“Galileo Galilei (GG) on the Ground-GGG”: experimental results and perspectives

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Abstract

The GGG differential accelerometer is made of concentric coaxial test cylinders weakly coupled in the horizontal plane and spinning in supercritical regime around their symmetry axis. GGG is built as a full scale ground based prototype for the proposed “Galileo Galilei-GG” space experiment aiming to test the equivalence principle (EP) to $10^{-17}$ at room temperature. We report measured $Q$ values of 95000 at 1.4 Hz, and expect even better ones at typical spin frequencies of a few Hz. An EP violation signal in the field of the Sun would appear as a low frequency displacement in the horizontal plane of the laboratory, and it can be separated out from a much larger whirl motion of the test masses at their natural differential frequency. So far we have managed to reduce the amplitude of this whirl to about 0.1 µm. We discuss how to improve these results in view of the very high accuracy GG experiment in space, and/or to reach a $10^{-13}$ sensitivity in the lab which would allow us to either confirm or rule out recent predictions of violation to this level.

Keywords: Equivalence principle; Quality factor measurements; Rotordynamics

1. Introduction

A fast rotating differential accelerometer made of weakly coupled concentric and self centering test cylinders, has been designed to be flown inside the small “Galileo Galilei”-GG satellite with the purpose of testing the equivalence principle (EP) to 1 part in $10^{17}$ at room temperature, see [1,2] and references therein. EP experiments in low Earth orbit take advantage of the stronger signal (by 3 orders of magnitude) for orbiting test masses, and the absence of weight (allowing the test masses to be very weakly suspended and coupled). Two other proposed missions, STEP [3] and µSCOPE [4], also aim to test the equivalence principle in space. The goals are $10^{-15}$ for µSCOPE and $10^{-17}$–$10^{-18}$ for STEP (by running the experiment at very low temperature). Both the STEP and µSCOPE accelerometers are sensitive only along the symmetry axis of the test cylinders and are designed to modulate the signal by rotation around an axis in the plane per-
perpendicular to it. Instead, the GG accelerometer spins around the symmetry axis (which appears to be the natural choice) and is sensitive in the plane perpendicular to it. In addition to preserving the 2D dimension of a possible EP violation signal in the orbital plane of the satellite, and to make fast rotation possible, a 2D accelerometer allows a full scale 1-g version of the instrument to be designed and tested in the laboratory. Indeed, if the spin/symmetry axis is used to suspend the accelerometer against local gravity, its plane of sensitivity lies in the horizontal plane where it could detect the signal of a possible violation of the equivalence principle. The “GG on the Ground”-GGG accelerometer is a full scale prototype of the one proposed for flight with GG (see [5] for details). We report the experimental measurements which confirm the main novel features of the GGG accelerometer as predicted from the theoretical analysis of its dynamical behavior, and the measured quality factors which demonstrate that the accelerometer is suitable for high accuracy EP tests. We also report the current sensitivity of the instrument and discuss how to improve it to demonstrate the feasibility of the GG experiment in space to $10^{-17}$ and to perform a ground test to $10^{-13}$.

The latter test would improve the present best results [6,7] by one order of magnitude, enough to either confirm or rule out recent violation predictions based on string theory [8].

2. Design and main features of the GGG differential accelerometer

Experimental tests of the equivalence principle are tests of the most direct experimental consequence of the “Principle”, namely the universality of free fall (UFF) by which in a gravitational field all bodies fall exactly the same independently of their mass or composition. UFF experiments require two test bodies of different composition in the gravitational field of a source mass (e.g., the Earth or the Sun). The bodies must be arranged to form a differential accelerometer and to accommodate a read-out system in between them to sense the effects of differential forces. In the GG accelerometer design for space the differential nature of the instrument is obtained in two ways. First, by arranging the test bodies (concentric, coaxial, hollow cylinders) like in a beam balance with the beam along the spin/symmetry axis of the cylinders and very weak coupling in the plane perpendicular to it (the plane of sensitivity). Second, by means of a capacitance read-out which is sensitive primarily to differential displacements of the centers of mass of the test cylinders relative to one another (it may sense common mode displacements too, but only to second order).

