“Galileo Galilei” (GG)
Test of the Equivalence Principle with a Small Spinning Satellite: The Stabilization of its Weakly Coupled Masses

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Abstract

GALILEO GALILEI (GG) is a project for a small, non-cryogenic, satellite mission aimed at testing the Equivalence Principle to 1 part in 10^{17}. The key novel features of the experiment are that the signal of a possible violation of equivalence is modulated at relatively high frequency by spinning the read out capacitance sensors, together with the test bodies and the entire satellite, so as to substantially reduce the impact of *“1/f”* noise, and that the GG spinning bodies are coupled by extremely weak, high quality, mechanical suspensions so as to be sensitive to tiny differential forces and to make use of the self-centring effect in weakly coupled rotors. Construction and testing of a ground prototype is underway at LABEN laboratory in Florence. A Pre-Phase-A Study of GG has been carried out by the italian space industries ALENIA SPAZIO and LABEN in collaboration with the proposing scientists under ASI (Agenzia Spaziale Italiana) funding [1]. In this paper we focus on the issue of how the GG weakly coupled rotors can be stabilized without affecting the expected sensitivity.

1 Introduction

The scientific objective of the GALILEO GALILEI (GG) small satellite mission is to test a fundamental *“Principle”* of modern Physics — the *Equivalence Principle* (EP) formulated by Einstein, generalizing Galileo’s and Newton’s work— to 1 part in 10^{17}, the best ground experiments [2] with laboratory bodies having found no violation to 1 part in 10^{12}. Testing the *Universality of Free Fall*, that Galileo pioneered at the beginning of the 17^{th} century, is the most direct experimental test of the *Equivalence Principle*, in fact not a *Principle* but a basic property of gravitation, the first discovered and yet most intriguing physical interaction. Unlike all other tests of *General Relativity*, which check its consequences and predictions, an experiment on the *Equivalence Principle* tests the foundations of General Relativity and of all metric theories of gravity alike, it probes the very essence of gravity and its uniqueness among the fundamental forces of Nature. A high accuracy, unquestionable, experimental result on the *Equivalence Principle* —whether it is violated or confirmed— will be a crucial asset for the Physics of the next century.

The crucial advantage for EP testing in an Earth orbit is that the driving signal due to *the gravitational field of the Earth* (long range) is given by the entire value of its gravitational acceleration. For a spacecraft orbiting at $520\,km$ altitude this amounts to $GM_{\oplus}/a^2 \simeq 840\,cm/s^2$ (G is the universal constant of gravity, $M_{\oplus}$ the mass of the Earth and a the orbital radius of the satellite) as opposed to a maximum value of $\simeq 1.69\,cm/s^2$ due to the field of the Earth at 45° latitude on the ground, and $\simeq 0.6\,cm/s^2$ due to the field of the Sun. Another main advantage of space is not so much the absence of seismic noise, but *weightlessness*: the gravitational attraction of the Earth is largely compensated by the centrifugal force due to the orbital motion of the spacecraft so the main $1\,g \simeq 10^3\,cm/s^2$ local acceleration of gravity is absent. On board the GG spacecraft the largest acceleration is about a factor 10^8 smaller than $1\,g$, which means that it is possible to suspend $100\,kg$ there with the same suspensions that one would use on Earth for $1\,milligram$.

The GG satellite is shown in Fig. 1 (and its section through the spin axis in Fig. 2). It has cylindrical symmetry, it is small ($1\,m$ by $1.3\,m$)

Figure 1: The GG small spacecraft with bar antennas (one foldable); solar cells are shown on the left ($1\,m$ diameter, $1.3\,m$ height). Total expected mass is $250\,kg$.

and stabilized by single axis rotation around the axis of maximum moment of inertia. The spin rate is about $5\,Hz$ and the spin axis is close to the normal to the orbit plane (precise values are not required). Since it is highly desirable to avoid attitude manoeuvres and to make the satellite stabilization totally passive, the inclination of the orbit plane over the equator of the Earth should be close to zero. An almost circular, almost equatorial low altitude ($\simeq 520\,km$) orbit is the current baseline. There are no strict requirements on the orbit injection parameters. The angular phase of the spacecraft and its rotation rate are measured by EES (Earth Elevation Sensor). Essentially no active attitude control is required.

