International Journal of Modern Physics D
Vol. 16, No. 12A (2007) 2259–2270

EXPERIMENTAL VALIDATION OF A HIGH ACCURACY TEST OF THE EQUIVALENCE PRINCIPLE WITH THE SMALL SATELLITE "GALILEO GALILEI"

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Received 16 November 2006
Revised 22 November 2006
Communicated by S. G. Turyshev

The small satellite "Galileo Galilei" (GG) has been designed to test the equivalence principle (EP) to 10^{-17} with a total mass at launch of 250 kg. The key instrument is a differential accelerometer made up of weakly coupled coaxial, concentric test cylinders rapidly spinning around the symmetry axis and sensitive in the plane perpendicular to it, lying at a small inclination from the orbit plane. The whole spacecraft spins around the same symmetry axis so as to be passively stabilized. The test masses are large (10 kg each, to reduce thermal noise), their coupling is very weak (for high sensitivity to differential effects), and rotation is fast (for high frequency modulation of the signal). A 1 g version of the accelerometer ("Galileo Galilei on the Ground" — GGG) has been built to the full scale — except for coupling, which cannot be as weak as in the absence of weight, and a motor to maintain rotation (not needed in space due to angular momentum conservation). GGG has proved: (i) high Q; (ii) auto-centering and long term stability; (iii) a sensitivity to EP testing which is close to the target sensitivity of the GG experiment provided that the physical properties of the experiment in space are going to be fully exploited.

Keywords: General relativity; space physics; experimental gravity.

1. Introduction

Laboratory tests of the equivalence principle (EP) allow the experimental results to be checked beyond question. The best such results have been obtained in a remarkable series of experiments using slowly rotating torsion balances[1] that have found no violation to 10^{-12} and slightly better.

It is known that an experiment performed inside a spacecraft orbiting at low altitude around the Earth can aim at improving these results, in the gravitational field of the Earth, by several orders of magnitude. The first proposed satellite experiment is STEP,[2] aiming at an EP test to 1 part in 10^{18}. STEP requires a cryogenic accelerometer, an actively controlled three-axis stabilized or slowly rotating spacecraft, with a total mass at launch of about 1 ton. A scaled-down, noncryogenic version of STEP has been designed, named μSCOPE,[3] to fly in 2009–2010. Abandoning cryogenics has allowed the total μSCOPE mass to be reduced to one-fourth of the STEP's mass, for an expected EP test to 1 part in 10^{15}. The concept of the STEP and μSCOPE experiments is outlined in Fig. 1.

"Galileo Galilei" (GG)[4] is a proposed space experiment to test the EP at room temperature with a total mass at launch close to that of μSCOPE ($\simeq$250 kg) but aiming at an EP test competitive with STEP, namely to 1 part in 10^{17}. We can convincingly argue that ultimately this is made possible by one single change in the experiment design from that of STEP and μSCOPE (see Fig. 2). Such an apparently simple change and its far-reaching consequences for EP testing in space are outlined in Sec. 2. Section 3 describes the GGG prototype[6–8] that we have built with the support of INFN (Istituto Nazionale di Fisica Nucleare), and reports the experimental results and current sensitivity. Finally, Sec. 4 shows the implications of the sensitivity obtained in the lab for the target of the space mission by analyzing the physical properties of the space environment and how the experiment, because of the way it has been designed, will benefit from it.

In February 2006 the GG mission was included in the National Aerospace Plan (PASN) of ASI (Agenzia Spaziale Italiana) for the next three years (see *ASI Plan*,[9] p. 47).

2. Peculiarities of the GG Experiment in Space

The GG experiment is designed to measure the relative acceleration of two test masses in free fall in the gravity field of the Earth. Thus GG tests the universality of free fall (UFF), whereby all bodies fall with the same acceleration regardless of their mass and composition, which is a direct consequence of the EP. A space mission can reach a sensitivity much higher than a ground experiment because,

