

**Fundamental limitations to high-precision tests of the universality of free fall by dropping atoms**

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Tests of the universality of free fall and the weak equivalence principle probe the foundations of general relativity. Evidence of a violation may lead to the discovery of a new force. The best torsion balance experiments have ruled it out to  $10^{-13}$ . Cold-atom drop tests have reached  $10^{-7}$  and promise to do 7 to 10 orders of magnitude better, on the ground or in space. They are limited by the random shot noise, which depends on the number  $N$  of atoms in the clouds (as  $1/\sqrt{N}$ ). As mass-dropping experiments in the nonuniform gravitational field of Earth, they are sensitive to the initial conditions. Random accelerations due to initial condition errors of the clouds are designed to be at the same level as shot noise, so that they can be reduced with the number of drops along with it. This sets the requirements for the initial position and velocity spreads of the clouds with given  $N$ . In the STE-QUEST space mission proposal aiming at  $2 \times 10^{-15}$  they must be about a factor 8 above the limit established by Heisenberg’s uncertainty principle, and the integration time required to reduce both errors is 3 years, with a mission duration of 5 years. Instead, offset errors at release between position and velocity of different atom clouds are systematic and give rise to a systematic effect which mimics a violation. Such systematic offsets must be demonstrated to be as small as required in all drops, i.e., they must be kept small by design, and they must be measured. For STE-QUEST to meet its goal they must be several orders of magnitude smaller than the size—in position and velocity space—of each individual cloud, which in its turn must be at most 8 times larger than the uncertainty principle limit. Even if all technical problems are solved and different atom clouds are released with negligible systematic errors, still these errors must be measured; and Heisenberg’s principle dictates that such measurement lasts as long as the experiment. While shot noise is random, hence its reduction becomes apparent as more and more drops are performed, the systematic effect due to offset errors at release must be identified through its specific known signature, and measured in order to be distinguished with certainty from the signal. This requires many well designed measurements to be performed—each to the target precision—for it to be ruled out as a source of violation. Ways may be pursued in order to mitigate the limitations identified here.

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General relativity (GR) rests on the experimental fact that in a gravitational field all bodies fall with the same acceleration regardless of their mass and composition. This is known as the universality of free fall (UFF), and it is also referred to as the weak equivalence principle (WEP) though it is by no means a *principle* of physics but rather a fact of nature that all experiments, from Galileo till the present time, have confirmed [1]. Experimental evidence of a violation would result in a scientific revolution, because in such a case either GR must be amended or a new force of nature (*fifth force*) is at play.

UFF-WEP in the field of Earth has been tested with macroscopic proof masses of different composition by dropping them from a height and by suspending them on a torsion balance. The dimensionless parameter  $\eta = \Delta a/a$  which quantifies a violation is obtained by measuring the differential acceleration  $\Delta a$  of the proof masses relative to each other ( $\Delta a$  is the physical observable quantity) as they fall with an average acceleration  $a$  towards the Earth ( $a$  is referred to as *driving acceleration*). If UFF-WEP holds,  $\eta = 0$ ; the smaller the value of  $\eta$ , the more sensitive is the experimental test. In the field of Earth the driving acceleration is  $\simeq 9.8 \text{ m s}^{-2}$  for mass dropping tests and  $\simeq 1.69 \times 10^{-2} \text{ m s}^{-2}$  at most (at  $45^\circ$  latitude) for proof masses suspended on a torsion balance. For the same sensitivity to differential accelerations, mass dropping tests would yield a smaller value of  $\eta$ , i.e., a better UFF-WEP test, by almost a factor of 600.

In spite of this big advantage, drop tests have measured  $\eta \simeq 7 \times 10^{-10}$  [2] while slowly rotating torsion balances [3] have done 4 orders of magnitude better, reaching  $\eta \simeq 10^{-13}$ , and finding no violation. These figures show a higher sensitivity to differential accelerations of the torsion balance as compared to mass dropping apparatus by about 4 million times. The parameter  $\eta$  is named Eötvös parameter in honor of the Hungarian physicist Roland von Eötvös, who first used the torsion balance for testing the universality of free fall.

If UFF holds the proof masses fall with the same acceleration. However, if their centers of mass at initial time happen to be located at different heights relative to the center of mass of the source body, a classical differential acceleration arises due to the fact that the gravitational field is not uniform (there is a nonzero gravity gradient, or tidal differential acceleration), which mimics a violation. Drop test [2] is unique (and the best) among drop tests in that the proof masses are coupled to one another as two halves (made of Al and Cu, respectively) of a single vertical disk, as suggested by Polacco. During the fall the disk is sensitive only to differential accelerations between the centers of mass of its two halves. Should the Earth attract them differently (*violation*), the disk would rotate about the horizontal axis, an effect that the authors could measure very precisely by means of a modified Michelson laser interferometer in which the two arms terminate on two corner-cube reflectors mounted on the rim of the disk. Ideally, this is a null experiment: no differential effect  $\Rightarrow$  no signal. However, it still depends upon release errors (the disk cannot

be dropped with exactly zero rotation rate), which turn out to be the limiting factor, as the authors themselves show [2].

