"Galileo Galilei-GG": design, requirements, error budget and significance of the ground prototype

A.M. Nobili [a,b,*], D. Bramanti [a], G.L. Comandi [a,b], R. Toncelli [a,b], E. Polacco [b], M.L. Chiofalo [a,b]

[a] *Space Mechanics Group, Department of Mathematics, University of Pisa, Via F. Buonarroti, I-56127 Pisa, Italy*
[b] *INFN, Sezione di Pisa, Via F. Buonarroti, I-56127 Pisa, Italy*

Abstract

"Galileo Galilei-GG" is a proposed experiment in low orbit around the Earth aiming to test the equivalence principle to the level of 1 part in 10^{17} at room temperature. A unique feature of GG, which is pivotal to achieve high accuracy at room temperature, is fast rotation in supercritical regime around the symmetry axis of the test cylinders, with very weak coupling in the plane perpendicular to it. Another unique feature of GG is the possibility to fly 2 concentric pairs of test cylinders, the outer pair being made of the same material for detection of spurious effects. GG was originally designed for an equatorial orbit. The much lower launching cost for higher inclinations has made it worth redesigning the experiment for a sun-synchronous orbit. We report the main conclusions of this study, which confirms the feasibility of the original goal of the mission also at high inclination, and conclude by stressing the significance of the ground based prototype of the apparatus proposed for space.
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1. Introduction

The equivalence principle (EP) stated by Galileo, reformulated by Newton and reexamined by Einstein to become the founding principle of General Relativity, can be tested from its most direct consequence: the universality of free fall (UFF), whereby all bodies fall with the same acceleration regardless of their mass and composition ($\eta \equiv \Delta a/a = 0$). The most ac-

curate EP experiments have been carried out on the ground with test bodies of different composition suspended on a torsion balance. In the case of Be and Cu it was found $\eta(\text{Be}, \text{Cu}) = (-1.9 \pm 2.5) \times 10^{-12}$ [1]. In [2], the differential acceleration between test cylinders of "earth's core" and "moon/mantle" composition in the gravitational field of the Sun ($a_\odot \simeq 0.6 \text{ cm s}^{-2}$) is reported with a 1σ statistical uncertainty $\Delta a_\odot = 5.6 \times 10^{-13} \text{ cm s}^{-2}$, hence $\Delta a_\odot/a_\odot \simeq 9.3 \times 10^{-13}$.

Test bodies in low Earth orbit are subject to a driving gravitational (and inertial) acceleration much stronger than on torsion balances on the ground, by about 3 orders of magnitude. Moreover, the absence

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* Corresponding author.
 E-mail address: nobili@dm.unipi.it (A.M. Nobili).

of weight is ideal in small force experiments. There is therefore general agreement on the fact that a very high accuracy test of the equivalence principle can be achieved only by flying the test masses inside a spacecraft in low Earth orbit. It is also agreed that the test bodies should be weakly coupled, concentric, co-axial cylinders, and that they should rotate (the faster the better) for signal modulation. The "Galileo Galilei" (GG) space experiment [3] aims to reach 10^{-17}, which is highly competitive with μSCOPE's goal (10^{-15}) [4]. STEP's goal is the same as that of GG [5], or even 1 order of magnitude more ambitious [6] but the spacecraft is much more massive and the experiment must be performed close to absolute zero rather than at room temperature as GG.

Both GG and STEP would be able to check a possible EP violation predicted by Fischbach et al. [7] at the 10^{-17} level by rigorous calculation of higher order weak interactions, should gravity couple anomalously to weak interaction energy. Beyond the standard model, within string theory, recent work [8] predicts much stronger a deviation, to the level of 10^{-12} for test bodies made of Cu and Be or Pt and Ti. A modest improvement over current torsion balance laboratory tests should be sufficient to either confirm or rule out this prediction.

2. The signal and the accelerometers

Testing the UFF requires two masses of different composition, arranged to form a differential accelerometer, and a read-out system in between them. In GG the test bodies are concentric, co-axial, hollow cylinders weakly coupled like in a beam balance with the beam directed along the symmetry axis, so as to be sensitive to differential accelerations acting between the bodies in the x, y plane perpendicular to it (the weaker the coupling, the higher the sensitivity). Coupling and balancing allow common mode effects to be rejected. Two capacitance bridges in between the test cylinders read their relative displacements (caused by differential accelerations) in the plane of sensitivity. The better the mechanical balance of the bridge capacitance plates halfway in between the test cylinders, the more insensitive is the read-out to common mode effects. Thus, the differential nature of the accelerometer is ensured both by the suspension and by the read-out.
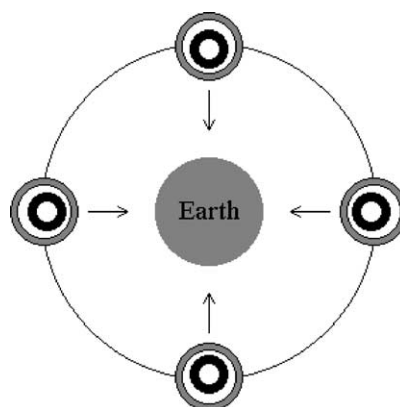

Fig. 1. Section across the spin/symmetry axis of the GG outer and inner test cylinders (of different composition) as they orbit around the Earth inside a co-rotating, passively stabilized spacecraft (not shown). The centers of mass of the test cylinders are shown to be displaced towards the center of the Earth as in the case of a violation of the equivalence principle in the field of the Earth (indicated by the arrows). The signal is therefore at the orbital frequency, and is modulated at the spin frequency of the system with respect to the center of the Earth (2 Hz nominal). The figure is not to scale (taken from [9]).

