

THE FAST ROTATING “GGG” DIFFERENTIAL ACCELEROMETER FOR TESTING THE EQUIVALENCE PRINCIPLE: CURRENT STATE AND ANALYSIS OF SEISMIC DISTURBANCES

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Measurements performed with the fast rotating “Galileo Galilei on the Ground – GGG” differential accelerometer are reported. They show the validity of the main novel features of this instrument which was built as a full scale prototype for the proposed “Galileo Galilei-GG” space experiment aiming to test the equivalence principle to 10^{-17} at room temperature. GGG can also aim to test the equivalence principle to 10^{-13} in the lab. The effects of terrain tilts and local horizontal disturbances are analyzed showing how they can be reduced below the required level.

1 The GGG differential accelerometer: design and current sensitivity

A fast rotating differential accelerometer made of weakly coupled concentric and self centering test cylinders, has been designed to be flown inside the small “Galileo Galilei”-GG¹ satellite with the purpose of testing the equivalence principle to 1 part in 10^{17} at room temperature. The accelerometer is sensitive in 2 dimensions in the plane perpendicular to its spin/symmetry axis. Because of this feature, it has been possible to design a version of it (“GG on the Ground”-GGG²) to be fully tested in the laboratory: if the spin/symmetry axis is used to suspend the accelerometer against local gravity, its plane of sensitivity lies in the horizontal plane where it could detect the signal of a possible violation of the equivalence principle (Fig. 1). Appropriate cardanic suspensions can withstand gravity along the vertical and also weakly couple the test cylinders in the horizontal plane. The coupling vertical beam is enclosed inside the rotation shaft by means of 3 cardanic suspensions: the central one to suspend the whole system, the top and down ones for the outer and inner test cylinder respectively (see Fig. 1). The relative displacements between the centers of mass of the test cylinders are detected by a differential capacitance read-out. The system spins at frequencies of a few Hz, higher than its natural frequencies. This allows the construction offset errors to be reduced during rotation (self-centering in supercritical regime). It also makes the suspension deformations (and consequent losses) take place at the spin frequency; since this is large, losses are much smaller than they would otherwise be. In Fig. 2 we report the measured values of the quality factor inverse of losses) at the natural frequencies of the system. The largest measured value is $Q=95000$ at 1.4 Hz.

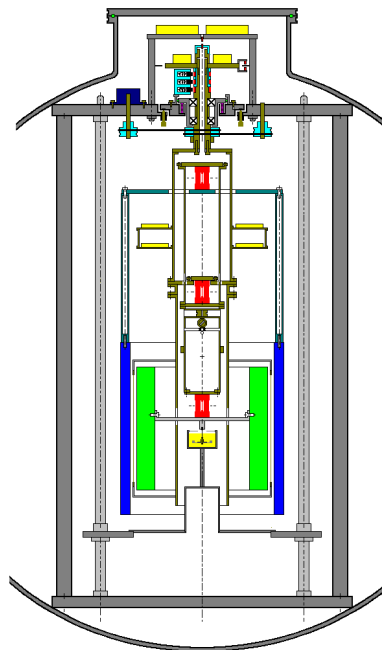


Figure 1: Schematic design of the GGG apparatus (section through the spin/symmetry axis of the system.) The concentric, coaxial test cylinders (green and blue) weigh 10 kg each. The enclosing vacuum chamber has 1 m diameter. Three cardanic suspensions (in red) are shown at the center, top and bottom of the arm which couples the test cylinders thus forming a balance with a vertical beam. So far the spin rate has been of a few Hz with a natural period of oscillation of the test cylinders relative to one another of 10 to 15 sec.

