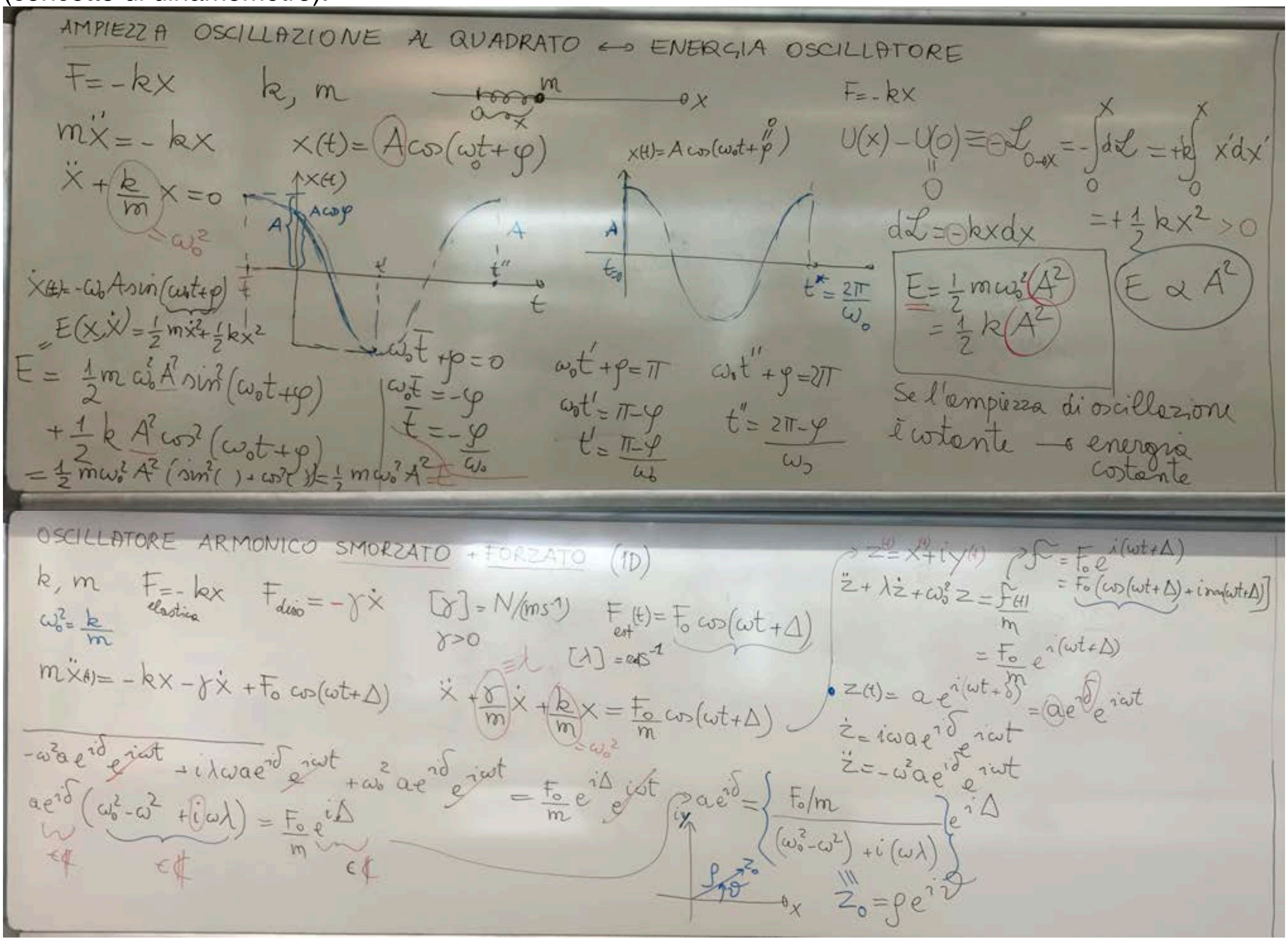
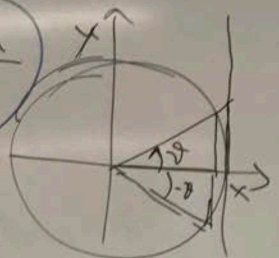
S7 - Settima settimana, lezioni del 28 e 30 marzo 2017

Ampiezza di oscillazione al quadrato ed energia di un oscillatore.
Oscillatore armonico smorzato e forzato, soluzione mediante numeri complessi, significato della ampiezza alla risonanza, relazione con la dissipazione.
Esercizio: oscillatore formato da due masse accoppiate con una molla; calcolo della frequenza naturale e casi particolari (masse uguali; caso di una massa molto minore dell'altra).
Esercizio: posizione di equilibrio di un oscillatore in presenza della accelerazione locale di gravità (concetto di dinamometro).



