{i\vartheta}e^{-i\vartheta} = e^{(i\vartheta - i\vartheta)} = e^0 = 1$



$z\bar{z} = 1^2 = 1$

$e^{i\alpha}e^{i\beta} = e^{i\alpha+i\beta} = e^{i(\alpha+\beta)}$

$= \cos(\alpha+\beta) + i\sin(\alpha+\beta)$

$(\cos\alpha + i\sin\alpha)\cdot(\cos\beta + i\sin\beta)$

$=$

$\cos\alpha\cos\beta + i\cos\alpha\sin\beta + i\sin\alpha\cos\beta - \sin\alpha\sin\beta =$

$= (\cos\alpha\cos\beta - \sin\alpha\sin\beta) + i(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$

$$\begin{cases}\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta\end{cases}$$

$e^{i\alpha}e^{i\beta} = e^{i(\alpha+\beta)}$

$\frac{e^{i\alpha}}{e^{i\beta}} = e^{i\alpha}\left(\frac{1}{e^{i\beta}}\right) = e^{i\alpha}e^{-i\beta} = e^{i(\alpha-\beta)}$

$(e^{i\alpha})^{i\beta} = e^{i\alpha i\beta} = e^{-\alpha\beta}$

$e^0 = 1$

$e^1 = e$