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Passive Vibration Isolation in a Spinning Spacecraft

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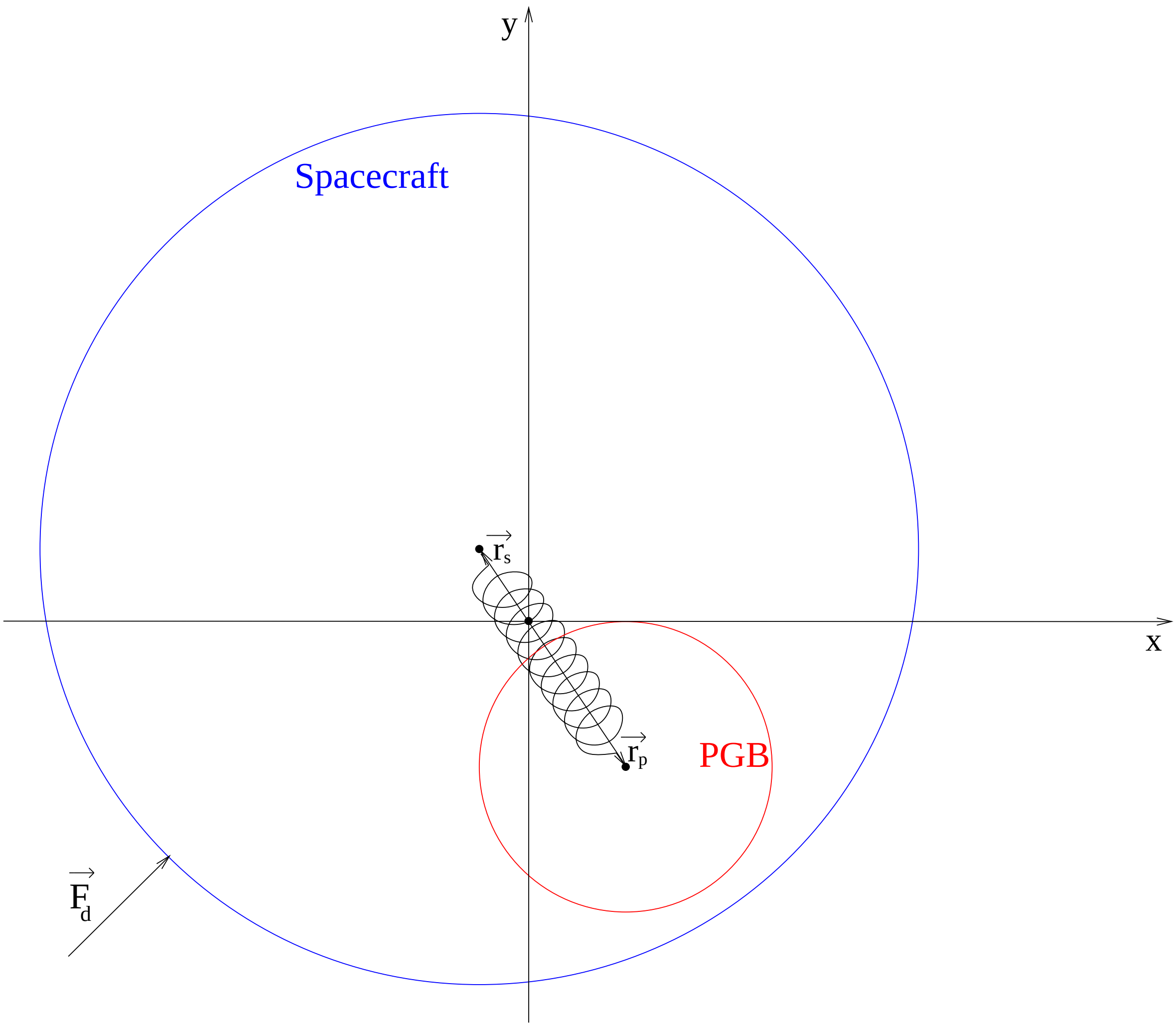
Transfer Function in the Inertial Reference Frame

