Einstein's comprehensive 1907 essay on relativity, part I

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As an important sequel to his first paper on relativity, Einstein's extensive discussion of the subject in the 1907 issue of *Jahrbuch der Radioaktivität und Elektronik*, including a first exposition of his embryonic ideas on gravitation, is of intrinsic as well as considerable historical interest. In this, the first of three parts dealing with Einstein's essay, a translation is presented of the Introduction and of Part I, which is concerned with relativistic kinematics.

A. INTRODUCTION

Two years after the appearance of his first paper on the theory of relativity,[1] Einstein published in the then prestigious *Jahrbuch der Radioaktivität und Elektronik* an extensive survey article on the subject,[2] entitled "Über das Relativitätsprinzip und die aus demselben gezogene Folgerungen."[3] In addition to a renewed painstaking analysis of the basic kinematic principles of the theory of relativity, and a treatment of relativistic mechanics and thermodynamics inspired by Planck's work on the subject of the same year, the memoir includes a note on the clash with conventional causality of assuming the existence of signals faster than light, an extension of Einstein's earlier discussions concerning the inertia of energy, and, most importantly, a first step towards the eventual creation of the general theory of relativity in the brilliant idea of an intimate link between "acceleration" and gravitation, as suggested by the proportionality between inertial and gravitational mass.

Some of the topics treated in the memoir and the methods employed there are even today not devoid of at least educational interest. But its greatest interest by far is certainly historical. The Introduction, in particular, contains valuable information bearing on the history of special relativity, when considered alongside the relevant material in Ref. 1. As for the history of general relativity, the significance of Einstein's first inspirational flashes, which in the course of a personal scientific odyssey culminated in his remarkable gravitational equations, is of course abundantly clear.

It is because of this considerable historical interest presented by the memoir that a translation of the Introduction and of the first and last parts together with a concise modernized rendition of its other parts appeared worth undertaking. Nevertheless, a few words of explanation are in order why there is, in fact, any need of an English rendition when the chief significance of this essay today is, after all, only to historians of science. What must be stressed here is that in addition to the professional historians of science, who are naturally in a position to avail themselves of the original material at first hand, there are many scientists who for one reason or another have serious but subsidiary historical interests and who may not find it feasible to go directly to the original work. Moreover, the comments presented in the footnotes that relate to the ideas or mathematical steps in the memoir, as well as the modernized rendition, may be helpful not only to the latter group but to some members of the former group as well.

The present paper consists of a translation of the Introduction and of Part I of Einstein's essay. As in the earlier work,[4] of which the present is a sequel, it is endeavored to strike some reasonable balance between a measure of adherence to prevailing English idiom and as close a reproduction of the original in content, style, and even punctuation, as seemed possible. Unlike Ref. 4, the present translation reproduces the numbering of formulas as found in the original. Thus the only changes that are introduced in the translation are in the mathematical notation of a few formulas (for typographical convenience) and in the addition of a few footnotes. The latter are numbered by Arabic numerals, while the footnotes of the original paper are indicated by lowercase Roman letters.

B. TRANSLATION OF THE INTRODUCTION AND THE KINEMATIC PART OF EINSTEIN'S PAPER

The Newtonian equations of motion retain their form upon a transformation to a new coordinate system which is in a state of uniform translational motion relative to the original system, according to the equations

$$x' = x - vt, \quad y' = y, \quad z' = z.$$

As long as one held to the opinion that all of physics could be built upon the Newtonian equations of motion, one could entertain no doubt that the laws of nature come out the same when referred to any one of a family of coordinate systems which move uniformly (free of acceleration) with respect to each other.[5] That independence from the state of motion of the employed coordinate system,[5] which will henceforth be called "the principle of relativity," appeared however all at once open to question through the brilliant confirmations which the electrodynamic theory of H. A. Lorentz has experienced.[a] That theory is founded, namely, on the assumption of a stationary immobile ether: its fundamental equations are not so constructed that they go over into equations of the same form upon application of the above transformation equations.

Since the emergence of that theory one had to expect that one would succeed in demonstrating an influence of the motion of the Earth relative to the luminiferous ether upon optical phenomena. As is known, Lorentz proved indeed in that work that according to his basic assumptions no influence of the relative motion upon the path of a [light][6] ray was to be expected, as long as one restricts oneself in the calculation to terms in which the ratio v/c of that relative velocity to the velocity of light in vacuum enters only in the first power. But the negative result of the Michelson and Morley experiment[b] showed that in a particular case an effect of second order (proportional to v^2/c^2) was also absent, even though according to the foundations of Lorentz's theory it ought to have been noticeable in the experiment.

It is known that that contradiction between theory and experiment was formally removed by the assumption of H. A. Lorentz and G. F. Fitzgerald, according to which moving bodies experience a certain contraction in the direction of their motion. This *ad hoc* assumption appeared however as just an artificial device for saving the theory; the Michelson and Morley experiment had made it quite clear that phenomena obey the principle of relativity also then, when that could not be foreseen from Lorentz's theory. It appeared thus that Lorentz's theory had to be abandoned again, and replaced by a theory whose foundations agreed with the principle of relativity, since such a theory made it possible to foresee the negative result of the Michelson and Morley experiment directly.

Surprisingly, however, it turned out that it was only necessary to grasp the concept of time sharply enough in order to get around the above difficulty. It required only the recognition that the auxiliary quantity introduced by H. A. Lorentz, and called by him "local time," can be defined as simply "time." If one adheres to the indicated definition of time, then the basic equations of Lorentz's theory accord with the principle of relativity, provided only the above transformation equations are replaced by transformation equations that agree with the new time concept. The hypothesis of H. A. Lorentz and G. F. Fitzgerald appears then as a necessary consequence of the theory. Only the idea of a luminiferous ether as the carrier of electric and magnetic forces does not fit in with the theory presented here; for electromagnetic fields do not appear here as states of some kind of matter, but rather as independently existing objects, on a par with matter, and sharing with the latter the characteristic of inertia.

In what follows it is endeavored to present an integrated survey of the investigations which have arisen to date from combining the theory of H. A. Lorentz and the theory of relativity.

In the first two parts of this work are treated the kinematic foundations of the theory of relativity and their application to the basic equations of the Maxwell–Lorentz theory; I am following here the investigations[c] of H. A. Lorentz (Versl. Kon. Akad. v. Wet., Amsterdam 1904), and A. Einstein (Ann. d. Phy. **16,** 1905).

In the first section,[7] in which application is made exclusively of the kinematic foundations of the theory of relativity, I have also treated some optical problems (the Doppler effect, aberration, the dragging of light through moving bodies); my attention has been drawn to the possibility of such a treatment by an oral communication and a paper (Ann. d. Phys. **23,** 989, 1907)[8] of Mr. M. Laue, and by a paper (in need of correction, to be sure) of Mr. J. Laub (Ann. d. Phys. [**23,** 738], 1907).

The third part contains a development of the dynamics of the material point (the electron). For the derivation of the equations of motion I employ the same method as in my above-mentioned work. Force is defined as in Planck's paper. Also taken from this work are the transformations of the equations of motion of a material particle, which make it possible to bring out so clearly the analogy of these equations with the equations of motion of classical mechanics.

The fourth part deals with the general conclusions concerning the energy and momentum of physical systems, to which one is led by the theory of relativity. These have been developed in the following original papers:

> A. Einstein, Ann. d. Phys. **18,** 639, 1905, and Ann. d. Phys. **23,** 371, 1907; as well as M. Planck, Sitzungsber. d. Kgl. Preuss. Akad. d. Wissensch. XXIX, 1907.

They are, however, derived here in a new way, which—as appears to me—allows us to recognize especially clearly the connection between those applications and the foundations of the theory. Also treated here is the dependence of entropy and temperature on the state of motion[9]; on entropy I follow entirely the last cited paper of Planck, while I define the temperature of moving bodies as Mr. Mosengeil does in his paper on moving cavity radiation.[d]

The most important result of the fourth part concerns the inertial mass of energy. This result suggests the question whether energy also possesses gravitational mass. The further question forces itself whether the principle of relativity is restricted to nonaccelerated moving systems. In order not to leave these questions completely out of the discussion, I have added in this essay a fifth part, which includes a new relativistic-theoretic view on acceleration and gravitation.

I. Kinematic part

1. The principle of the constancy of the velocity of light. Definition of time. The principle of relativity

In order to be able to describe any given physical process, it must be possible for us to label spatially and temporally the changes occurring at individual points of space.

For the spatial labeling of a process occurring in a spatial element with infinitely short duration (point-event), we require a Cartesian coordinate system, i.e., three mutually perpendicular and rigidly connected rigid rods together with a rigid unit measuring rod.[e] Geometry permits the determination of the position of a point or of the place of a point-event in terms of three measure-numbers (coordinates x,y,z).[f] For the temporal labeling of a point-event we employ a clock, which is at rest relative to the coordinate system and in whose immediate vicinity the event occurs. The time of the point-event is defined by the simultaneous reading of the clock.

We consider clocks, at rest relative to the coordinate system, arranged at many points. These are all to be equivalent; i.e., the difference of the readings of any two such clocks are to remain unaltered, when they are arranged near each other. If we imagine these clocks stationed in any manner, then provided they are arranged with sufficiently small separations, the ensemble of clocks allows the temporal labeling of an arbitrary point-event—namely, by means of the adjacent clock.

The sum total of these clock-readings does not, however, provide us as yet with a "time," as it is needed for the purposes of physics. We require in addition a rule according to which these clocks are to be set with respect to each other.

We assume now that *the clocks can be so regulated, that the propagation velocity in empty space of every light ray—when measured with these clocks—is everywhere equal to a universal constant c,* provided the coordinate system is not accelerated.[10] If A and B are two points occupied by clocks at rest in the coordinate system and at a distance r apart, and if t_A is the reading of the clock at A when a light ray propagating through a vacuum in the direction AB reaches the point A, and t_B is the reading of the clock at B when the light ray arrives at B, then regardless of the state of motion of the light source or of other bodies, one always has

$$r/(t_B - t_A) = c.$$

That the assumption just made, which we shall call the "principle of the constancy of the velocity of light," is actually satisfied in nature, is not at all self-evident, but it is made probable—at least for a coordinate system in a definite state of motion—by the experimental confirmations of Lorentz's theory,[g] which is based on the assumption of an absolutely stationary ether.[h]

The aggregate of the readings of all the clocks that have been set according to the foregoing specification, and which we can imagine to be at rest at the individual points relative to the coordinate system, we call the time belonging to the coordinate system under consideration, or briefly, the time of this system.

The coordinate system under consideration together with a unit measuring rod and the clocks which serve to establish the time of the system, we call "reference system S." We suppose that the natural laws are established with respect to the system S, which is at first, say, at rest relative to the Sun. Subsequently the system is accelerated for a time by some outside causes, finally reaching again a state of nonaccelerated motion. How will the laws of nature turn out when the processes are referred to a coordinate system[11] S which is now in a different state of motion?

Regarding this question we now make the simplest imaginable assumption, and one that is also suggested by the Michelson and Morley experiment: *The laws of nature are independent of the state of motion of the system of reference, at least if the latter is without acceleration.*[10]

The following discussion will be based on this assumption, which we call the "principle of relativity," as well as on the aforestated principle of the constancy of the velocity of light.

2. General remarks concerning space and time

1. We consider a number of unaccelerated rigid bodies that move alike (i.e., are mutually at rest). According to the principle of relativity we conclude that the laws governing the possible mutual spatial arrangements of these bodies do not change with the change in their common state of motion. From this it follows that the laws of geometry always determine the positional possibilities of rigid bodies in the same way, independently of their common state of motion.[12] Assertions about the shape of a body moving without acceleration[10] have therefore immediate sense. We shall call the shape of a body in the described sense its "geometric shape." The latter is obviously not dependent on the state of motion of a system of reference.

2. According to the definition of time given in Sec. 1 a statement about time has sense only with respect to a reference system in a definite state of motion. It is therefore to be expected (and will be confirmed in what follows) that two distant point-events, which are simultaneous with respect to a reference system S, are in general not simultaneous with respect to a reference system S' in a different state of motion.

3. Let a body consisting of the material points P move in some manner relative to a reference system S. At the time t of S each material point P possesses a definite position in S, i.e., it coincides with a definite point Π at rest relative to S. The aggregate of the positions of the points Π relative to the coordinate system of S, and the aggregate of the mutual positional relations of the points Π, we call respectively the *position*[13] and the *kinematic shape*[13] of the body with respect to S at the time t. If the body is at rest relative to S, then its kinematic shape and its geometric shape are identical.

It is clear that an observer at rest relative to a reference system S, can only ascertain the kinematic shape of a body that is moving relative to S, but not its geometric shape.

In the sequel we shall not usually differentiate explicitly between geometric and kinematic shape; an assertion of geometric content is concerned with the kinematic or the geometric shape, respectively, according as it does or does not relate to a reference system S.