High frequency modulation of the expected signal— for the reduction of "$1/f$" electronic and mechanical noise—is obtained by spinning the accelerometer around the symmetry axis (the beam of the balance): a cylindrical spacecraft encloses, in a nested configuration, a cylindrical cage with the test cylinders inside, and is stabilized by rotation around the symmetry axis. Once the spacecraft has been given the required rate of rotation at the beginning of the mission (2 Hz nominal with respect to the center of the Earth), no motor is needed in space. Hence, the space experiment is not affected by noise from the motor, contrary to what happens with rotating apparata in ground based laboratories where the motor and its noise are a serious matter of concern. As shown in Fig. 1, an EP violation in the field of the Earth would generate a signal of constant amplitude (for zero orbital eccentricity) whose direction always points to the center of the Earth, hence changing orientation with the orbital period of the satellite. The read-out, also rotating with the system, will therefore modulate an EP violation signal at its spin frequency with respect to the Earth.

The expected signal benefits from the spacecraft orbiting the Earth at low altitude. Having selected 520 km for GG, an orbit inclination of 97.5° ensures

that the spacecraft follows the annual motion of the Sun (sun-synchronous orbit) and makes it possible to use a high latitude, low cost, Russian launcher for orbit injection. By maintaining the spin/symmetry axis of the spacecraft within about 10 degrees from the orbit normal, there is almost no degradation of the signal along both directions in the sensitivity plane of the accelerometer.

For the GG space experiment we have designed 2 concentric differential accelerometers, the inner one for EP testing (with cylinders of different composition) and the outer one for zero check (with cylinders made of the same material). They are sensitive in the x, y plane perpendicular to the symmetry axis which is also the (natural) axis of rotation, so as to provide frequency modulation of the expected signal. Fig. 2 shows a section through the spin/symmetry axis of the accelerometers, and a detailed description is given in the caption. In order to provide an intermediate stage of isolation between the spacecraft and the test cylinders the accelerometers of Fig. 2 are not suspended directly from the spacecraft, but instead from the so-called PGB ("pico gravity box") laboratory: a cylindrical structure which is mechanically suspended from the spacecraft along its symmetry axis (see Fig. 4) so as to provide weak coupling in the plane perpendicular to the axis while being more stiff along it (as in the case of the test cylinders).

In GG the test cylinders are suspended mechanically. However, whatever the nature of the suspensions, there will always be a non-zero offset vector $\vec{\varepsilon}$ from the spin axis (in the reference frame fixed with the system) due to construction and mounting errors. The equilibrium position vector of the centre of mass of the suspended body, for given angular spin frequency ω_s, is given by the equation:

$$\vec{r}_{\text{eq}} = \frac{1}{1 - (\omega_s/\omega_n)^2} \cdot \vec{\varepsilon}, \tag{1}$$

where ω_n is the natural frequency of the suspended mass. The system will spin at a frequency either below or above the natural one. From (1), it follows that in the first case the equilibrium position will be farther away from the spin axis than the original offset ε, while in the second case equilibrium will take place closer than ε to the spin axis. The phenomenon is known as auto-centering in super critical rotation. Test masses used for EP testing naturally require

$\omega_s > \omega_n$, because they must be weakly coupled (i.e., with low natural frequency for better sensitivity to differential forces) and in rapid rotation (i.e., with high modulation frequency for better reduction of "$1/f$" noise). In space, thanks to the absence of weight, the suspensions can be extremely weak, so that $\omega_s \gg \omega_n$. In this case it is:

$$\vec{r}_{\text{eq}} \simeq -\left(\frac{\omega_n}{\omega_s}\right)^2 \cdot \vec{\varepsilon} \tag{2}$$

which shows that extremely good auto-centering will be achieved (the equilibrium position vector, like the original offset vector, is fixed with the rotor); it also shows that, in order for the system to reach its equilibrium position on the opposite side with respect to the offset vector $\vec{\varepsilon}$, it must have 2 degrees of freedom. Indeed, it is well known that 1D systems are highly unstable if spinning at frequencies above the natural one [10].

As compared to EP testing accelerometers which have only one sensitive axis (the symmetry axis of the test cylinders), the 2D accelerometer of the GG mission has 3 advantages:

 (i) it retains the same dimensionality as the physical problem, which is a 2-body problem in the gravitational field of the Earth;
 (ii) the plane of sensitivity being perpendicular to the symmetry axis of the test cylinders, the symmetry axis is also the axis of rotation which is the natural choice, and the spin/modulation frequency can be larger than the natural one, providing auto-centering and better reduction of "$1/f$" noise;
(iii) it doubles the amount of output data for any given integration timespan.