In Sec. 2 we recall the main features of the GG test of the Equivalence Principle and how the signal is modulated at the spin frequency of $5\,Hz$ ($\simeq 3 \cdot 10^4$ times higher than the orbital frequency). A general presentation of the mission is given in [3]. Details on the GG experiment, the calibration and balancing procedure, the analysis of all known perturbing effects and the expected sensitivity can be found in [1]. In the remaining of this paper we concentrate on dissipation in the mechanical suspensions of the GG bodies, on the slow instabilities that this dissipation gives rise to, and on how these can be actively controlled and the system stabilized so as to allow high accuracy EP testing.

2 High Frequency Signal Modulation

In GG the Equivalence Principle is tested by testing the *Universality of Free Fall* for two concentric hollow cylinders of different composition in the gravitational field of a given source mass (the Earth). Even in space a high sensitive test requires to be sensitive to extremely small relative displacements. For test bodies orbiting around the Earth the signal is always directed towards the centre of the Earth, hence it is necessarily at the orbit frequency: $\nu_{orb} = 1.75 \cdot 10^{-4}\,Hz$ at the altitude of GG. Low frequency signals are affected by *"1/f"* noise which seriously limits the integration time; furthermore numerous perturbing effects, both gravitational and non gravitational, do act at orbit frequency too. The answer to this problem is to modulate the signal at a higher frequency, the higher the better. One could rotate the sensors only (capacitance plates) located in between the test masses to detect the relative displacements, or, which is simpler, rotate the whole system, masses plus capacitance plates, with symmetry/spin axes perpendicular to the orbit plane (see Fig. 3). In addition, if the rotors are weakly coupled, weakly enough to respond by an appreciable amount to a small differential force such as the one of an EP violation, their centres of mass will oscillate one with respect to the other at their natural frequency; the weaker the coupling, the larger the displacement. This would happen also in absence of spin. What the spin of the sensors provides is a modulation of the signal at the spin frequency. The sensors detect a time changing relative distance of the form:

$$\Delta x = \Delta x_{EP} \cos(\omega_s t + \phi_{EP}) \cdot \mathcal{F} \qquad (1)$$

where Δx_{EP} is the expected displacement due to EP violation (constant in value), ω_s is the spacecraft spin angular velocity (with respect to the centre of the Earth), ϕ_{EP} is the known phase of the EP violation signal, and the factor $\mathcal{F} = \cos\theta + \sin\theta\cos(\omega_{orb} t + \phi)$ depends on the angle θ between the spin axis of the satellite