3. Transformation of coordinates and time

Let S and S' be equivalent coordinate systems, i.e., these systems possess unit measuring rods of equal length and clocks running at the same rate, when compared in a state of relative rest. It is then clear that every law of nature which holds with respect to S, holds also with respect to S' in exactly the same form, provided S and S' are at rest with respect to each other. The principle of relativity requires the same perfect agreement also when S' is in a state of uniform translational motion relative to S.[14] Thus, in particular, we must obtain the same value for the velocity of light in vacuum, relative to both coordinate systems.

Let an event[15] be determined relative to S by the variables x,y,z,t, and relative to S' by the variables x',y',z',t', where S and S' are free of acceleration and move with respect to each other. What are the equations that obtain between the two sets of variables?

We can say at once that these equations must be linear in the stated variables, since this is demanded by the homogeneity properties of space and time. From this it follows in particular that the coordinate planes of S'—referred to the reference system S—are uniformly moving planes; although in general these planes will not be mutually perpendicular. However, if we choose the position of the x'-axis so that the latter—when referred to S—has the same direction as that of the translational motion of S' with respect to S, then it follows from symmetry considerations that the coordinate planes of S' referred to S must be mutually perpendicular. In particular, we shall choose, as we may, the positions of the two coordinate systems so that the x-axis of S coincides lastingly with the x'-axis of S', and that the y'-axis of S' referred to S is parallel to the y-axis of S. We shall, further, choose as the time origin of both systems the instant of coincidence of the origins of coordinates; the linear transformation equations we are seeking are then homogeneous.

We now conclude immediately from our knowledge of the position of the coordinate planes of S' relative to S, that every pair of the following set of equations is equivalent:

$$x' = 0 \text{ and } x - vt = 0; \quad y' = 0 \text{ and } y = 0;$$
$$z' = 0 \text{ and } z = 0.$$

Hence three of the sought transformation equations are of the form:

$$x' = a(x - vt), \quad y' = by, \quad z' = cz.$$

Since the velocity of propagation of light in empty space equals c with respect to both reference systems, the two equations

$$x^2 + y^2 + z^2 = c^2t^2$$

and

$$x'^2 + y'^2 + z'^2 = c^2t'^2$$

must be equivalent. From this and from the above-found expressions for x',y',z' one concludes after a simple calculation, that the sought transformation equations must be of the form:

$$t' = \phi(v)\beta[t - (v/c^2)], \quad x' = \phi(v)\beta(x - vt),$$
$$y' = \phi(v)y, \quad z' = \phi(v)z,$$

where we have set

$$\beta = [1 - (v/c)^2]^{-1/2}.$$

We shall now determine the function of v which still remains undetermined. If we introduce a third reference system S'', that is equivalent to S and S', moves with the velocity $-v$ relative to S', and is oriented relative to S' as S' is to S, we obtain by a double application of the equations arrived at above

$$t'' = \phi(v)\phi(-v)t, \quad x'' = \phi(v)\phi(-v)x,$$
$$y'' = \phi(v)\phi(-v)y, \quad z'' = \phi(v)\phi(-v)z.$$

Since the origins of coordinates of S and S'' remain in coincidence, and since the axes have the same orientation and the systems are "equivalent," therefore this transformation is the identity,[i] so that

$$\phi(v)\phi(-v) = 1.$$

Since, moreover, the relationship between y and y' cannot depend on the sign of v,

$$\phi(v) = \phi(-v).$$

Therefore,[j] $\phi(v) = 1$, and the transformation equations read

$$t' = \beta[t - (v/c^2)x], \quad x' = \beta(x - vt),$$
$$y' = y, \quad z' = z, \tag{1}$$

where

$$\beta = [1 - (v/c)^2]^{-1/2}.$$

If one solves the equations (1) with respect to x,y,z,t, one obtains the same equations, except that the "primed" quantities are replaced by the corresponding "unprimed" quantities and conversely, and v is replaced by $-v$. This also follows directly from the principle of relativity and from the consideration that S is in a state of uniform translation with respect to S', with velocity $-v$ in the direction of the X'-axis.

In general, we obtain according to the principle of relativity a correct relationship between "primed" quantities (defined with respect to S') and "unprimed" quantities (defined with respect to S), or between quantities of only one of these types, when we replace the unprimed by the primed symbols and conversely, and also replace v by $-v$.

4. Consequences from the transformation equations that concern rigid bodies and clocks

1. Consider a body at rest relative to S'. Let x_1',y_1',z_1' and x_2',y_2',z_2' be the coordinates of two of its material points referred to S'. Between the coordinates x_1,y_1,z_1 and x_2,y_2,z_2 of these points relative to S, there obtain at each time t of S, according to the above-derived transformation equations, the relations

$$x_2 - x_1 = [1 - (v/c)^2]^{1/2}(x_2' - x_1'),$$
$$y_2 - y_1 = y_2' - y_1', \quad z_2 - z_1 = z_2' - z_1'. \tag{2}$$

The kinematic shape of a body considered to be in a state of uniform translation depends thus on its velocity relative to the reference system; namely, by differing from its geometric shape in being contracted in the direction of the relative motion in the ratio $1:[1 - (v/c)]^{1/2}$. A relative motion of reference systems with superluminal velocity is incompatible with our principles.

2. Suppose that there is a clock at rest at the origin of coordinates of S', which runs ν_0 times faster than the clocks employed in the systems S and S' for the measurement of time, i.e., this clock executes ν_0 periods during a time in which the reading of a clock which is at rest relative to it and is of the nature of the clocks employed in S and S' for the measurement of time, increases by one unit. How fast does the first-mentioned clock run as viewed from S?

The clock under consideration completes each of its cycles at the instants $t_n' = n/\nu_0$, where n runs through the set of integers, and $x' = 0$ for this clock. From this one obtains, with the aid of the first two transformation equations, the values

$$t_n = \beta t_n' = \beta n/\nu_0$$

for the instants t_n when, as viewed from S, the clock completes each of its cycles. As viewed from S, the clock executes therefore $\nu = \nu_0/\beta = \nu_0[1 - (v/c)^2]^{1/2}$ periods per unit of time; or: a clock moving uniformly with the velocity v relative to a reference system, runs when viewed from that system, more slowly in the ratio $1:[1 - (v/c)^2]^{1/2}$ than the same clock when at rest relative to this reference system.

The formula $\nu = \nu_0[1 - (v/c)^2]^{1/2}$ admits of a very interesting application. Mr. J. Stark has shown last year[k] that the ions forming the canal rays emit line spectra, by observing a shift of spectral lines which he interpreted as a Doppler effect.

Since we may well consider the oscillational process corresponding to a spectral line as an intra-atomic process whose frequency its determined solely by the ion, we can consider such an ion as a clock of definite frequency ν_0, which frequency is obtainable, e.g., by examination of the light emitted by identical ions that are all at rest relative to the observer. The above consideration shows then that the influence of the motion on the light frequency to be ascertained by the observer is not yet completely accounted for by the Doppler effect. The motion reduces in addition the (apparent) proper frequency of the emitting ions according to the above relationship.[1]

5. *Addition theorem of velocities*

Let a point move uniformly relative to the system S' according to the equations

$$x' = u_x't', \quad y' = u_y't', \quad z' = u_z't'.$$

Replacing x',y',z',t' by x,y,z,t by means of the transformation equations (1), one obtains x,y,z as functions of t, and hence also the velocity components[16] u_x,u_y,u_z of the point with respect to S. There result thus the equations [correcting a trivial misprint]

$$u_x = \frac{u_x' + v}{1 + (vu_x'/c^2)}, \quad u_y = \frac{[1-(v/c)^2]^{1/2}}{1 + (vu_x'/c^2)} u_y',$$

$$u_z = \frac{[1-(v/c)^2]^{1/2}}{1 + (vu_x'/c^2)} u_z'. \tag{3}$$

The parallelogram law of velocities holds therefore only in first approximation. Setting

$$u^2 = u_x^2 + u_y^2 + u_z^2, \quad u'^2 = u_x'^2 + u_y'^2 + u_z'^2,$$

and denoting by α the angle between the x'-axis (v) and the direction of motion of the point relative to S' (u'), then [correcting a dimensionally obvious misprint]

$$u = [(v^2 + u'^2 + 2vu'\cos\alpha) - (vu'\sin\alpha/c)^2]^{1/2}/[1 + (vu'\cos\alpha/c^2)].$$

If both velocities (v and u') have the same direction, then:

$$u = (v + u')/[1 + (vu'/c^2)].$$

From this equation it follows that from the superposition of two velocities that are smaller than c, there always results a velocity smaller than c. For if we set $v = c - k$, $u' = c - \lambda$, where k and λ are positive and smaller than c, then:

$$u = c(2c - k - \lambda)/(2c - k - \lambda + k\lambda c^{-1}) < c.$$

It follows further that the superposition of the velocity of light c and a "subluminal velocity" yields again the velocity of light c.[17]

From the addition theorem of velocities results the further interesting consequence, that no action can exist which can be utilized for arbitrary signaling and which has a propagation speed greater than that of light in vacuum. In fact, suppose a material strip extended along the x-axis of S, relative to which a certain action can be propagated with the speed W (as judged from the material strip), and let observers who are at rest relative to S be situated both at the point $x = 0$ (point A) and at the point $x = \lambda$ (point B). Let the observer at A send signals to the observer at B by means of the aforementioned action, through the material strip, which is not at rest but moves with the speed v ($< c$) in the direction of the *negative* x-axis. The signal is then, according to the first of equations (3), carried from A to B with the speed $(W - v)/[1 - (Wv/c^2)]$. The time T required for this is therefore [correcting a trivial misprint]

$$T = \lambda[1 - (Wv/c^2)]/(W - v).$$

The speed v can take on any value smaller than c. If therefore, as we have assumed, $W > c$, we can always choose v so that $T < 0$. This result signifies that we must consider as possible a transmission mechanism that allows the intended action to precede the cause. Although from a purely logical point of view this result does not contain, in my opinion, any contradiction, yet it clashes so much with the character of our whole experience, that the impossibility of the assumption $W > c$ appears thereby to be sufficiently proven.

6. *Applications of the transformation equations to some optical problems*

Let the light-vector of a plane light wave propagating in vacuum be proportional to $\sin\omega[t - (lx + my + nz)/c]$ when referred to the system S, and to $\sin\omega'[t' - (l'x' + m'y' + n'z')/c]$ when referred to S'. The transformation equations developed in Sec. 3 require that the following relations hold between the quantities $\omega,l,m,n,$ and ω',l',m',n':

$$\omega' = \omega\beta\left(1 - \frac{lv}{c}\right), \quad l' = \frac{l - (v/c)}{1 - (lv/c)}$$

$$m' = \frac{m}{\beta[1 - (lv/c)]}, \quad n' = \frac{n}{\beta[1 - (lv/c)]}. \tag{4}$$

We shall interpret the formula for ω' in two different ways, according as we consider the observer as moving and the (infinitely distant) light source as stationary, or conversely the first as stationary and the latter as moving.

1. If an observer moves with the speed v relative to an infinitely distant light source of frequency ν, so that the connecting line "light source-observer" forms the angle ϕ with the velocity of the observer, referred to a coordinate system which is at rest relative to the light source, then the frequency ν' of the light as perceived by the observer is given by the equation

$$\nu' = \nu[1 - (v/c)\cos\phi]/[1 - (v/c)^2]^{1/2}.$$

2. If a light source which has the frequency ν_0 when referred to a comoving system, moves so that the connecting line "light source-observer" forms the angle ϕ with the velocity of the light source, referred to a system at rest relative to the observer, then the frequency ν perceived by the observer is given by the equation

$$\nu = \nu_0[1 - (v/c)^2]^{1/2}/[1 - (v/c)\cos\phi]. \tag{4a}$$

The last two equations express the Doppler principle in its general form. The last equation enables us to recognize how the observed frequency of the light emitted (resp. absorbed) by canal rays depends on the velocity of the ions forming the rays and on the direction of sighting.

If one further denotes by ϕ,ϕ' the respective angles between the wave normal (direction of the ray) and the direction of the relative motion of S' with respect to S (i.e., and [the directions of] the x- axis or x'-axis),[18] then the equation for l' assumes the form

$$\cos\phi' = [\cos\phi - (v/c)]/[1 - (v/c)\cos\phi].$$

This equation shows the influence of the relative motion of the observer on the apparent place of an infinitely distant light source (aberration).

We shall now investigate how fast light propagates in a medium moving in the direction of the light ray. Let the medium be at rest relative to the system S' and let the light-vector be proportional to sin $\omega'[t' - (x'/V')]$ and sin $\omega[t - (x/V)]$ respectively, according as the process is referred to S' or to S. The transformation equations yield

$$\omega = \beta\omega'[1 + (v/V')], \quad \omega/V = \beta(\omega'/V')[1 + (V'v/c^2)].$$

Here V' is to be considered as a function of ω' known from the theory of optics of stationary bodies. By dividing the two equations we obtain

$$V = (V' + v)/[1 + (V'v/c^2)],$$

an equation which could have been also obtained by direct application of the addition theorem of velocities.[m] In case V' can be taken as known, the problem is solved completely by the last equation. But when only the frequency (ω) referred to the "stationary" system S can be taken as known, as e.g., in the case of the known experiment of Fizeau, then one has to apply the above two equations together with the connection between ω' and V', in order to determine the three unknowns ω', V', and V.