The only well-known disadvantage of rotation at frequencies above the natural one is the onset of whirl motions, at the natural frequencies of the system, around the equilibrium position. Whirl is due to losses in the suspensions (the smaller the losses, the slower the growth rate of whirl) and needs to be damped to prevent instability, but it can be separated to recover the equilibrium position thanks to the fact that the whirl frequencies of the system are known [12,13, Chapter 6].
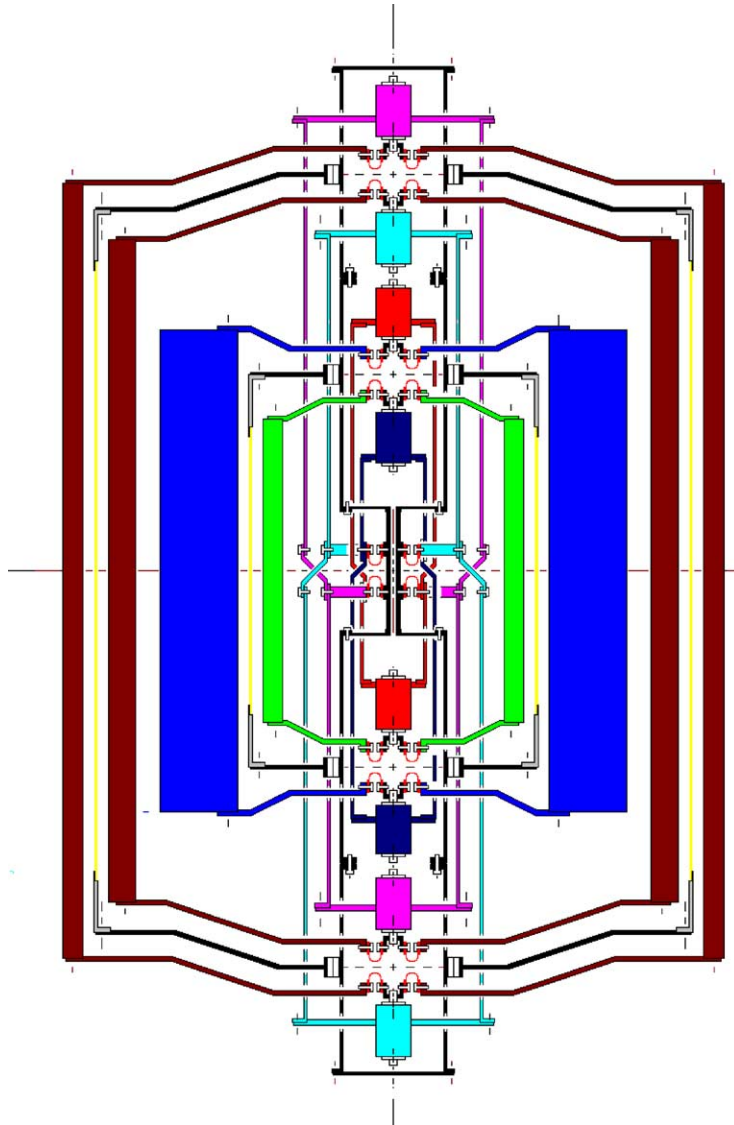
Fig. 2. Section through the spin axis of the differential accelerometers of the proposed GG mission for testing the equivalence principle in low Earth orbit. (Figure is in colour on the web.) There are 4 test cylinders (weighing 10 kg each), one inside the other, all centered at the same point (nominally, the center of mass of the spacecraft) forming 2 differential accelerometers: the inner one for EP testing (cylinders of different composition; shown in green and blue, respectively) and the outer one for zero check (cylinders made of the same material; both shown in brown). In each accelerometer the 2 test cylinders are coupled to form a beam balance by being suspended at their top and bottom from the 2 ends of a coupling arm made of 2 concentric tubes (each tube suspends one test cylinder at each end, which makes it asymmetric top/down; however, the two of them together form a symmetric coupling). All 4 tubes (2 for each coupling arm) are suspended at their midpoints from the same suspension shaft (the longest vertical tube in figure). In all cases the suspensions are ⋃-shape (or ⋂-shape) thin strips (shown in red), to be carved out of a solid piece of CuBe. At each connection there are 3 of them, at 120° from one another (the planar section in figure shows 2 for explanatory purposes only). There are capacitance plates (connected to the suspension shaft) for the read-out of differential displacements in between each pair of test cylinders (shown as yellow lines in section). The 8 small cylinders drawn along the symmetry axis are inchworms for the fine adjustment of the lengths of the coupling arms in order to center each test mass on the center of mass of the spacecraft. The whole system is symmetric around the spin axis as well as top/down. The two accelerometers are both centered at the center of mass of the spacecraft in order to reduce common mode tidal effects and improve the reliability of the zero check. (Taken from [11].)
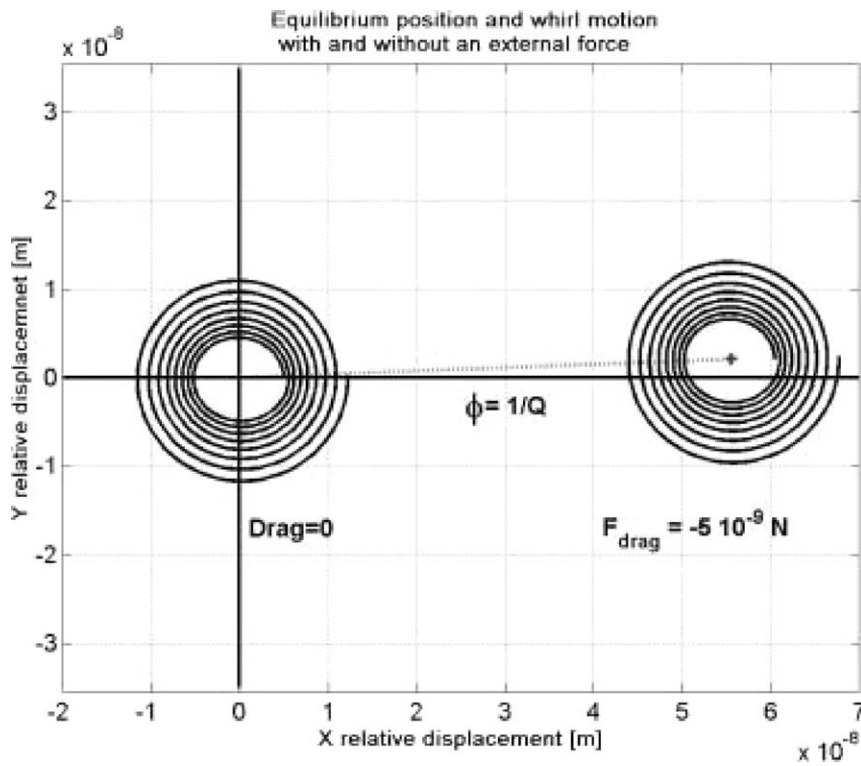
Fig. 3. Relative motion—in the non-rotating reference frame—of two coupled test cylinders spinning at frequency larger than the natural coupling one. The figure shows the plane of motion in two cases: with (right) and without (left) the inertial force deriving from an external residual drag acting on the spacecraft. It shows how the effect of an external force (air drag in this case), is that of displacing the equilibrium position of the system. A small phase lag appears due to energy losses in the suspensions (i.e., finite quality factor Q). Due to these losses whirl motion at the natural frequency of the system arises in either case around the corresponding equilibrium position.