Consider a spacecraft of cylindrical symmetry whose symmetry axis is also the axis of maximum moment of inertia. Inside the spacecraft is the PGB (Pico Gravity Box) laboratory, also of cylindrical symmetry and with the axis of maximum moment of inertia coinciding (within manufacture/mounting errors) with the symmetry axis of the spacecrfat. The PGB is mechanically coupled to the spacecraft (along the sysmmetry axis) by means of mechanical springs with low stiffness k in all directions. Let the spacecraft and PGB be rigidly locked to one another during launch and until they have reached the nominal spin angular velocity Ω around the principal axis of inertia of the system. Let the *cruise phase*, under the effect of a non–gravitational force F_d (e.g. due to the atmospheric drag), begin at time $t = 0$, once the rigid lockers have been unlocked and the initial transient phase has been completed. Since we refer here to the GG space experiment we consider only the motion of the spacecraft/PGB system in the x, y plane perpendicular to the spin axis, which is the plane of the expected signal; we know that tilting torques are negligible, that differential rotations in azimuth can be dealt with and that motions along the *z-spin* axis are sufficiently small (see *"GG Pre Phase A Report"*, ASI, September 1996)

In an inertial reference frame whose centre of mass coincides, at $t = 0$, with the centre of mass of the spacecraft/PGB system the equations of motion read:

$$\begin{cases} m_s\ddot{\vec{r}}_s = -k(\vec{r}_s - \vec{r}_p) - c_r(\dot{\vec{r}}_s - \dot{\vec{r}}_p) + c_r\vec{\Omega} \times (\vec{r}_s - \vec{r}_p) - c_{nr}(\dot{\vec{r}}_s - \dot{\vec{r}}_p) + \vec{F}_d \\ m_p\ddot{\vec{r}}_p = -k(\vec{r}_p - \vec{r}_s) - c_r(\dot{\vec{r}}_p - \dot{\vec{r}}_s) + c_r\vec{\Omega} \times (\vec{r}_p - \vec{r}_s) - c_{nr}(\dot{\vec{r}}_p - \dot{\vec{r}}_s) \end{cases}$$

where m_s is the mass of the spacecraft, m_p the mass of the PGB, c_r and c_{nr} are the *rotating* and *non rotating* damping coefficients respectively, the position vectors and velocities lie in the x, y plane and subscripts s and p refer to the spacecraft and the PGB respectively. c_r is known as *rotating* damping coefficient because it is due to *friction inside rotating parts* of the system.



c_{nr} is known as *non-rotating* damping coefficient because it is due to *friction inside non-rotating parts* of the system. Note that c_r and c_{nr} refer to two physically different types of energy dissipation in the rotor and do not depend on the reference frame (e.g., fixed or rotating) used to describe the problem (see Paper I, §6). In GG there is no damping due to friction between rotating and non rotating parts (referred to as *friction in the bearings*) because there is no motor, there are no bearings and there are no non–rotating parts. Friction in the bearings is present in ground based rotating machines and the corresponding torque tends to slow down the machine (were not for the presence of the motor). Note that in the equations of motion of ground based rotors the effect of friction in the bearings does not enter either, because the motor supplies the energy required in order to maintain the spin rate of the system at its nominal value. Friction in the bearings is therefore not relevant for these calculations, neither in GG nor in ground machines and should not be confused with friction between non–rotating parts (expressed by the non–rotating damping coefficient c_{nr}) which has a stabilizing effect. As for the c_{nr} coefficient used in the following, it should be read as automatically multiplied by a safety factor ξ_s slightly larger than 1 ($\xi_s = 1.2$).