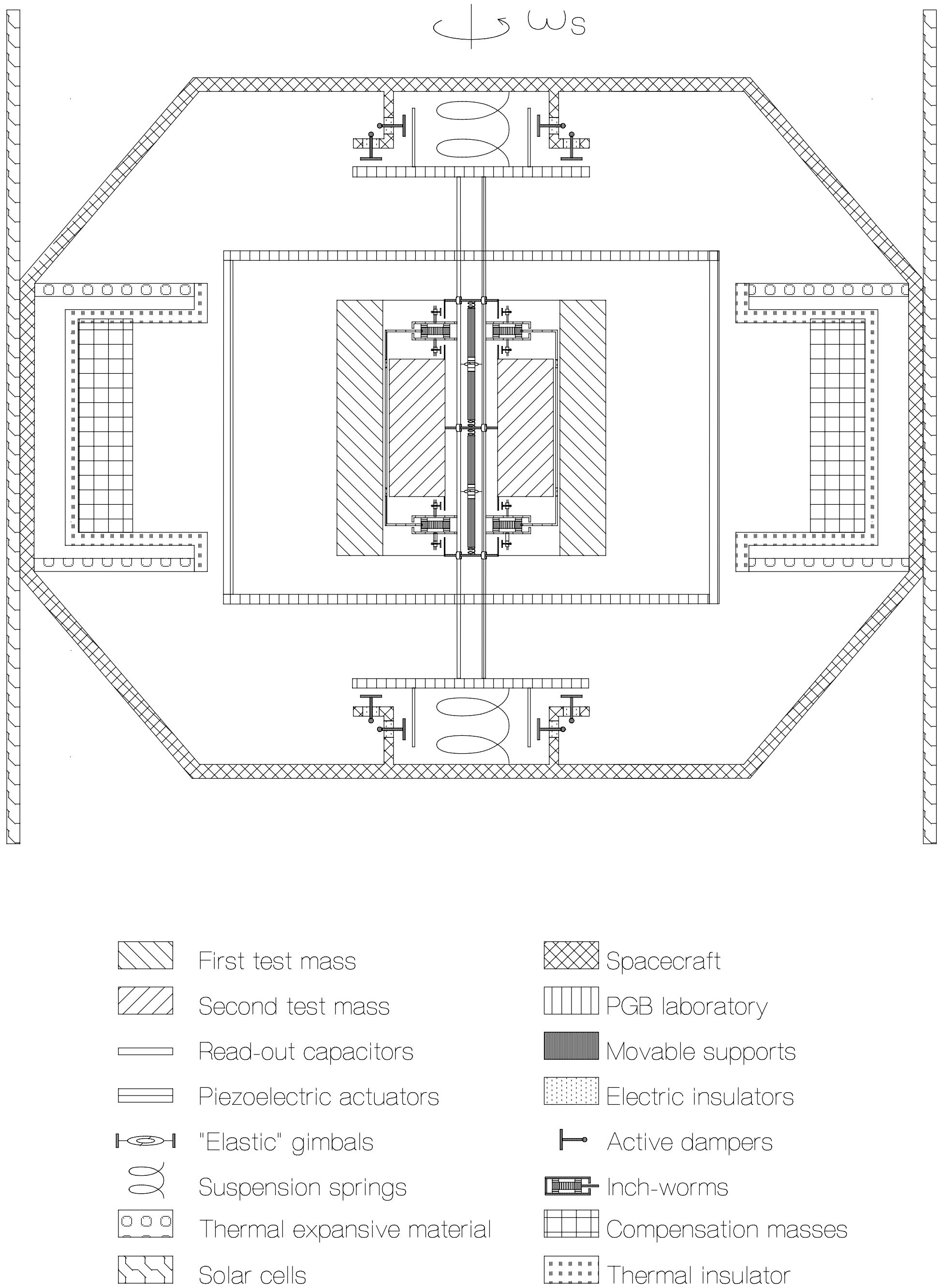

Figure 2: Section through the spin axis of the GG spacecraft.

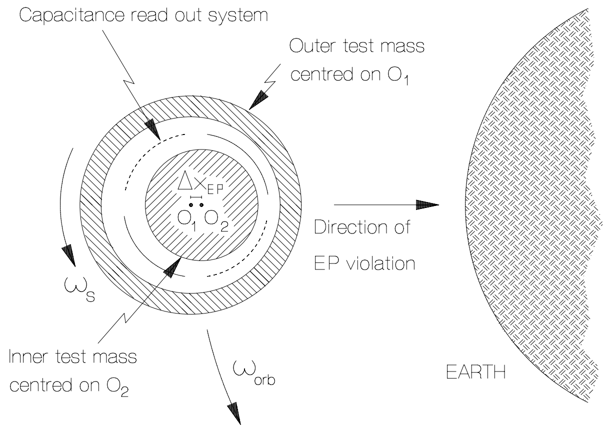

Figure 3: Section across the spin axes of the test bodies with their centres of mass displaced by a distance Δx_{EP} because of an Equivalence Principle violation in the field of the Earth. The centres of mass are fixed in their diplaced configuration while the bodies rotate independently around O_1 and O_2 respectively. The relative displacement vector always points towards the centre of the Earth and it is therefore detected by the capacitors at their spinning frequency with respect to the Earth.

and the orbit normal, and is maximum ($\mathcal{F} = 1$) for a spin axis perpendicular to the orbit plane. Fast rotation has other important advantages: many disturbing effects give only DC output signals, and temperature perturbations, dissipation and thermal noise, are reduced. Mechanical suspensions allow the test masses to be electrically grounded, hence eliminate the major source of electrostatic perturbations which can seriously affect small force gravitational experiments.

The elastic properties assumed in the baseline dynamical model determine the natural frequency for differential oscillations of the test masses to be $\simeq 1.2 \cdot 10^{-2}\, rad/s$. This value is confirmed by numerical simulations. From this, the displacement to be expected in response to an EP violation (*differential*) signal at the level of 10^{-17} is obtained. We get $\Delta x_{EP} \simeq 5.8 \cdot 10^{-11}\, cm$, which can be detected by the capacitance sensors located halfway between the test bodies [1].