Would a relative displacement of the test bodies caused by an external force be wiped out by auto-centering in supercritical rotation as it happens for the original offset ε? The answer is "No", because the offset vector is fixed in the rotating frame of the system while an external force, such as a possible violation of the equivalence principle, or the disturbing effect due to air drag acting on the spacecraft inside which the test masses are suspended, give rise to a displacement of the equilibrium position of the bodies in the non-rotating reference frame. In the presence of such a force, whirl motion will take place around the displaced position of equilibrium. This phenomenon has been simulated numerically and the result is plotted in Fig. 3 which shows (in the non-rotating reference frame) whirl motion in the absence of an external force, as well as around a displaced equilibrium position as caused by the effect of air drag.

The read-out consists of two pairs of capacitance plates located halfway in between the test cylinders and forming two capacitance bridges in the x, y directions of the plane perpendicular to the spin/symmetry axis. A differential force acting between the cylinders will displace their centers of mass and unbalance the bridges, thus generating an output voltage signal. Ideally, the bridges should sense only differential forces. In practice, they can be rejected only to some extent: the better is the mechanical centering of the plates in between the test cylinders, the more effective is the rejection of common mode forces. As reported in Section 4, a capacitance read-out—which can operate at room temperature—is adequate to the task of the GG mission.

3. The spacecraft and the orbit

The GG spacecraft is designed around the accelerometers and it is meant to provide the rotation of the system around its symmetry axis. It is therefore an axi-symmetric spacecraft passively stabilized by rotation around its axis of maximum moment of inertia (Fig. 4). At 520 km altitude, a sun-synchronous orbit requires an inclination of 97.5° over the equator. The orbit is almost circular. (See [14] for details.)

The sensitive plane of the accelerometers should lie in the plane of the expected signal, that is in the orbital plane. The spin/symmetry axis should therefore be normal to the orbit plane. However, while the spin axis is almost unaffected by external torques and therefore remains fixed in space, regression of the nodes of an inclined orbit due to the flattening of the Earth makes the orbit normal precess around the axis perpendicular to the equator (with a 1 year period in the case of a sun-synchronous orbit). As a result, a spin axis originally aligned with the orbit normal would no longer be so as time goes by. However, it can be shown that if the spin axis stays within about ±10°

from the orbit normal, the expected signal is only very slightly diminished (along only one component) with respect to its maximum value. Therefore, the GG spacecraft is equipped with cold gas thrusters (see Fig. 4) to be used to realign its spin axis along the orbit normal every about 20 days of data taking. For the spacecraft to maintain its cylindrical symmetry and its center of mass not to be affected by attitude maneuvers, two tanks have been designed, both of toroidal shape, to be located one above and one below the center of mass. During attitude maneuvers all the masses suspended inside the spacecraft are locked using inchworms placed around their central coupling arms—see Fig. 2).