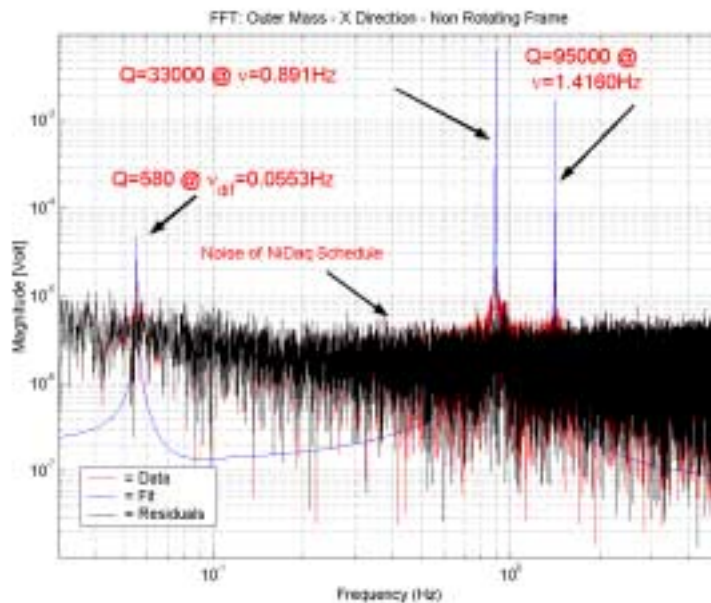


Figure 2: Resulting quality factors of the GGG system at the natural frequencies (at zero spin) obtained by measuring the oscillation decay of the system. The blue curve is the FFT of the fitted output data.

Once in supercritical rotation, the test cylinders show whirl motions at frequencies close to their natural frequencies (in the non rotating horizontal plane of the laboratory) and, primarily, a whirl motion relative to one another at the natural frequency of differential oscillations. The experimental results reported in Fig. 3 (after coordinate transformation to the non rotating reference system) show that whirl motion can be

identified and separated out, so that a low frequency, smaller differential signal can be detected (a violation signal in the field of the Sun would follow its daily motion). In Fig. 4 we show that low frequency residual noise is of about $1.5 \mu\text{m}$.