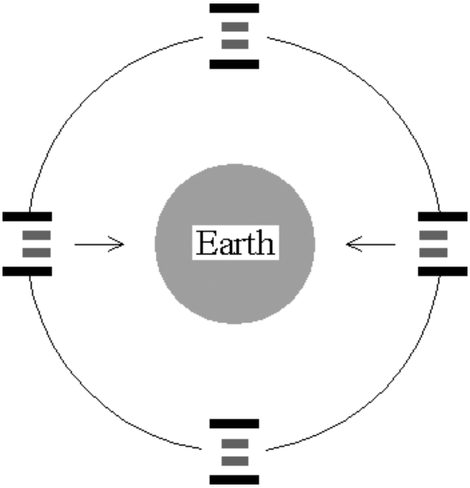

Fig. 1. Concept of the STEP and μSCOPE space experiments to test the equivalence principle. The figure (not to scale) shows a section in the plane along the symmetry axis of two coaxial, concentric test cylinders made of different materials in orbit around the Earth, the symmetry axis lying in the orbit plane. Along this axis the cylinders are weakly coupled, while coupling is stiff in the other directions. Together they form a 1D accelerometer sensitive along the symmetry axis. If the EP were violated — hence one cylinder would be attracted by the Earth more than the other — a differential acceleration would appear as indicated by the arrows, namely at the orbital frequency of the spacecraft. In order to separate the frequency of the signal from the orbital one, at which many disturbances occur, and to up-convert the signal to a higher frequency for reduction of $1/f$ noise, the whole satellite enclosing the accelerometer is actively rotated around an axis perpendicular to the orbit plane (hence to the sensitive/symmetry axis). In so doing, the sensitive axis, and the test cylinders around it, are physically rotated with respect to the Earth as if the accelerometer were still and the Earth were rotating around it. The physical system is a forced oscillator, the forcing effect being at the rotation frequency of the spacecraft with respect to the center of the Earth.

in the case of EP violation, test bodies in low Earth orbit are subjected to an acceleration from the Earth which is more than a thousand times larger than that due to the Sun acting on torsion balances on the ground. Another main advantage of space is weightlessness, which makes it possible to use extremely weak suspensions resulting in a large response to EP violation.

In GG two test masses of different composition are arranged to form a differential accelerometer (Fig. 3, right). The test masses (10 kg each) are concentric, coaxial, hollow cylinders. These two masses are mechanically coupled by attaching them at their top and bottom to two ends of a coupling arm by using flexible lamellae. The coupling arm is made up of two concentric tubes similarly attached at their midpoints to a single shaft. This assembly preserves the overall symmetry of the apparatus, when the two parts of the arm are taken together. The masses are mechanically coupled through the balance arm such that they are free to move in the transverse XY plane, and all of them taken together form the physical system. The masses oscillate in a two-dimensional harmonic potential defined by