Since the spin/symmetry axis of the spacecraft is maintained near the axis perpendicular to the sun-synchronous orbit, solar cells for power generation are located on the surface of a dish facing the sun (see Fig. 4). This dish serves also the purpose of shielding the spacecraft body—a compact, 1 m size structure in the shape of a spinning top enclosing the accelerometers—from sunlight, so as to reduce the effects of thermal disturbances on the experiment.
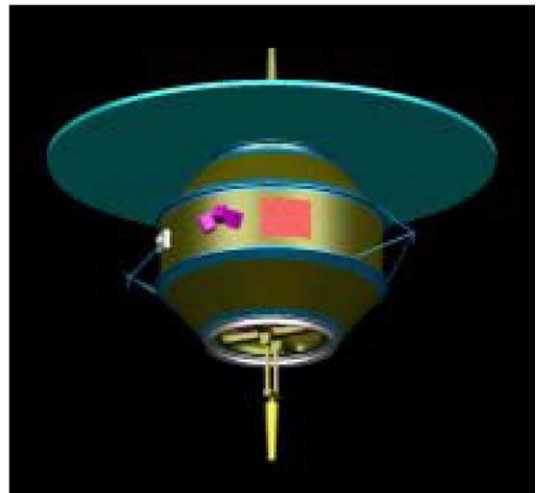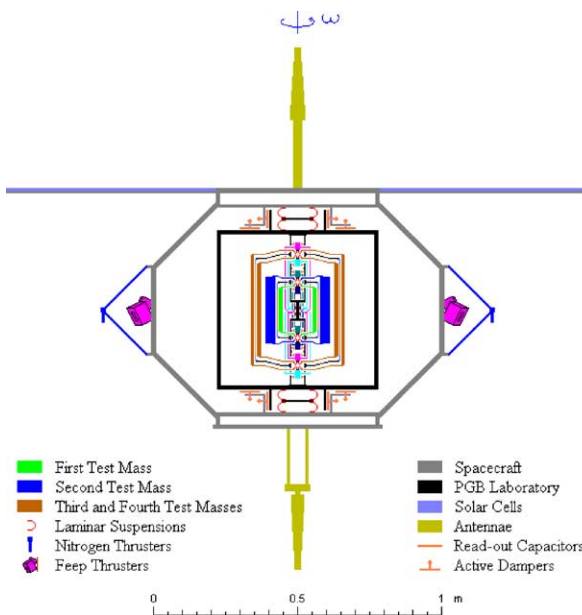

Fig. 4. The GG spacecraft as it has been designed for flight in high inclination, sun-synchronous orbit. (Figure is in colour on the web.) On the right-hand side is a 3D view, while on the left is a section along the spin/symmetry axis—showing the PGB laboratory and the accelerometers inside the spacecraft. The section and the legenda give details on the main parts of the spacecraft and the experimental apparatus. The total mass is 280 kg, the orbit is almost circular, has an altitude of 520 km and an inclination of 97.5° (sun-synchronous orbit).

The largest disturbing acceleration experienced by the accelerometers is due to the effect of residual air drag acting on the spacecraft and not on the test masses suspended inside it, thus resulting in an inertial acceleration equal and opposite to the acceleration caused by air drag on the spacecraft. Moreover, the largest and most "dangerous" air drag effect is due to its "along track" component, which has the same orbital frequency as the signal and differs from it only in phase (the signal is in the radial satellite—center of the Earth direction). The inertial acceleration resulting from air drag—and in general from non-gravitational forces acting on the spacecraft—are in principle the same on the test bodies in each accelerometer. They are known as "common mode" effects and should not produce any differential signal to compete with the target differential signal of an equivalence principle violation. However, this would be so only in the ideal case that the suspensions of the test cylinders in the accelerometers were perfectly identical and the capacitance bridges of the read-out were perfectly balanced, i.e., under conditions of perfect "common mode rejection". In the GG space experiment the strategy chosen is for air drag (and non-gravitational effects) to be *partially compensated* by the spacecraft drag-free control system, and *partially rejected* by the accelerometers themselves. In this way, the burden of reducing to an acceptable level this very large effect is shared between the spacecraft and the experimental apparatus, each of them being given a reasonable task.

Common mode rejection relies on the coupled suspension of the test cylinders and the capacitance differential read-out in between them, and on well established in-flight balancing procedures. Drag compensation requires the spacecraft to be equipped with thrusters and an appropriate control system to force the spacecraft itself to follow the motion of an undisturbed test mass inside it. Since drag compensation must be active during data taking, there are severe limitations on the disturbances it produces which make ordinary impulsive thrusters not suitable. Finely tunable proportional thrusters based on field emission electric propulsion (FEEP) appear to be the best choice, also because of their high specific impulse and consequent need of only a negligible mass of propellant. The test mass which drives the drag-free control system is the PGB (see Fig. 4, left), whose motion relative to the spacecraft in the plane perpendicular to the symmetry

axis is read by two capacitance bridges. In terms of frequency, drag must be compensated in a narrow frequency range around the orbital one, in order to reduce its component along track. For this purpose, a control based on a notch filter has been tested in numerical simulations of the GG system and found to be effective [13, Chapter 6]. The PGB can provide the required driving signal to the drag control system because the orbital frequency around which drag must be compensated is below its natural frequency above which disturbances acting on the spacecraft are attenuated (see Fig. 5)

The transfer function of the PGB, given in Fig. 5, shows that effects at the orbital frequency are unaffected by the PGB suspension. They are sensed by the capacitance read-out in between the PGB and the spacecraft through the relative displacements they produce between the two, and these measurements serve as input to the drag free control. Note that the expected signal too is at the orbital frequency (see Fig. 1), hence, it is not attenuated. Instead, the figure shows that disturbances at the spin frequency of the spacecraft (in the non-rotating frame) are significantly reduced. Such disturbances are due primarily to the FEEP thrusters used for drag compensation, because in order to compensate for the effect of drag at the orbital frequency of the spacecraft around the Earth (in the non-rotating reference frame) while spinning with the spacecraft itself, they must fire at the spin frequency relative to the center of the Earth (2 Hz). Since this is also the modulation frequency of the expected signal, its attenuation by the PGB by about 5 orders of magnitude is a considerable advantage for the experiment.