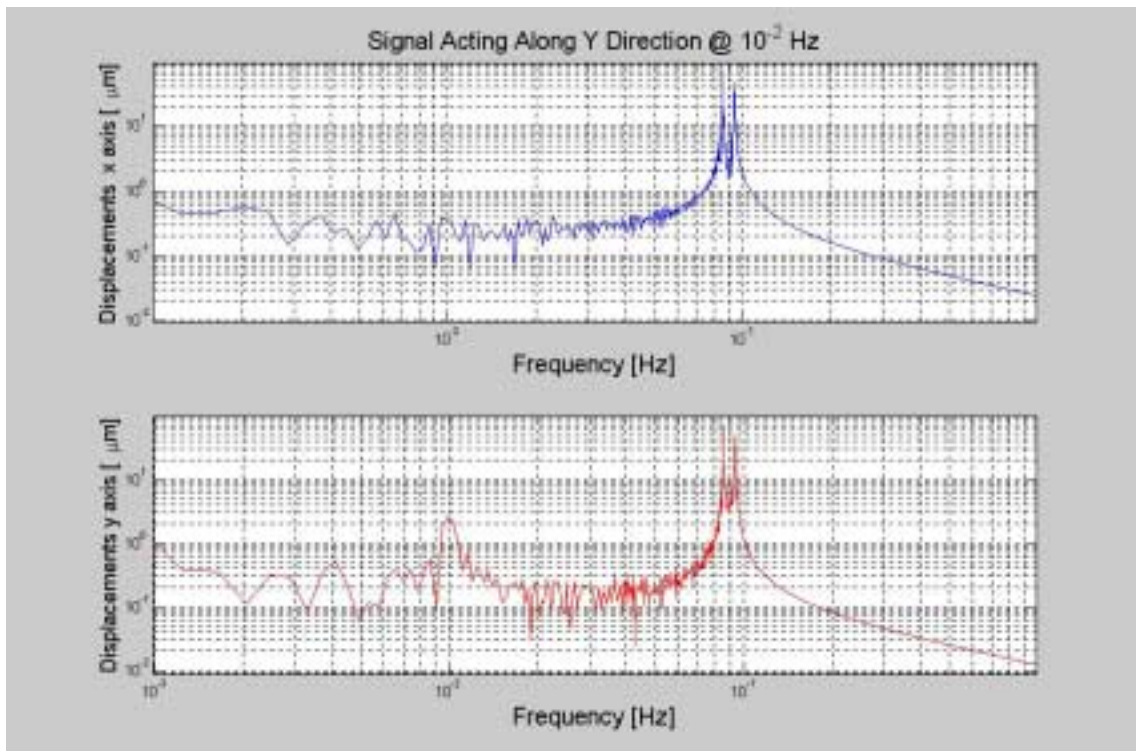


Figure 3: A signal applied in the y direction at 0.01 Hz is recovered from the output data though about 400 times smaller than the whirl (more than $100 \mu\text{m}$) at about 0.1 Hz (system spinning at 2 Hz).

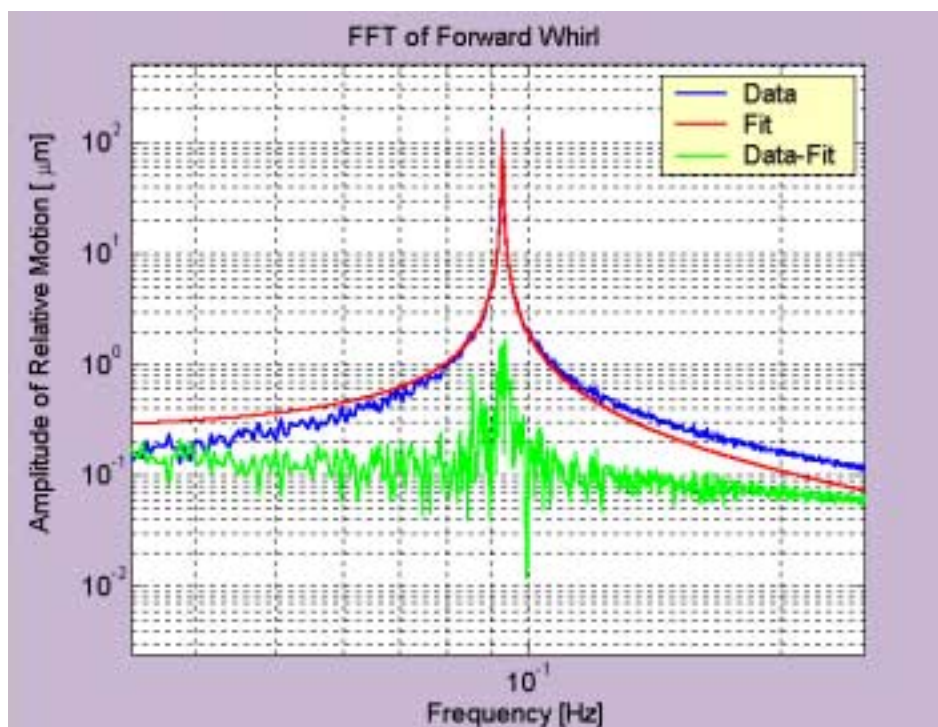


Figure 4: Measured and fitted whirl signal. Residual noise at lower frequencies is a few $10^{-1} \mu\text{m}$ (2 Hz spin).

In more recent measurements performed with an improved and more accurate control system, the amplitude of whirl motion has been reduced to 0.2 μm , down from 100 μm reported in Figs. 3 and 4. According to the current plan of the GGG experiment as recently funded by INFN, we expect to reach a low frequency stability of 10^{-10} m by the year 2005, corresponding to a sensitivity in testing the equivalence principle in the field of the Sun of 1 part in 10^9 .

