On the universality of free fall, the equivalence principle and the gravitational redshift

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At the birth of space age...

- 1957: Sputnik launched
- 1958: *Science Advisory Committee* appointed by Eisenhower advises to set up NASA (founded in 1958). Publishes the *Introduction to outer space*, divides scientific objectives in 4 categories: Early, Later, Still later, And much later still, puts Physics 1st in the Early category and writes:

  "Physicists are anxious to run one crucial and fairly simple gravity experiment as soon as possible. This experiment will test an important prediction made by Einstein's general theory of relativity, namely that a clock will run faster as the gravitational field around it is reduced." (GP-A...)

- Pound & Rebka measurement of the Grav Redshift yet to come (Mössbauer effect demonstrated in 1958, Nobel prize in 1961; Pound & Rebka paper to appear in 1959)
- Gravitational redshift listed as one of the 3 crucial tests of GR.

  Evidence from Mercury's perihelion advance and light deflection widely accepted (though not conclusive yet...)

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Does a measurement of the gravitational redshift really test GR? (I)

- Schiff (AJP, Jan 1960): Grav redshift can be derived solely from WEP/UFF and Special Relativity (both very well tested) and is not a test of GR:

  "Terrestrial or satellite experiments that would go beyond supplying corroborative evidence for the equivalence principle and special relativity would be very difficult to perform, and would, for example require a frequency standard with an accuracy somewhat better than 1 part in 10^{18}"

- Questioned by Dicke (AJP, same issue): Experimental evidence of UFF/WEP for ordinary matter does not necessarily apply to clocks

- Schiff's note added in proof (AJP, same issue):

  "The Eötvös experiments show with considerable accuracy that the gravitational and inertial mass of normal matter are equal. This means that the ground state Hamiltonian for this matter appears equally in the inertial mass and in the interaction of this mass with a gravitational field. It would be quite remarkable if this could occur without the entire Hamiltonian being involved in the same way, in which case a clock composed of atoms whose motions are determined by this Hamiltonian would have its rate affected in the expected manner by a gravitational field".

  Formalized by Thorne as the *"Schiff conjecture"* (PRD 1973) after a strong argument with Schiff in Nov 1970 and Schiff's death in Jan 1971

Does a measurement of the gravitational redshift really test GR? (II)

- Dicke changed his mind (*The Theoretical Significance of Experimental Relativity*, 1964):

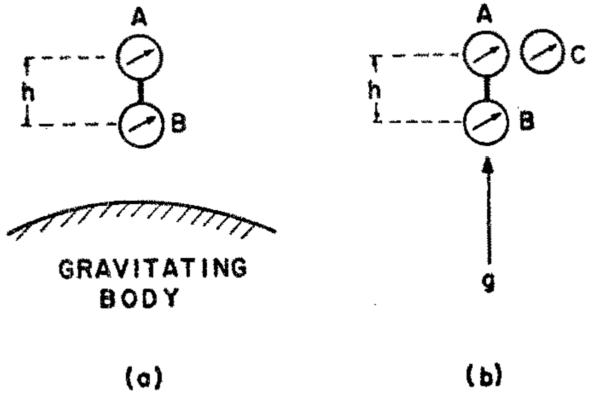
"The red shift can be obtained from the null result of the Eötvös experiment, mass energy equivalence, and the conservation of energy in a static gravitational field and static coordinate system."... "While this experiment may not be the most important of relativity experiments, it is interesting, and I should like to discuss briefly the experiment of one of my students, J. Brault, on the redshift of solar lines."

- Nordtvedt (PRD, 1975), 1 one year before GP-A is launched:

WEP violation might affect clocks more strongly than ordinary masses, depending on amount of energy rearranged in generating the frequency standard. For H maser clock (as in GP-A) he estimated a stronger violation possibly by $\simeq 10^4$.

WEP/UFF confirmed to $10^{-11} - 10^{-12}$ *by Dicke & Braginsky* $\Rightarrow$ *GP-A should measure gravitational redshift to better than* $10^{-7} - 10^{-8}$ *...no way...*

Schiff's derivation of the gravitational resdshift (I)



(a) Two identically-constructed clocks, A and B, are at rest in a gravitational field
(b) The gravitating body is replaced by an upward acceleration g of clocks A and B in empty space; a stationary clock C is used to compare their frequencies

If WEP/UFF hold for all bodies, including clocks, (a) and (b) are "locally" equivalent ($h \ll R_\oplus$, tides are negligible)

Schiff chooses (b): clock C ticks with period T; clock A passes by clock C with velocity v_A and period T_A; clock B passes by clock C with velocity v_B and period T_B

Time dilation of Special Relativity ensures:

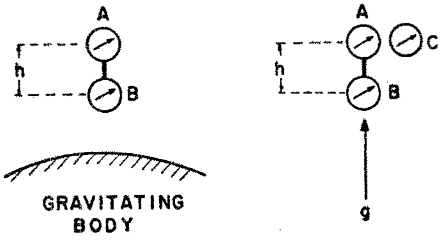
$$T_A = \frac{T}{\sqrt{1-\frac{v_A^2}{c^2}}} \, , \; T_B = \frac{T}{\sqrt{1-\frac{v_B^2}{c^2}}}$$

$$(v_A = \sqrt{2gh} \;\; v_B = \sqrt{4gh})$$

To first order in gh/c^2:

$$T_B \simeq T_A\left(1+\frac{gh}{c^2}\right)$$

Schiff's derivation of the gravitational resdshift (II)



(a) (b)

If h is not negligible (non uniform gravitational field) Schiff reasons in (a) and replaces gh with the gravitational potential difference between clock A and clock B at distance r_A, r_B from the center of mass of the gravitating body:

$$\Delta\Phi = -\frac{GM}{r_A} + \frac{GM}{r_B} > 0$$

Requires to perform a series of inter-comparisons between a number of identical clocks arranged in such a way that the gravitational field is nearly uniform from one to the next. Then (to first order):

$$\frac{\Delta\nu}{\nu} \simeq \frac{\Delta\Phi}{c^2}$$

Clock B at "lower altitude" is red-shifted w.r.t. clock A at "higher altitude"

Requires only WEP and Special Relativity; it does not require conservation of energy nor mass-energy equivalence

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Measurements of the gravitational redshift as (poor) tests of WEP/UFF

...but if WEP is violated, even if clocks are identical, they are attracted by the source body with a gravitational acceleration different then in case of no violation:

$$M_g = M_i(1+\eta_e) \qquad m_g = m_i(1+\eta_c)$$

$$g' = g(1+\eta\mathcal{O}(\eta^2)) \quad (g = \frac{GM_i}{r^2} \;\; \eta = \eta_e + \eta_c)$$

$$\Delta\Phi' = \Delta\Phi(1+\eta+\mathcal{O}(\eta^2))$$

Since clocks are equally attracted, Schiff's argument still holds, and the right expression for gravitational redshift is

$$\left(\frac{\Delta\nu}{\nu}\right)_\eta = \frac{\Delta\Phi}{c^2}\left(1+\eta+\mathcal{O}(\frac{\Delta\Phi}{c^2})\right)$$

2^{nd} order terms must be included because $\eta \lesssim 10^{-13}$ is even smaller ($\Delta\Phi/c^2 \simeq 4.3\cdot 10^{-10}$ in GP-A and $6\cdot 10^{-10}$ in STE-QUEST)

... but nobody dares to propose such a good measurement of gravitational redshift (STE-QUEST aims at 10^{-7}; even Müller, Peters & Chu 2010 claim $7\cdot 10^{-9}$)

Why tests of UFF/WEP can be more accurate than measurements of gravitational redshift by so many orders of magnitude?

UFF/EP tests can be performed as *null experiments*.

$$\eta = \frac{\Delta a}{a}$$

is derived from the differential acceleration Δa of the test bodies freely falling with average acceleration a.
If they are coupled so that the experiment measures directly the differential acceleration, this gives η directly: no experiment signal, no violation (to the level of noise); the smaller the signal (or the noise), the better the test.

No precise prediction must be made to which the measured signal should be compared in order to obtain the physical quantity of interest

A measurement of gravitational redshift is an *absolute measurement*. GP-A result (PRL 1980) is:

$$\left(\frac{\Delta\nu}{\nu}\right)_{GP-A} = [1 + (2.5 \pm 70) \cdot 10^{-6}] \cdot \left(\frac{\varphi_s - \varphi_e}{c^2} - \frac{|\vec{v}_s - \vec{v}_e|^2}{c^2} - \frac{\vec{r}_{se} \cdot \vec{a}_e}{c^2}\right)$$

The measured frequency shift had to be compared with the sum of the 3 terms at the right, whose values depend on various physical quantities, some of which to be measured during the experiment itself.
It is only by comparing the theoretical prediction and the measured shift that the authors could establish the ratio $[1 + (2.5 \pm 70) \cdot 10^{-6}]$.

No wonder it took them 4 years to publish the results of an experiment that lasted only about 2 hours!

Consider Müller, Peters & Chu measurement of gravitational redshift...

Peters, Chung & Chu (1999) measured g (falling Cs atoms, atom interferometry):

$$\frac{\Delta g_{Cs}}{g} = 3 \cdot 10^{-9}$$

and compared it with g measured by FG-5 absolute gravimeter nearby (falling CCR glass, laser interferometry)

They concluded:

$$\eta_{Cs-glass} = (7 \pm 7) \cdot 10^{-9}$$

Müller, Peters & Chu (2010) measured the gravitational redshift from the same data.