By applying the Fourier transform to the equations of the previous system we obtain the following linear system of equations:

$$
\begin{cases}
-\omega^2 \hat{x}_s = -\omega_{0s}^2(\hat{x}_s - \hat{x}_p) + i\omega \frac{c_r + c_{nr}}{m_s}(\hat{x}_s - \hat{x}_p) - \frac{c_r}{m_s}\Omega(\hat{y}_s - \hat{y}_p) + \frac{F_{dx}}{m_s} \\
-\omega^2 \hat{y}_s = -\omega_{0s}^2(\hat{y}_s - \hat{y}_p) + i\omega \frac{c_r + c_{nr}}{m_s}(\hat{y}_s - \hat{y}_p) + \frac{c_r}{m_s}\Omega(\hat{x}_s - \hat{x}_p) + \frac{F_{dy}}{m_s} \\
-\omega^2 \hat{x}_p = -\omega_{0p}^2(\hat{x}_p - \hat{x}_s) + i\omega \frac{c_r + c_{nr}}{m_p}(\hat{x}_p - \hat{x}_s) - \frac{c_r}{m_p}\Omega(\hat{y}_p - \hat{y}_s) \\
-\omega^2 \hat{y}_p = -\omega_{0p}^2(\hat{y}_p - \hat{y}_s) + i\omega \frac{c_r + c_{nr}}{m_p}(\hat{y}_p - \hat{y}_s) + \frac{c_r}{m_p}\Omega(\hat{x}_p - \hat{x}_s)
\end{cases}
$$

The solutions of this system are:

$$
\begin{cases}
\hat{x}_s = \frac{F_{dx}[k - m_p\omega^2 - i\omega(c_r + c_{nr})][m_p m_s \omega^2 - (k - i\omega(c_r + c_{nr}))(m_s + m_p)] - F_{dy} c_r \Omega m_p^2 \omega^2 - F_{dx}(c_r\Omega)^2(m_s + m_p)}{[(k - i\omega(c_r + c_{nr}))(m_s + m_p) - m_p m_s \omega^2]^2 \omega^2 + (c_r\Omega)^2(m_s + m_p)^2\omega^2} \\
\hat{y}_s = \frac{F_{dy}[k - m_p\omega^2 - i\omega(c_r + c_{nr})][m_p m_s \omega^2 - (k - i\omega(c_r + c_{nr}))(m_s + m_p)] + F_{dx} c_r \Omega m_p^2 \omega^2 - F_{dy}(c_r\Omega)^2(m_s + m_p)}{[(k - i\omega(c_r + c_{nr}))(m_s + m_p) - m_p m_s \omega^2]^2 \omega^2 + (c_r\Omega)^2(m_s + m_p)^2\omega^2} \\
\hat{x}_p = \frac{F_{dx}[k - i\omega(c_r + c_{nr})][m_p m_s \omega^2 - (k - i\omega(c_r + c_{nr}))(m_s + m_p)] + F_{dy} c_r \Omega m_p m_s \omega^2 - F_{dx}(c_r\Omega)^2(m_s + m_p)}{[(k - i\omega(c_r + c_{nr}))(m_s + m_p) - m_p m_s \omega^2]^2 \omega^2 + (c_r\Omega)^2(m_s + m_p)^2\omega^2} \\
\hat{y}_p = \frac{F_{dy}[k - i\omega(c_r + c_{nr})][m_p m_s \omega^2 - (k - i\omega(c_r + c_{nr}))(m_s + m_p)] - F_{dx} c_r \Omega m_p m_s \omega^2 - F_{dy}(c_r\Omega)^2(m_s + m_p)}{[(k - i\omega(c_r + c_{nr}))(m_s + m_p) - m_p m_s \omega^2]^2 \omega^2 + (c_r\Omega)^2(m_s + m_p)^2\omega^2}
\end{cases}
$$

The transfer function for the *GG spacecraft-PGB* system is defined as:

$$
tf = \frac{\sqrt{|\hat{x}_p|^2 + |\hat{y}_p|^2}}{\sqrt{|\hat{x}_s|^2 + |\hat{y}_s|^2}}
$$