4. Requirements and error budget

In order to be sensitive to differential effects in the plane perpendicular to the spin/symmetry axis, the test cylinders of each accelerometer (see Fig. 2) are weakly coupled to one another. With the suspensions as designed, the natural differential period is 540 s. Instead, all suspensions are stiff along the axis (with a natural period of 30 s) as well as in response to forces acting on both masses in the accelerometer (common mode effects). These mechanical features have been chosen for best sensitivity to differential
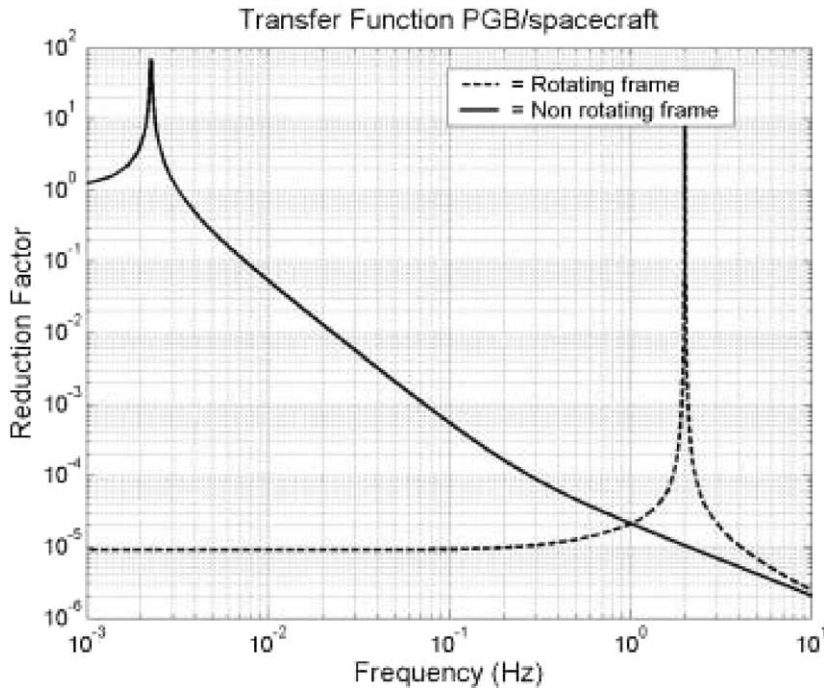
Fig. 5. Transfer function of the PGB laboratory (enclosing the accelerometers), suspended inside the spacecraft. The frequency of natural oscillations in the plane perpendicular to the spin/symmetry axis is 1/360 Hz. The transfer function is shown in the reference frame rotating with the system and also in the non-rotating one. In the non-rotating frame, any effect at frequencies below the natural one (the threshold frequency) is essentially unaffected, while above the threshold, disturbances are attenuated (the higher the frequency, the better the attenuation). These facts can be seen at the corresponding frequencies also in the rotating frame. In the rotating frame the peak near the spin frequency is larger than 1 because it shows (within the resolution of the figure) the transfer function at two nearby frequencies. These are the spin frequency ± the natural one, and at the natural frequency disturbances are amplified depending on the mechanical quality factor of the suspensions (see the transfer function in the non-rotating frame). It is well known that a better attenuation is obtained with lower quality factors. In this case the value of the quality factor is 90.

forces in the plane, while minimizing the effects of common mode forces in the same plane as well as those of all disturbances along the axes. In particular, the goal of testing the equivalence principle to 1 part in 10^{17} in the gravitational field of the Earth requires to detect the effect of a differential acceleration of $a_{EP} \simeq 8.4 \times 10^{-17}$ m s^{-2} (pointing to the center of the Earth as in Fig. 2), which amounts to a relative displacement between the test cylinders of the inner accelerometer of 0.6 pm.

The main requirements which need to be fulfilled in order to reach the mission goal are concerned with: mechanical balance of the test cylinders; drag compensation; mechanical balance of the capacitance bridges; temperature variations (in space and time); damping of whirl motions and quality factor at the spin frequency.

Each accelerometer is conceptually a beam balance with the beam along the symmetry axis. Ideally, it should be insensitive to common mode forces in the x, y plane of sensitivity perpendicular to it. Perfect rejection is obviously impossible, and we require that all common mode forces in the plane are rejected by a factor $\chi_{CMR} = 1/10^5$. Much better rejection than this is achieved with ordinary balances on the ground where the common mode force (local gravity) is many orders of magnitude stronger than the largest common mode force (due to residual air drag) acting on the GG test cylinders. The balancing procedure relies on the capacitance bridges in between the test cylinders as sensors and the inchworms on the accelerometer's coupling arms (see Fig. 2) as actuators. Once balancing is completed, the inchworms can be switched off so as not to disturb the measurements. For the resid-

ual effect of air drag at the orbital frequency and in the plane of sensitivity we require a compensation factor of $1/10^4$, using the capacitance bridges between PGB and spacecraft as sensors and FEEP thrusters as actuators. As a result of both compensation and rejection, the residual differential effect of air drag on the test masses of the accelerometers is 10^9 times smaller than its original value, which for the GG spacecraft and orbit is $a_{\text{drag}} \leqslant 2 \times 10^{-7}$ m s^{-2} (worst case). This means that the disturbance due to air drag is larger than the signal by a factor 2.4 at most, and can anyway be distinguished from it because of the large phase difference between the two. The amount of drag effect remaining after compensation by FEEP thrusters gives a common mode effect on the test masses of the accelerometers, which—if the capacitance plates of the read-out are not perfectly balanced in between the test cylinders (i.e., the gaps on the two sides are not equal)—results in a spurious differential signal. For it to be a few times smaller than the target signal the unbalance must be (with a 5 mm gap) of a few μm, which is not a stringent requirement. We also require drag compensation by a factor 1/400 along the spin/symmetry axis (at the orbital frequency) in order to reduce the separation between the centers of mass of the test cylinders along this axis (see below).