Again, if G and G' are the group velocities referred to S and S' respectively, then by the addition theorem of velocities

$$G = (G' + v)/[1 + (G'v/c^2)].$$

Since the connection between G' and ω' is provided by the theory of optics of stationary bodies,[n] and since ω' is calculable from ω according to the foregoing, therefore the group velocity G can be computed also when we are given only the frequency of the light referred to S along with the nature and the velocity of motion of the body.

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[a] H. A. Lorentz, *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern,* Leiden, 1895. New edition Leipzig 1906.

[b] A. A. Michelson and E. W. Morley, Amer. Journ. of Science (3) **34,** p. 333, 1887.

[c] The pertinent investigations of E. Cohn also come into consideration, but I have not made here any use of the latter.

[d] Kurd von Mosengeil, Ann. d. Phys. **22,** 867, 1907.

[e] Instead of "rigid" bodies one could just as well speak here and in the sequel of solid bodies free of deforming forces.

[f] In addition one requires auxiliary rods (rulers, compasses).

[g] H. A. Lorentz, *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern.* Leiden 1895.

[h] It is to be noted especially that this theory yields a result for the drag coefficient (Fizeau's experiment) in agreement with experience.

[i] This conclusion is based on the physical assumption that the length of a measuring rod as well as the rate of a clock do not suffer any lasting change by being set in motion and then brought back to rest.

[j] Obviously $\phi(v) = -1$ does not enter into consideration.

[k] J. Stark, Ann. d. Phys. **21,** 401, 1906.

[l] Cf. Sec. 6, Eq. (4a).

[m] Cf. M. Laue, Ann. d. Phys. **23,** 989, 1907.

[n] We have namely [correcting a misprint], $G' = V'/[1 + (dV'/d\omega')/V']$.

[1] A. Einstein, Ann. Phys. (Leipzig) **17,** 891 (1905).

[2] A. Einstein, Jahrb. Radioakt. Elektron. **4,** 411 (1907). Corrections in A. Einstein, Jahrb. Radioakt. Elektron. **5,** 98 (1908).

[3] "On the Principle of Relativity and the Conclusions Drawn Therefrom."

[4] H. M. Schwartz, Am. J. Phys. **45,** 18 (1977).

[5] Implicit is here the tacit assumption that the coordinate systems are "inertial." The same elliptic expression appears also in Ref. 1, but there the explicit reference to this restriction on the coordinate systems is made at the start of the paper (where reference is made to *systems in which the laws of mechanics hold good*).

[6] *Brackets* are used here, as in Ref. 4, to enclose added clarifying words or symbols and, in general, material not found in the original.

[7] The correct word here is "part." However, the original word is "Abschnitt," namely, "section"—an obvious (and trivial) inaccuracy.

[8] Obviously, in these citations, as in all others to follow, the last number represents the year.

[9] One understands here, of course, the state of motion of the physical systems under observation, relative to a specified coordinate system.

[10] That is, not accelerated relative to an inertial reference frame. See Ref. 5.

[11] There is possibly a prime missing from the letter S.

[12] The phrase "state of motion" here and in the sequel represents obviously an abbreviation for the more complete statement: "state of uniform rectilinear motion relative to an inertial reference frame."

[13] Italics not in the original.

[14] Omission of mention at this point that S is assumed to be an inertial reference system is reminiscent of a similar omission in Einstein's second formulation of the principle of relativity in Ref. 1, Sec. 2.

[15] We translate henceforth "Punktereignis" simply as "event."

[16] The original contains an apparent minor slip of the pen, namely, the replacement here and on the fourth line of p. 423 of the letter u by the letter w (the latter occurring in the same connection in Ref. 1).

[17] This paragraph is essentially identical in content with the corresponding material in Sec. 5 of Ref. 1, and footnote 29 of Ref. 4 is also relevant here.

[18] This phrase [in the original: *Nennt man ferner ϕ bezw. ϕ' den Winkel zwischen der Wellennormalle (Strahlrichtung) und der Richtung der Relativbewegung von S' gegen S (d.h. mit der x- bezw. x'-Achse)*] is obviously incomplete. One must add: "as measured in S,S' respectively." [The formula that follows and Eq. (4a) are obviously equivalent to the second and first of Eqs. (4).]

Einstein's comprehensive 1907 essay on relativity, part II

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This continuation of the English rendition of Einstein's 1907 essay on relativity, of which the first part appeared in the June 1977 issue of this Journal, is devoted to Parts II-IV of the essay, dealing with the relativistic treatments of electrodynamics, optics, mechanics, and thermodynamics. The original text of these parts covers 27 printed pages. However, owing to the nature of the subject matter and the character of the original exposition, it was possible to reproduce here the essential content of the original text intact in terms of a free rendition, using modern notation, and retaining all the original formulas with their numbering, and also including direct translations of all passages of possible historical interest. Mathematical amplifications of a few of the key derivations in the original text are presented in added footnotes.

A. INTRODUCTION

This is the second part of a three-part English rendition of Einstein's 1907 memoir on relativity.[1] As noted in the introductory remarks to the first part,[2] in addition to possessing obvious historical interest, Einstein's memoir is also of considerable didactic interest. This is particularly true of the portion of the memoir presented here, consisting of Parts II–IV, which contain concise and instructive discussion of the relativistic treatment of a number of basic topics in electrodynamics, optics, mechanics, and thermodynamics.

Because Parts II–IV of Einstein's paper are, with only a few possible exceptions,[3] mainly important for the physical results and the methods of their derivation, their essential content can be represented accurately, as is done here, without resorting to a complete direct translation.[4] But in those instances where any doubt existed at all whether an original idea would be strictly conveyed by a free rendition, or where possible historical interest might attach to the original phraseology, the corresponding passages or phrases have been translated fully, and these are enclosed in quotation marks. On the other hand, the free rendition of the rest of the material, including the employment of direct vector notation, has resulted in increased compactness in the presentation, without any essential loss in clarity.

Except for the notation, all the formulas in the original text are reproduced together with their original numbering; and Eqs. (1)–(4) and Secs. 1–6 to which reference is made in this paper, are contained in Ref. 2. In a few instances, for convenience of reference, originally unnumbered formulas have been indicated by asterisks. All original footnotes are labeled by lower-case Roman letters, and the added footnotes by Arabic numerals. The latter are largely concerned with expanding or clarifying mathematical steps in derivations presented in the original text.

B. SUMMARY OF PARTS II–IV OF EINSTEIN'S 1907 MEMOIR

II. Electrodynamic Part

7. Transformation of the Maxwell–Lorentz Equations[5]

The equations

$$(\rho \mathbf{u} + \partial \mathbf{E}/\partial t)/c = \nabla \times \mathbf{H}, \tag{5}$$

$$\partial \mathbf{H}/c\partial t = -\nabla \times \mathbf{E}, \tag{6}$$

where[6] **E**,**H** are, respectively, the vectors of electric and magnetic "field-strength," and

$$\rho = \nabla \cdot \mathbf{E} \tag{*}$$

is the "4π-fold density of electricity,"[7] together with the "assumption that the electric charges are immutably bound to small rigid bodies (ions, electrons), form the foundation of Lorentz's electrodynamics and optics of moving bodies."

The reference systems S and S' being defined as in Sec. 3, and taking Eqs. (5) and (6) [as well as Eqs. (*) and (**)] to hold with respect of S, an application of the transformation Eqs. (1) connecting S and S', yields equations relative to S' of the same form as Eqs. (5) and (6), i.e.,

$$(\rho' \mathbf{u}' + \partial \mathbf{E}'/\partial t')/c = \nabla' \times \mathbf{H}', \tag{5'}$$

$$\partial \mathbf{H}'/c\partial t' = -\nabla' \times \mathbf{E}', \tag{6'}$$

provided [introducing now the current notation: $\beta \equiv v/c$, $\gamma \equiv (1-\beta^2)^{-1/2}$][8]

$$E'_x = E_x, \quad E'_y = \gamma(E_y - \beta H_z), \quad E'_z = \gamma(E_z + \beta H_y). \tag{7a}$$

$$H'_x = H_x, \quad H'_y = \gamma(H_y + \beta E_z), \quad H'_z = \gamma(H_z - \beta E_y). \tag{7b}$$

$$\rho' = \nabla' \cdot \mathbf{E}' = \gamma(1 - v u_x/c^2)\rho. \tag{8}$$

$$u'_x = (u_x - v)/(1 - u_x v/c^2),$$
$$u'_y = u_y/\gamma(1 - u_x v/c^2), \; u'_z = u_z/\gamma(1 - u_x v/c^2). \tag{9}$$

In arriving at this conclusion, one must show that a resulting arbitrary factor depending on v which can be applied to $\mathbf{E}'$ and $\mathbf{H}'$, is necessarily unity, and this "can be easily shown in a way similar to that used in Sec. 3 in connection with the function $\phi(v)$."

The above result combined with the principle of relativity implies that $\mathbf{E}'$,$\mathbf{H}'$ represent, respectively, the vectors of "electric and magnetic field-strength referred to S'." Moreover, comparison of Eqs. (3) and (9) shows that $\mathbf{u}'$ is the velocity of the electric particles relative to S', and hence ρ' is the density of electricity with respect to S'. "The electrodynamic foundation of the Maxwell–Lorentz theory agrees, thus, with the principle of relativity."

In interpreting Eqs. (7a), it should be observed that in accordance with the principle of relativity, and the manner of determining with the aid of Coulomb's law the magnitude of a given electric charge when at rest relative to S or relative to S', it can be concluded that the two respective magnitudes must be identical. "This conclusion is based moreover on the assumption, that the magnitude of an electric quantity is independent of its prior course of motion."

Equations (7a) and (7b) show that there is no absolute significance attaching to **E** and **H** separately, but that their relative roles in a given electromagnetic phenomenon depend in general on the choice of reference system. In particular, the "electromotive" forces acting on a charge which is moving in a magnetic field can be represented by *electric* forces referred to the *rest frame* of the charge.

In agreement with the result obtained earlier concerning the invariance of the magnitude of an electric charge, we find from Eq. (8) that if a charged body is at rest relative to S', so that its total charge e' is given by the integral $\int \rho' dx'dy'dz'/4\pi$, and if e is the total charge of the body at a fixed time t of S, then

$$e' = e.$$

In fact, by Eq. (1) it follows that for t constant, $dx'dy'dz' = \gamma dxdydz$, while by Eq. (8) applied to the present case (when $u_x = v$), $\rho' = \rho/\gamma$.

"With the aid of Eqs. (1), (7), (8), and (9), all problems in the electrodynamics and optics of moving bodies, for which an essential role is played only by velocities and not by accelerations, can be reduced to a series of problems in the electrodynamics and optics of stationary bodies."

As an optical application, let us consider a plane wave of light in vacuum, having in S the representation

$$\mathbf{E} = \mathbf{E}_0 \sin\Phi, \quad \mathbf{H} = \mathbf{H}_0 \sin\Phi, \quad \Phi = w[t - (\mathbf{n}\cdot\mathbf{x}/c)],$$

and hence by Eqs. (1) and (7), having in S' the representation,

$$E'_x = E^0_x \sin\Phi', \quad E'_y = \gamma(E^0_y - \beta H^0_z)\sin\Phi',$$
$$E'_z = \gamma(E^0_z + \beta H^0_y)\sin\Phi',$$
$$H'_x = H^0_x \sin\Phi', \quad H'_y = \gamma(H^0_y + \beta E^0_z)\sin\Phi',$$
$$H'_z = \gamma(H^0_z - \beta E^0_y)\sin\Phi',$$
$$\Phi' = w'[t' - (\mathbf{n}'\cdot\mathbf{x}'/c)].$$

That the wave normal and the electric and magnetic field vectors in S' are mutually perpendicular, and the latter two have equal magnitudes, is a direct consequence of Eqs. (5′) and (6′). The results stemming from the equality of Φ and Φ' have already been discussed in Sec. 6. We now determine the amplitude A' and the state of polarization of the wave in S'.