The mechanical coupling between the GG spacecraft and the PGB is $k = 20\,dyn/cm$, with a natural frequency of oscillation $\omega_\circ \simeq 2\pi \cdot 0.004\,rad/s$. The mass of the spring is neglected. The total energy dissipation due to rotating friction is expressed by the quality factor Q. When in supercritical rotation, we use for the coefficient of rotating damping: $c_r \simeq (1/Q) \cdot k/\Omega$ (supercritical rotating damping), which would be incorrect if the system were dominated by a large amount of rotating viscous damping. We have shown in Paper I (Appendix) that in such a case the corresponding coefficient of rotating damping in the equations of motion is given by $(c_r)_v = (1/Q_v) \cdot k/\omega_\circ$ <u>with</u> $Q_v \ll \Omega/\omega_\circ$. Since this is certainly not the GG case, a coefficient c_r of the previous form (i.e. $\propto 1/\omega_\circ$) can <u>never</u> be used in combination with a value for the viscous quality factor $Q_v \ll \Omega/\omega_\circ$ ($Q_v \ll \Omega/\omega_\circ \simeq 1.25 \cdot 10^3$ in the GG spacecraft–PGB case), but only with values $Q_v \geq \Omega/\omega_\circ$ (in point of fact it is $Q_v \gg \Omega/\omega_\circ$ for very small viscous rotating damping and $Q_v \simeq \Omega/\omega_\circ$ for an intermediate amount; see Paper I, Appendix). To the contrary, the ESTEC Technical Reoport on GG (see its Appendix provided on October 7 1996 at ESA HQ during the GG presentation) uses $(c_r)_v = (1/Q_v) \cdot k/\omega_\circ$ with $Q = 10$, thus implicitly making the assumption that the GG system be dominated by a very large amount of rotating viscous friction. There is no physical grounds for such an assumption because rotating damping in the GG system comes essentially from energy dissipated inside the tiny springs when deformed at the spin frequency of $5\,Hz$ and any contribution due to dissipation in the rotating active dampers is very small because of the very small damping forces that they are required to provide (see Paper I, Appendix and §3). Among the transfer functions shown below we have computed also the case with $Q = 100$; $Q \simeq 100$ is the value measured experimentally for a PGB prototype spring that we have manufactured carrying 3 wires for signal transmission (see *"GG Pre Phase A Report"*, ASI, September 1996).