All mechanical balancing will be affected by temperature variations. Since there are about 20 days available for data taking between two successive attitude maneuvers, we require that temperature variations be small enough not to destroy the balancing of the system for that span of time. Temperature time variations must be such that $\dot{T} < 0.1$ K/day, the requirement being set by the mechanical balance of the capacitance bridges, which are affected by the differential thermal expansion of the test masses and bridge frame. Variation of the suspensions stiffness with the temperature are not relevant. Along the z spin/symmetry axis it must be $\Delta T/\Delta z < 4$ K/m, and this requirement is set by the mechanical balance of the test cylinders since it is affected by the expansion/contraction of the coupling arms. Passive thermal isolation is sufficient to avoid temperature variations larger than these, and no active thermal control is needed. Temperature constraints are not very demanding in GG because its rapid rotation averages out azimuthal temperature variations and makes the radiometer effect negligible; much more demanding

constraints need to be satisfied in case of slow rotation of the test cylinders [9,11]. During eclipses, when the satellite happens to go in and out of the Earth's shadow, different heating of the outer shell of the spacecraft as compared to the internal apparatus (which is thermally isolated) would produce a differential rotation rate due to changes in the moment of inertia and conservation of angular momentum. This is avoided by means of a small mass compensation system based on a photo-diode sensor to detect the phase lag between the outer and inner part of the spacecraft, and inchworm actuators to displace little masses and compensate moment of inertia changes; the masses required are of a few grams because changes of moment of inertia caused by temperature variations are very small.

Whirl motions (as shown in Fig. 3) of all suspended bodies are damped by means of capacitance sensors/actuators. In the non-rotating frame whirls have the frequencies of natural oscillations (slow), while the sensors/actuators spin fast with the whole system (2 Hz). The spacecraft is equipped with Earth elevation sensors to measure its state of rotation in order to perform the coordinate transformation between the rotating and non-rotating frame which is needed for an accurate reconstruction and damping of the whirl motion [13, Chapter 6]. The growth rate of whirls is determined by losses in the system, essentially in the mechanical suspensions as they undergo deformations at the frequency of spin. The time constant of the growth is $(Q_s/\pi) \cdot T_w$, where Q_s is the quality factor at the frequency of spin and T_w the natural period of the whirl. The force required to damp the whirl is a fraction Q_s of the mechanical coupling force [12]. In GG the requirement is $Q_s = 20000$ (at 2 Hz), which laboratory tests have shown to be achievable: we have recently measured 30000 at 0.9 Hz and about 100000 at 1.4 Hz with the "GG on the Ground-GGG" laboratory prototype. With a Q of at least 20000, whirl growth is so slow that data taking can be performed between successive damping, thus avoiding any disturbance at all from damping forces. In order to reconstruct the position of relative equilibrium of the test cylinders in the non-rotating reference frame, as affected by a low frequency differential force (like an EP violation at the orbital frequency around the Earth) whirl motion at the natural frequency of oscillation can be separated out. Tests with the laboratory prototype demonstrate that a

low frequency differential effect can be detected even in the presence of a much larger whirl [15].

The error budget of the space experiment is performed keeping in mind that both the frequency and phase of the expected signal are well known: once the high frequency signal modulation due to the spin rate of the spacecraft has been eliminated by coordinate transformation to the non-rotating system, the signal must appear as a differential displacement at the orbital frequency, and always pointing to the Earth.

The most dangerous perturbing effects are therefore those which are close to the signal both in frequency and phase. There are two such effects: the Earth monopole coupling to higher mass moments of the test bodies and the radiometer effect. The first is due to the fact that the test bodies are not monopoles; they have non-zero higher mass moments, and the monopole mass moment of the Earth will couple differently to them giving rise to a differential force. Being due to the Earth,—which is the also the source mass of a possible violation of equivalence—this effect cannot in any way be distinguished from the signal. For a given spacecraft altitude and a given target in EP testing, the dominant mass moment of the test cylinders (quadrupole) must be small enough for this effect to be below the signal. The values required (about 0.01) are realistic to obtain by test mass machining. The radiometer effect is caused by the residual gas pressure in the presence of temperature gradients across the test masses generated by the infrared radiation from the Earth. In GG temperature gradients are averaged out by the fast rotation and the radiometer effect is negligible even at room temperature [9,16].

At the same frequency as the signal but, with a phase difference of about 90°, we have the inertial force caused by residual air drag acting on the outer surface of the spacecraft along its orbit. With the requirements given above for drag compensation and common mode rejection, the residual differential acceleration due to air drag is 2–3 times larger than the signal (worst case) and can be separated from it thanks to the large phase difference.

At twice the orbital frequency there is the tidal effect due to a non-zero separation between the centers of mass of the test cylinders along the spin/symmetry axis whenever it is not exactly aligned with the orbit normal [13, Chapter 2.2]. With a compensation

of non-gravitational forces (mostly solar radiation pressure) along the spin axis by 1/400, and with a common mode rejection in that direction of 1/50 (by suspensions machining only) this tidal effect is almost one order of magnitude smaller than the signal.