2.1 Compensation and/or Rejection of Air Drag Perturbations

The residual atmosphere at the satellite orbiting altitude, and the radiation from the Sun, act on the outer surface of the GG spacecraft but not on the test bodies suspended inside (Fig. 2). Since gyroscopic effects are negligible (due to the very high spin energy of all the GG rotors), these non–gravitational effects appear as inertial accelerations on the centres of mass of the bodies, equal and opposite to the acceleration acquired by the spacecraft. Most of the effect (along–track) is at the satellite orbiting frequency around the Earth; smaller, low frequency variations are also to be expected.

It is easy to check that the air drag perturbing acceleration along–track, as well as the acceleration due to solar radiation pressure, are larger than the expected signal by several orders of mangnitude. However, unlike the signal (which is *differential*), these perturbations are in principle *common mode*, i.e. they would give no differential force provided that the two masses were equal and identically suspended. Since identical suspensions are not realistic, a good strategy is to mechanically couple the test masses and balance them like in an ordinary balance, so as to reject common mode forces leaving only a much smaller differential effect to compete with the signal. This procedure is known as *Common Mode Rejection (CMR)*. However, the smaller the non–gravitational force, the easier will be the balancing and the better the CMR achieved. It is therefore desirable, although not a *conditio sine qua non*, that a space mission to test the Equivalence Principle be, to some extent, *drag–free*, i.e. capable to partially compensate for the effect of non gravitational forces such as air drag and solar radiation pressure. Indeed, originally GG was designed as a *non drag-free* satellite; it also accomodated three pairs of test masses rather than one [4].

In GG drag compensation is performed with FEEP (*Field Emission Electric Propulsion*) thrusters. Their advantages for high precision missions in Fundamental Physics devoted to the detection of very small forces, are numerous: high specific impulse, negligible amount of propellant (few grams of Caesium for a few months mission duration), no moving parts, fine electric tuning and consequent high level of proportionality. By comparison, He thrusters (whose proportionality is ensured mechanically rather than electrically) would need, for the same purpose, a very large amount of propellant whose perturbing effect on the experiment becomes a problem in itself.

2.2 Abatement of Platform Noise

The signal of an EP violation for test bodies orbiting around the Earth would be at their orbital frequency. With the capacitance sensors (in fact the whole GG satellite) spinning at $5\,Hz$ *with respect to the Earth* the EP signal is detected at their spinning frequency. So is the perturbing signal from the main along-track component of the residual atmospheric drag, because it also acts at the orbital frequency of the spacecraft around the Earth. As for the effects due to air density fluctuations at low frequencies, they will be close to the signal. These perturbations, as well as the signal, are not attenuated by mechanical suspensions because their frequencies are too low even for a very weak suspension in absence of weight. Low frequency *non-gravitational* perturbations can be reduced either by active drag compensation or by *Common Mode Rejection*, or both; as recalled in Sec. 2.1, in GG we rely on a combination of both active drag compensation, by means of FEEP thrusters, and *Common Mode Rejection* by the balanced suspension of the test masses [1].