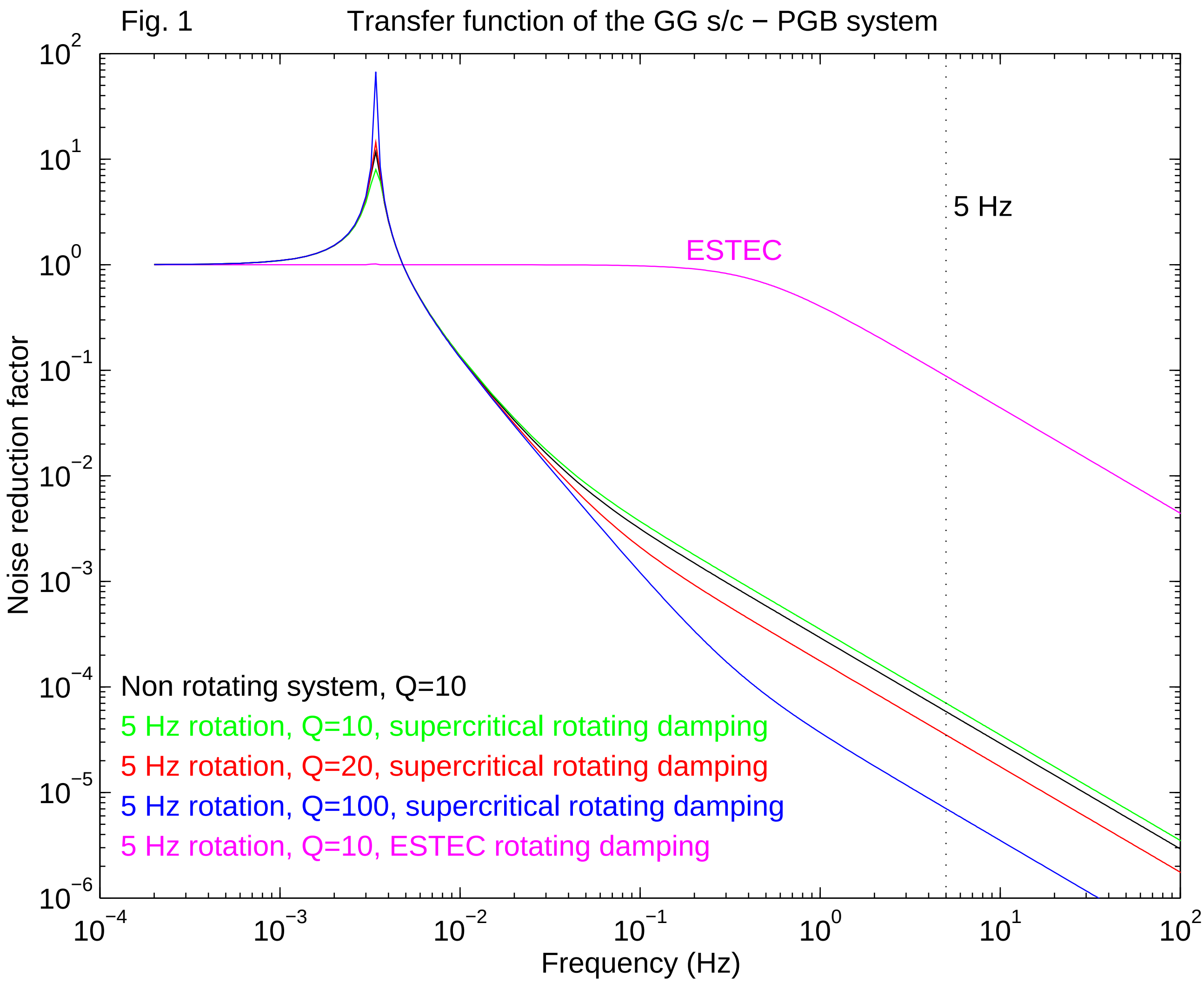


Fig. 1. *Transfer function of the GG spacecraft-PGB system in the inertial reference frame for the cases of zero spin rate and supercritical rotation at* $5\,Hz$*. At zero spin rate (black curve) the well known behaviour of passive noise attenuators on the ground is recovered, where the height of the peak at the natural frequency is about* Q*. In supercritical rotation we obtain the green, red and blue curves (for* $Q = 10$*,* 20 *and* 100 *respectively) showing that the system is effective in attenuating vibrations at the GG spin/signal frequency of* $5\,Hz$ *(the higher the* Q *the better the attenuation, which is peculiar of supercritical rotation). This means attenuation of disturbances which act at* $5\,Hz$ *w.r.t. the fixed frame, i.e. which are DC or* $10\,Hz$ *w.r.t. the rotating frame. The system is obviously transparent to disturbances at frequencies below the natural one; in particular it is transparent to effects which are DC w.r.t. the fixed frame (i.e. at* $5\,Hz$ *w.r.t. the rotating one). The "ESTEC" curve corresponds to a system in supercritical rotation dominated by a very large amount of rotating viscous friction (see Paper I, Appendix); such a system would be almost ineffective in attenuating vibrational noise.*