To this end we take the wave normal **n** parallel to the x,y plane and choose at first the axes in S so that,

$$E^0_x = 0, \quad E^0_y = 0, \quad E^0_z = A,$$
$$H^0_x = -A\sin\Phi, \quad H^0_y = -A\cos\Phi, \quad H^0_z = 0,$$

ϕ being the angle between **n** and the x axis. Then (correcting an obvious misprint),

$$E'_x = 0, \quad E'_y = 0, \quad E'_z = \gamma(1 - \beta\cos\phi)A\sin\Phi',$$
$$H'_x = -A\sin\phi\sin\Phi', \quad H'_y = \gamma(-\cos\phi + \beta)A\sin\Phi',$$
$$H'_z = 0.$$

Hence,

$$A' = \gamma(1 - \beta\cos\phi)A. \qquad (10)$$

"This relation holds obviously also in the special case when the *magnetic* field is perpendicular to the directions of the relative motion and of the wave normal. Since by the superposition of these two special cases we can construct the general case, it follows that the relation (10) has general validity when we introduce a new reference system S', and that the angle between the plane of polarization and the plane parallel to the directions of the relative motion and of the wave normal, is the same in both reference systems."[9]

III. Mechanics of a Material Point (of an Electron)

8. *Derivation of the Equations of Motion of a (Slowly Accelerated) Material Point or Electron*

"Let a particle supplied with an electric charge ϵ (to be called 'electron' in the sequel) move in an electromagnetic field; we assume the following about its law of motion:

"If at a given instant of time the electron is at rest with respect to a (nonaccelerated) system S', then its motion relative to S' during the next instant proceeds according to the equations"[10]

$$m(d^2\mathbf{x}'/dt'^2) = e\mathbf{E}', \qquad (*)$$

where m [μ in the original] is "a constant which we call the mass of the electron." Applying Eqs. (1) and (7a), and using the fact that at our initial instant we have in S the equations $\dot{x} \equiv dx/dt = v$, $\dot{y} = \dot{z} = 0$, we find at first in a few simple steps that

$$m\gamma^3\ddot{x} = eE_x, \quad m\gamma\ddot{y} = e(E_y - \beta H_z),$$
$$m\gamma\ddot{z} = e(E_z + \beta H_y). \qquad (**)$$

Then, writing $q^2 = |\dot{\mathbf{x}}|^2$, replacing v in β by $\dot{x}$, and introducing into "the appropriate places [of (**)] the terms obtained by cyclic interchange from" $\dot{x}H_z/c$ and $-\dot{x}H_y/c$, "which vanish in the considered special case," we obtain generally[11]:

$$\frac{d}{dt}\left[m\dot{\mathbf{x}}\Big/\left(1 - \frac{q^2}{c^2}\right)^{1/2}\right] = \mathbf{K}, \qquad (11)$$

where

$$\mathbf{K} = e\{\mathbf{E} + [(\dot{\mathbf{x}} \times \mathbf{H})/c]\}. \qquad (12)$$

The vector **K** will be called "the *force* acting on the material point." When q^2/c^2 can be neglected, Eq. (11) shows that **K** goes over into the Newtonian force, and it will be seen in Sec. 9 that in relativistic mechanics this vector generally performs the functions of *force* in classical mechanics.

Equation (11) will be retained also when the force is not of electromagnetic origin. "In this case equations (11) do not have any physical content, but have to be considered as equations of definition of force."[12]

9. *Motion of a Mass Point and Mechanical Principles*

When Eqs. (5) and (6) are multiplied scalarly by $\mathbf{E}/4\pi$ and $\mathbf{H}/4\pi$, respectively, and the results added and integrated, one obtains the *energy conservation* equation

$$\int(\rho/4\pi)\mathbf{u}\cdot\mathbf{E}\, d^3x + dE_e/dt = 0, \qquad (13)$$

where

$$E_e = (1/8\pi) \int (\mathbf{E}^2 + \mathbf{H}^2) d^3x,$$

"is the electromagnetic energy of the space [i.e., region] under consideration," and the integrand in Eq. (13) is the electromagnetic energy absorbed by the electrical substance per unit volume and unit time.

In particular, the contribution of an electron to the first term of Eq. (13) is $e\mathbf{E} \cdot \dot{\mathbf{x}}$, where $\mathbf{E}$ is the external electric field, i.e., the field excluding that due to the electron itself. By Eq. (12) this expression equals $\mathbf{K} \cdot \dot{\mathbf{x}}$. "Thus the vector $\mathbf{K}$ designated as 'force' in the preceding section bears the same relation to work done as in Newtonian mechanics." On the other hand, by Eq. (11),[13]

$$\int \mathbf{K} \cdot \dot{\mathbf{x}}\, dt = \gamma_q mc^2 + \text{const}, \qquad (14)$$

introducing [in this rendition] the abbreviation

$$\gamma_q \equiv (1 - q^2/c^2)^{-1/2}.$$

Thus, the equations of motion (11) are consistent with the principle of mechanical energy conservation, with the right-hand side of Eq. (14) representing "the kinetic energy of the material point (the electron)."

That they are also consistent with the principle of conservation of momentum is seen by taking the vector products of Eqs. (5) and (6) by $\mathbf{H}/4\pi$ and $-\mathbf{E}/4\pi$, respectively, then adding and integrating "over a region on whose boundaries the field strengths vanish"[14]

$$\frac{d}{dt} \int (1/4\pi c)(\mathbf{E} \times \mathbf{H}) d^3x + \int (\rho/4\pi)[\mathbf{E} + (\mathbf{u} \times \mathbf{H})/c] d^3x = 0, \qquad (15)$$

or, according to Eq. (12),

$$\frac{d}{dt} \int (1/4\pi c)(\mathbf{E} \times \mathbf{H}) d^3x + \sum \mathbf{K} = 0. \qquad (15a)$$

"If the charges are bound to freely moving material points (electrons)," then by applying Eqs. (12) and (11) to the second integral in Eq. (15), we obtain the equation

$$\frac{d}{dt}\left[\int \frac{1}{4\pi c} (\mathbf{E} \times \mathbf{H}) d^3x + \sum \gamma_q m \dot{\mathbf{x}} \right] = 0. \qquad (15b)$$

Thus the expression $\boldsymbol{\xi} = \gamma_q m \dot{\mathbf{x}}$ "plays the role of the momentum of a material point, and according to Eq. (11), as in classical mechanics," $d\boldsymbol{\xi}/dt = \mathbf{K}$.

From Eq. (11) it is also seen immediately that the equations of a particle can be put in the Lagrangian form [using here Einstein's notation for the Lagrangian]

$$(d/dt)(\partial H/\partial \dot{\mathbf{x}}) = \mathbf{K},$$

where

$$H = -mc^2/\gamma_q + \text{const}.$$

They can also be expressed in terms of Hamilton's principle [using Einstein's symbols again here and in the sequel, except for retaining "m" and vector notation]

$$\int_{t_0}^{t_1} (dH + A) dt = 0; \quad A = \mathbf{K} \cdot \partial \mathbf{x}, \quad \text{the virtual work.}$$

Upon introducing the momentum vector

$$\boldsymbol{\xi} = \partial H/\partial \dot{\mathbf{x}},$$

and the kinetic energy of the particle[15]

$$L = mc^2[1 + (\xi^2/m^2c^2)]^{1/2} + \text{const},$$

the Hamiltonian canonical equations assume the form

$$d\boldsymbol{\xi}/dt = \mathbf{K}, \quad d\mathbf{x}/dt = \partial L/\partial \boldsymbol{\xi}.$$

10. On the Possibility of an Experimental Check of the Theory of Motion of the Material Point. Kaufmann's Investigation

There is presented here (on pp. 436–439) a discussion of Kaufmann's experiments [published in Ann. Phys. **19**, 487 (1906)] on the deflection of β rays of "radium-bromide" by constant electric and magnetic fields, with a reproduction of diagrams of the apparatus and of the deflection graph. With reference to the latter Einstein remarks:

"Considering the difficulty of the investigation one might be inclined to take the agreement [with the theory of relativity] as adequate. However, the existing deviations are systematic and substantially outside the limits of error of Kaufmann's investigation. That M. Kaufmann's calculations are free of mistakes follows from their agreeing throughout with the results obtained by M. Planck using a different method of calculation.[a]

"Whether the systematic deviations have their basis in an as yet unrecognized source of error or in the fact that the foundations of the theory of relativity do not correspond to the facts, can be decided with certainty only when a greater variety of observational material becomes available.

"It should be observed further that the theories of electron motion of Abraham[b] and of Bucherer[c] yield curves which fit the observed curves substantially better than the curves deduced from the theory of relativity. But it is my opinion that scant plausibility attaches to those theories, because their basic assumptions which concern the mass of the moving electron are not suggested by theoretical systems that encompass wider complexes of phenomena."

IV. On the Mechanics and Thermodynamics of Systems

11. On the Dependence of Mass upon Energy

We consider a physical system surrounded by an enclosure that is impermeable to radiation, which is floating freely in space "and is subject to no other forces except the action of electric and magnetic forces in the surrounding space." By Eq. (13), the total energy thus absorbed by the system [in the time interval (t_1,t_2)—not indicated explicitly in the original paper] is

$$\int_{t_1}^{t_2} dE = \int_{t_1}^{t_2} dt \int (\rho/4\pi)\mathbf{E}_a \cdot \mathbf{u}\, d^3x,$$

where $\mathbf{E}_a$ denotes the exterior electric field. Applying the inverses of the transformations (7a), (8), and (9), and using the fact that the Jacobian $\partial(x_i',t')/\partial(x_i,t) = 1$, one finds[16]

$$\int_{t_1}^{t_2} dE = \gamma \int_{t_1}^{t_2} \int (\rho'/4\pi)\{\mathbf{E}_a' \cdot \mathbf{u}' + v[E_x'^{(a)} + (\mathbf{u}' \times \mathbf{H}_a')_x/c]\} d^3x'\, dt'.$$

By the principle of relativity and application of Eq. (12) it follows then "in easily understood notation" that

$$dE = \gamma dE' + \gamma v \int \sum K_x'\, dt'. \qquad (16)$$

Suppose now that our system is at rest as a whole relative to S', and that its parts move relative to S' so slowly that the squares of their velocities relative to S' can be neglected with respect to c^2. It then follows by Newtonian mechanics that $\Sigma K'_x = 0$ for every t'. However, the integral in Eq. (16) does not necessarily vanish, since the time interval is taken between t_1 and t_2. But if it is assumed that no external forces act on the system before t_1 and after t_2, then this integral will certainly vanish, so that

$$dE = \gamma dE'. \qquad (*)$$

From this result it can be deduced that "the energy of a (uniformly) moving system, that is not under the influence of external forces," can be written as

$$E = [m + (E_0/c^2)]c^2\gamma_q, \qquad (16a)$$

[γ_q defined in Sec. 10, Eq. (*)], where m is the mass of the physical system such as occurs in Eq. (14),[17] $E_0 \equiv E'$ (the subscript 0 indicating, henceforth, quantities of the physical system referred to a *comoving* coordinate frame), and $\mathbf{q}$ is the velocity of translation of this system relative to S. In fact, E depends on E_0 and $\mathbf{q}$, and by Eq. (*), $\partial E/\partial E_0 = \gamma_q$, so that $E = \gamma_q E_0 + \phi(\mathbf{q})$. "The case when $E_0 = 0$, i.e., when the energy of the moving system is a function of the *velocity q alone,* we have already investigated in Secs. 8 and 9." [See Eq. (14).] "We obtain thus Eq. (16a), where we omit the integrating constant."[18] Comparison of Eqs. (16a) and (14) shows that our physical system behaves, as far as concerns the dependence of its energy upon its velocity of translation, as a particle of mass M given by the formula

$$M = m + (E_0/c^2). \qquad (17)$$

"This result is of extraordinary theoretical importance, for in it the inertial mass and the energy of a physical system appear as similar things. A mass μ is equivalent, as regards inertia, to a quantity of energy μc^2. Since we can dispose of the zero point of E_0 arbitrarily, we are not at all in a position to differentiate without arbitrariness between a 'true' and an 'apparent' mass of the system. It appears far more natural to consider every inertial mass as a store of energy."

As to the practical possibility of checking this result, it appears for the present to be negligible. "The decrease, e.g., in the mass of a system, which gives up 1000 gram-calories, amounts to 4.6×10^{-11} g." Indeed "during the radioactive decay of a substance enormous amounts of energy are freed." But "Mr. Planck writes concerning this" that from the measurements of Precht [Ann. Phys. **21,** 599 (1906)] it can be concluded that the mass equivalent of the energy emitted by a gram-atom of radium even during a year is only 0.012 mg, and is thus still too small a part of the mass of the substance to be detected. But (according to Einstein) one need not exclude the possibility that "radioactive processes will become known for which a considerably larger percentage of the mass of the original atom is converted into energy of different radiations than is the case for radium . . ."

"In the preceding it is tacitly assumed that such a change of mass can be measured by the instrument usually employed for the measurement of masses, namely, the balance; and that therefore the relation

$$M = m + (E_0/c^2)$$

holds not only for inertial mass, but also for gravitational mass; or in other words, that under all circumstances inertia and gravitation due to a system are strictly proportional. We should, therefore, also have to assume, for example, that cavity radiation possesses not only inertia, but also weight. But that proportionality between inertial and gravitational mass holds without exception for all bodies within the accuracy so far attained, so that until the contrary is proved its general validity must be accepted. We shall moreover encounter in the last section of this essay a new argument in support of the assumption."