The frequency affected by gravitational redshift is the Compton frequency $\omega_C = m\hbar/c^2$ (m the rest mass of Cs atom). Gravitational redshift is recovered from the atom interferometry signal – which contains the local gravitational acceleration g – with g measured by the absolute gravimeter nearby (they need it in order to recover the frequency shift from the measured phase shift). They report:

$$\beta = (7 \pm 7) \cdot 10^{-9}$$

Since the mass-energy content is the full mass-energy of freely falling Cs atoms (as in 1999!) $\Rightarrow$ no Nordtvedt amplification factor $\Rightarrow$ it is a test of UFF/WEP :

$$\left(\frac{\Delta\nu}{\nu}\right)_\eta = \frac{\Delta\Phi}{c^2}\left(1 + \eta + \mathcal{O}(\frac{\Delta\Phi}{c^2})\right)$$

with

$$\eta_{Cs-glass} = (7 \pm 7) \cdot 10^{-9}$$

thus, assuming the procedure is correct, they are getting exactly the same result on UFF/WEP test as in 1999

Comparison of gravitational redshift with clocks of different internal structure..

The frequency rate of any clock on the ground is affected by the gravity field of the Sun. Since the solar potential varies over the year due to the eccentricity of the Earth's orbit, the frequency rate of the clock undergoes an annual variation due to the gravitational redshift from the Sun. Should two nearby clocks of different internal structure be affected differently by the gravity field of the Sun, a difference will appear in their annual frequency shifts. Clocks farther apart on the surface of the Earth can also be compared.

Ashby et al (PRL, 2007): Over a timespan of 7 years compared the frequencies of four H masers at NIST (USA) with one Cs fountain clock in the same lab, and also with three more Cs fountain clocks in Europe (in Germany, France and Italy).
The result is that the annual variation of the gravitational potential of the Sun produces on all pairs of clocks the same frequency shift to $1.4 \cdot 10^{-6}$, despite their different structure and also different location on the surface of the Earth.

In space STE-QUEST should take into account additional terms (to order $1/c^2$ and $1/c^3$) due to the motion of the clocks. Should a discrepancy be found, interpretation would be hard and disputable.

Is there a "quantum" test of UFF/WEP? (I)

In the first big jump after Eötvös (by almost 3 orders of magnitude!) Dicke's group tested Al and Au. In 1964 Dicke wrote:

"...gold and aluminum differ from each other rather greatly in several important ways. First, the neutron to proton ratio is quite different in the two elements, varying from 1.08 in aluminum to 1.5 in gold. Second, the electrons in aluminum move with non-relativistic velocities, but in gold the k-shell electrons have a 15 per cent increase in their mass as a result of their relativistic velocities. Third, the electromagnetic negative contribution to the binding energy of the nucleus varies as Z^2 and represents 1/2 per cent of the total mass of a gold atom, whereas it is negligible in aluminum. In similar fashion, the virtual pair field around the gold nucleus would be expected to represent a far bigger contribution to the total energy than in the case of aluminum. Also, the virtual pion field, and other virtual fields, would be expected to be different in the two atoms.

We would conclude that in most physical aspects gold and aluminum differ substantially from each other and that the equality of their accelerations represents a very important condition to be satisfied by any theory of gravitation."

What matters are the physical properties of atoms, not how many are there and/or how they are manipulated and/or how the signal is read. As long as there is no difference in the mass-energy content of the atoms being tested depending on the kind of test, these are technicalities... very important and interesting "per se", but still technicalities...

Is there a "quantum" test of UFF/WEP? (II)

Storey & Cohen–Tannoudji (1994) demonstrated that in a uniform gravitational field the quantum propagator of any object is determined by the action along the classical path. Unnikrishnan (2002) easily extended this result to show that same is true for accelerated objects in free space.

$\Downarrow$

The outcome of any test of WEP performed in a classical set-up will hold in a quantum context.

What is needed is a very high sensitive test of UFF/WEP with materials composition as widely different as possible to increase chance of violation. This is where new physics might come from, and at a relatively low cost.

The best "quantum" test of WEP is at 10^{-7} (Fray et al., PRL 2004), involves atoms differing by 2 neutrons only and prospects for improvement in space are not so bright (no gain in signal, very long integration time needed...)

Tests with macroscopic masses have reached 10^{-13}, have ample material choice, signal in space increases by about 3 orders of magnitude $\Rightarrow 10^{-15}$ with μSCOPE is within reach; GG can push it to 10^{-17} without cryogenics.