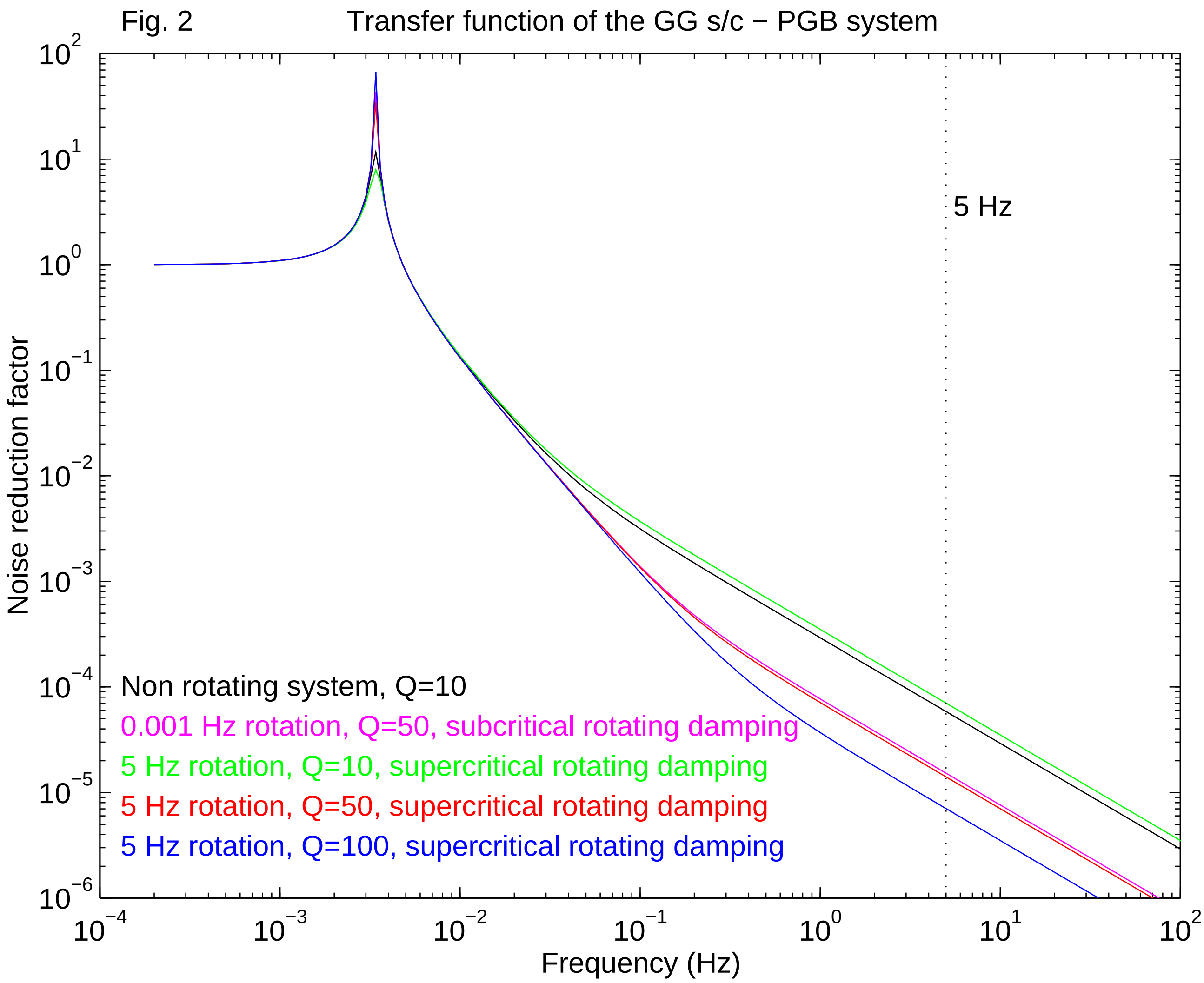


Fig. 2. *Transfer function of the GG spacecraft-PGB system in the inertial reference frame at zero spin rate, in supercritical and in subcritical rotation at* $5\,Hz$ *and* $0.001\,Hz$ *respectively. The coefficient of rotating damping is* $c_r \simeq (1/Q)(k/\Omega)$ *for the supercritical case and* $c_r \simeq (1/Q)(k/\omega_\circ)$ *for the subcritical one, which is correct for the GG system. The black curve shows the transfer function at zero spin rate and* $Q = 10$*. The violet curve shows the transfer function of the system in slow subcritical rotation with* $Q = 50$*; the peak at natural frequency has height about* Q*. The green, red and blue curves refer to the cases of fast supercritical rotation with* $Q = 10$, 50 *and* 100 *respectively; noise reduction of disturbances acting at* $5\,Hz$ *(w.r.t. the fixed frame) is apparent, a higher* Q *giving a better attenuation.*