12. Energy and Momentum of a Moving System

We return to the physical system discussed at the beginning of Sec. 11, and apply to the external fields $\mathbf{E}_a$, $\mathbf{H}_a$, the considerations that led to Eq. (15), thus obtaining the relation

$$\frac{d}{dt}\left[\int \frac{1}{4\pi c}(\mathbf{E}_a \times \mathbf{H}_a)_x\, d^3x\right] + \int \frac{\rho}{4\pi}\left[\mathbf{E}_a + \frac{1}{c}(\mathbf{u} \times \mathbf{H}_a)\right]_x d^3x = 0.$$

"We will now assume that the law of conservation of momentum has general validity. Then the part of the second term of this equation, extended over the [interior of the] envelope of the system, must be representable as the derivative with respect to time of a quantity G_x completely determined by the instantaneous state of the system, which we shall designate as the X-component of the momentum of the system." By applying Eqs. (1), (7)–(9), we find as in Sec. 11 (and using the same elliptic notation) that

$$\int dG_x = \gamma \int \int \frac{\rho'}{4\pi}\left\{\left[\mathbf{E}'_a + \frac{1}{c}(\mathbf{u}' \times \mathbf{H}'_a)\right]_x + \frac{\beta}{c}\mathbf{E}'_a \cdot \mathbf{u}'\right\} d^3x'dt'.$$

Hence,

$$dG_x = (\gamma\beta/c)dE' + \gamma \textstyle\int \sum K'_x\, dt'. \qquad (18)$$

With considerations analogous to those underlying the discussion following Eq. (16), the last term in Eq. (18) can be taken as null, and one can conclude that when no external forces act on the system, so that its momentum depends only on the energy E_0 of the system relative to the comoving coordinate frame and on the velocity $\mathbf{q}$ of translation of the latter, one has the vector relation[19]

$$\partial \mathbf{G}/\partial E_0 = \gamma_q \mathbf{q}/c^2 \qquad [\gamma_q \equiv (1 - q^2/c^2)^{-1/2}].$$

By integration and reference to Eq. (15b), it follows then by steps analogous to those leading to Eq. (16a) that

$$\mathbf{G} = \gamma_q[m + (E_0/c^2)]\mathbf{q}, \qquad (18a)$$

a result consistent with the analogous result Eq. (16a).

When no restriction is made on the existence of external forces, the last term in Eq. (18) can no longer be dropped, since the time integration is between the limits t_1, t_2, but the calculation can be made tractable by first breaking up the time interval of integration into three parts $[(t_1/\gamma) - (\beta x'/c), t_1/\gamma]$, $(t_1/\gamma, t_2/\gamma)$, $[t_2/\gamma, (t_2/\gamma) - (\beta x'/c)]$, and introducing the assumption that the forces vary little in periods that are of the order of magnitude of $\beta x'/c$ (otherwise "under the application of the basic assumptions employed here, we could not speak at all of an energy or a momentum of the system."[d]) In the second interval $\Sigma K'_x = 0$, and in the first and third, by our assumption, K'_x can be taken as constant. Hence,

$$\int \Sigma K'_x\, dt' = (\beta/c)(\Sigma x'K'_x)_1 - (\beta/c)(\Sigma x'K'_x)_2 = -(\beta/c)d(\Sigma x'K'_x),$$

and the solutions of Eqs. (16) and (18) consist in the addition to Eqs. (16a) and (18a) of the respective terms

$$-\gamma_q(q^2/c^2)\Sigma\delta_0 K_{0\delta}, \quad -\gamma_q(\mathbf{q}/c^2)\Sigma\delta_0 K_{0\delta},$$

[yielding Eqs. (16b), (18b), respectively, in the original text], "where $K_{0\delta}$ is the component of the force in the direction of motion referred to a comoving reference system, and δ_0 represents the distance as measured in this system between the point of application of that force and a plane perpendicular to the direction of motion."

In particular, if the external force is a homogeneous normal pressure acting on the surface of our system, so that

$$\Sigma\delta_0 K_{0\delta} = -p_0 V_0, \tag{19}$$

where according to our notational convention p_0, V_0 are the pressure and the volume of the system relative to a comoving reference system,[20] then

$$E = \gamma_q[mc^2 + E_0 + (q^2/c^2)p_0V_0], \tag{16c}$$

$$\mathbf{G} = \gamma_q[m + (E_0 + p_0V_0)/c^2]\mathbf{q}. \tag{18c}$$

13. Volume and Pressure of a Moving System. Equations of Motion

It is an immediate consequence of Eq. (2) that

$$V = V_0/\gamma_q, \tag{20}$$

where V is the volume of the system relative to S. To find the connection between p and p_0, we find first the Lorentz transformation of the components of a force of any type. In view of the last assertion in Sec. 8, it suffices to consider the special case of an electromagnetic force.[e] By a consideration of a charge e that is at rest in S', it follows by a straightforward inspection of Eqs. (12) and (7a) that

$$K'_x = K_x, \quad K'_y = \gamma K_y, \quad K'_z = \gamma K_z. \tag{21}$$

Suppose now that s' is a surface element at rest relative to S' and $\mathbf{n}'$ is the unit vector normal to it and directed towards the interior of the body. The components of the corresponding force (in S') are then (putting $K_x \equiv K_1$, etc.)

$$K'_i = p's'n'_i \equiv p's'_i \quad (i = 1,2,3).$$

But by Eqs. (2),

$$s'_1 = s_1, \quad s'_j = \gamma s_j \quad (j = 2,3).$$

Hence, combining these two sets of equations and Eq. (21), we find at once, since $K_i = ps_i$, that $p' = p$, i.e.,

$$p = p_0. \tag{22}$$

Equations (16c), (20), and (22) enable us to represent E_0, V_0, p_0 in terms of E, V, p, and $\mathbf{q}$. In particular Eq. (18c) can be written as[21]

$$\mathbf{G} = \mathbf{q}(E + pV)/c^2 \qquad \text{[corrected]}, \tag{18d}$$

"which equation in conjunction with the equation

$$d\mathbf{G}/dt = \sum \mathbf{K},$$

expressing the principle of conservation of momentum, determine completely the translational motion of the system as a whole," provided $\Sigma\mathbf{K}$ and the set E,p,V, or an equivalent set, are known functions of t.

14. Examples

Consider a physical system consisting of electromagnetic radiation enclosed in a "massless" container. If no external forces act on the system, then by Eqs. (16a) and (18a),

$$E = \gamma_q E_0, \quad \mathbf{G} = \gamma_q E_0 \mathbf{q}/c^2 = E\mathbf{q}/c^2.$$

But if the container walls are "perfectly flexible and extensible," so that an outside pressure must be applied to them to balance the pressure exerted by the radiation, then by Eqs. (16c) and (18c), and using the well-known formula [correcting an obvious misprint]

$$p_0 = E_0/3V_0,$$

we have,

$$E = \gamma_q[1 + (\mathbf{q}^2/3c^2)]E_0, \quad \mathbf{G} = \gamma_q(4E_0/3c^2)\mathbf{q}.$$

"We consider, further, the case of an electrically charged massless body. If no external forces act upon it, we can apply again Eqs. (16a) and (18a)." We then find the first set of equations of this section.

"Of these values a part derives from the electromagnetic field, and the rest from the massless body under the action of the forces arising from its charge."[22]

15. Entropy and Temperature of Moving Systems

The following results are presented for the transformation of entropy η, heat Q, and absolute temperature T:

$$\eta = \eta_0, \tag{25}$$

$$dQ = dQ_0/\gamma_q, \tag{26}$$

$$T = T_0/\gamma_q. \tag{27}$$

For the proof of Eq. (25), Einstein cites verbatim the words of Planck.[23] Then using the known expression

$$dQ = dE + pdV - \mathbf{q}\cdot d\mathbf{G} \tag{23}$$

for the heat received by a body, and the known thermodynamic relation

$$dQ = Td\eta \tag{24}$$

holding for reversible processes, an application of Eqs. (16c), (18c), (20), and (22) yields Eqs. (26) (since $dQ_0 = dE_0 + p_0dV_0$), and hence Eq. (27).

16. Dynamics of Systems and the Principle of Least Action

Since in the paper cited in Sec. 15 Planck obtains results identical to those obtained in this work, but on the basis of the principle of least action instead of the principles of conservation of energy and momentum employed in this work, it is of interest to establish the connection between these principles.

Consider, then, a system whose state is determined by the variables, $\mathbf{q}$, V, and T. For reversible processes the two conservation principles imply the relations

$$dE = \mathbf{F}\cdot d\mathbf{x} - pdV + Td\eta \tag{28}$$

and

$$\mathbf{F} = d\mathbf{G}/dt, \tag{29}$$

where $\mathbf{F}$ is the resultant force acting on the system.[24] But

$$\mathbf{F}\cdot d\mathbf{x} = \mathbf{F}\cdot \dot{\mathbf{x}}dt = \dot{\mathbf{x}}\cdot d\mathbf{G} = d(\dot{\mathbf{x}}\cdot\mathbf{G}) - \mathbf{G}\cdot d\dot{\mathbf{x}},$$

and $\dot{\mathbf{x}} = \mathbf{q}$. Hence,

$$d(-E + T\eta + \mathbf{q}\cdot\mathbf{G}) = \mathbf{G}\cdot d\dot{\mathbf{x}} + pdV + \eta dT.$$

Since the left-hand side of this equation is the complete differential of the Lagrangian H of our system, it yields, using Eq. (29), the equations

$$\frac{d}{dt}\frac{\partial H}{\partial \dot{\mathbf{x}}} = \mathbf{F}, \quad \frac{\partial H}{\partial V} = p, \quad \frac{\partial H}{\partial T} = \eta.$$

These are the relations derived by Planck with the aid of the principle of least action, which have served as his starting point. [Planck, *loc. cit.*, p. 549, Eqs. (6) and (7).]

ACKNOWLEDGMENTS

The author is indebted to the referees for helpful suggestions, and to Dr. Otto Nathan for permission to publish this rendition.

[a]Cf. M. Planck, Verhandl. d. Deutschen. Phys. Ges. **VIII.** Jahrg. Nr. 20, 1906; **IX.** Jahrg. Nr. 14, 1907.

[b]M. Abraham, Gött. Nachr. 1902.

[c]A. H. Bucherer, *Math. Einführung in die Elektronentheorie*, p. 58, Leipzig 1904.

[d]Cf. A. Einstein, Ann. d. Phys. **23**, § 2, 1907. [Both in this paper, which develops under restricted conditions the essential reasoning underlying the discussions in Secs. 11 and 12, as well as in these sections, the integral expressions for the total force on the charges of the system are replaced by *sums*, reflecting adherence to the idea of the atomicity of matter.]

[e]This circumstance provides also the justification for the procedure employed in the preceding investigations, which consisted in our introducing only interaction of a *purely electromagnetic* kind between the system under consideration and its environment. The results hold quite generally.

[1]A. Einstein, Jahrb. Radioakt. Elektron. **4**, 411 (1907); Corrections, *ibid.* **5**, 98 (1908).

[2]H. M. Schwartz, Am. J. Phys. **45**, 512 (1977).

[3]Notably, the remarks bearing on the inertia of energy.

[4]The interested professional historian of science will naturally in any case consult the original text.

[5]This appropriate designation of Eqs. (5), (6), (*), and the tacitly assumed equation $\nabla\cdot\mathbf{H} = 0$ [Eq. (**)] occurs here perhaps for the first time. In his first paper on relativity, Einstein refers to them as "Maxwell–Hertz equations with convection currents taken into account" [A. Einstein, Ann. Phys. **17**, 891 (1905), p. 916]. Why is Eq. (**) not included in the Maxwell–Lorentz set? The noninclusion of Eq. (*) in the *displayed* set of these equations would suggest that perhaps Einstein simply took Eq. (**) for granted, because he considered the nonexistence of magnetic monopoles as a universally established fact. As is shown in Ref. 8, Eq. (**) is involved in the deduction of Eqs. (7a) and (7b).

[6]The symbol **B**, employed in H. M. Schwartz, Am. J. Phys. **39**, 1287 (1971), in a similar context, is here interchangeable with **H**, and whereas the former is currently the more common symbol, the latter has stronger historical associations.

[7]So that despite appearances, it is Gaussian, and not Lorentzian units, which Einstein employs.