2 Relevance for a space test to 10^{-17}

A space version of such an instrument, to be used within the GG mission¹ in low Earth orbit, would take advantage of the stronger driving signal (8.4 ms^{-2} from the Earth at 520 km altitude in GG, instead of 0.006 ms^{-2} from the Sun in GGG). It would also allow much weaker suspensions due to absence of weight, and consequent higher sensitivity (which depends on the differential period squared) by a factor about 450. It can be argued that another factor of about 200 can be gained due to the absence of motor and motor/bearings noise, and thanks to the much higher symmetry of the space accelerometer (no 1-g preferential direction, hence much better rejection of common mode forces and consequent higher sensitivity to differential forces). Overall this amounts to about 8 orders of magnitude gain, thus making a 10^{-17} test in space a goal worth pursuing. In point of fact, the error budget of the GG space experiment, as developed within mission studies so far, has turned out to be compatible with this goal¹. The improvement over current best ground results^{3,4} would be of 5 orders of magnitude.

3 Seismic noise attenuation for a ground test to 10^{-13}

Very recently, predictions of violation have been reported⁵ at levels close to the current best results, so that even a slight improvement on those experiments on the ground (i.e. to reach the 10^{-13} level) would be significant. The GGG rotating differential accelerometer can be used to test the Equivalence Principle in the gravitational field of the Sun to 1 part in 10^{13} . In GGG this goal requires to detect low frequency (24-hr) relative displacements of the test cylinders of 10^{-13} m, which in turn requires to reduce daily seismic disturbances below this level.

If the terrain where the apparatus is located undergoes low frequency tilts of amplitude α we need to evaluate the effects of such tilts on the experiment and reduce them if necessary. In the presence of a laminar cardanic suspension providing a stiffness k in the horizontal plane the tilt angle β at equilibrium is determined by the condition that the restoring force of the suspension equals the horizontal component of the local acceleration of gravity arising because of the tilt. The equilibrium equation is:

$$mg\beta = k\ell(\alpha - \beta) \quad (\sin \alpha \approx \alpha, \sin \beta \approx \beta) \quad (1)$$

from which the ratio β/α is derived by which the original tilt angle α is reduced because of the suspension:

$$\frac{\beta}{\alpha} \approx \frac{k\ell}{mg} \left(1 + \frac{k\ell}{mg} \right)^{-1} \quad (2)$$

If $k\ell/mg \ll 1$ so that terms of order $(k\ell/mg)^2$ or higher can be neglected, we get that tilts are reduced by the factor:

$$\frac{\beta}{\alpha} \approx \frac{k\ell}{mg} \ll 1 \quad (3)$$

In terms of the acceleration acting on the mass m , in absence of the cardanic suspension, the tilt angle is α and the mass is subject to the *gravitational* acceleration $g\alpha$ in the horizontal plane. With the suspension and the tilt angle β the corresponding acceleration is $g\beta$ (in point of fact, there is also a vertical component, but it is negligible for small tilts). Therefore, the acceleration on the test mass too is reduced, in the presence of the suspension, by the same ratio β/α given by Eq. 3 as the tilt angle.

Let us now consider the case in which, instead of being subject to a terrain tilt, the system is subject to a horizontal disturbing acceleration with the same (low) frequency as the tilt, and amplitude $a = \alpha g$. As a result, the test mass is subject to an *inertial* acceleration equal and opposite to the disturbing one, which defines the direction of a new local vertical as the vectorial sum $\vec{g} - \vec{a}$ forming an angle $\alpha = a/g$ with the original vertical (defined by the direction of local gravity in absence of the disturbing acceleration). In the presence of a suspension providing a stiffness k in the horizontal plane, equilibrium is reached at a different angle γ with the original vertical, where the acceleration $g\gamma - a$ acting on the test mass ($\sin \gamma \approx \gamma$, $\cos \gamma \approx 1$) is balanced by the restoring force of the suspension according to the equation:

$$mg\gamma - ma = -k\ell\gamma \quad (4)$$

The equilibrium angle with the suspension is therefore:

$$\gamma \approx \frac{a}{g} \left(1 + \frac{k\ell}{mg} \right)^{-1} \approx \frac{a}{g} \left(1 - \frac{k\ell}{mg} \right) \quad (5)$$