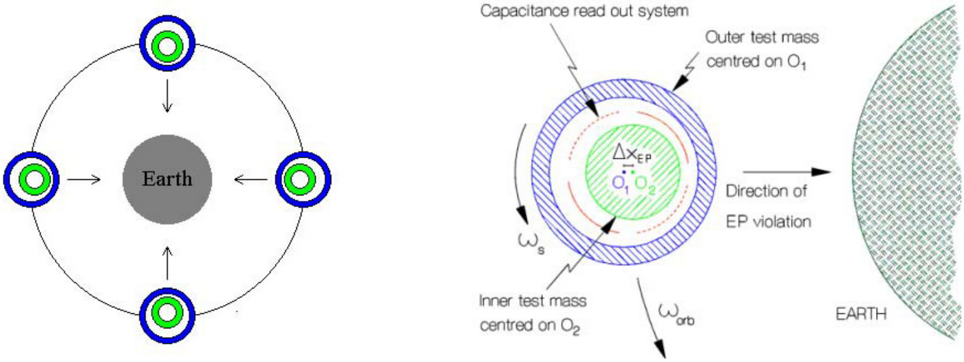

Fig. 2. (*Left*): Concept of the GG space experiment to test the equivalence principle. As compared to the STEP and μSCOPE scheme shown in Fig. 1, the symmetry axis of the test cylinders is simply turned by 90° to become perpendicular to the orbit plane. In order to sense an EP violation effect from the Earth the cylinders are now weakly coupled in the orbit plane and stiff along the symmetry axis, thus forming a 2D accelerometer. The figure (not to scale) shows a section across the symmetry axis, which is also the symmetry axis of a capacitance bridge readout placed between the test cylinders and of the whole satellite (neither of them shown). By making it the axis of the maximum moment of inertia and the spin axis of the whole satellite, the satellite is passively stabilized without requiring active attitude control. If EP were violated, and hence one cylinder would be attracted by the Earth more than the other, all along the orbit their centers of mass would be displaced toward the center of the Earth, as indicated by the arrows. (*Right*): Enlargement from the previous figure showing (not to scale) the test cylinders and the capacitance readout at one given position along the orbit. They all spin at angular velocity ω_s while orbiting the Earth at $\omega_{\rm orb}$. In the case of EP violation the centers of mass of the test bodies are displaced from one another by a vector $\Delta\mathbf{x}_{\rm EP}$. Under the (differential) effect of this new force the test masses, which in this plane are weakly coupled by mechanical suspensions, reach equilibrium at a displaced position where the new force is balanced by the weak restoring force of the suspension, while the bodies rotate independently around O_1 and O_2 respectively. The vector of this relative displacement has a constant amplitude (for zero orbital eccentricity) and points to the center of the Earth (the source mass of the gravitational field). The signal is therefore modulated by the capacitors at their spinning frequency with respect to the center of the Earth $\omega_{s\oplus} \equiv \omega_s - \omega_{\rm orb}$.

the suspension springs at the ends of the balance arm while free-falling around the Earth. A differential acceleration of the masses would thus give rise to a displacement of the equilibrium position in the XY plane. The displacement of the test masses is sensed by two sets of capacitance plates located between the test cylinders, one set for each orthogonal direction (X and Y). Each set of capacitance plates is placed in an AC bridge configuration such that a displacement of the masses causes an imbalance of the bridge and is thus converted into a voltage signal. When the physical system is mechanically well balanced it is insensitive to "common mode" accelerations. In addition, the capacitance bridges are predominantly sensitive to differential displacements. Thus, the differential nature of the accelerometer is ensured both by the dynamics of the physical system and by the displacement transducer.

The goal of testing the EP to 1 part in 10^{17} in the gravitational field of the Earth requires the detection of a differential acceleration $a_{\rm EP} \simeq 8.4 \cdot 10^{-17}\,{\rm m/s^2}$.