[8]The proof is sketched out in Einstein's first paper on relativity (reference in Ref. 5), Secs. 6 and 9. Section 6 treats the transformation of Maxwell's equations in free space under the Lorentz transformations (1). The deduction of the fourth (and similarly the first) of the last set of equations on p. 908 of this paper, is not as immediate as that of the other equations of the set, and use of Eq. (**) is essential, as follows (setting $x_0 = ct$ and $\beta = v/c$): $\partial H_x/\partial x_0 = \gamma(\partial/\partial x_0' - \beta\partial/\partial x')H_x = \partial E_y/\partial z - \partial E_z/\partial y$; $\partial H_x/\partial x_0' = (\partial E_y/\partial z' - \partial E_z/\partial y')/\gamma + \beta\partial H_x/\partial x'$ (since $y' = y$, $z' = z$); $\partial H_x/\partial x' = \gamma(\partial/\partial x + \beta\partial/\partial x_0)H_x = \gamma[-\partial H_y/\partial y - \partial H_z/\partial z + \beta(\partial E_y/\partial z - \partial E_z/\partial y)]$, using Eqs. (**) and (6); hence, $\partial H_x/\partial x_0' = \gamma[(\gamma^{-2} + \beta^2)(\partial E_y/\partial z' - \partial E_z/\partial y') - \beta(\partial H_y/\partial y' + \partial H_z/\partial z')] = \gamma[\partial(E_y - \beta H_z)/\partial z' - \partial(E_z + \beta H_y)/\partial y']$. Relation (8) follows directly, using Eqs. (7a), (5), and (*): $\rho' = \nabla'\cdot\mathbf{E}' = \gamma(\partial/\partial x + \beta\partial/\partial x_0)E_x + \gamma[\partial(E_y - \beta H_z)/\partial y + \partial(E_z + \beta H_y)/\partial z] = \gamma\nabla\cdot\mathbf{E} + \gamma\beta(-u_x/c)\rho = \gamma(1 - vu_x/c^2)\rho$. Then Eqs. (9) are obtained from Eqs. (5′), (7a), (7b), (5), (*), and (8); for instance, $\rho' u_x'/c = \partial H_z'/\partial y' - \partial H_y'/\partial z' - \partial E_x'/\partial x_0' = \gamma[\partial(H_z - \beta E_y)/\partial y - \partial(H_y + \beta E_z)/\partial z - (\partial/\partial x_0 + \beta\partial/\partial x)E_x] = \gamma[-\beta\nabla\cdot\mathbf{E} + \rho u_x/c] = \gamma\rho(u_x/c - \beta)$, or, $u_x' = \gamma(u_x - v)\rho/\rho' = (u_x - v)/(1 - u_x v/c^2)$.

[9]This statement may not appear so immediately obvious. The general validity of Eq. (10) is, however, readily obtained from the general form of the transformation of $(\mathbf{E},\mathbf{H})$ under a Lorentz transformation [as given, e.g., in H. M. Schwartz, *Introduction to Special Relativity* (McGraw–Hill, New York, 1968), Eq. (v), p. 304 (where $\mathbf{B} \equiv \mathbf{H}$)]. The fact that $\mathbf{E}\cdot\mathbf{H} = 0$, $\mathbf{E}^2 = \mathbf{H}^2$ imply the same relations for the primed quantities is immediately checked, and in a few simple steps one finds for $\mathbf{E}'^2$ the expression $\gamma^2\mathbf{E}^2\{1 + \beta^2[1 - \cos^2(\boldsymbol{\beta},\mathbf{E}) - \cos^2(\boldsymbol{\beta},\mathbf{H})] - 2\beta \cos(\boldsymbol{\beta},\mathbf{n})\}$, which confirms Eq. (10), since $1 - \cos^2(\boldsymbol{\beta},\mathbf{E}) - \cos^2(\boldsymbol{\beta},\mathbf{H}) = \cos^2(\boldsymbol{\beta},\mathbf{n}) \equiv \cos^2\phi$. Again, the cosine of the angle between the planes in question, taking the "polarization plane" as defined in optics, is given by the expression $[(\mathbf{n}\times\mathbf{H})/|\mathbf{n}\times\mathbf{H}|]\cdot(\boldsymbol{\beta}\times\mathbf{n})/|\boldsymbol{\beta}\times\mathbf{n}|$, which reduces since $\mathbf{n} = (\mathbf{E}\times\mathbf{H})/E^2$ (remembering that $\mathbf{E}\cdot\mathbf{H} = 0$, $E = H$), to $(-\mathbf{E}/E)\cdot(\boldsymbol{\beta}\cdot\mathbf{H}\mathbf{E} - \boldsymbol{\beta}\cdot\mathbf{E}\mathbf{H})/E^{-1}[(\boldsymbol{\beta}\cdot\mathbf{H})^2 + (\boldsymbol{\beta}\cdot\mathbf{E})^2]^{1/2} = -(\boldsymbol{\beta}\cdot\mathbf{H})[(\boldsymbol{\beta}\cdot\mathbf{H})^2 + (\boldsymbol{\beta}\cdot\mathbf{E})^2]^{-1/2}$. This expression is form invariant under Lorentz transformations, since $\boldsymbol{\beta}\cdot\mathbf{H} = \boldsymbol{\beta}\cdot\mathbf{H}'$, $\boldsymbol{\beta}\cdot\mathbf{E} = \boldsymbol{\beta}\cdot\mathbf{E}'$ [*loc. cit.*, p. 304, Eq. (vi)].

[10]In the original form: $\mu d^2x_0'/dt'^2 = \epsilon X'$, $\mu d^2y_0'/dt'^2 = \epsilon Y'$, $\mu d^2z_0'/dt'^2 = \epsilon Z'$. The term "unaccelerated system" is of course synonymous here with the term "inertial system." The subscript 0, which is dropped in his general Eq. (11), is introduced by Einstein provisionally to indicate the specialization under which Eqs. (**) are obtained.

[11]These equations were first obtained by M. Planck, Verh. Deutsch. Phys. Ges. **8**, 136 (1906). Einstein refers in the introductory remarks of his essay to Planck's work but not to the paper; and Einstein's proof, based on symmetry considerations, is his own. As the latter proof, so Planck's proof also, is only sketched out in his 1906 paper. It can be presented more fully with the aid of Eqs. (3–29a) and (v) on pp. 52 and 304, respectively, of the reference contained in Ref. 9. With the (temporary) notation, $x_0 \equiv ct$, $f \equiv df/dx_0$, the first of the referred equations yields after a few obvious reductions: $d^2\mathbf{x}'/dx_0'^2 = \{(1 - \boldsymbol{\beta}\cdot\dot{\mathbf{x}})\ddot{\mathbf{x}} + \boldsymbol{\beta}\cdot\ddot{\mathbf{x}}[\dot{\mathbf{x}} - \gamma\boldsymbol{\beta}/(\gamma + 1)]\}/\gamma^2(1 - \boldsymbol{\beta}\cdot\dot{\mathbf{x}})^3$, $\boldsymbol{\beta}\cdot d^2\mathbf{x}'/d^2x_0' = \boldsymbol{\beta}\cdot\ddot{\mathbf{x}}/\gamma^3(1 - \boldsymbol{\beta}\cdot\dot{\mathbf{x}})^3$. Then by the second of the referred equations, Eq. (*), and the identity $\boldsymbol{\beta}\cdot\mathbf{E}' = \boldsymbol{\beta}\cdot\mathbf{E}$, one finds: $\gamma e[\mathbf{E} + (\boldsymbol{\beta}\times\mathbf{H})] = e\mathbf{E}' - [e(1 - \gamma)\boldsymbol{\beta}\cdot\mathbf{E}'/\beta^2]\boldsymbol{\beta} = mc^2\{d^2\mathbf{x}'/dx_0'^2 - [(1 - \gamma)(\boldsymbol{\beta}\cdot\mathbf{d}^2\mathbf{x}'/dx_0'^2)/\beta^2]\boldsymbol{\beta}\} = mc^2[(1 - \boldsymbol{\beta}\cdot\dot{\mathbf{x}})\ddot{\mathbf{x}} + \boldsymbol{\beta}\cdot\ddot{\mathbf{x}}\dot{\mathbf{x}}]/\gamma^2(1 - \boldsymbol{\beta}\cdot\dot{\mathbf{x}})^3$. We may now identify $\boldsymbol{\beta}$ with $\dot{\mathbf{x}}$, with the result [retaining the symbol γ for $(1 - \dot{\mathbf{x}}^2)^{-1/2}$]: $e[\mathbf{E} + (\dot{\mathbf{x}}\times\mathbf{H})] = mc^2(\gamma\ddot{\mathbf{x}} + \gamma^3\dot{\mathbf{x}}\cdot\ddot{\mathbf{x}}\dot{\mathbf{x}}) \equiv mc^2 d(\gamma\dot{\mathbf{x}})/dx_0$, which is seen to be identical with Eqs. (11) and (12), when account is taken of the difference in notation (and in particular, in the meaning of the dot operator).

[12]This statement, reflecting Mach's philosophical position in mechanics, is of obvious historical interest, though what Einstein means here by lack of *physical content* (*physikalischen Inhalt*) can only be conjectured.

[13]Since $\dot{\mathbf{x}}\cdot[d(\gamma_q\dot{\mathbf{x}})/dt] \equiv c^2 d\gamma_q/dt$. In the second line above Eq. (14) the dots are missing from x,y,z, in the original. This is one of a large number of obvious minor misprints (not corrected in the second reference in Ref. 1), which will not be pointed out further.

[14]A similar restriction, insuring the vanishing of surface integrals arising in an integration-by-parts step, applies of course also in the derivation of Eq. (13).

[15]The quantity $|\xi|$ is represented in the original by the already actively employed symbol ρ.

[16]This result is readily checked for arbitrary direction of $\mathbf{v}$ by the use of the inverse of the first of Eqs. (v) in the reference indicated in Ref. 9. When it is observed that $(\rho c,\rho\mathbf{u})$ is a four vector when ρ denotes the

charge density as measured in S [see Eq. (7-5) in the reference in Ref. 9], and one uses the inverse of Eq. (3-29a) of the cited reference, one finds: $\rho \mathbf{u} \cdot \mathbf{E} = \rho'\{\mathbf{u}' + [\gamma c + (\gamma - 1)\beta^{-2}\beta \cdot \mathbf{u}']\beta\} \cdot \{\gamma[\mathbf{E}' - (\beta \times \mathbf{H}')] + [(1 - \gamma)/\beta^2]\beta \cdot \mathbf{E}'\beta\} = \gamma\rho'\{\mathbf{u}' \cdot \mathbf{E}' + \beta \cdot [c\mathbf{E}' + (\mathbf{u}' \times \mathbf{H}')]\}$.

[17]Actually, the *rest mass* (or *proper mass*) of the system, but the idea was still represented in the essay simply by the word "mass."

[18]Setting the integration constant of Eq. (14) here equal to zero is thus at this stage only an assumption, but certainly a natural and fundamental assumption.

[19]The symbols q and G in the original are obviously meant to represent vector quantities, as is, for instance, apparent from Eq. (18) with the last term set equal to zero.

[20]It should be noted that in all the derivations in this section it is tacitly assumed that the physical systems are such that one can speak of a unique translational velocity $\mathbf{q}$ of a system, as in the case of a classically rigid body. This condition is explicit in the pertinent part of footnote d.

[21]The original equation contains a strangely overlooked misprint, namely, the right-hand side contains the added term μq (i.e., $m\mathbf{q}$ in the present notation). But clearly m is already contained in E. Explicitly: By Eqs. (16c) and (20), $E = (mc^2 + E_0)\gamma_q + (q/c)^2\gamma_q^2 pV$, $E_0 + p_0V_0 = (E/\gamma_q) - (q/c)^2\gamma_q pV - mc^2 + \gamma_q pV = (E + pV)/\gamma_q - mc^2$; hence, $\gamma_q[m + (E_0 + p_0V_0)/c^2] = (E + pV)/c^2$, and Eq. (18c) implies Eq. (18d) as presented here.

