

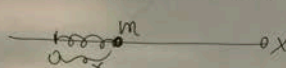
Ampiezza di oscillazione al quadrato ed energia di un oscillatore.

Oscillatore armonico smorzato e forzato, soluzione mediante numeri complessi, significato della ampiezza alla risonanza, relazione con la dissipazione.

Esercizio: oscillatore formato da due masse accoppiate con una molla; calcolo della frequenza naturale e casi particolari (masse uguali; caso di una massa molto minore dell'altra).

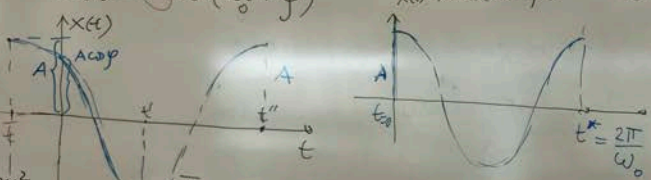
Esercizio: posizione di equilibrio di un oscillatore in presenza della accelerazione locale di gravità (concetto di dinamometro).

AMPIEZZA OSCILLAZIONE AL QUADRATO ↔ ENERGIA OSCILLATORE

$F = -kx$ k, m  $F = -kx$

$m\ddot{x} = -kx$ $x(t) = A\cos(\omega_0 t + \varphi)$ $x(t) = A\cos(\omega_0 t + \varphi)$ $U(x) - U(0) = \int_0^x -kx' dx' = +\frac{1}{2} kx^2 > 0$

$\ddot{x} + \frac{k}{m}x = 0$ $\omega_0^2 = \frac{k}{m}$ $dL = -kx dx$ $E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} kx^2$ $E \propto A^2$

$\dot{x}(t) = -\omega_0 A \sin(\omega_0 t + \varphi)$  $E = \frac{1}{2} m \omega_0^2 A^2 \sin^2(\omega_0 t + \varphi) + \frac{1}{2} k A^2 \cos^2(\omega_0 t + \varphi)$

$E = \frac{1}{2} m \omega_0^2 A^2 (\sin^2(\omega_0 t + \varphi) + \cos^2(\omega_0 t + \varphi)) = \frac{1}{2} m \omega_0^2 A^2$

$\omega_0 t' + \varphi = \pi$ $\omega_0 t'' + \varphi = 2\pi$
 $\omega_0 t' = \pi - \varphi$ $t' = \frac{\pi - \varphi}{\omega_0}$
 $\omega_0 t'' = 2\pi - \varphi$ $t'' = \frac{2\pi - \varphi}{\omega_0}$

Se l'ampiezza di oscillazione è costante → energia costante

OSCILLATORE ARMONICO SMORZATO + FORZATO (1D)

k, m $F = -kx$ $F_{diss} = -\gamma \dot{x}$ $[x] = N/(ms^{-1})$ $F_{ext}(t) = F_0 \cos(\omega t + \Delta)$

$\omega_0^2 = \frac{k}{m}$ $\gamma > 0$ $\lambda = \frac{\gamma}{m}$ $[x] = ms^{-1}$

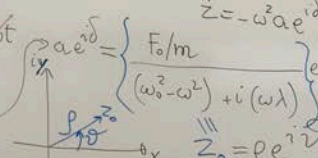
$m\ddot{x}(t) = -kx - \gamma \dot{x} + F_0 \cos(\omega t + \Delta)$ $\ddot{x} + \frac{\gamma}{m} \dot{x} + \frac{k}{m} x = \frac{F_0}{m} \cos(\omega t + \Delta)$

$z = x + iy(t)$ $F = F_0 e^{i(\omega t + \Delta)}$
 $\ddot{z} + \lambda \dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i(\omega t + \Delta)}$
 $= \frac{F_0}{m} e^{i\omega t} e^{i\Delta}$
 $z(t) = a e^{i(\omega t + \delta)} = a e^{i\delta} e^{i\omega t}$
 $\dot{z} = i\omega a e^{i\delta} e^{i\omega t}$
 $\ddot{z} = -\omega^2 a e^{i\delta} e^{i\omega t}$

$- \omega^2 a e^{i\delta} e^{i\omega t} + i\omega a e^{i\delta} e^{i\omega t} + \omega_0^2 a e^{i\delta} e^{i\omega t} = \frac{F_0}{m} e^{i\Delta} e^{i\omega t}$

$a e^{i\delta} (\omega_0^2 - \omega^2 + i\omega\lambda) = \frac{F_0}{m} e^{i\Delta}$

$a e^{i\delta} = \frac{F_0/m}{(\omega_0^2 - \omega^2) + i(\omega\lambda)}$

$z_0 = p e^{i\delta}$ 

$$z_0 = \rho e^{i\vartheta} \quad \frac{1}{z_0} = \frac{1}{\rho e^{i\vartheta}} = \frac{1}{\rho} e^{-i\vartheta} = \frac{(\omega_0^2 - \omega^2) + i\omega\lambda}{F_0/m}$$

$$\frac{1}{\rho^2} = \frac{(\omega_0^2 - \omega^2)^2 + \omega^2\lambda^2}{F_0^2/m^2}$$

$$\Rightarrow \rho^2 = \frac{F_0^2/m^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\lambda^2}$$