Transfer Function in the Rotating Reference Frame

Let us calculate the transfer function of the GG spacecraft–PGB system which would be measured by an observer rotating with the system at its angular speed $\Omega = 2\pi 5\, rad/s$. Instead of re-casting the equations of motion in the body fixed reference system, we can obtain the spectral coordinates in the rotating system from the inertial coordinates in the following way:

$$\begin{cases} x_p^R(t) = x_p(t)\cos(\Omega t) + y_p(t)\sin(\Omega t) \\ y_p^R(t) = -x_p(t)\sin(\Omega t) + y_p(t)\cos(\Omega t) \\ x_s^R(t) = x_s(t)\cos(\Omega t) + y_s(t)\sin(\Omega t) \\ y_s^R(t) = -x_s(t)\sin(\Omega t) + y_s(t)\cos(\Omega t) \end{cases}$$

with $x_i(t)$, $y_i(t)$ the inertial coordinates $(i = p, s)$ and $x_i^R(t)$, $y_i^R(t)$ the body fixed coordinates. The spectral coordinates are obtained operating the Fourier transform of the $x_i^R(t)$, $y_i^R(t)$. Using $\cos(\Omega t) = (\exp(i\Omega t) + \exp(-i\Omega t))/2$ and $\sin(\Omega t) = i(\exp(-i\Omega t) - \exp(i\Omega t))/2$ it is easy to find:

$$\begin{cases} \hat{x}_p^R = [\hat{x}_p(\omega+\Omega) + \hat{x}_p(\omega-\Omega) + i\hat{y}_p(\omega-\Omega) - i\hat{y}_p(\omega+\Omega)]/2 \\ \hat{y}_p^R = [\hat{y}_p(\omega+\Omega) + \hat{y}_p(\omega-\Omega) + i\hat{x}_p(\omega-\Omega) - i\hat{x}_p(\omega+\Omega)]/2 \\ \hat{x}_s^R = [\hat{x}_s(\omega+\Omega) + \hat{x}_s(\omega-\Omega) + i\hat{y}_s(\omega-\Omega) - i\hat{y}_s(\omega+\Omega)]/2 \\ \hat{y}_s^R = [\hat{y}_s(\omega+\Omega) + \hat{y}_s(\omega-\Omega) + i\hat{x}_s(\omega-\Omega) - i\hat{x}_s(\omega+\Omega)]/2 \end{cases}$$

The transfer function is defined as:

$$tf^R = \frac{\sqrt{|\ \hat{x}_p^R\ |^2 + |\ \hat{y}_p^R\ |^2}}{\sqrt{|\ \hat{x}_s^R\ |^2 + |\ \hat{y}_s^R\ |^2}}$$