And the deviation from the *new* local vertical is

$$\beta \approx \frac{a}{g} \cdot \frac{k\ell}{mg} = \alpha \cdot \frac{k\ell}{mg} \quad (6)$$

The effect of a suspension with horizontal stiffness k is therefore to make the test mass tilt from the *new* local vertical only by the same small angle β as in the case of a terrain tilt by the angle $\alpha = a/g$ (in the same approximation in which terms of the order of $(k\ell/mg)^2$ or higher are neglected). In terms of the acceleration acting on the test mass (in the horizontal, sensitivity plane), this is $g\gamma - a$ which, in the presence of the suspension, amounts to $a(k\ell/mg)$. This means a reduction, with respect to the local disturbing acceleration a acting at the top of the system, by the ratio $k\ell/mg \ll 1$, just as in the case of the terrain tilts. Therefore, because of the equivalence between inertial and gravitational mass which at this level can be assumed to be valid, local terrain tilts cannot be distinguished from horizontal disturbing accelerations. Since in GGG the signal is a relative displacement of the test cylinders around the local vertical, as its direction changes because of horizontal seismic accelerations the beam of the balance will follow it, but these absolute displacements are not relevant for the GGG measurements while those relative to it are reduced by the suspension just like tilts.

Low frequency tilts in the vicinity of the GGG apparatus have been monitored with the ISA tiltmeter/accelerometer. Daily effects turn out to have an amplitude of about 10^{-6} rad (corresponding to horizontal accelerations of about 10^{-6} g). The resulting effect on the GGG test cylinders is to give rise to relative displacements (in the horizontal plane of the laboratory) at the same frequency and with an amplitude of about $4 \cdot 10^{-7}$ m (the suspension arm relevant for the relative displacements of the test cylinders being about 0.4 m long). The goal of testing the Equivalence Principle to 10^{-13} with GGG requires to detect low frequency (24-hr) relative displacements of the test cylinders of 10^{-13} m, which in turn requires daily seismic disturbances to be reduced below this level. This can be done partly actively and partly passively. Active reduction is done using as sensor a tiltmeter placed inside the vacuum chamber at the top of the GGG

frame (not rotating), and as actuators a set of PZTs (also not rotating, at 120° in the horizontal plane around the symmetry axis, providing tilts of the apparatus through vertical displacements). The tiltmeter currently installed can detect tilts of 10^{-9} rad. At the location of the PZTs the arm length with respect to the symmetry axis is about 0.1 m, and therefore the vertical effect of such tilts would be of about 10^{-10} m, which they can correct by applying a voltage of the order of a mV. If successful, this control would leave a residual relative displacement of the test cylinders of about $4 \cdot 10^{-10}$ m.

A further reduction by about 4 orders of magnitude, down to $4 \cdot 10^{-14}$ m, which would bring the effects of tilts and horizontal disturbances well below the target signal, can be obtained using a passive cardanic suspension and the lever effect. If k_s is the intrinsic elastic constant in any direction of the horizontal plane, of a laminar suspension with strip length λ placed at the top of a suspension arm of length ℓ , because of the lever effect the resulting elastic constant in the horizontal plane is $k = (\lambda/\ell)^2 k_s$. With the reduction factor in the tilt angle as given by Eq. 3, it follows that, with $\lambda \approx 5 \cdot 10^{-3}$ m, $\ell \approx 0.5$ m, $m \approx 40$ kg, $k_s \approx 8 \cdot 10^2$ N/m, the reduction is $\beta/\alpha \approx 10^{-4}$, as required. The laminar suspensions currently used in GGG have $\lambda_{GGG} \approx 5 \cdot 10^{-3}$ m, $k_{sGGG} \approx 10^3$ N/m, thus indicating that it is possible to achieve the required passive attenuation so that, overall, seismic disturbances would not impair a 10^{-13} test of the equivalence principle on the ground with the GGG differential accelerometer.

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