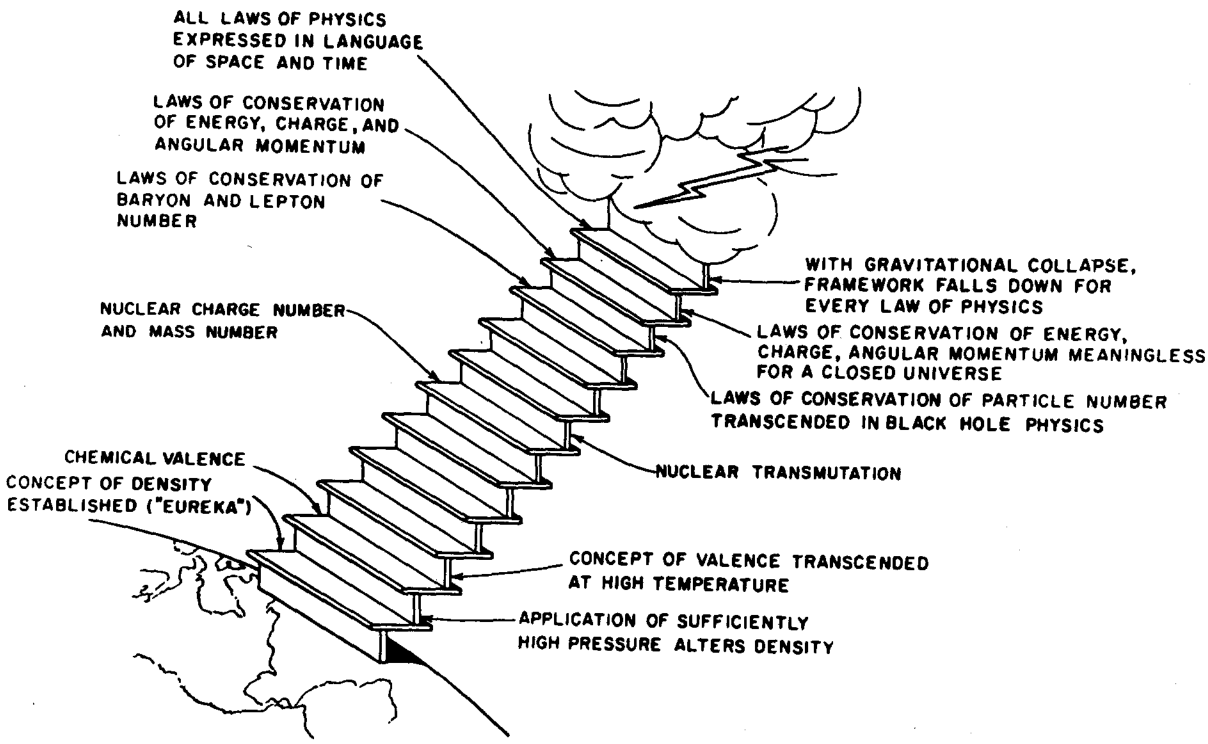
[22]Einstein refers here in a footnote to pp. 373–379 of his paper, footnote d.

[23]In Planck's essay (referred to by Einstein towards the end of his introductory remarks), entitled "Zur Dynamik bewegter Systeme," on p. 552, Einstein uses the symbol η for Planck's S, and his quotation contains an obvious misprint that consists in reversing the sense of an inequality.

[24]These results are obtained directly from Eqs. (23) and (24).

MUTABILITY: THE STAIRCASE OF LAW, AND LAW TRANSCENDED

—John Archibald Wheeler



Einstein's comprehensive 1907 essay on relativity, part III

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This is the concluding part of the English rendition of Einstein's 1907 essay on relativity, of which part I appeared in the June 1977 issue of this Journal and part II in the September 1977 issue. It consists of a direct translation of the last part of the essay, part V, entitled "Principle of Relativity and Gravitation," and of a few added footnotes.

A. INTRODUCTION

Einstein's 1907 essay on relativity[1] does not appear to be widely known. Yet, as noted in the Introduction to the first part of the present English rendition of this essay,[2] it is of substantial interest both on didactic and historic grounds. Its didactic value, relating to the treatment of a number of basic topics in Special Relativity, is particularly in evidence in the portion of Einstein's essay that is dealt with in the second part of the present rendition.[3] Its historical importance is associated mainly with the genesis of special relativity, and also with the genesis of general relativity. Part V, the last part of Einstein's 1907 essay, contains Einstein's first published expression of his initial highly important seminal ideas on the latter subject. It is translated here, as far as seemed feasible, verbatim, with a few added mainly explanatory notes.

It may perhaps not be amiss to point out here that the latter notes, as well as those presented in the other two parts of the present rendition, and in the partial translation of Einstein's first paper on relativity,[4] have for their principal aim only the facilitating of a close reading of the respective fundamental papers of Einstein, whether historically or pedagogically motivated.[5]

As in the previous parts of this rendition all the original footnotes are labeled by lower-case roman letters, and the added footnotes by arabic numerals.

B. TRANSLATION OF THE GRAVITATIONAL PART OF EINSTEIN'S 1907 MEMOIR

V. Principle of Relativity and Gravitation

17. Accelerated Reference System and Gravitational Field

Until now we have applied the principle of relativity—i.e., the assumption that the laws of nature are independent of the state of motion of the reference system—only to *nonaccelerated* reference systems. Is it conceivable that the principle of relativity holds also for systems which are accelerated with respect to each other?

This is not really the place for the exhaustive treatment of this subject. Since it forces itself, however, on the mind of anyone who has followed the previous applications of the principle of relativity, I shall not refrain here from taking a position on the question.

We consider two systems of motion, Σ_1 and Σ_2. Suppose Σ_1 is accelerated in the direction of its X axis, and γ is the magnitude (constant in time) of this acceleration. Suppose Σ_2 is at rest,[6] but situated in a homogeneous gravitational field, which imparts to all objects an acceleration $-\gamma$ in the direction of the X axis.

As far as we know, the physical laws with respect to Σ_1 do not differ from those with respect to Σ_2; this derives from the fact that all bodies are accelerated alike in the gravitational field. We have therefore no reason to suppose in the present state of our experience that the systems Σ_1 and Σ_2 differ in any way, and will therefore assume in what follows the complete physical equivalence of the gravitational field and the corresponding acceleration of the reference system.[7]

This assumption extends the principle of relativity to the case of uniformly accelerated translational motion of the coordinate system. The heuristic value of the assumption lies therein that it makes possible the replacement of a homogeneous gravitational field by a uniformly accelerated reference system, the latter case being amenable to theoretical treatment to a certain degree.

18. Space and Time in a Uniformly Accelerated Reference System

We consider first a body whose individual material points possess relative to the nonaccelerated reference system S, at a fixed time t of S, a certain acceleration but no velocity. What influence does this acceleration γ have on the shape of the body with respect to S?

If such an influence exists, it will consist in a dilatation of constant ratio in the direction of the acceleration, and possibly in the two directions perpendicular to this direction[8]; since an influence of another kind is precluded by considerations of symmetry. Those dilatations arising from the acceleration (if they exist at all) must be even functions of γ; and they can be thus disregarded when one restricts oneself to the case when γ is so small that terms of the second and higher powers in γ may be neglected.[9] Since we wish to confine ourselves in the sequel to this case, we do not have therefore to assume any influence of the acceleration on the shape of a body.

We consider now a reference system Σ which is uniformly accelerated relative to the nonaccelerated reference system S in the direction of the X axis of the latter. Let the clocks and the measuring rod of Σ, when examined at rest, be the same as the clocks and the measuring rod of S. Let the origin of coordinates of Σ move along the X axis of S, and let the axes of Σ remain parallel to those of S. There exists at every instant a nonaccelerated reference system S', whose coordinate axes coincide with the coordinate axes of Σ at that instant (for a fixed time t' of S'). If a point-event occurring at this time t' has the coordinates ξ,η,ζ with respect to Σ, then

$$x' = \xi, \quad y' = \eta, \quad x' = \zeta,$$

since by the foregoing discussion we must not assume any influence of the acceleration upon the shape of the measuring bodies used in measuring ξ,η,ζ. Let us imagine further that at this time t' of S' the clocks of Σ are so adjusted that their reading at this instant is t'. What can we say about the rate of the clocks in the next time element τ?

First we have to bear in mind that a specific influence of the acceleration upon the rate of the clocks does not enter into consideration, since it would have to be of the order of γ^2. Moreover, since we can neglect, as being of order τ^2, the influence upon the rate of the clocks of the velocity attained during τ as well as of the path traveled by the clocks relative to those of S' during the time τ, therefore for the time element τ the readings of the clocks of Σ are fully replaceable[10] by the readings of the clocks of S'.

It follows from the foregoing discussion, that in the time element τ light in vacuum propagates with the universal velocity c relative to Σ, if we define simultaneity in the system S' which is instantaneously at rest relative to Σ, and apply for the measurement of time and lengths, clocks and measuring rods which are the same as those used in the measurement of time and lengths in nonaccelerated systems. The principle of the constancy of the velocity of light can thus be applied also here for the definition of simultaneity, provided one confines oneself to very small light paths.

We imagine now that the clocks of Σ are set in the indicated manner at that time $t = 0$ of S when Σ is momentarily at rest relative to S. The totality of the readings of the clocks of Σ so set, shall be called the "local time" σ of the system Σ. As one recognizes immediately, the physical significance of the local time σ is as follows. If one utilizes this local time σ for the temporal labeling of processes occurring at individual space elements of Σ, then the laws obeyed by those processes cannot depend on the position of the particular spatial element, i.e., on its coordinates, if one employs at the different spatial elements not only the same clocks, but the same measuring devices as well.

On the other hand, we must not consider the local time σ as simply the "time" of Σ, because, in fact, two events taking place at two different points of Σ are not simultaneous in the sense of the above definition when their local times σ are equal to each other. Since, namely, any two clocks of Σ are synchronous with respect to S at the time $t = 0$, and undergo the same motion, they remain continuously synchronous with respect to S. On this account, according to Sec. 4,[11] they are not synchronous with respect to a reference system S' that is momentarily at rest relative to Σ and in motion relative to S, and therefore, according to our definition, neither are they synchronous with respect to Σ.

We define now the "time" τ of the system Σ as the totality of those readings of the clock located at the origin of coordinates of Σ, which are simultaneous, in the sense of the above definition, with the events to be temporally labelled.[a]

We will now find the relationship obtaining between the time τ and the local time σ of an event. From the first of Eqs. (1)[11] it follows that two events are simultaneous with respect to S', and hence also with respect to Σ, when

$$t_1 - (vx_1/c^2) = t_2 - (vx_2/c^2),$$

where the indices refer to the one and to the other point-event, respectively. We restrict ourselves at first to the consideration of such short times,[b] that all terms containing second or higher powers of τ or of v can be discarded; then, by reference to Eqs. (1) and (2),[11] we have to set

$$x_2 - x_1 = x_2' - x_1' = \xi_2 - \xi_1;$$
$$t_1 = \sigma_1, \quad t_2 = \sigma_2; \quad v = \gamma t = \gamma\tau,^{12}$$

so that we obtain from the above equations:

$$\sigma_2 - \sigma_1 = \gamma\tau(\xi_2 - \xi_1)/c^2.$$

If we place the first point-event at the origin of coordinates, so that $\sigma_1 = \tau$ and $\xi_1 = 0$, we obtain upon dropping the index for the second point-event,

$$\sigma = \tau[1 + (\gamma\xi/c^2)]. \qquad (30)$$

This equation is valid, to begin with, when τ and ξ lie below certain bounds. It holds obviously for arbitrarily large τ, if the acceleration γ is constant with respect to Σ,[13] because then the connection between σ and τ must be linear. For arbitrarily large ξ Eq. (30) does not hold. In fact, since the choice of the origin of coordinates cannot influence the relation in question, one concludes that Eq. (30) must be replaced in all strictness by the equation

$$\sigma = \tau e^{\gamma\xi/c^2}.$$

We shall, however, retain formula (30).

According to Sec. 17, Eq. (30) is to be applied also to a coordinate system in which a homogeneous gravitational field is acting. In this case we have to set $\Phi = \gamma\xi$, where Φ denotes the gravitational potential, so that we obtain

$$\sigma = \tau[1 + (\Phi/c^2)]. \qquad (30a)$$

We have defined two kinds of time for Σ. Which of the two definitions do we have to utilize in the different cases? Let us suppose that at each of two places of different gravitational potential ($\gamma\xi$) there exists a physical system, and that we wish to compare the physical quantities associated with them. To this end, we shall clearly proceed most naturally as follows. We betake ourselves with our measuring devices first to the one physical system, and carry out our measurements there; and then we betake ourselves with our measuring devices to the other system to carry out here the identical measurements. If the measurements yield the identical results in both places, we shall designate the two physical systems as "identical." Among the mentioned measuring devices there exists a clock, with which we measure the local time σ. From this it follows that for the definition of physical quantities at a given place of the gravitational field, we quite naturally utilize the time σ.

But if one deals with a phenomenon that necessitates the simultaneous consideration of objects situated at places of different gravitational potential, then we must employ the time τ in the terms where the time appears explicitly (i.e., not only in the definition of physical quantities); since otherwise the simultaneity of the events would not be expressed by the identity of the values of their time. Since in the definition of the time τ one does not employ an arbitrarily chosen instant, but rather a clock situated at an arbitrarily chosen place, the laws of nature, when one uses the time τ, cannot vary therefore with the time, but may well vary with the place.

19. Influence of the Gravitational Field Upon Clocks

If at a point P of the gravitational field Φ there is situated a clock which indicates the local time, then according to Eq.

(30a) its indications are $1 + (\Phi/c^2)$ greater than the time τ, i.e., it runs $1 + (\Phi/c^2)$ faster than an identically constructed clock situated at the origin of coordinates. Suppose that an observer situated anywhere in space ascertains in some manner the indications of the two clocks, e.g., by optical means. Since the time interval $\Delta\tau$, which elapses between the instant of an indication of one of the clocks and its being perceived by the observer, is independent of τ, the clock at P runs for an observer situated anywhere in space $1 + (\Phi/c^2)$ times faster than the clock at the origin of coordinates. It is in this sense that we can say that the process taking place within the clock—and more generally, every physical process—proceeds at a rate which is the faster the greater the gravitational potential of the place where it occurs.