Both these features are retained in GGG at 1-g. Like in space, the design is that of a beam balance with the beam along the local vertical, which is also the spin/symmetry axis of the test bodies. These are concentric, coaxial hollow cylinders with the same 10 kg mass as in space. Appropriate cardanic suspensions are used such that they can withstand gravity along the vertical while also weakly coupling the test cylinders in the horizontal plane, for best sensitivity to differential forces. The coupling vertical beam is enclosed inside the rotation shaft by means of 3 such suspensions: the central one to suspend the whole system, the top and down ones for the outer and inner test cylinder respectively (see Fig. 1). The relative displacements between the centers of mass of the test cylinders are detected by a differential capacitance read-out in all similar to the one designed for flight. As in the design for space, the system spins in supercritical regime, i.e., at frequencies (of a few Hz) higher than its natural frequencies which allows self centering and reduced mechanical and electronic noise. The main difference with respect to the experiment in space is the need of a motor and of bearings (which are well-known sources of noise) to provide the rotation of the system. To the contrary, in space, once the whole satellite has been spun to the required rate, this is maintained by conservation of angular momentum and no motor or bearings are needed. Another important difference is that on the ground the spin/symmetry axis of the accelerometer is also the direction of local gravity, a force which exceeds any other force acting on the system by far hence imposing a top/down asymmetry in the accelerometer design, as it is apparent by comparing the GGG accelerometer shown in Fig. 1 with the GG accelerometer shown in Fig. 2 of Ref. [1].

The theoretical analysis of the GGG dynamical system allows us to predict its natural frequencies of oscillation in the plane of sensitivity of the instrument (the horizontal plane), to be compared with their
Fig. 1. Section through the spin axis of the GGG differential accelerometer inside the vacuum chamber. (Figure is in colour on the web.) VC: vacuum chamber; MO: motor (drawn in brown); x: ball bearings; OR: O-rings; AD: annular dishes with the read-out electronics; CP: capacitance plates; OD: optical device; \( m_i \): inner test mass (green); \( m_o \): outer test mass (blue); LS: laminar suspensions (orange); \( m_a \): coupling arm (cyan); ST: suspension tube (yellow). The open circle indicates the position of the bodies center-of-mass CM. The drawing is to scale and the inner diameter of the vacuum chamber is 1 m. Also: \( L_o = 38 \text{ cm} \); \( L_a = 19 \text{ cm} \); \( L_i = 4.5 \text{ cm} \); \( \Delta L \equiv 0 \text{ cm} \); \( l = 0.5 \text{ cm} \).
measured values. The sensitivity of the instrument to
the effect of differential forces (such as the effect of
an EP violation) increases with the natural frequency
of differential oscillation of the test cylinders one with
respect to the other to power $-2$, and this frequency
can be reduced by using the force of gravity to provide
a negative spring.

In Fig. 2 this frequency is just below 0.1 Hz,
but it can be further reduced. The figure shows the
comparison between the theoretical and the predicted
values of the 3 natural frequencies at zero spin rate
in the X and Y directions of the horizontal/sensitivity
plane, indicating that the values of these frequencies
are known beforehand. Once in rotation, the values
of the natural frequencies slightly change depending
on the spin rate, and these changes can also be
predicted theoretically. Fig. 3 shows that all measured
values of the natural frequencies lie on the predicted
lines. For each spin rate, the supercritical regime lies
below the 45$^\circ$ resonance line. In this regime, due to
inevitable losses in the system, at the slightly changed
values of the natural frequencies the system develops
whirl motions (see Section 3). More details on the
simulation program that we have developed in order
to predict the dynamical behavior of the GGG system
(and possibly improve its sensitivity by appropriate
changes in the design) are given in [9,10].

3. Quality factor, whirl control and sensitivity

In order to reduce thermal noise and to improve
sensitivity, an accelerometer devoted to testing the
equivalence principle should have quality factors $Q$
(inverse of loss factors in the system) of values as
high as possible. An important advantage of rotation
in supercritical regime (spin frequency higher than the natural ones) comes from the well-known fact that in this regime the suspensions are deformed at the spin frequency of the system, not at their natural ones; hence, losses occur at this frequency (which is the highest in the system), and they are known to decrease with frequency. We can therefore design the system so as to have a very weak coupling of the test cylinders, hence, a very low natural frequency for differential oscillations of their centers of mass one with respect to the other (for best sensitivity to differential forces), and yet obtain a high quality factor by spinning at high frequency. Moreover, since rotation provides the modulation of the signal, high spin rate also means high modulation frequency and reduced “1/f” noise.