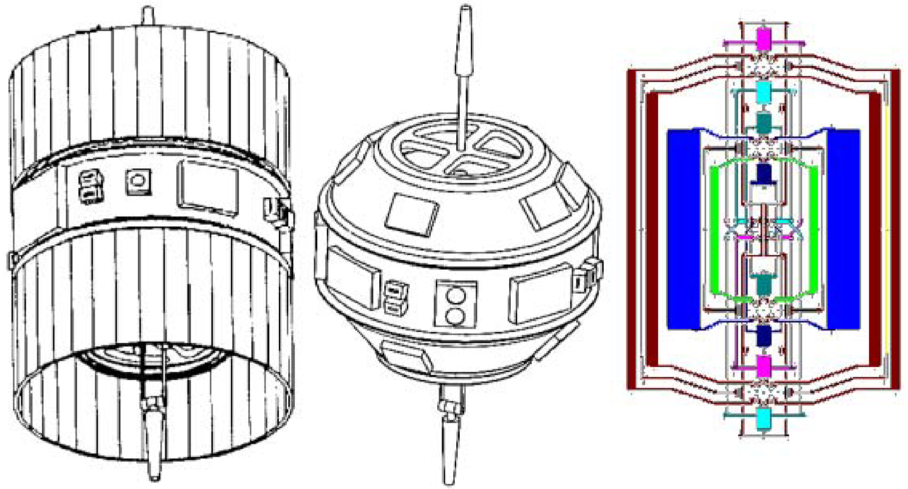

Fig. 3. The GG satellite with solar panels (*left*) and without (*center*). GG is a compact 1-m-diameter structure in the shape of a spinning top stabilized passively by one-axis rotation. Its total mass is 250 kg and its orbit is low (520 km altitude), almost circular and almost equatorial. Inside the “spinning top” (center) — through an intermediate corotating attenuation stage known as PGB — is located the key instrument (*right*) for testing the equivalence principle in the gravitational field of the Earth. It consists of four test cylinders (10 kg each), one inside the other, forming two differential accelerometers: the inner one for EP testing (cylinders of different composition; shown in green and blue respectively) and the outer one for zero check (cylinders made of the same material; both shown in brown). In each accelerometer the two test cylinders are coupled to form a beam balance, as described in Sec. 2. Note that: (i) the whole system is symmetric around the spin axis as well as top/down; (ii) the two accelerometers are both centered at the center of mass of the spacecraft (unlike other proposed space experiments) in order to reduce tidal effects and improve reliability of the zero check; (iii) mechanical suspensions provide electric grounding and passive discharging; (iv) cryogenics is not required.

To achieve this sensitivity it is necessary: (i) that the test masses are very weakly coupled to each other (otherwise the displacement signal resulting from such a tiny acceleration is too small to detect); (ii) that the signal (at the orbital frequency) is up-converted to a higher frequency, the higher the better, to reduce $1/f$ noise. In the GG accelerometer, once unlocked in orbit, the target acceleration signal, a_{EP}, would generate a displacement $\Delta x_{\mathrm{EP}} \simeq 0.6\,\mathrm{pm}$ pointing to the center of the Earth. As shown in Fig. 2 (right), by spinning the satellite and the enclosed accelerometer, with its displacement transducer, around their common symmetry axis, the EP violation displacement signal is *modulated* at the spin frequency of the system relative to the center of the Earth: $\omega_{s\oplus} \equiv \omega_s - \omega_{\mathrm{orb}}$.

It is to be noted that this signal modulation could in principle be achieved by spinning the displacement transducer *only*, and not the test cylinders themselves (though it would not be wise), which means that there is no forcing of the coupled cylinders due to rotation. Instead, in Fig. 1, where the rotation axis is perpendicular to the sensitive axis of the accelerometer, the whole accelerometer must rotate — faster than it orbits around the Earth — in order to up-convert the signal to a higher frequency. Therefore, it will necessarily respond as a forced oscillator, with natural angular frequency ω_n, forced at the spin frequency $\omega_{s\oplus}$, just as if the accelerometer is sitting still and the Earth is rotating around it at $\omega_{s\oplus}$. This fact limits the spin frequency to be smaller than the natural coupling one ($\omega_{s\oplus} < \omega_n$) because a forcing signal at a frequency higher than the natural one would be attenuated by the factor $(\omega_{s\oplus}/\omega_n)^2$. Instead, very sensitive EP tests require *both* weak coupling (i.e. small ω_n) *and* fast rotation (i.e. high $\omega_{s\oplus}$). With its novel design (see Fig. 2), GG can satisfy both these needs, a property which is unique to this experiment since the limitation reported above holds also on the ground for EP tests with rotating torsion balances.

Once the spacecraft has been given the required rate of rotation at the beginning of the mission (2 Hz with respect to the center of the Earth), no motor or ball bearings are needed inside the satellite. In fact, all parts of the apparatus and the satellite corotate around a common symmetry axis. Since the satellite is not constrained to spin slowly, a spin speed which optimizes the stability of the satellite can be chosen. In this way the spacecraft is also passively stabilized by rotation around its symmetry axis and no active attitude control is required for the entire duration of the space mission. This passive stabilization results in a reduction in the total mass, complexity, cost and (last but not least) acceleration noise on the sensitive accelerometer (Fig. 3, right), which is the heart of the experiment.

Due to the very weak coupling between the masses and rapid spin, the GG system is a rotor in the supercritical regime and supercritical rotors are known to auto-center even if fabrication and mounting errors give rise to departures from ideal cylindrical symmetry. The only disadvantage of spinning at frequencies above the natural oscillation frequencies of the rotor is the onset of whirl motions. These occur at the natural frequencies of the system as orbital motion of the masses around the equilibrium position. Whirl arises due to losses in the suspensions (the smaller the losses, the slower the growth rate of whirl) and needs to be damped to prevent instability. With a Q of at least 20,000, which laboratory tests have shown to be achievable, whirl growth is so slow that experimental runs can be performed between successive damping cycles, thus avoiding any disturbance from damping forces.