In the torsion balance the proof masses are coupled and there is a position of equilibrium which depends on the design and construction properties of the balance and its suspension fiber, not on the initial conditions. At 1g only forces that act on the proof masses along directions which are not parallel to each other (as it would happen in case of violation) do affect the balance by causing a nonzero torque along the fiber—which is aligned with the local gravitational acceleration  $\vec{g}$ —thus displacing the equilibrium position. The Earth's gravity gradient gives a spurious torque along the fiber only by coupling to imperfections in the geometry of the balance, which results in a difference in the directions of its effect on the proof masses [4].

At a first glance totally free proof masses appear as the best choice to test if they fall with exactly the same acceleration in the field of Earth, both on the ground and in space. Since the proof masses must be sensitive to extremely small differential forces, the weaker their coupling the better, and one may be tempted to push weak coupling to the limit of no coupling at all (i.e., totally free masses). For macroscopic masses this issue has been thoroughly investigated by many authors [5–9], including Blaser [5], who was the leading scientist of the STEP (satellite test of the equivalence principle) studies when this proposal was selected by ESA (European Space Agency) as a medium size candidate mission two times, first in collaboration with NASA and then as an ESA only mission [10]. All studies [5–9] have demonstrated beyond question that drop tests are not at all the best choice, on the ground as well as in space, because the initial condition errors in combination with gravity gradient result in a large systematic effect which severely limits these experiments.

As shown in [8], a nice favorable case is that of the Moon and the Earth freely falling in the field of the Sun (with an average acceleration  $g_{\odot} \simeq 6 \times 10^{-3} \text{ m s}^{-2}$ ), the Moon's orbit being measured by laser ranging to corner-cube reflectors left by astronauts on its surface. In this case the gravity gradient from the Sun is very small thanks to its very large distance ( $d_{\oplus\odot} \simeq 1.5 \times 10^{11} \text{ m}$ ) yielding  $\gamma_{\odot} \simeq \frac{2g_{\odot}}{d_{\oplus\odot}} \simeq 1.3 \times 10^{-11} g_{\odot}/\text{m}$ . Thus, laser ranging with centimeter precision has been able to reach  $10^{-13}$  [11,12] (compatible with  $\frac{\gamma_{\odot}}{g_{\odot}} 10^{-2} \simeq 1.3 \times 10^{-13}$ ). One order of magnitude improvement is expected with the APOLLO laser ranging system at millimeter level [13] once the physical model has been improved accordingly. Beyond that, it will be extremely hard to overcome the effect of gravity gradient and initial condition errors.

In recent years, several tests of UFF-WEP have been performed by dropping atoms in light pulse atom interferometers [14–16]. They have reached  $\eta \simeq 10^{-7}$ , a factor 140 worse than drop test [2] and 6 orders of magnitude worse than the torsion balance test [3], but scientists promise to do many orders of magnitude better, on the ground [17] or in space [18–21]. An additional cold atom test, also to  $\eta \simeq 10^{-7}$ , has been published in 2014 [22] but it is not considered in this work because it is not based on a mass dropping approach, hence initial condition errors as discussed here do not apply to it.

The effect of initial condition errors as studied so far [5–9] was concerned only with macroscopic proof masses. In this work we revisit the issue to include atoms dropped in

light pulse atom interferometers, motivated by the fact that the number of atoms at detection is very small compared to Avogadro's number, and therefore the extremely small mass of the falling bodies plays a key role when considering limitations on position and velocity errors imposed by Heisenberg's position-momentum uncertainty principle.

The paper is organized as follows.

Section II recalls the basic mathematical formulas for the effect of initial condition errors in the measurement of the Earth's gravitational acceleration with atom interferometers. Section III analyzes the random and systematic effects of initial condition errors in cold-atom drop tests of the universality of free fall. Section IV shows how Heisenberg's principle limits such tests (due to the small number of atoms if compared to Avogadro's number) and points out the difference in dealing with initial condition errors versus shot noise, because the latter is random while the former give rise also to a systematic differential acceleration which mimics the signal and must therefore be distinguished from it.

In the last section we briefly summarize the results and conclude that at present cold-atom drop tests of UFF-WEP are not competitive with results achieved on the ground by torsion balances and with goals pursued in space, all using macroscopic proof masses and none being based on a mass dropping approach. We also offer some indications as to how the limitations due to initial condition errors outlined in this work can be mitigated (or avoided) in order to improve the present level of UFF-WEP tests with cold atoms.

## II. EFFECTS OF EARTH'S GRADIENT AND INITIAL CONDITION ERRORS

In 1995, by monitoring the motion of a freely falling corner-cube retroreflector with a laser interferometer, scientists were able to measure the absolute value of the local gravitational acceleration  $g$  to  $\Delta g/g \simeq 1.1 \times 10^{-9}$  [23].

In 1999 the absolute value of  $g$  was measured to  $\Delta g/g \simeq 3 \times 10^{-9}$  by dropping cesium atoms in a light pulse atom interferometer [24,25]. In this case the key optical elements of the interferometer (beam splitters and mirrors) are implemented by using stimulated Raman transitions between atomic hyperfine ground states (see [26] for details). The atomic wave packet is split, redirected, and finally recombined via atom-light interactions. The phase that the atoms acquire during the interferometer sequence is proportional to the gravitational acceleration  $g$  that they are subjected to. At present shot noise is the limiting factor, and it is proportional to  $1/\sqrt{N}$ , with  $N$  the number of atoms at detection. Being random, shot noise is expected to decrease as  $1/\sqrt{n}$ , with  $n$  the number of drops. In a well designed experiment any other noise source that needs to be reduced as  $1/\sqrt{n}$  should not exceed the shot noise limit. If so, the number of drops required to reduce shot noise to the target precision will bring all other noise sources below the target too.