It is therefore apparent that the supercritical regime is extremely well suited for accelerometers aiming to test the equivalence principle.

Quality factors at the natural frequencies can be measured, for the whole system, at zero spin rate, by exciting oscillations at these frequencies and measuring the decay in the oscillation amplitude. Fig. 4 reports measurements performed in 2002 at the 0.9 Hz natural frequency, yielding value of 16450. In 2003, with improved suspensions, we have obtained, at about the same frequency a higher $Q$ value (33000), as shown in Fig. 5. The same figure shows measured $Q$ values at the other 2 natural frequencies (1.4 Hz and 0.08 Hz). As expected, the $Q$ value increases with the frequency (losses are smaller at higher frequencies),

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**Fig. 3.** So-called “Campbell diagram” for the GGG rotor. It gives the natural frequencies of the system (in the non-rotating reference frame of the laboratory) as function of the spin rate of the rotor. (Figure is in colour on the web.) The blue (solid) lines have been predicted within our simulation program of the system (circles on these lines are computed including also a realistic dissipation, i.e., losses in the system, to show that dissipation does not affect the natural frequencies); the red crosses are the measured values, and they clearly confirm the predicted ones. The supercritical regime is easily identified below the red dashed line at 45° inclination (see [9] for details).
Fig. 4. Measurement of the quality factor of the GGG system at its natural frequency of 0.9 Hz. (Figure is in colour on the web.) The system (at zero spin) is excited at this frequency and the decay in oscillation amplitude is measured. The decay turns out to be compatible with a $Q$ value of 16450. The run refers to the GGG system set up as in the year 2002.

Fig. 5. Resulting quality factors of the GGG accelerometer at the natural frequencies (at zero spin) as obtained by measuring the oscillation decay of the system. (Figure is in colour on the web.) The blue curve is the FFT of the fitted output data. The runs refer to an improved system set up (with improved cardanic suspensions) of June 2003. Note the higher $Q$ value at about 0.9 Hz as compared to the value reported in Fig. 4.
reaching the value of 95000 at 1.4 Hz. Since the spin rate is 2 Hz and above, we expect that the relevant $Q$ in supercritical regime will be even better (higher) than this value. With cardanic suspensions of rather complex shape (see Fig. 1), which are therefore not easy to manufacture, the measured $Q$ is indeed higher than we had expected. It is worth noting that the GG space mission studies have been carried out assuming, for the mission target of testing the equivalence principle to $10^{-17}$, a $Q$ value of 20000 (see [1] and references therein). The ground measurements reported in Fig. 5 indicate that such an assumption is in fact rather conservative.

In supercritical rotors losses at the spin frequency are also relevant for the growth rate of whirl motions that such rotors are known to develop, once in supercritical rotation, at their natural frequencies in the non-rotating system. For instance, the centers of mass of the GGG test cylinders do develop an orbital motion in the horizontal plane of the laboratory at the natural frequency of differential oscillations around their position of relative equilibrium, which is determined by external differential forces (see simulation, Fig. 3 of Ref. [1] for the case in space). Such a whirl motion grows in amplitude at a rate which depends on the $Q$ of the system at the spin frequency: the higher the $Q$ at this frequency, the slower the growth rate of the whirl. More precisely, rotordynamics predicts that whirl grows with a (negative) $Q$ equal and opposite to the $Q$ of the system at its spin frequency. In GGG whirls are controlled actively by means of capacitance sensors/actuators with a control scheme which is proportional to the tangential whirl velocity. Measurements of the relative displacements of the test cylinders show, after coordinate transformation to the non-rotating reference system, a controlled whirl motion at a differential frequency of 0.08 Hz: the whirl radius has been reduced from a few hundred µm to about 0.1 µm (see the FFT plot of Fig. 6 at this whirl frequency). In order to detect the effect of a low frequency differential force (such as in the case of a 24 hr EP violation signal in the field of the Sun), the corresponding displacement between the centers of mass should be separated out from the whirl and also emerge from the residual low frequency noise, mostly seismic noise. An example of recovery of an

Fig. 6. FFT of the relative displacements of the test cylinders in the Y direction of the horizontal plane in the non-rotating reference system. The relevant whirl at the natural frequency of 0.08 Hz has been reduced to about 0.1 µm. The effect of a differential force at lower frequency must be separated out and emerge from the low frequency residual noise (see Fig. 7).
applied signal at frequency below whirl frequency is shown in Fig. 7, and indicates that recovery is possible even though the applied force produces a displacement much smaller than the whirl radius. However, this was possible in the run of Fig. 7 where the residual low frequency noise was also much smaller than the whirl. In Fig. 6 (which reports more recent measurements) the whirl radius is smaller than in Fig. 7 by about 3 orders of magnitude, but residual low frequency noise is not correspondingly smaller. In fact we have reasons to think that this is local noise due to the vacuum chamber opening/closing system, which can be fixed.