Instead, vibrational noise at the spin frequency of the sensors (or close to it), i.e. noise which acts at $5\,Hz$ w.r.t. the Earth, hence at $10\,Hz$ or DC in the rotating frame, will be effectively attenuated by mechanical suspensions. We do that by locating the test bodies inside a laboratory (that we call Pico-Gravity-Box, PGB) mechanically suspended inside the spacecraft by means of weak helicoidal springs. As seen in the fixed frame, the system is transparent to DC and low frequency effects (like the signal, the residual atmospheric drag and its low frequency fluctuations...) but is very efficient in attenuating vibrational noise above its threshold frequency, particularly around the spin frequency. The transfer function of the system, when viewed in the rotating frame, shows a sharp peak of value 1 at $5\,Hz$ (Fig. 4), meaning that the system is perfectly transparent at the signal frequency. In this way the signal is not affected, in amplitude, by the spin; also the low frequency drag effects (of the fixed frame) are up-converted to high frequency with no amplification (and no reduction either, of course). The only difference, and indeed big advantage, with respect to the non rotating case being that the detecting instruments work much better at higher frequency. So the sharp peak at $5\,Hz$ is a key feature of GG. But how can the peak at 5 Hz be so sharp? Simply thanks to the fact that the PGB provides good attenuation at its sides, at lower and higher frequencies, i.e. around $10\,Hz$ and $0\,Hz$ (w.r.t the rotating frame). This means good attenuation of perturbations which are at $5\,Hz$ w.r.t the fixed frame. The need to attenuate these perturbations should not be neglected. Although in space we obviously don't need a motor, we cannot forget that the FEEP thrusters will act at about $5\,Hz$ (to reduce the main along track effect of drag at the orbit frequency, and also its low frequency components). Since the FEEP thrusters compensate low frequency drag effects while spinning at $5\,Hz$, any mismatches and imperfections in their firing will give rise to spacecraft perturbations at $5\,Hz$ w.r.t. the fixed frame (hence at $10\,Hz$ and $0\,Hz$ in the rotating frame). Sonic noise of the GG spacecraft structure is peaked at much higher frequency, but a tail at the spin/signal modulation frequency should not be excluded, and will be attenuated.

Fig. 4 shows the transfer function (in the rotating frame) for a *quality factor* Q (see Eq. (2) below) of the PGB suspensions of 100. It is worth to point out that a Q value higher than this could be obtained, were it necessary in order to reduce noise around the spin/signal modulation frequency (so as to increase the sharpness of the peak shown in Fig. 4). The lower Q value of the PGB suspension springs (as compared to the springs which suspend the test bodies; see Sec. 3.1), is due to the fact that they have to carry wires and their insulation; experimental measurements of the dissipation (in the ground test for horizontal oscillations at $\simeq 5\,Hz$) in the PGB prototype springs, carrying the required number of wires and insulation, have yielded $Q \simeq 90$. An alternative solution is possible with helicoidal springs made of separate wires insulated at the clamping; these would have a lower dissipation and therefore a higher Q.

The weak mechanical suspensions of the PGB, which are possible only thanks to weightlessness, provide an effective, passive means of isolation from the (relatively) high frequency vibrations

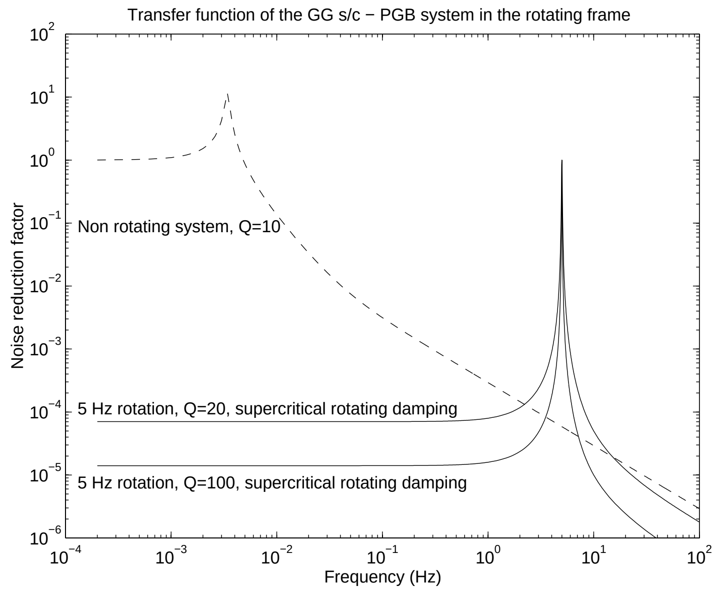

Figure 4: Transfer function of the GG spacecraft-PGB system in the rotating frame. If the spin rate is put to zero (dashed curve) the transfer function goes back to the one typical of a non rotating system (with $Q = 10$ in this case). In supercritical rotation two curves are plotted for two different values of Q (20 and 100, top and bottom curve respectively). It is a peculiarity of supercrtical rotation that the higher the Q, the better the noise reduction. The peak with value 1 at the spinning frequency shows that the passive noise attenuator does not reduce vibrations at very low frequency w.r.t. the fixed frame, particularly the DC ones; the observer corotating with the system sees these DC perturbations as $5\,Hz$, and finds that the attenuator does not reduce them, or better that it is transparent to $5\,Hz$ effects. Perturbations which are seen at $5\,Hz$ by the non rotating observer (and attenuated), have frequencies $0\,Hz$ and $10\,Hz$ for the body fixed observer, and in fact he too finds that they are attenuated.

around the spin/signal modulation frequency.