At the natural frequency of differential oscillation of the test masses (1/540 s) there is a residual whirl motion of their centers of mass which gives rise to a tidal effect from the Earth at the whirl frequency. However, it can be proven that it does not affect the position of relative equilibrium around which whirl motion takes place (see [17]). It causes a small deformation of the whirl orbit which circulates with the motion of the spacecraft around the Earth, does not accumulate in time and does not prevent recovery of the equilibrium position by separation of the whirl motion. Similarly, the whirl orbit is also affected by resonant drag effects due to air granularities along the spacecraft orbit around the Earth. In this case too the equilibrium position is not affected, the deformation of the whirl orbit circulates with the orbital period, it does not accumulate with time and can be separated out.

The accelerometers are designed to be sensitive in the plane perpendicular to the spin/symmetry axis and more stiff along it. However, there is a modest drag compensation requirement along the spin axis because a center of mass separation along it will generate (in the presence of a tilt angle with respect to the orbit normal) a tidal effect in the sensitive plane whose frequency is close to that of the signal.

Mechanical suspensions allow the test masses to be electrically grounded, thus avoiding the need to measure the amount of accumulated charge and to discharge the masses, which inevitably disturbs the measurements. Residual so-called "patch effects" are known to be small and slowly moving. Moreover, their presence can be checked by changing sign to an applied known electric potential corresponding to the resolution achieved: since the force is proportional to the square of the potential, the resulting effect must be the same to rule out a patch effect potential at that level. Requirements on magnetic impurities and magnetic susceptibility for the test masses can be met.

Rotation of the whole system together makes many effects coming from local (fixed) disturbances, (such as local mass anomalies or parasitic capacitances) to become DC signals, and therefore not an issue.

Finally, thermal noise, is compatible with the goal of the experiment thanks to the high frequency of spin, to the high Q of the system and the large mass of the test cylinders (10 kg test bodies compensate for working at 300 K rather than at a few K but with masses of 100 g).

5. Significance of the ground prototype

Since the GG differential accelerometer has two degrees of freedom it is possible to design a ground version of it to be fully operated and tested at 1 g. In the ground version the plane of sensitivity of the accelerometer lies in the horizontal plane of the laboratory while the third dimension is used to suspend the system against local gravity. In this way it can detect the horizontal component of a possible violation of equivalence either in the field of the Earth (a differential force constant in the North–South direction) or in the field of the Sun (a differential force following its 24 hr motion). This is the "Galileo Galilei on the Ground-GGG" prototype, which has been extensively described in [11] and whose recent results are given in [15]. The GGG accelerometer is designed to have the same features as the one proposed for flight, essentially: weak coupling, high frequency supercritical rotation and differential capacitance read-out. However, there are several very important differences to bear in mind.

The main difference is due to the special character of the third dimension—that of local gravity—which makes it impossible for the GGG design to be as symmetric as in space (see Fig. 2), and limits its capability to reject common mode effects. In addition, while no motor is needed in space once the spacecraft has been brought to its nominal rotation rate, GGG requires a motor and its bearings, which are a relevant source of noise. Frequency modulation by fast rotation can be at higher frequency than in space, but while the entire spacecraft spins together with the test bodies and therefore any local mass anomaly gives rise to a DC effect which does not affect (as long as it is constant in time) the modulated signal, nearby mass anomalies in the laboratory and its vicinity give effects which are directly competing with a possible violation of the equivalence principle in the field of the Earth. In order to separate them out, these effects need

to be measured and compensated, as in the rotating torsion balance experiment by [1]. In GGG the test bodies are rotors in a non-inertial reference frame (because of the diurnal rotation of the Earth) and the resulting gyroscopic effects would be in the North–South direction like an equivalence principle violation in the field of the Earth and indistinguishable from it, see [11]. However GGG can be used to detect a violation of equivalence in the field of the Sun, because of its 24 hr period, and the driving signal is only slightly weaker than in the field of the Earth. Local mass anomalies are not a problem in this case but the 24 hr component of the tidal effect from the Sun, not being perfectly rejected, would leave a residual differential force which must be separated by measurements at different declinations of the Sun around the equinoxes.

A 24 h signal requires local seismic noise at this frequency (tilts and horizontal accelerations) to be attenuated. This can be done first actively (using a tilt-meter as sensor and PZTs as actuators to maintain the verticality of the rotation axis) and then passively, by means of a cardanic suspension of the whole system. As long as horizontal accelerations are considered, it is worth noticing that in GGG only displacements with respect to the local vertical are relevant, and they can be passively attenuated with a cardanic suspension whose stiffness is only slightly weaker than that of the suspensions currently in use for the test bodies.

Whirl motions need to be damped both on the ground and in space, but in space they can only be damped actively with capacitance sensors/actuators which are fixed in the rotating frame of the whole system. In GGG active damping can be performed also in the non-rotating frame, which is what has been realized so far.

An obvious advantage of the ground experiment is the absence of drag.

Given the stronger driving signal in space (by 3 orders of magnitude), the weaker coupling (and consequent higher sensitivity which can be achieved in absence of weight), the better symmetry of the accelerometer in space and the absence of noise from the motor, it can be convincingly argued that even a relatively modest GGG test of the equivalence principle in the field of the Sun can generate confidence in the capability of the GG space experiment to reach its goal. The space experiment will benefit from sev-

eral key features developed and tested in GGG: the read-out and data analysis, the beam-balance of the accelerometer and its rotation in supercritical regime, the weak and high Q mechanical suspensions, the capacitive active control of whirl motions.

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