$z_0 = \rho e^{i\vartheta}$

$\frac{1}{z_0} = \frac{1}{\rho e^{i\vartheta}} = \frac{1}{\rho} e^{i(-\vartheta)} = \frac{(\omega_0^2-\omega^2)+i\omega\lambda}{F_0/m}$

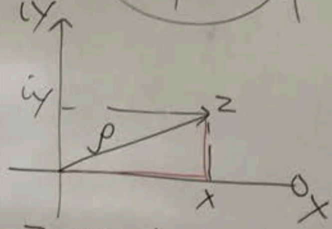


$\frac{1}{\rho^2} = \frac{(\omega_0^2-\omega^2)^2+\omega^2\lambda^2}{F_0^2/m^2} \Rightarrow \rho^2 = \frac{F_0^2/m^2}{(\omega_0^2-\omega^2)^2+\omega^2\lambda^2}$

$tg(-\vartheta) = \frac{\omega\lambda}{\omega_0^2-\omega^2}$

$\parallel$

$-tg\,\vartheta \to tg\,\vartheta = -\frac{\omega\lambda}{\omega_0^2-\omega^2}$



$z(t) = a e^{i\delta} e^{i\omega t} = z_0 e^{i\Delta} e^{i\omega t} = \rho e^{i\vartheta} e^{i\Delta} e^{i\omega t}$

$= \rho e^{i(\omega t+\Delta+\vartheta)}$

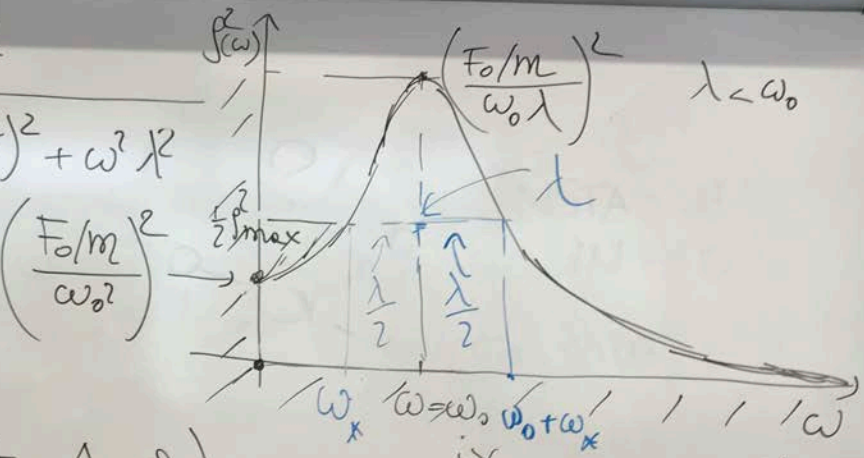
$z = x + iy$

$\bar{z} = x - iy$

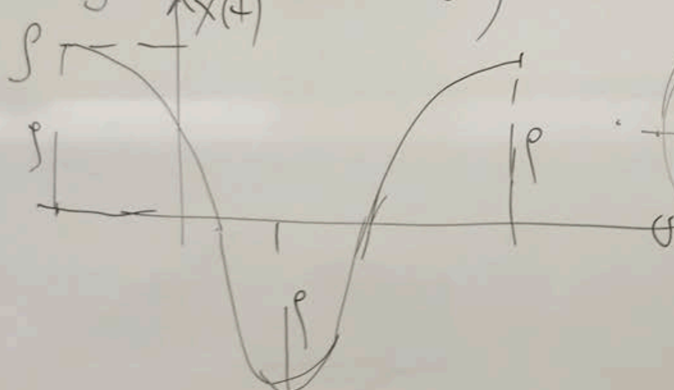
$z\bar{z} = x^2 + y^2$

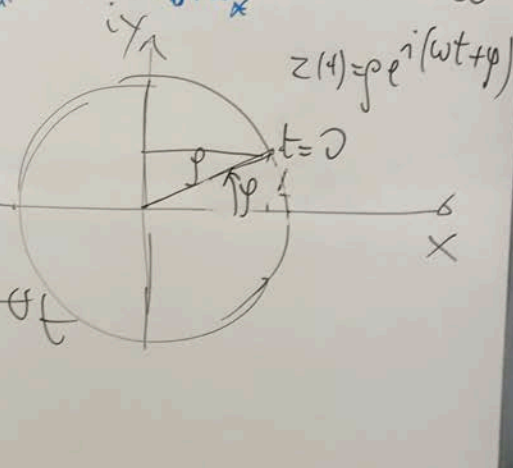
$z(t) = \underbrace{\left\{\frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+\omega^2\lambda^2}}\right\}}_{=\rho} \underbrace{e^{i(\omega t+\Delta+\vartheta)}}_{\text{adimensionale}} \to x(t) = \rho\cos(\omega t+\Delta+\vartheta)$