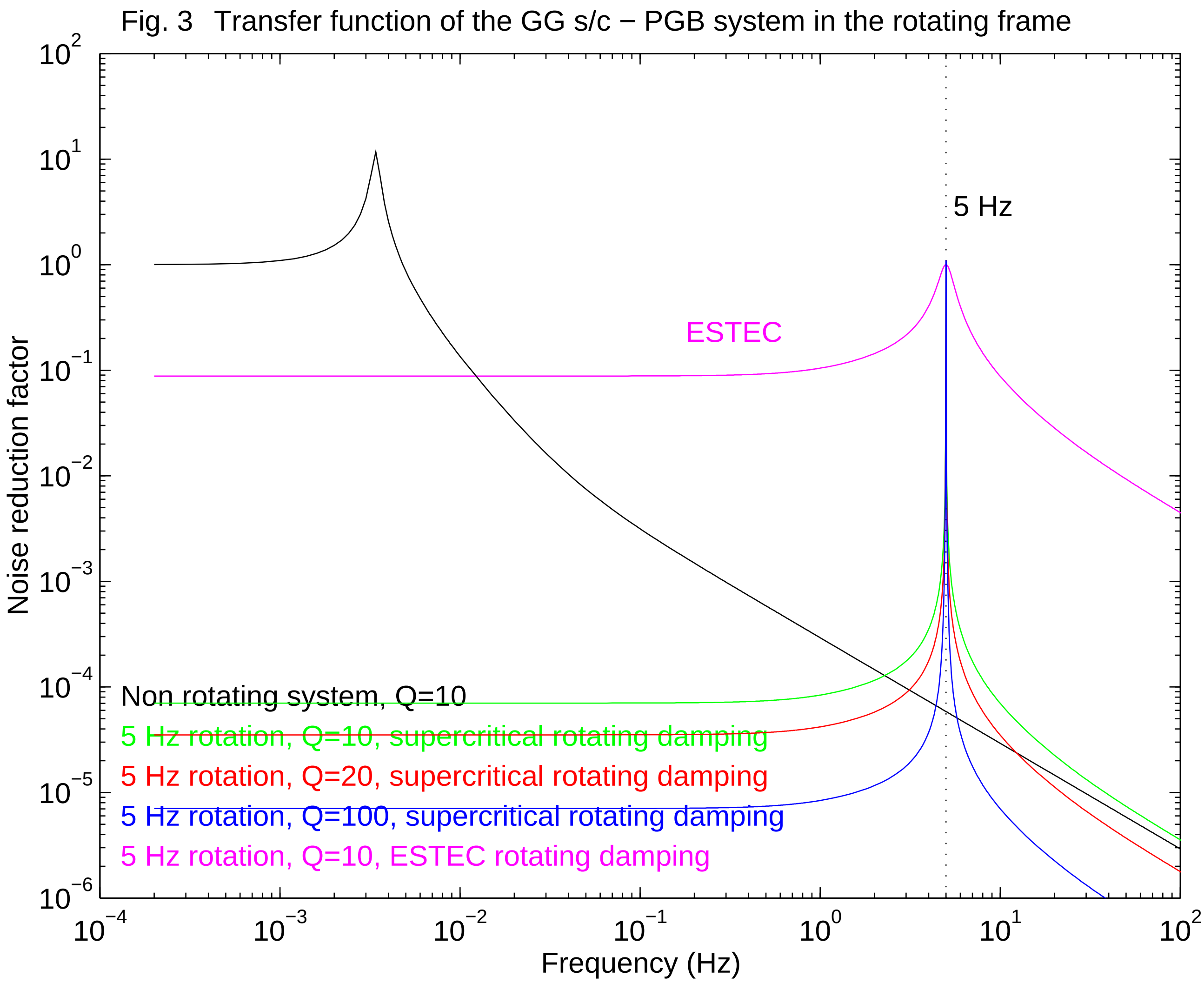


Fig. 3 *Transfer function of the GG spacecraft-PGB system in the reference frame corotating with the system at zero spin rate and in supercritical rotation. At zero spin rate (black curve) the transfer function is obviously the same as in Fig. 1. In supercritical rotation (with the same coefficient* c_r *as used in the inertial reference frame) we obtain the green, red and blue curves, respectively for* $Q = 10, 20$ *and* 100*. The peak at the spinning frequency shows that the passive noise attenuator* <u>*cannot change its properties*</u> *just because we look at it in the rotating frame. It cannot reduce vibrations at very low frequency w.r.t. the inertial frame, particularly the DC ones; the observer corotating with the system sees these DC perturbations as* $5\,Hz$*, and finds that the attenuator cannot reduce them, or better that it is transparent to* $5\,Hz$ *effects, where* $tf^R = 1$*. Perturbations which are seen at* $5\,Hz$ *by an inertial observer (and attenuated), have frequencies* $0\,Hz$ *and* $10\,Hz$ *for the body fixed observer, and in fact he too finds that they are attenuated. Like in Fig. 1, if we make the ESTEC assumption that the system be dominated by a very large amount of rotating viscous damping, we find that it is almost ineffective as noise attenuator.*