The largest disturbing acceleration experienced by the accelerometer is due to the effect of residual air drag acting on the spacecraft and not on test masses suspended inside it. This inertial acceleration, resulting from air drag and in general from nongravitational forces acting on the spacecraft, is in principle the same on both the test bodies. Ideally, common mode effects should not produce any differential signal to compete with the target differential signal of an EP violation. In reality, they can only be partially rejected. In the GG space experiment the strategy chosen is for air drag to be partially compensated by the drag-free control system, and partially rejected by the accelerometer itself. Drag compensation requires the spacecraft to be equipped with appropriate thrusters and a control system to force the spacecraft to follow the motion of an undisturbed test mass inside it at (and close to) the frequency of the signal.

Realistic error budget and numerical simulations of the GG experiment carried out within Phase A mission studies funded by ASI[4] are consistent with an EP test to 1 part in 10^{17}.

The novel design of GG has allowed us to build the full scale "Galileo Galilei on the Ground" (GGG) prototype of the satellite experiment, in which the basic physical principles as well as all the associated technology are tested.

3. GGG Prototype: Design, Experimental Results and Sensitivity

GGG[5–8] mimics the design of GG in every possible way within the constraints set by local gravity. At 1 g, unlike in space, (i) the test masses (10 kg each) need to be supported against local gravity, which breaks the symmetry of the accelerometer along the Z axis; (ii) thicker suspensions are needed, which reduce the time period of the natural oscillation of the balance; (iii) bearings (ball bearings in our case) and a motor are needed to maintain a constant rotation speed, which inevitably conveys some noise to the test bodies; (iv) only the accelerometer rotates, everything else around being a potential source of disturbance (primarily axis tilts).

As shown in Fig. 4, GGG consists of two concentric, coaxial hollow cylinders suspended from a balancing arm as in a beam balance, the beam being vertical and coinciding with the common symmetry axis. This assembly is supported by a hollow shaft, at the midpoint of the balancing arm, through a third cardanic joint. The suspension is arranged such that the masses are free to move in the horizontal plane confined by a weak harmonic potential given by the spring constants of the three suspensions. If a force in the horizontal plane acts on one of the masses, and not on the other, it results in a deviation of the masses from their equilibrium position which is sensed by two sets of capacitance plates, as in GG. If a gravitational interaction with an external body results in such a displacement (and other "classical" differential couplings are ruled out), it would constitute a violation of the EP.

In the GGG experiment, the Sun is the source mass of an expected EP violation and therefore the deviation (if any) would occur at a period of 24 h. If $\eta \equiv \Delta a/a$ is the difference in acceleration between the two suspended masses, normalized to the average acceleration that they experience toward the Sun, their displacement from the equilibrium position is given by $\Delta x = \eta a/\omega_n^2$, where ω_n is the natural frequency of oscillation of the masses relative to each other in the harmonic potential of the balance. The centers of mass of the suspended bodies coincide (unlike ordinary balances), as in GG, to minimize classical tidal effects. The shaft is supported on ball bearings such that the accelerometer can be rotated around the symmetry axis to modulate the displacement signal, as in GG. The microstepping motor, driven by very stable clock, is weakly coupled to the rotor in order to minimize the noise conveyed to the rotor from the motor.