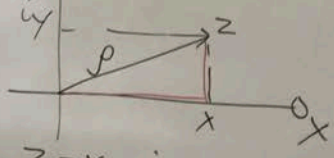
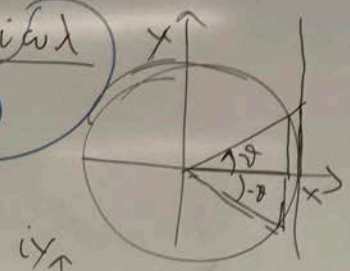
$$\tan(-\vartheta) = \frac{\omega\lambda}{\omega_0^2 - \omega^2}$$

$$\Rightarrow \tan\vartheta = -\frac{\omega\lambda}{\omega_0^2 - \omega^2}$$

$$z(t) = a e^{i\delta} e^{i\omega t} = z_0 e^{i\Delta} e^{i\omega t} = \rho e^{i\vartheta} e^{i\Delta} e^{i\omega t} = \rho e^{i(\omega t + \Delta + \vartheta)}$$

$$z(t) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + \omega^2\lambda^2}} e^{i(\omega t + \Delta + \vartheta)}$$

$$x(t) = \rho \cos(\omega t + \Delta + \vartheta)$$

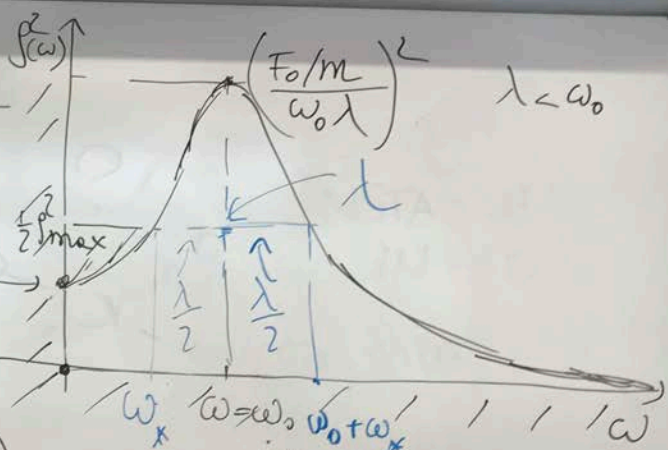


$$z = x + iy$$

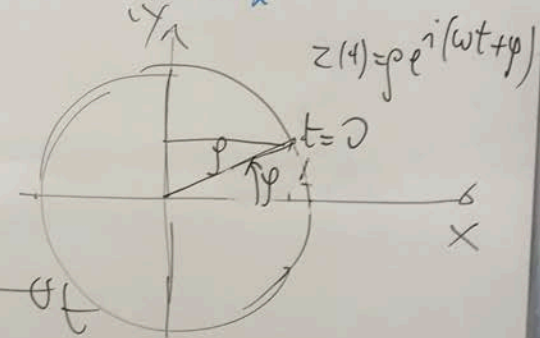
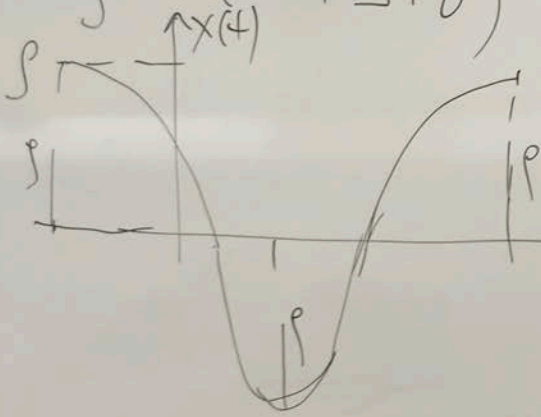
$$\bar{z} = x - iy$$

$$z\bar{z} = x^2 + y^2$$

$$\rho^2(\omega) = \frac{F_0^2/m^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\lambda^2}$$



$$x(t) = \rho \cos(\omega t + \Delta + \vartheta)$$

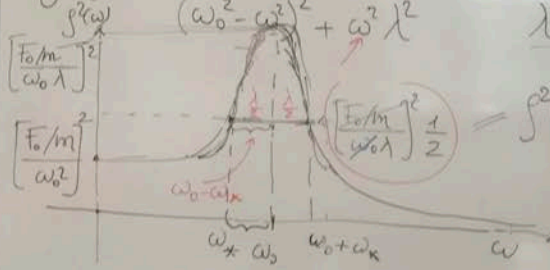


Esercizio (molle forzato smorzato)