The gravity gradient of Earth, combined with the initial position and velocity of the falling atoms, gives rise to a systematic spurious effect on their acceleration (hence on the measured phase shift) which cannot be neglected if one aims at measuring  $g$  to about  $10^{-9}$ . For atoms falling in an atom interferometer this effect has been calculated by [25–27]

following the tutorial [28]. Only the contribution to first order in the gravity gradient  $\gamma$  is relevant and in [24] it has been reported to be

$$\Delta g = \gamma \left( \frac{7}{12} g T^2 - v_0 T - z_0 \right), \quad (1)$$

where  $T$  is the time interval between successive light pulses (up to 160 ms in this experiment),  $\gamma \simeq 3 \times 10^{-7} g/m$  is the gravity gradient in the laboratory,  $z_0$  and  $v_0$  are the initial position and velocity of the atom. Note that because of a misprint, in [24] the second term on the right-hand side of (1) reads  $-\gamma v_0$ , while it should be multiplied by  $T$ .

For a free falling point mass (including one single atom) whose initial conditions—nominally zero—have errors  $\Delta z_0, \Delta v_0$  (in the direction to the center of mass of Earth) the first order tidal acceleration at the height of fall  $z(t)$  is

$$\Delta g(t) = -2 \frac{GM_\oplus}{R_\oplus^3} z(t) = \gamma \left( \frac{1}{2} g t^2 - \Delta v_0 t - \Delta z_0 \right), \quad (2)$$

with  $M_\oplus, R_\oplus$  the mass and radius of Earth,  $G$  the universal constant of gravity, and

$$\gamma = g \frac{2}{R_\oplus} \simeq 3.14 \times 10^{-7} g/m \quad (3)$$

the gravity gradient of Earth whose numerical value is as given by [24] and reported in (1).

It is interesting to note that the disturbing acceleration (1) computed for atoms falling in the atom interferometer differs from (2) in the coefficient of the quadratic term by  $\frac{1}{12} \gamma g T^2$ . As pointed out by [29], this discrepancy is due to the fact that in the atom interferometer the acceleration of the atoms is measured as a second difference of their positions at times 0,  $T$ , and  $2T$  when—during their ballistic flight—they are subjected to light pulses (see Sec. 2.1.3 of Peters' Ph.D. thesis [26]). If we take (2) and integrate twice in order to get the position, we obtain three position terms proportional to  $t^4, t^3$ , and  $t^2$ , respectively. We then compute the acceleration by defining it as the second position difference at times 0,  $T$ , and  $2T$ . We find that for the  $t^2$  and  $t^3$  terms the second position difference is the same as the corresponding second time derivative, while this is not so for the  $t^4$  position term. In this case it is easy to see that the second position difference yields  $\frac{7}{12} \gamma g T^2$  instead of  $\frac{1}{2} \gamma g T^2$ , with an acceleration difference by  $\frac{1}{12} \gamma g T^2$  [30]. It is apparent that this difference has nothing to do with the “quantum mechanics” versus “classical mechanics” approach. As stated in [26] (Sec. 2.1.3): *...this type of measurement is not intrinsically “quantum mechanical.” ...We can simply ignore the quantum nature of the atom and model it as a classical point particle that carries an internal clock and can measure the local phase of the light field.*

The acceleration difference discussed above does not affect cold-atom drop tests of the universality of free fall, because by taking the difference of the free fall accelerations of two different atom species or isotopes—which is the physical quantity to be measured when testing UFF—the term quadratic with time in the acceleration (1) cancels out. It does not cancel out in experiments to measure the absolute value of  $g$ , such as [24], in which case the systematic effect (1) due to initial condition errors in combination with the Earth's gradient had to

be considerably reduced in order to measure the absolute value of  $g$  to  $3 \times 10^{-9}$ . The authors report a careful systematic error analysis that required them to perform many measurements of  $g$  at different heights. By fitting the measurement data to the predicted curve the gradient error could be identified, measured, and, to that extent, subtracted (as shown by the authors in their table of systematic effects).

Cold-atom drop tests of the universality of free fall have been proposed and investigated by the European Space Agency [18–21] to be performed in space at low Earth altitude. In these proposals two overlapped clouds of different isotopes fall in a dual-isotope interferometer (DII). The free fall acceleration is measured simultaneously for each cloud. By computing their difference, the acceleration of interest  $\Delta g = \eta g(h)$  is derived.