Though the measurements reported in Fig. 6 clearly show the potentiality of the GGG novel design for detecting the effect of very small low frequency differential forces in the horizontal plane, they have also identified an important issue which requires immediate attention. In spite of the high $Q$ values measured at zero spin rate (due to the suspensions only), the growth rate of whirl once in supercritical rotation is indeed much faster than expected by high $Q$, indicating that, during rotation, much bigger losses take place in the system beside the ones in the suspensions. A source of “rotating damping” (the kind of damping which is known in rotordynamics to produce whirl instability) may be due to the rubber O-ring used to transmit rotation from the motor (in its offset location) to the rotating suspension shaft/tube (see Fig. 1). This problem can be solved by eliminating the O-ring altogether, i.e., by locating the motor on the spin axis with care to have a hollow shaft for optical transmission of digitised data from the read-out to the computer outside the vacuum chamber. Other possible causes of spurious losses are under investigation in order to obtain the $Q$ values measured in absence of rotation.
4. Conclusions

A space version of the fast rotating GGG differential accelerometer presented here, to be used within the GG mission ([1] and references therein) in low Earth orbit, would take advantage of the stronger driving signal (8.4 m s\(^{-2}\) from the Earth at 520 km altitude in GG, instead of 0.006 m s\(^{-2}\) from the Sun in GGG). It would also allow much weaker suspensions due to absence of weight, and consequent higher sensitivity (which depends on the differential period squared) by a factor of about 450. It can be argued that another factor of about 200 can be gained due to the absence of motor and motor/bearings noise, and thanks to the much higher symmetry of the space accelerometer (no 1-g preferential direction, hence much better rejection of common mode forces and consequent higher sensitivity to differential forces). Overall this amounts to about 8 orders of magnitude gain, thus making a \(10^{-17}\) test in space a goal worth pursuing. In point of fact, the error budget of the GG space experiment, as developed within mission studies so far, has turned out to be compatible with this goal [1]. The improvement over current best ground results [6,7] would be of 5 orders of magnitude.

Very recently, predictions of violation have been reported [8] at levels close to the current best results, so that even a slight improvement on those experiments on the ground (to reach the \(10^{-13}\) level) would be able to either confirm or rule out these predictions. The GGG rotating differential accelerometer can be used to test the equivalence principle in the gravitational field of the Sun to 1 part in \(10^{13}\). In GGG this goal requires to detect low frequency (24 hr) relative displacements of the test cylinders of \(10^{-13}\) m, which in turn requires to reduce daily seismic disturbances by 7 orders of magnitude with respect to daily tilts measured so far. This can be done partly actively and partly passively. Active reduction is done using as sensor a tiltmeter placed inside the vacuum chamber at the top of the GGG frame (not rotating), and as actuators a set of PZTs (also not rotating, at 120° in the horizontal plane around the symmetry axis, providing tilts of the apparatus through vertical displacements). 3 orders of magnitude reduction can be obtained in this way. A further reduction by about 4 orders of magnitude, down to \(4 \times 10^{-14}\) m, which would bring the effects of tilts and horizontal disturbances well below the target signal, can be obtained using a passive cardanic suspension and the lever effect (see [11] for details). As discussed in [11], the main reason why such a significant reduction of seismic noise at very low frequency is possible is in the very nature of the GGG differential accelerometer. Since the test cylinders are arranged as in a vertical beam balance, the observable of interest in GGG are the relative displacements of the test cylinders relative to it, not the absolute location of the beam which indeed undergoes much larger displacements by following the local vertical in its seismic disturbed motion. This is not the case in VIRGO-like apparatus used to detect gravitational waves with interferometric techniques. The suspended test masses being the mirrors of the interferometric system, their absolute displacements (unless one could make it possible that both mirrors undergo exactly the same displacements) must fulfill a rather stringent requirement coming from the need that locking is preserved (see [12]).

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References