Now there exist "clocks," which are to be found at places of different gravitational potential and whose rates can be controlled very precisely; these are the generators of spectral lines. It follows from the above discussion[c] that the light coming from the surface of the Sun, which arises from such a generator, possesses a wavelength that is greater by about a two-millionth part than that of the light generated by identical material on the surface of the Earth.

20. *Influence of Gravitation Upon Electromagnetic Processes*

If we refer an electromagnetic process at a given instant to a nonaccelerated reference system S' that is momentarily at rest relative to the reference system Σ, which is accelerated as above, then by Eqs. (5) and (6)[14] we have the equations

$$\left(\rho' u'_x + \frac{\partial X'}{\partial t'}\right)\Big/ c = \frac{\partial N'}{\partial y'} - \frac{\partial M'}{\partial z'}, \quad \text{etc.,}$$

and

$$\frac{\partial L'}{c\partial t'} = \frac{\partial Y'}{\partial z'} - \frac{\partial Z'}{\partial y'}, \quad \text{etc.}$$

According to the above, we can immediately identify the quantities ρ', u', X', L', x', etc., referred to S' with the corresponding quantities ρ, u, X, L, ξ, etc., referred to Σ, as long as we confine ourselves to an infinitely short time,[d] which is infinitely close to the time of relative rest of S' and Σ. Moreover, we have to replace t' by the local time σ. However, we may not simply set

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial \sigma},$$

because a point at rest with respect to Σ, to which the equations transformed to Σ are to be referred, changes its velocity relative to S' during the time element $dt' = d\sigma$. To this change there corresponds according to Eqs. (7a) and (7b)[14] a temporal change in the field components which are referred to Σ. We have therefore to set

$$\frac{\partial X'}{\partial t'} = \frac{\partial X}{\partial \sigma}, \quad \frac{\partial Y'}{\partial t'} = \frac{\partial Y}{\partial \sigma} + \frac{\gamma N}{c}, \quad \frac{\partial Z'}{\partial t'} = \frac{\partial Z}{\partial \sigma} - \frac{\gamma M}{c},$$
$$\frac{\partial L'}{\partial t'} = \frac{\partial L}{\partial \sigma}, \quad \frac{\partial M'}{\partial t'} = \frac{\partial M}{\partial \sigma} - \frac{\gamma Z}{c}, \quad \frac{\partial N'}{\partial t'} = \frac{\partial N}{\partial \sigma} + \frac{\gamma Y}{c}.$$

The electromagnetic equations referred to Σ read thus, to begin with,

$$\left(\rho u_\xi + \frac{\partial X}{\partial \sigma}\right)\Big/ c = \frac{\partial N}{\partial \eta} - \frac{\partial M}{\partial \zeta}$$
$$\left(\rho u_\eta + \frac{\partial Y}{\partial \sigma} + \frac{\gamma N}{c}\right)\Big/ c = \frac{\partial L}{\partial \zeta} - \frac{\partial N}{\partial \xi},$$
$$\left(\rho u_\zeta + \frac{\partial Z}{\partial \sigma} - \frac{\gamma M}{c}\right)\Big/ c = \frac{\partial M}{\partial \xi} - \frac{\partial L}{\partial \eta},$$
$$\frac{\partial L}{c\partial \sigma} = \frac{\partial Y}{\zeta \xi} - \frac{\partial Z}{\partial \eta},$$
$$\left(\frac{\partial M}{\partial \sigma} - \frac{\gamma Z}{c}\right)\Big/ c = \frac{\partial Z}{\partial \xi} - \frac{\partial X}{\partial \zeta},$$
$$\left(\frac{\partial N}{\partial \sigma} + \frac{\gamma Y}{c}\right)\Big/ c = \frac{\partial X}{\partial \eta} - \frac{\partial Y}{\partial \xi}.$$

We multiply these equations by $1 + (\gamma\xi/c^2)$, and set for short

$$X^* = X[1 + (\gamma\xi/c^2)], \quad Y^* = Y[1 + (\gamma\xi/c^2)], \quad \text{etc.,}$$
$$\rho^* = \rho[1 + (\gamma\xi/c^2)].$$

Upon neglecting terms of second degree in γ, we obtain then the equations

$$\left(\rho^* u_\xi + \frac{\partial X^*}{\partial \sigma}\right)\Big/ c = \frac{\partial N^*}{\partial \eta} - \frac{\partial M^*}{\partial \zeta},$$
$$\left(\rho^* u_\eta + \frac{\partial Y^*}{\partial \sigma}\right)\Big/ c = \frac{\partial L^*}{\partial \zeta} - \frac{\partial N^*}{\partial \xi}, \qquad (31a)$$
$$\left(\rho^* u_\zeta + \frac{\partial Z^*}{\partial \sigma}\right)\Big/ c = \frac{\partial M^*}{\partial \xi} - \frac{\partial L^*}{\partial \eta}.$$

$$\frac{\partial L^*}{c\partial \sigma} = \frac{\partial Y^*}{\partial \zeta} - \frac{\partial Z^*}{\partial \eta},$$
$$\frac{\partial M^*}{c\partial \sigma} = \frac{\partial Z^*}{\partial \omega} - \frac{\partial X^*}{\partial \zeta}, \qquad (32a)$$
$$\frac{\partial N^*}{c\partial \sigma} = \frac{\partial X^*}{\partial \eta} - \frac{\partial Y^*}{\partial \xi}.$$

From these equations one sees first how the gravitational field influences static and stationary phenomena. The regularities that are obtained are the same as in the gravitation-free field, except for the substitution of $X[1 + (\gamma\xi/c^2)]$, etc., for X, etc., and $\rho[1 + (\gamma\xi/c^2)]$ for ρ.

Moreover, in order to survey the course of nonstationary states we employ the time τ for terms that involve differentiation with respect to time, as well as for the definition of the velocity of electricity, i.e., we set according to Eq. (30),

$$\frac{\partial}{\partial \tau} = \left(1 + \frac{\gamma\xi}{c^2}\right)\frac{\partial}{\partial \sigma}$$

and

$$w_\xi = [1 + (\gamma\xi/c^2)]u_\xi.$$

We obtain thus

$$\left(\rho^* w_\xi + \frac{\partial X^*}{\partial \tau}\right)\Big/ c[1 + (\gamma\xi/c^2)] = \frac{\partial N^*}{\partial \eta} - \frac{\partial M^*}{\partial \zeta}, \text{ etc.,} \qquad (31b)$$

and

$$\left(\frac{\partial L^*}{\partial \tau}\right)\Big/ c\,[1 + (\gamma\xi/c^2)] = \frac{\partial Y^*}{\partial \zeta} - \frac{\partial Z^*}{\partial \eta}, \text{ etc.} \qquad (32b)$$

These equations, too, are of the same form as the corresponding ones in the acceleration-free or gravitation-free space, but here instead of c there appears the quantity

$$c[1 + (\gamma\xi/c^2)] = c[1 + (\Phi/c^2)].$$

It follows from this that the light rays that are not propagated in the direction of the ξ axis are bent by the gravitational field; as is easily seen, the change in direction per centimeter of light path amounts to $(\gamma/c^2)\sin\phi$, where ϕ is the angle between the direction of the gravitational force and that of the light ray.

By means of these equations and those which connect field strengths and electric currents at a given place according to the theory of optics of stationary bodies, it is possible to ascertain the influence of the gravitational field upon optical phenomena for stationary bodies. It should be borne in mind in this connection that those equations from the optics of stationary bodies are valid for the local time σ. Unfortunately, the influence of the Earth's gravitational field is according to our theory so slight (because of the smallness of $\gamma x/c^2$), that there exists no prospect for a comparison of the results of the theory with experience.

If we multiply Eqs. (31a) and (32a) successively by $X^*/4\pi, \cdots, N^*/4\pi$ and integrate over infinite [i.e., all of] space, we obtain, using our previous notation:

$$\int [1 + (\gamma\xi/c^2)]^2(\rho/4\pi)(u_\xi X + u_\eta Y + u_\zeta Z)d\omega + \int [1 + (\gamma\xi/c^2)]^2(1/8\pi) \times \frac{\partial(X^2 + Y^2 + \cdots + N^2)}{\partial\sigma} d\omega = 0.$$

$\rho(u_\xi X + u_\eta Y + u_\zeta Z)/4\pi$ is the energy η_σ conveyed to the matter per unit volume and unit local time σ; when this energy is measured by means of the measuring devices located at the place in question. Hence by Eq. (30), $\eta_\tau = \eta_\sigma[1 + (\gamma\xi/c^2)]$ is the energy (similarly measured) conveyed to the matter per unit volume and per unit of time τ. $(X^2 + Y^2 + \cdots + N^2)/8\pi$ is the electromagnetic energy ϵ per unit volume—similarly measured. If we bear in mind, further, that according to Eq. (30) we have to set $\partial/\partial\sigma = [1 - (\gamma\xi/c^2)]\partial/\partial\tau$, we obtain

$$\int [1 + (\gamma\xi/c^2)]\eta_\tau\, d\omega + \frac{d}{d\tau}\int [1 + (\gamma\xi/c^2)]\epsilon\, d\omega = 0.$$

This equation expresses the principle of the conservation of energy and contains a very remarkable result. An energy or a transport of energy which has the respective value $E = \epsilon\, d\omega$ or $E = \eta\, d\omega\, d\tau$, when measured at a given spot, contributes to the energy integral in addition to the value E corresponding to its quantity, also a value $E\gamma\xi/c^2 = E\Phi/c^2$ corresponding to its position. To every energy E there belongs thus in the gravitational field an energy of position, which is as large as the energy of position of a "ponderable" mass of magnitude E/c^2.

The law deduced in Sec. 11,[14] that to a quantity of energy E there belongs a mass of magnitude E/c^2, holds thus not only for the inertial, but also for the gravitational mass, provided the assumption introduced in Sec. 17 is valid.

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[a]The symbol "τ" is thus employed here in a different sense than previously.

[b]Thereby there arises also, according to (1),[11] a certain restriction regarding the values of $\xi = x'$.

[c]By assuming that Eq. (30a) holds also for a nonhomogeneous gravitational field.

[d]This restriction does not impair the domain of validity of our results, since by the nature of things, the laws to be derived cannot depend on the time.

[1]A. Einstein, Jahrb. Radioakt. Elektronik **4**, 411 (1907). Corrections by Einstein in Jahrb. Radioakt. Elektronik **5**, 98 (1908).

[2]H. M. Schwartz, Am. J. Phys., **45**, 512 (1977).

[3]H. M. Schwartz, Am. J. Phys. **45**, 811 (1977).

[4]H. M. Schwartz, Am. J. Phys. **45**, 18 (1977).

[5]The work contained in Refs. 2–4, as well as in the author's rendition of Poincaré's Rendiconti paper on relativity that appeared in this journal [**39**, 1287 (1971); **40**, 862 (1972); **40**, 1282 (1972)], were in fact undertaken after the initiation of an historical study, entitled *Lorentz, Poincaré, Einstein, and the Special Theory of Relativity* (that it is planned to complete shortly), which pointed out a need for readier access to the above-mentioned works of Poincaré and of Einstein on special relativity. Similarly, the present paper is in part motivated by an interest in investigating certain intriguing questions in the genesis of the general theory of relativity.

[6]Rest and *acceleration* with respect to an *inertial frame* is, of course, tacitly assumed.

[7]This bold intuitive extrapolation is of course a remarkable characteristic of Einstein's youthful genius.

[8]Rather than "in the two directions perpendicular to this direction" [in the original: . . . in den beinden dazu senkrechten Richtungen . . .] what was intended is, of course, "in any direction perpendicular to this direction" (reflecting the cylindrical symmetry about the direction of acceleration).

[9]What is to be taken as "small" for the dimensional quantity γ is apparent from subsequent discussion.

[10]The original word "nutzbar" is replaced in the second reference of Ref. 1 by the word "ersetzbar."

[11]See Ref. 1 or Ref. 2.

[12]At the end of the second reference in Ref. 1, Einstein states that he is prompted by a communication from Planck to clarify the notion of "uniform acceleration" in the new kinematics; and that this is to be understood here as the acceleration relative to the instantaneous rest system of the body under consideration. He concludes that the "relation $v = \gamma t$ holds only in the first approximation; but this suffices, since only terms linear with respect to t or τ need to be considered here."

[13]This restriction would be, of course, already contained in the title of the section, if there existed no ambiguity in the use here of the term "acceleration." But, actually, such an ambiguity does exist (see footnote 12). For an explicit relativistic definition of *uniform acceleration*, see, e.g., W. Pauli, *Theory of Relativity* (Pergamon, New York, 1958), Sec. 26; or H. M. Schwartz, *Introduction to Special Relativity* (McGraw-Hill, New York, 1968), Eq. (ii), p. 83.

[14]See Ref. 1 or Ref. 3. As in Ref. 2, but not in Ref. 3, Einstein's notation is retained here throughout. The symbols X, Y, Z and L, M, N represent the respective components of the electric and magnetic field intensities.