In absence of weight the leading term measured for each free falling atom cloud is the inertial acceleration arising because of nongravitational forces acting on the outer surface of the spacecraft. This inertial acceleration is huge compared to the target, but common to both clouds, and therefore—if the instrument is properly designed—it can be rejected. If not, it must be compensated by drag-free control of the spacecraft, which requires a proof mass unaffected by nongravitational forces (to the level of drag compensation), a sensor to measure its motion relative to the spacecraft, and thrusters (with the necessary amount of propellant) to make the spacecraft follow the proof mass. With  $^{85}\text{Rb}, ^{87}\text{Rb}$  a rejection factor of  $4 \times 10^8$  is postulated [21]. At present the best measured rejection factors are 550 for  $^{85}\text{Rb}$  and  $^{87}\text{Rb}$  [15] and 303 for  $^{87}\text{Rb}$  and  $^{39}\text{K}$  [31], showing that an improvement by about 6 orders of magnitude is needed to meet the requirement.

We mention for completeness that on the ground in addition to the gradient (3) there is also a gradient of the centrifugal acceleration. It is due to the Earth's daily rotation at angular velocity  $\omega_\oplus$ , and it adds a factor  $\leq \omega_\oplus^2 \sim 5.4 \times 10^{-10} g/m$ , which in the  $g$  measurement [24,25] is negligible.

At low Earth altitude  $h$  the gravity gradient is

$$\gamma_{\text{space}} = \frac{2}{(R_\oplus + h)} g(h)/m, \quad (4)$$

$g(h)$  being the Earth's gravitational acceleration at altitude  $h$ . Unless the spacecraft attitude is fixed in inertial space the centrifugal gradient must be added too, which is 1/2 of the gravity gradient (4).

### III. RANDOM AND SYSTEMATIC INITIAL CONDITION ERRORS IN COLD-ATOM DROP TESTS OF UFF

From now on we consider the effect of initial condition errors (ICE) in cold-atom drop tests of UFF-WEP, and neglect the term quadratic in time because in the differential acceleration of two clouds dropped simultaneously it cancels out.

Let us start with one single atom freely falling in the presence of the Earth's gradient  $\gamma$ . If it has been released at time  $t = 0$  with ICE  $\Delta z_0, \Delta v_0$  (in modulus) the resulting error at time  $t$  in its measured free fall acceleration is

$$\Delta g(t)_{\text{ICE singleatom}} = \gamma (\Delta z_0 + \Delta v_0 t), \quad (5)$$

where  $\gamma$  is (3) on the ground and (4) in space.

If  $N$  atoms with these ICE are released together, random velocities abate with  $\sqrt{N}$ , and random position errors are  $\sqrt{N}$  smaller too, hence the error in the acceleration measured at time  $t$  is

$$\Delta g(t)_{\text{ICE singlecloud}} = \gamma \left( \frac{\Delta z_0}{\sqrt{N}} + \frac{\Delta v_0}{\sqrt{N}} t \right). \quad (6)$$

In the case of a DII, in which the free fall accelerations of two atom clouds are measured independently, each one with a random error (6), the random error in their acceleration relative to each other (*differential*) is a factor  $\sqrt{2}$  times larger.

Due to the random nature of noise (6), it can be reduced by performing many drops, as long as they are uncorrelated. With  $n$  such drops the sigmas of the center of mass position and velocity at initial time, i.e.,  $\Delta z_0/\sqrt{N}$  and  $\Delta v_0/\sqrt{N}$  will further decrease as  $1/\sqrt{n}$ .

In a DII there is also a contribution to the differential acceleration due to position and velocity offset errors  $\Delta z_{0,\text{rel}}, \Delta v_{0,\text{rel}}$  (in the direction to the center of mass of the Earth) between the center of mass position and velocity of the two clouds at release relative to each other. They arise because of inevitable differences in trapping and releasing different isotopes or species—and are therefore systematic—yielding a systematic differential acceleration:

$$\Delta g(t)_{\text{ICE offsets}} = \gamma(\Delta z_{0,\text{rel}} + \Delta v_{0,\text{rel}} t), \quad (7)$$

which mimics a violation.

Note that an error in the component of the relative velocity along the orbit does also result in a relative position error in the radial direction (i.e., the direction of a violation signal in the field of the Earth) because different along track velocities give rise to different orbital radii, the orbital velocity being inversely proportional to the square root of the orbital radius.

It is mandatory to demonstrate that the measured signal is not due to the systematic error (7).

A known solution adopted in drop tests with macroscopic proof masses consists in dropping masses of the same composition using the same apparatus and performing an experiment as similar as possible to the real one: since in this case there must be no violation, the sensitivity measured sets the level of the UFF test that can be claimed with this experiment. This check has been done very rigorously in the case of drop test [2], and it led to establishing that UFF could not be tested better than  $\simeq 7 \times 10^{-10}$  because errors in releasing the vertical disk could not be reduced below this level. A null test of this type cannot be done with free identical atoms as test masses, because identical atoms cannot be distinguished. As suggested by [32], one should make the two atom clouds slightly different (e.g., by dropping the same atom in different metastable states), with a difference that allows them to be distinguished in the atom interferometry measurement, but that is negligible for the sought for UFF violation signal.

An effective alternative when the signature of a systematic effect is known as in this case, is to perform a number of measurements—each to the target precision—in appropriately modified experimental conditions such that the systematic effect can be separated out based on its known dependence on the physical parameters involved. In so doing the systematic effect is separated out and measured, so that its contribution to the signal of interest can be firmly identified and possibly

reduced below the target. Such a careful analysis of systematics (e.g., as reported by [24] for the absolute measurement of  $g$ ) obviously requires the integration time needed to complete one single measurement (by reducing random errors below the target) to be short enough so that systematics can be separated from the signal in a realistic time span. This is the case of the torsion balance experiments.