3 The GG Bodies as Weakly Coupled Rotors

If the concentric spinning test cylinders of the GG experiment were free, totally unconstrained, rotors, each one would spin smoothly with the centre of mass perfectly aligned on the spin axis, and the centre of mass would respond freely to any applied force, albeit small. A constrained rotor is indeed very close to a free one provided its spin angular velocity ω_s is larger (possibly much larger) than its natural frequency of oscillation ω_n ("supercritical" rotation). Which amounts to saying that the mechanical suspension is weak, possibly very weak; as it is indeed the case in space thanks to weightlessness. In this case the centre of mass of the rotor reaches equilibrium by aligning itself very accurately on the spin axis with an extremely small rotation radius; more importantly, if external forces are applied to it, it responds simply by moving to another equilibrium position while still spinning around its own axis.

"Supercritical" rotation is commonly used on the ground for rotating machines to exploit the effect of self alignment. If the centre of mass of the suspended body is located, because of inevitable errors in construction and mounting, with an offset $\vec{\epsilon}$ from the rotation axis, equilibrium is established on the opposite side of the unbalance vector $\vec{\epsilon}$ (fixed in the rotating frame) with respect to the rotation axis, where the centrifugal force due to rotation and the restoring elastic force of the spring equal each other. It can be shown that this happens at a distance from the spin axis smaller than the original unbalance by a factor $(\omega_n/\omega_s)^2$. In space, due to the absence of weight (i.e. small ω_n) this ratio can be very small. In GG it is about 10^{-6}. Thus, it is a physical characteristic of the system that an equilibrium position exists, fixed in the rotating frame, very precisely aligned on the spin axis. In the presence of a force the equilibrium position changes but the body continues to spin around its own axis in the displaced position. If

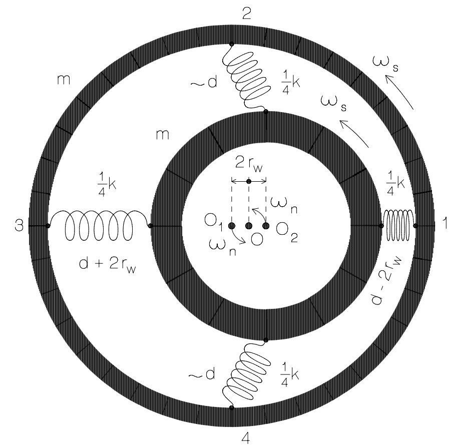

Figure 5: Mathematical model of a rotor made of two bodies, each of mass m, coupled by weak springs. The coupling constant is k and the natural frequency of oscillation ω_n. Both bodies are spinning at the same angular velocity ω_s around their respective centres of mass O_1 and O_2. In turn, O_1 and O_2 are *whirling* around the centre of mass O of the whole system, at a distance r_w from it, and at the natural angular velocity ω_n. In the GG case $\omega_n \simeq 10^{-3}\omega_s$.

the rotor is made, as in the GG case, by four concentric cylindres all weakly suspended, each of them will have its own rotation axis.

3.1 Dissipation, Whirl Motions and Destabilizing Forces

No matter how good is the mechanical quality of the suspensions and how accurately they are clamped, as the system rotates they will undergo deformations and, in this process, will dissipate energy. The only energy that can be dissipated is the spin energy of the rotor, which means that the spin angular velocity must decrease. As a consequence, the spin angular momentum also decreases, and since the total one must be conserved, the motion of the two bodies one around the other (in the same direction as the rotational motion) will grow in amplitude. Which is what is observed, and is referred to in the literature on Rotordynamics as *whirling motion.* Friction between *rotating* parts of the system (i.e. friction inside the suspension springs) is the physical cause of the whirl motion, and it is referred to as *rotating damping.* Except in the case of rotating machines with very viscous bearings, the rotor whirls at essentially its natural frequency (with respect to a fixed frame of reference). In GG there is no motor, there are no bearings, no fluids, no oils, no greases; only carefully clamped suspensions of high mechanical quality (particularly for the test bodies) which undergo only minute deformations. Hence, the GG bodies whirl at their natural frequencies of oscillation.