m, k $[\gamma] = N/m$ $F_{diss} = -\gamma \dot{x}$ $\lambda = \frac{\gamma}{m}$

1D $F(t) = F_0 \cos(\omega t + \Delta)$ $\omega_0 = \sqrt{\frac{k}{m}}$

$f^2(\omega) = \frac{F_0^2/m^2}{(\omega_0^2 - \omega^2)^2 + \omega^2 \lambda^2}$

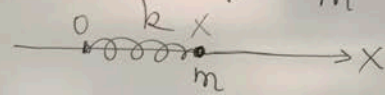


→ approssimazione: $\omega \approx \omega_0$
 $\lambda < \omega_0$

$f^2(\omega_0) = \frac{F_0^2/m^2}{4\omega_0^2 \lambda^2} = \frac{F_0^2/m^2}{4(\omega_0 - \omega_0)^2 + \lambda^2}$
 $a^2 - b^2 = (a+b)(a-b)$

$f^2(\omega) = \frac{F_0^2/m^2}{[(\omega_0 - \omega)(\omega_0 + \omega)]^2 + \omega^2 \lambda^2} \approx \frac{F_0^2/m^2}{4\omega_0^2(\omega_0 - \omega)^2 + \omega_0^2 \lambda^2}$
 $f^2(\omega) \approx \frac{F_0^2/m^2}{4\omega_0^2} \frac{1}{(\omega_0 - \omega)^2 + \frac{\lambda^2}{4}}$
 $\Rightarrow \frac{1}{2\lambda^2} = \frac{1}{4(\omega_0 - \omega)^2 + \lambda^2}$
 $2\lambda^2 = 4(\omega_0 - \omega)^2 + \lambda^2$
 $\lambda^2 = 4(\omega_0 - \omega)^2$
 $\omega_0 - \omega = \pm \frac{\lambda}{2} \Rightarrow \omega_0 - \omega = \frac{\lambda}{2}$

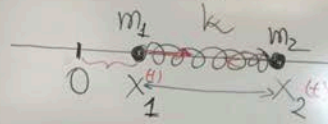
Esercizio $\ddot{x} = -\frac{k}{m}x$



$\omega_0 = \sqrt{\frac{k}{m}}$

$\xi \equiv x_2 - x_1$

$\ddot{\xi} = -k \left(\frac{m_1 + m_2}{m_1 m_2} \right) \xi = -\frac{k}{\mu} \xi$



$\mu = \frac{m_1 m_2}{m_1 + m_2}$

$\ddot{\xi} = -\frac{k}{\mu} \xi$

$\omega'_0 = \sqrt{\frac{k}{\mu}}$

$m_1 \ddot{x}_1 = +k(x_2 - x_1)$
 $m_2 \ddot{x}_2 = -k(x_2 - x_1)$
 $\ddot{x}_1 = \frac{k}{m_1} (x_2 - x_1)$
 $\ddot{x}_2 = -\frac{k}{m_2} (x_2 - x_1)$
 $\ddot{x}_2 - \ddot{x}_1 = -k \left(\frac{1}{m_2} + \frac{1}{m_1} \right) (x_2 - x_1)$

i) $m_1 = m_2 = m$ $\mu = \frac{m^2}{2m} = \frac{m}{2}$
 $\ddot{\xi} = -\frac{2k}{m} \xi$

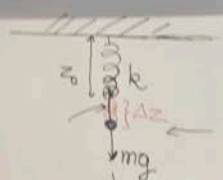
$\omega'_0 = \sqrt{\frac{2k}{m}}$

ii) $m_2 \gg m_1$ $\left(\frac{m_1}{m_2} \right) \ll 1$

$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1 m_2}{m_2 \left(\frac{m_1}{m_2} + 1 \right)} = \frac{m_1}{1 + \epsilon} \approx m_1 (1 - \epsilon) \approx m_1$
 $\omega'_0 = \sqrt{\frac{k}{m_1}}$

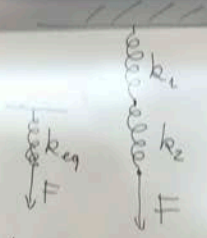
$(1+\epsilon)^{-1} = \frac{1}{1+\epsilon} = 1 - \epsilon + \epsilon^2 - \dots$
 $-1/(1+\epsilon)^2$

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$



posizione di equilibrio

$$mg = k \Delta z \rightarrow \Delta z = \frac{mg}{k}$$



kequivalente?

$$F = k_1 \Delta z_1 \Rightarrow \Delta z_1 = \frac{F}{k_1}$$

$$F = k_2 \Delta z_2 \Rightarrow \Delta z_2 = \frac{F}{k_2}$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

$$\Delta z_{tot} = \Delta z_1 + \Delta z_2 = F \left(\frac{1}{k_1} + \frac{1}{k_2} \right)$$

$$F = k_{eq} \Delta z_{tot}$$

$$\Delta z_{tot} = \frac{F}{k_{eq}}$$