So far cold-atom drop tests of the universality of free fall have been performed on the ground [14–16] reaching  $\eta \simeq 10^{-7}$ , a factor 140 worse than drop test [2] and 6 orders of magnitude worse than the torsion balance test [3]. With  $\gamma \simeq 3.14 \times 10^{-7} g/m$ , the effect of initial condition errors is not a limitation at this level.

A cold-atom drop test has been proposed in 2007 [17] aiming initially at  $\eta = 10^{-15}$  and eventually at  $10^{-17}$ , to be performed inside a 10-m-tall vacuum chamber built at Stanford University. They have imaged single clouds [33] of  $^{87}\text{Rb}$  atoms with  $N = 4 \times 10^6$  atoms, 200  $\mu\text{m}$  initial radius, 2 mm/s initial velocity spread,  $T = 1.15$  s, and a reported shot noise limit  $\Delta g_{\text{sn}} \simeq 4 \times 10^{-12} g$ . With these values the contribution from ICE to the acceleration error (5) is reported to produce a phase shift of 0.18 rad (Table 1, term 5 in [33]) which, if compared to the phase shift of  $2.1 \times 10^8$  rad produced by the leading  $g$  term, yields  $\Delta g \simeq 8 \times 10^{-10} g$ . The authors are aware that the measurement is limited by seismic noise. Nevertheless, by comparing various portions of the imaged cloud and extracting correlated phase noise over many runs they succeed in reducing the phase shift noise by  $\simeq 100$ , thus inferring an acceleration sensitivity  $\Delta g \simeq 6.7 \times 10^{-12} g$ , close to the shot noise limit.

The Space Time Explorer and Quantum Equivalence Principle Space Test (STE-QUEST) proposal studied by ESA as candidate to a medium size mission [18,20,21] aims at a UFF test to  $\eta = 2 \times 10^{-15}$  by dropping atoms in a dual isotope  $^{85}\text{Rb}, ^{87}\text{Rb}$  interferometer in low Earth orbit at  $h \simeq 700$  km altitude [where  $g(h) \simeq 8 \text{ m s}^{-2}$  and  $\gamma(h) \simeq 2.83 \times 10^{-7} g(h)/\text{m}$ ]. It is expected to be limited by a random shot noise differential acceleration  $\Delta g_{\text{sn}} \simeq 3.66 \times 10^{-13} g(h)$  defined as (see [21], p.11)

$$\begin{aligned} \Delta g_{\text{sn}} &= \sqrt{2} \frac{1}{C} \frac{1}{k T^2} \frac{1}{\sqrt{N}} \\ &\simeq 2.93 \times 10^{-12} \text{ m s}^{-2} \simeq 3.66 \times 10^{-13} g(h), \end{aligned} \quad (8)$$

where  $\lambda = 780$  nm is the laser wavelength,  $k = \frac{8\pi}{\lambda}$  exploits the technique for enhancing the area of the interferometer,  $T = 5$  s is the free evolution time (time interval between subsequent light pulses), and  $C = 0.6$  is the contrast.

By performing  $n = 1.48 \times 10^5$  drops, uncorrelated and in the same experimental conditions, the random shot noise (8) can be reduced to  $9.5 \times 10^{-16} g(h)$ , which is a factor 2.1 below the target violation signal  $2 \times 10^{-15} g(h)$ . In the experiment design outlined in [21]—20 s repetition time, 0.5 h out of 16 h dedicated to the experiment at perigee—one measurement with this number of drops requires 3 years to be completed, within a total mission duration of 5 years.

For the random acceleration error (6) not to exceed the shot noise limit (8) the atom clouds are required to have  $N = 10^6$  atoms at detection with

$$300 \mu\text{m} \text{ initial radius, } 82 \mu\text{m s}^{-1} \text{ initial velocity spread.} \quad (9)$$

#### IV. HEISENBERG'S PRINCIPLE AND INTEGRATION TIME: SHOT NOISE VS SYSTEMATIC RELEASE ERRORS

Each test mass, as well as a single atom, must obey Heisenberg's uncertainty principle (HP), which states

$$\Delta p_0 \Delta z_0 \geq \frac{\hbar}{2}, \quad (10)$$

where  $\hbar = 1.054 \times 10^{-34}$  J s is the reduced Planck constant and the linear momentum  $\Delta p_0$  contains the mass of the body. For a single atom, because of its extremely small mass, HP requires (in the case of Rb)

$$\begin{aligned} (\Delta v_0 \Delta z_0)_{\text{HP atom}} &\geq \frac{\hbar}{2} \frac{1}{m_{\text{Rb}}} \geq \frac{\hbar}{2} \frac{1}{85.468 \times 10^{-3}} N_A \text{ m}^2/\text{s} \\ &\geq 3.7 \times 10^{-10} \text{ m}^2/\text{s}, \end{aligned} \quad (11)$$

where  $m_{\text{Rb}} = \frac{8.5 \times 10^{-2}}{N_A}$  kg is the mass of a Rb atom and  $N_A = 6.022 \times 10^{23}$  is Avogadro's number.