$\rho^2(\omega) = \frac{F_0^2/m^2}{(\omega_0^2-\omega^2)^2+\omega^2\lambda^2}$



$x(t) = \rho\cos(\omega t+\Delta+\vartheta)$





Esercizio (oscillatore forzato smorzato)

$m, k \quad [\gamma] = N/m \quad F_{diss} = -\gamma \dot{x} \quad \lambda = \frac{\gamma}{m}$

1D $\quad F(t) = F_0 \cos(\omega t + \Delta) \quad \omega_0 = \sqrt{\frac{k}{m}}$

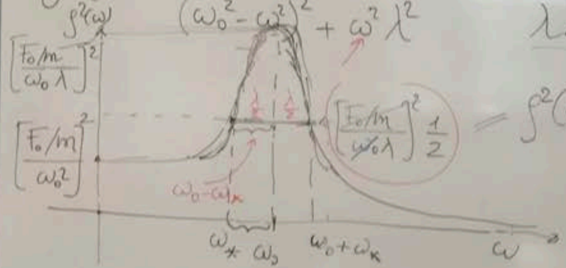
$$\rho^2(\omega) = \frac{F_0^2/m^2}{(\omega_0^2 - \omega^2)^2 + \omega^2 \lambda^2}$$

→ approssimazione: $\omega \simeq \omega_0$

$\lambda < \omega_0$

$$\rho^2(\omega) = \frac{F_0^2/m^2}{[(\omega_0-\omega)\underbrace{(\omega_0+\omega)}_{2\omega_0}]^2 + \underbrace{\omega^2}_{\omega_0}\lambda^2} \simeq \frac{F_0^2/m^2}{4\omega_0^2(\omega_0-\omega)^2 + \omega_0^2\lambda^2} \simeq$$

$$\rho^2(\omega) \simeq \frac{F_0^2/m^2}{4\omega_0^2} \cdot \frac{1}{(\omega_0-\omega)^2 + \frac{\lambda^2}{4}}$$



$$\left[\frac{F_0/m}{\omega_0 \lambda}\right]^2 \frac{1}{2} = \rho^2(\omega_*) = \frac{F_0^2/m^2}{4\omega_0^2} \cdot \frac{1}{(\omega_0-\omega_*)^2 + \frac{\lambda^2}{4}}$$

$$\Rightarrow \frac{1}{2\lambda^2} = \frac{1}{4(\omega_0-\omega_*)^2 + \lambda^2}$$

$$2\lambda^2 = 4(\omega_0-\omega_*)^2 + \lambda^2$$

$$\lambda^2 = 4(\omega_0-\omega_*)^2$$

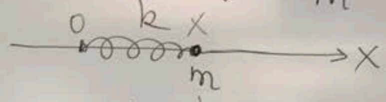
$$\Downarrow$$

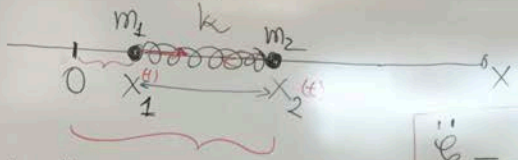
$$(\omega_0-\omega_*)^2 = \frac{\lambda^2}{4} \Rightarrow \omega_0 - \omega_* = \frac{\lambda}{2}$$

$$a^2 - b^2 = (a+b)(a-b)$$

Esercizio

$\ddot{x} = -\frac{k}{m} x \qquad \omega_0 = \sqrt{\frac{k}{m}}$





$\xi \equiv x_2 - x_1$

$$\ddot{\xi} = -k\left(\frac{m_1+m_2}{m_1 m_2}\right)\xi = -\frac{k}{\mu}\xi$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\boxed{\ddot{\xi} = -\frac{k}{\mu}\xi} \qquad \omega_0' = \sqrt{\frac{k}{\mu}}$$