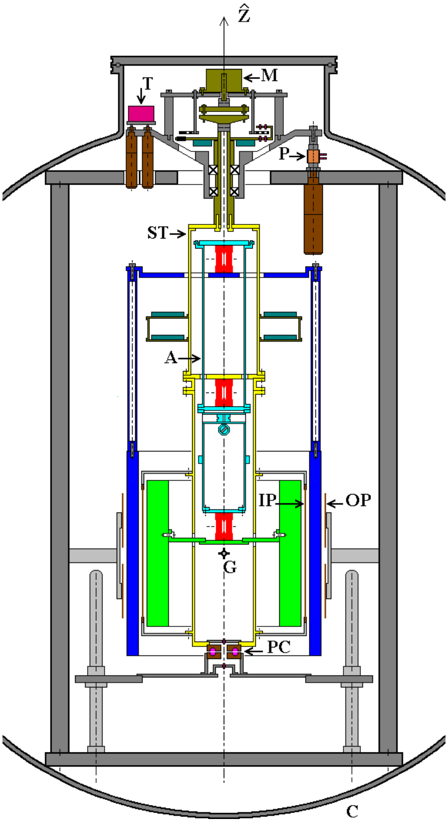

Fig. 4. Section through the spin axis $\hat{Z}$ of the GGG differential accelerometer inside the vacuum chamber. The drawing is to scale and the inner diameter of the vacuum chamber is 1 m. C — vacuum chamber; M — motor; x — ball bearings; ST — suspension tube; A — coupling (balance) arm, located inside the suspension tube, with its three laminar cardanic suspensions (in red); G — center of mass of the two-cylinder system (in blue the outer cylinder, in green the inner one; 10 kg each). IP are the internal capacitance plates of the differential motion detector, OP are the outer ones for whirl control and PC is the contactless inductive power coupler providing power to the electronics inside the rotor. T and P, at the top of the rotor, are the tilt meter and three PZT's (at 120° from one another; only one is shown) for automated control of low frequency terrain tilts.

Figure 5 shows, with a systematic series of measurement runs, the property of the GGG test cylinders to auto-center to reach a well-defined position of physical equilibrium which — for the given apparatus — is independent from initial conditions, as expected theoretically. Figure 6 reports some of the Q values measured with GGG, indicating that the values required for GG — with thinner suspensions — are realistic.

Figure 7 reports, in $m/\sqrt{\text{Hz}}$, residual relative displacements of the test cylinders in the horizontal, nonrotating plane, as measured between June 2005 and October 2006. We have acquired the ability to perform long runs during which

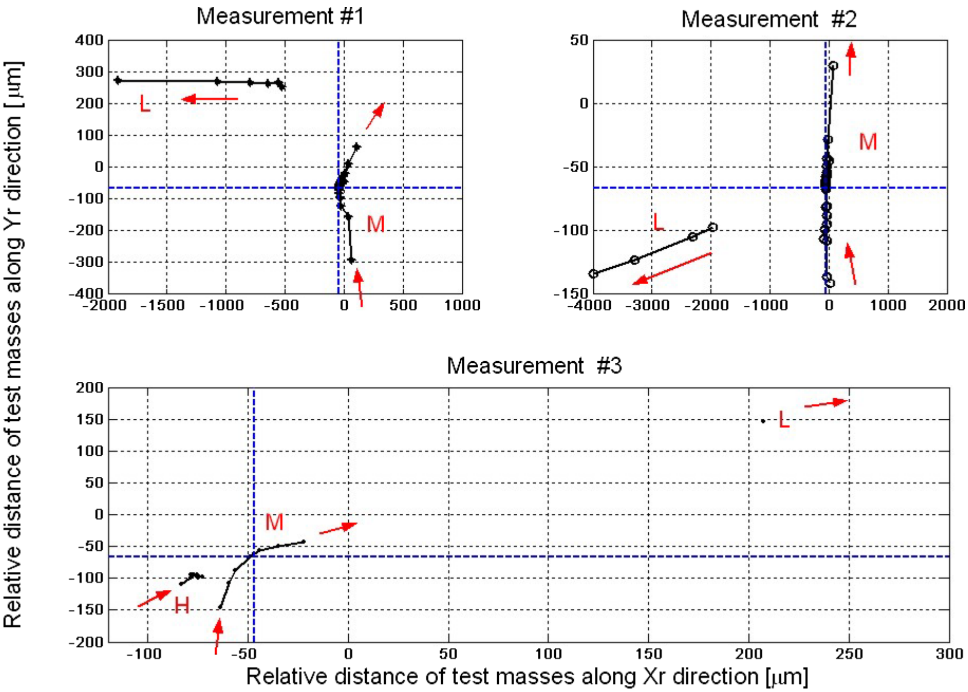

Fig. 5. Experimental evidence for auto-centering of the test cylinders in supercritical rotation. In the horizontal plane of the rotor, $X_r\,Y_r$, the centers of mass of the test cylinders approach each other as the spin frequency increases (along red arrow) from below the first resonance (L), to between the two resonances (M), to above both resonances (H). The equilibrium position reached is always the same (determined by the intersection of the two dashed lines), independently of initial conditions, as predicted theoretically. Each data point refers to a run of several hours.