For each cloud made of  $N = 10^6$  Rb atoms released together, the random errors on the initial center of mass velocity and position are reduced by  $\sqrt{N}$ . This is equivalent to a free mass with position error ( $\sqrt{N} \Delta z_0$ ) and momentum error ( $m_{\text{Rb}} \sqrt{N} \Delta v_0$ ), for which HP requires  $(Nm_{\text{Rb}} \Delta v_0 \Delta z_0)_{\text{HP freemass}} \geq \hbar/2$ , hence

$$\begin{aligned} (\Delta v_0 \Delta z_0)_{\text{HP freemass}} &\geq \frac{\hbar}{2} \frac{1}{85.468 \times 10^{-3}} \frac{N_A}{N} \text{ m}^2/\text{s} \\ &\geq 3.7 \times 10^{-16} \text{ m}^2/\text{s}. \end{aligned} \quad (12)$$

This is the ultimate limit, since it is the HP limit for a single, free Bose-Einstein condensate of  $N$  atoms, and as such a lower limit to the initial conditions of the real experiment (free atoms released from an optical trap).

In order to quantitatively assess the implications of Heisenberg's uncertainty principle in cold-atom drop tests of UFF we refer to the STE-QUEST proposed space experiment because it has been investigated within ESA for several years and literature is available that provides all the information needed for quantitative assessment [18,20,21]. A similar experiment was investigated by ESA (with participation by the author) to be performed on the International Space Station [19]. However, the target was less ambitious than for STE-QUEST, and the ESA report of that study is an unpublished draft. STE-QUEST aims at  $\eta = 2 \times 10^{-15}$ , a target that would improve the current best tests performed with macroscopic masses by a factor of 50, which makes the proposal worth considering despite the huge gap (by a factor of 50 million) which separates it from the level that atom interferometers have achieved so far [14–16,22].

In the STE-QUEST atom clouds (9) each atom obeys the HP limit (11) by a factor 66, that is the product of its position and velocity errors is above the uncertainty limit by a factor 66. Their centers of mass have position and velocity errors smaller by a factor  $\sqrt{N} = 10^3$ , hence they are above the HP limit (12) by the same factor 66. With a comparable share of error in position and velocity it means that each error is roughly a factor  $\sqrt{66} \simeq 8$  above its HP limited value. By comparison, position and velocity errors of the single clouds realized by [33] are a factor  $\sqrt{1.1 \times 10^3} \simeq 33$  above the HP limit.

In every drop the contribution (6) of random ICE to the acceleration difference between the clouds amounts, with the planned initial condition errors (9), to  $\sqrt{2} \Delta g_{\text{ICE singlecloud}} \simeq 2.84 \times 10^{-13} g(h)$ , which is slightly smaller, by a factor 1.3, than the expected shot noise limit (8). Thus, the same number of drops that need to be performed in order to reduce the shot noise a factor 2.1 below the target signal will reduce (also as  $1/\sqrt{n}$ ) the initial random errors—hence the random differential acceleration error  $\sqrt{2} \Delta g_{\text{ICE singlecloud}}$ —to a value 2.7 times smaller than the signal.

With the same number of atoms, were it possible to run the experiment with position and velocity errors at exactly the limit of Heisenberg's uncertainty principle, they would result in a random differential acceleration error a factor  $\sqrt{66} \simeq 8$  smaller, of  $3.5 \times 10^{-14} g(h)$ , which is the lowest possible limit and a factor 10 below the expected shot noise (8).

Let us now consider the systematic error (7).

In the list of STE-QUEST systematic errors the requirements set by the proposers for the offset errors  $\Delta z_{0 \text{ rel}}, \Delta v_{0 \text{ rel}}$  at release are (see [21], Table 4, first entry)

$$\Delta z_{0 \text{ rel}} = 1.1 \text{ nm}, \quad \Delta v_{0 \text{ rel}} = 0.31 \text{ nm/s}. \quad (13)$$

First of all, it is interesting to compare these requirements with the position offsets between the centers of mass as required in UFF-WEP experiments using macroscopic proof masses.

It was pointed out by [5] that GG [34] is the only proposed space experiment in which the test masses are coupled and motion occurs around a position of relative equilibrium independent from initial conditions (as in the case of the torsion balance). In GG the proof masses are coaxial cylinders rotating around the symmetry axis, weakly coupled in two dimensions (2D) (the plane perpendicular to the spin or symmetry axis) whose physical property of self-centering (starting from construction or mounting offsets of  $10 \mu\text{m}$ ) makes the gradient effect compatible with a test 200 times more sensitive than STE-QUEST, to  $\eta = 10^{-17}$ . In Microscope [35], to be launched in April 2016, the coaxial test cylinders are sensitive along the symmetry axis while rotation occurs along an axis perpendicular to it; the position offsets required by construction or mounting amount to  $20 \mu\text{m}$ , to be reduced to  $0.1 \mu\text{m}$  by offline data analysis (over various measurements) based on the specific, known signature of the gradient effect so as to bring it below a target which is two times more sensitive than that of STE-QUEST, i.e.,  $\eta = 10^{-15}$ .