$$\begin{cases} m_1 \ddot{x}_1 = +k(x_2 - x_1) \\ m_2 \ddot{x}_2 = -k(x_2 - x_1) \end{cases}$$

$$\begin{cases} \ddot{x}_1 = \frac{k}{m_1}(x_2 - x_1) \\ \ddot{x}_2 = -\frac{k}{m_2}(x_2 - x_1) \end{cases}$$

$$\ddot{x}_2 - \ddot{x}_1 = -k\left(\frac{1}{m_2} + \frac{1}{m_1}\right)(x_2 - x_1)$$

i) $m_1 = m_2 \equiv m \qquad \mu = \frac{m^2}{2m} = \frac{m}{2}$

$$\ddot{\xi} = -\frac{2k}{m}\xi \qquad \omega_0' = \sqrt{\frac{2k}{m}}$$

ii) $m_2 >> m_1 \qquad \underbrace{\left(\frac{m_1}{m_2}\right)}_{\varepsilon} << 1$

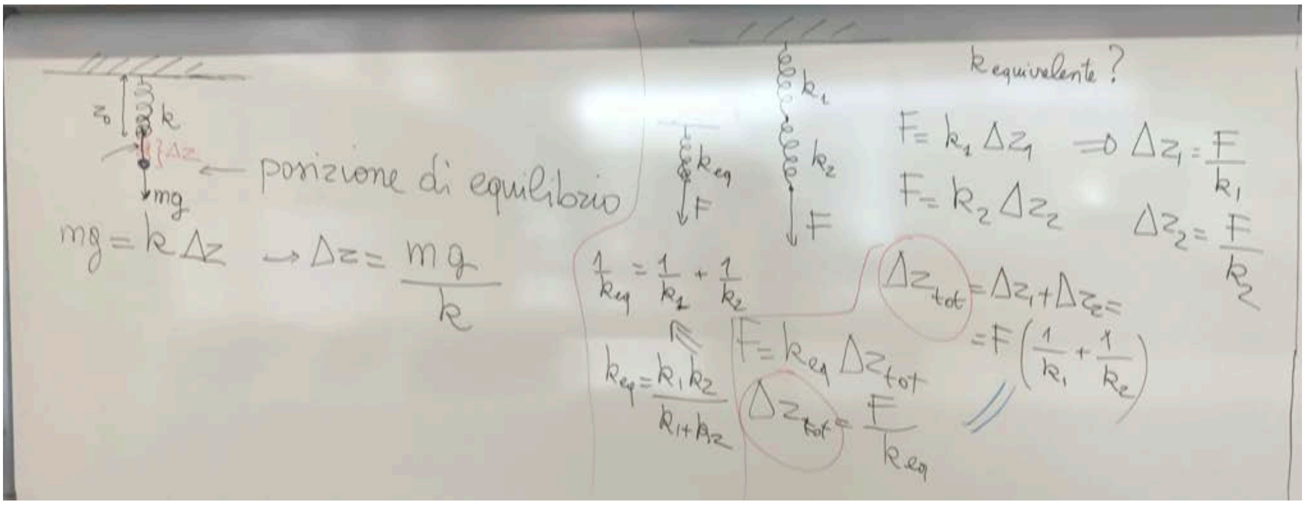
$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1 \cancel{m_2}}{\cancel{m_2}\left(\underbrace{\frac{m_1}{m_2}}_{\varepsilon} + 1\right)} = \frac{m_1}{1+\varepsilon} \simeq m_1(1 - \varepsilon) \simeq m_1 \quad \left(\varepsilon = \frac{m_1}{m_2}\right)$$

$$\omega_0' = \sqrt{\frac{k}{m_1}}$$

$$(1+\varepsilon)^{-1} = \frac{1}{1+\varepsilon} = 1 - \varepsilon$$

$$-1(1+\varepsilon)^{-2}$$

$$f(\overset{\varepsilon}{x}) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \dots$$



z_0
k
Δz
posizione di equilibrio
mg
$mg = k\Delta z \rightarrow \Delta z = \frac{mg}{k}$
k_{eq}
F
k_1
k_2
F
k equivalente?
$F = k_1 \Delta z_1 \Rightarrow \Delta z_1 = \frac{F}{k_1}$
$F = k_2 \Delta z_2 \quad \Delta z_2 = \frac{F}{k_2}$
$\Delta z_{tot} = \Delta z_1 + \Delta z_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$
$F = k_{eq} \Delta z_{tot}$
$\Delta z_{tot} = \frac{F}{k_{eq}}$
$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$
$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$