whirl (at $\simeq 0.076\,\text{Hz}$, $13.2\,\text{s}$ natural period) is accurately controlled and is not a limitation. For an EP experiment in the field of the Sun, i.e. at $1.16 \times 10^{-5}\,\text{Hz}$, the best result is $\simeq 2 \times 10^{-6}\,\text{m}/\sqrt{\text{Hz}}$, amounting — in 3.8 days of integration time — to $\simeq 3.5 \times 10^{-9}\,\text{m}$. With a natural frequency of $0.076\,\text{Hz}$, and an acceleration from the Sun of $6 \times 10^{-3}\,\text{m/s}^2$, this means a sensitivity $\eta_{\odot 3.8\,\text{days}} \simeq 1.3 \times 10^{-7}$, currently limited by the 16-bit ADC converter and by the residual noise due to terrain tilts and temperature variations. Increasing the natural period would improve the sensitivity as the period squared. We are working on that, though it is not easy at 1 g. Longer integration times are feasible (e.g. 10 months, with an improvement by a factor 10); a spin frequency closer to the nominal 2 Hz value of the GG satellite will be tested.

4. Relevance of GGG Current Sensitivity for the Experiment in Space

Let an accelerometer with the capabilities of GGG as of today be flown in a GG satellite (520 km altitude, $1.75 \times 10^{-4}\,\text{Hz}$ orbital frequency). From Fig. 7,

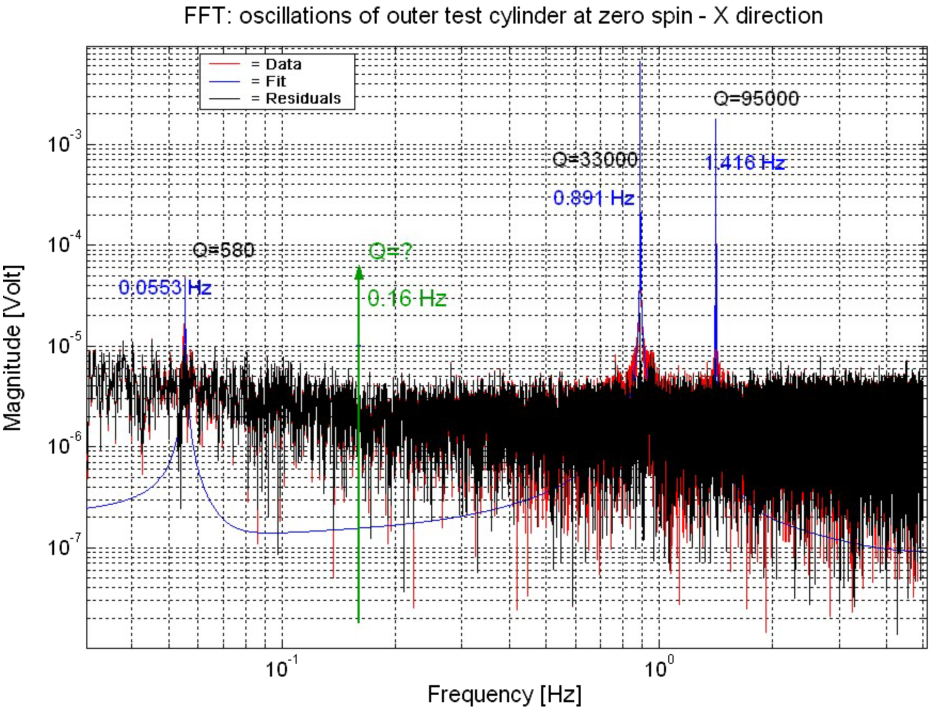

Fig. 6. Quality factors of the GGG accelerometer at its three natural frequencies, obtained by exciting them (at zero spin) and measuring the oscillation decay. In a later assembly, at the natural frequency of 0.08 Hz we have measured $Q = 3970$. In supercritical rotation the relevant Q is measured from the growth of whirl at the natural differential frequency of the test cylinders; at a spin frequency of 0.16 Hz (green line) we have measured 3020.

at 1.75×10^{-4} Hz it is sensitive to $10^{-6}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$, and hence — in 10 months of integration time, very compatible with the GG mission's duration — to relative displacements of 280 pm. With a natural frequency of 0.076 Hz (13.2 s period), and an acceleration from the Earth of $8.4\,\mathrm{m/s^2}$, this means an EP test in the field of the Earth to 7.5×10^{-12}, obtained by performing in space just as GGG in the lab today.