In STE-QUEST, if the requirements (13) are met in every drop (though they do not need to be measured to this level in every drop) the differential acceleration (7) is a factor 2.7 below the target signal.

It is crucially important to verify that the offsets between different atom clouds at release meet the requirements (13), and that they do meet them in all drops for the entire duration of the experiment. Assume that a fraction  $f < 1$  of the required number of drops  $n$  has initial offset errors that are larger than required by (13), to the extent that the resulting differential acceleration error (7) is  $k > 1$  times larger than in the remaining  $(1-f)n$  drops which meet (13). Then, the resulting average error in the differential

acceleration is

$$\overline{\Delta g} = \frac{[fn \cdot k\Delta g] + [(1-f)n \cdot \Delta g]}{n} = [f(k-1) + 1]\Delta g. \quad (14)$$

For instance, if a fraction  $f = 10\%$  of the drops have offsets that produce acceleration errors  $k = 21$  times larger than those produced in the remaining 90% of the drops in which (13) are met, the resulting systematic error (7) will be 3 times bigger, which is larger than the target signal and indistinguishable from it with a single measurement.

The size of the atom clouds at initial time (in position and velocity space) is the size of the trapped clouds. It is limited by HP (11) (see [36]), and the centers of mass position and velocity are limited by HP (12).

When confined, clouds of different isotopes or species have different size, because of the different physical properties of the atoms, including mass. In STE-QUEST the mass difference alone results in a size difference of  $3 \mu\text{m}$ . In current instruments, estimates of the offset errors at release are many orders of magnitude away from the requirements (13). In the UFF test [15] to  $\eta \simeq 10^{-7}$  the estimates reported are  $\Delta z_{0\text{rel}} = \pm 0.2 \text{ mm}$  and  $\Delta v_{0\text{rel}} \leq 6 \text{ mm/s}$ , which are about  $1.8 \times 10^5$  and  $1.7 \times 10^7$  times larger, respectively, than (13).

Nevertheless, there is no theoretical limit to the accuracy with which the centers of the two clouds can be made coincident. In principle, with enough care in preparing and releasing the trap, the offsets can be made to meet (13).

However, such preparation requires validation and only repeated measurements in the actually realized trap can provide it. In order to be excluded as the cause of any anomalous acceleration found in the experiment (violation?) the initial offsets must be measured, and the measurement is limited by HP (12), namely by the uncertainty limit in position and velocity of the center of mass of each cloud.

On the other hand, the required initial offsets (13) are well below this limit. Because of the extremely small mass of the clouds (even  $10^6$  atoms are very few compared to Avogadro's number) they are below the HP limit (12) by a factor  $\sqrt{1.1 \times 10^3} \simeq 33$ . Thus, the uncertainty principle prohibits the initial offsets to be measured to the required precision in a small number of drops.

Only by measuring them for the entire integration time of the UFF test, and by averaging over as many drops as required for the test, they can be proven to meet the requirements.

Should STE-QUEST aim at testing UFF only two times better than the present goal, to  $\eta = 10^{-15}$ , it would require an integration time 4 times longer, of 12 years, to complete one single measurement and to measure the initial offsets to the level required! In addition, the initial size and velocity spread of each cloud would have to be only a factor 4 above the HP limit. These facts explain why the target of STE-QUEST could not possibly be pushed to  $10^{-15}$  in order to make it competitive with Microscope.

Assuming that all technical problems are solved, and the initial offsets are negligibly small, still they must be measured and the integration time needed (for a given target) to reach the precision required is dictated by HP limit (12). This is a

fundamental limit and can be relaxed only by increasing the number  $N$  of atoms in the clouds (without increasing position and velocity errors in their centers of mass), which would as well reduce the shot noise limit.

Should the target of STE-QUEST be one order of magnitude less ambitious, to  $2 \times 10^{-14}$ , with the same shot noise, then initial condition errors could be 10 times larger, hence the clouds would be a much safer factor  $\sqrt{665} \simeq 82$  above Heisenberg's limit and the integration time would be a factor 100 shorter, requiring 11 days. Offset errors at release could also be 10 times larger. They would still be below the HP limit for each cloud (but only slightly, by a factor  $\sqrt{11} \simeq 3.3$ ), and their measurement would require the same integration time, but this would now be realistic and leave enough time for checking their systematic effect. At this level limitations would not be fundamental but mostly technical, as it is inevitable given the 5 million gap from the current state of the art. However, this goal would be 20 times less sensitive than Microscope's goal and only 5 times better than the current best tests of UFF-WEP, making the case for an expensive space mission extremely weak.

In its current design STE-QUEST proposes to measure the offsets at apogee (while drops to test UFF are performed at perigee), by producing atom clouds and imaging them in order to verify, based on their evolution, how far apart they were at release (see [21], p. 12). For this approach to work, one should demonstrate that systematic errors are the same in both cases, and that the accuracy of the imaging method is close to Heisenberg's limit.

The key difference between random and systematic errors must be kept clearly in mind. Once a random error has been reduced to a certain level the result is apparent, and if it has reached the design level the measurement is over. Instead, a systematic effect which is known to mimic the signal requires a number of different measurements, each of them to the target level, in order to verify its specific signature, i.e., the way it depends on some physical parameters, for it to be distinguished from the signal beyond doubts. It is well known that a very long integration time rules out the possibility of a careful check of systematic errors and questions the significance of an experimental result.

It has been suggested to cancel the Earth's gradient by placing a mass nearby. A reduced value of  $\gamma$  would reduce the systematic effect (7) and possibly make it irrelevant, at least when aiming at  $\eta = 2 \times 10^{-15}$ . Inside the small experimental region in which atoms are dropped the Earth's gradient is almost constant, but time varying depending on the orbital motion and the attitude of the spacecraft. Instead, the mass must be fixed (to avoid bigger problems), and very close by (at 50 cm more than 2 tons would be required). Hence, its gradient changes across the region but it is constant in time. As a result, the Earth's gradient would be either under compensated or over compensated, thus not solving the problem. On the ground this difficulty may be reduced because the Earth's gradient in the experimental region does not change with time (the laboratory does not move), and a large mass could be placed far away. However, with the atoms falling at 1g the experimental chamber inside which the Earth's gradient must be compensated is certainly larger (as in the 10 m evacuated tower at Stanford).

Another possibility may be worth investigating. Instead of using position and momentum as conjugate variables [subject to the uncertainty relation (10)] one may try to combine them in one single variable such that its error can be minimized at the expense of the error in its conjugate. This technique (known as *squeezing*) has been recently applied to shot noise in atomic clock measurements [37] with a reduction equivalent to increasing the number of atoms by a factor 100, i.e., equivalent to reducing the phase measurement noise, hence the acceleration shot noise, by a factor 10.

## V. CONCLUSIONS

In experiments to test the universality of free fall and the weak equivalence principle with free macroscopic masses, initial condition errors are a well known limitation [2,5,8]. Macroscopic masses have Avogadro's number on their side, while cold-atom drop tests are limited by Heisenberg's principle because of the vanishingly small mass of the atom clouds.

The proposed STE-QUEST space experiment [18,20,21] investigated by the European Space Agency needs 3 years to complete one single measurement of the universality of free fall to  $2 \times 10^{-15}$  within a total mission duration of 5 years. It is known that such a very long integration time is set by the need to reduce the random shot noise.

Here we have investigated the effects of initial condition errors in the presence of the Earth's gravity gradient, and in particular the systematic differential acceleration due to (systematic) offset errors at release between clouds of different isotopes or species. We have shown that:

(i) the requirements (13) on position and velocity offset errors at release for this systematic differential acceleration not to exceed the shot noise limit are—for the goal of STE-QUEST—a factor  $\sqrt{1.1 \times 10^3} \simeq 33$  below the limit set by Heisenberg's principle;

(ii) the systematic offset errors (7) must meet the requirements in all drops, and this must be ensured by measuring them, which requires—because of Heisenberg's principle limit for each cloud—the same integration time needed to reduce the random shot noise;

(iii) the systematic nature of the effect caused by offset errors at release demands—for the experiment to be reliable and its result to be acceptable—that more measurements are performed until it is possible to distinguish this effect from the target violation signal.

The integration time set by the uncertainty principle can be reduced only by increasing the number of atoms in the clouds (as long as this is done without increasing their position and velocity errors), which would as well reduce the shot noise.

The Achille's heel of light pulse atom interferometers in testing the universality of free fall to  $2 \times 10^{-15}$  and better by dropping atoms appears to be the extremely small mass of the atom clouds.

Were STE-QUEST aiming at a 10 times less ambitious goal, to  $2 \times 10^{-14}$ , it would not hit the fundamental limits outlined here. In this case the challenges would be mostly technical, in order to bridge the 5 million gap which separates this goal from the current  $10^{-7}$  level of UFF-WEP drop tests with cold atoms.

When aiming at  $2 \times 10^{-15}$ —like STE-QUEST in its current design—or better, ways may be pursued to overcome, or at least to alleviate, the limitations pointed out in this work.

Squeezing techniques can be investigated, which would reduce the effect of initial condition errors, thus allowing the corresponding requirements to be relaxed.

The very large number of drops needed seems inevitable; however, one might optimize the time needed to perform them both in space and on the ground.

Partial compensation of the Earth's gradient by means of an appropriate artificial mass nearby seems unrealistic in space but may be attempted on the ground with a tradeoff between the free fall time (hence the size of the experimental chamber) and a reduced gradient (hence relaxed requirements on initial condition errors).

A zero check by dropping the same atoms with some clever technique to allow them to be distinguished as suggested by [32] can be investigated on the ground. If well designed, no appreciable violation should occur and the experiment would reliably establish the limiting value of  $\eta$  that a cold-atom drop test of UFF can achieve.

Nonetheless, a UFF test to a few  $10^{-15}$  by dropping atoms appears to be hard. Cold-atom tests which are not based on mass dropping approach (such as [22]) should not be affected by initial condition errors and maybe worth closer attention by the community.

As at present, a comparison with space experiments using macroscopic masses and not based on the mass dropping approach shows that Microscope [35] (to be launched in April 2016) can make one measurement to  $10^{-15}$  in 1.4 days while GG [34] requires a few hours to reach  $10^{-17}$ ; the limitation being thermal noise at room temperature in both cases but in different frequency regions of the signal due to different up-conversion rates by means of rotation [38,39].

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