It is to be noted that in space the accelerometer will have *both* a lower platform noise *and* a better sensitivity. A *lower platform noise* than in GGG is due to the absence of terrain tilts, motor and bearings, resulting in an improvement by about a factor of 50. A *better sensitivity* requires us to weaken the coupling of the test cylinders; this can be done because the largest acceleration on GG (due to residual air drag) is 10^8 times smaller than local gravity on GGG. In fact, GG is designed with a natural period of 545 s[4] versus 13.2 s now in GGG, which means an improvement by a factor of 1700. For these improvements to take effect, the ADC converter will also need to be improved; as for thermal effects, they will be less

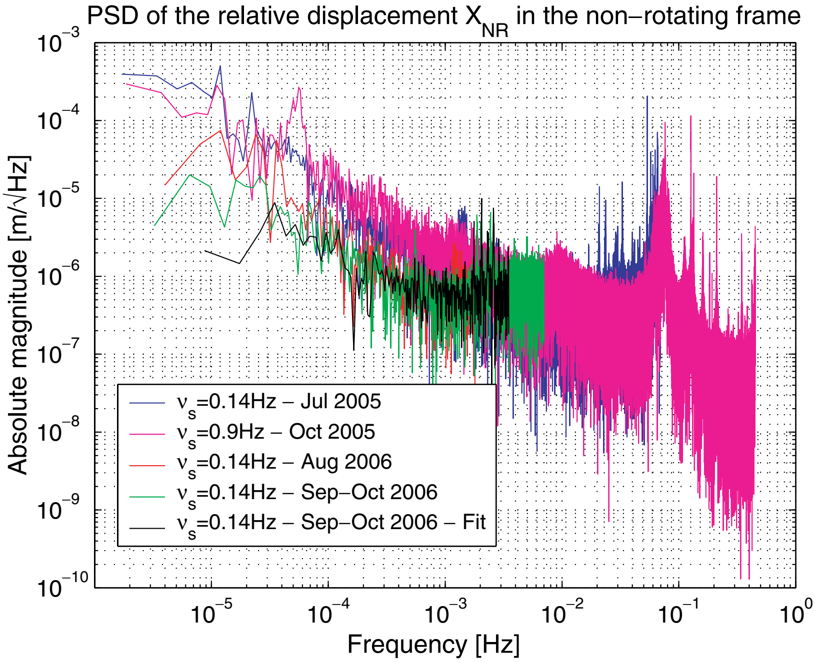

Fig. 7. Power spectral density (in $\mathrm{m}/\sqrt{\mathrm{Hz}}$) of the relative displacements of GGG test cylinders in the horizontal, nonrotating plane (X_{nr} direction). Whirl at the frequency $\simeq 0.076$ Hz is clearly well controlled. The best result (shown in black) was obtained after subtracting the displacements due to thermal expansions. The thermal expansion was modeled as a linear function of temperature. An EP violation signal from the Sun would occur at $1.16 \cdot 10^{-5}$ Hz (1 day). The orbital frequency of GG, at which an EP violation from the Earth would occur in space (see Fig. 2), is 1.75×10^{-4} Hz (5700 s).

severe because of the common rotation of the whole satellite, but require appropriate choices (e.g. carbon fiber structure, thermal insulation) already taken into account in GG mission studies.[4]

With such a reduced platform noise and an improved sensitivity made possible in space, the sensitivity reported above for GGG (7.5×10^{-10}) gets close to the GG mission target of testing the EP to 10^{-17}, which requires one to detect relative displacements of 0.6 pm at the satellite orbital frequency of 1.75×10^{-4} Hz.

Acknowledgment

GG studies have been funded by ASI and the GGG experiment by INFN.

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