In GG it is important that the test cylinders (of equal mass and different composition) be very weakly coupled in the plane perpendicular to the spin axis, so as to be sensitive to tiny differential forces. The mathematical model typically used in Rotordynamics literature to describe the system is shown in Fig. 5, where r_w is the radius of whirl of each body around the common centre of mass which, for the purpose of the present discussion on whirl motion, is shown to coincide with the equilibrium position ("perfect" self alignment).

Can the whirl motion be damped and the rotating system be stabilized?

If there is nothing else in the system but *rotating damping* there is nothing to prevent the amplitude of the whirl motion from growing, and therefore the system is unstable. In rotating machines on the ground whirl motions are usually damped by *non-rotating damping*, namely by sufficient friction occurring between two parts of the system, both non-rotating as shown in Fig. 6 (i.e. between two parts of the non-rotating supports, for example friction between the non-rotating part of the bearings and their fixed supports). This friction generates non-rotating damping forces which are effective in damping transversal translational oscillations of the rotor's axis of rotation (such as the whirl motions), and they do so without slowing down its rotation. In the ground rotors in which non–rotating damping is provided by tipping the non–rotating part of the bearing in oil, this is essentially viscous damping. *Non-rotating friction* should not

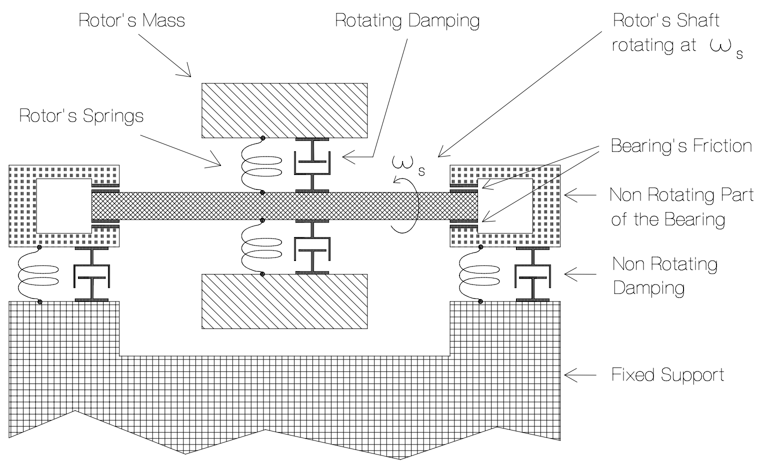

Figure 6: Sketch of a ground rotating machine showing where *rotating damping*, *non-rotating damping* and *friction in the bearings* are localized. The different rôles they play in the dynamics of the system are discussed in Sec. 3.1

be confused with *friction in the bearings*, also shown in Fig. 6. This is the friction (mostly viscous) between the rotating body and the non rotating parts, which is obviously effective in slowing down the rotor but almost completely ineffective at damping whirling motions (and also at producing them). An important advantage of the GG space experiment is the absence of bearings, hence of bearing friction at all. From a physical viewpoint the most important characteristic distinguishing the effects of *rotating* and *non–rotating friction* on one side from the effects of *friction in the bearings* on the other is that the former produce forces on the rotor, while the latter produces torques. They are therefore essentially independent from one another and interact only to a second order, namely because of construction errors, asymmetries, misalignments etc. From this it follows that if one were in fact using the forces generated by *friction in the bearings* in order to stabilize the whirling motions (rather than the forces due to *non-rotating friction*), he would inevitably need extremely large forces.

In the GG space experiment where there are no non–rotating parts an equivalent non–rotating damping must be provided by an active control system of sensors and actuators fixed with the rotating bodies. Before any such device can be designed, it is obviously necessary to establish the magnitude of the forces which destabilize the system and which will therefore need to be counteracted actively. The actual implementation of the active control forces in the real space mission, with realistic errors in all components of the control system, can only be faced once the magnitude of the destabilizing forces has been firmly established. In turn, this requires to firmly establish the amount of energy losses in the GG rotating system. The only components where there can be losses are the mechanical suspension springs and the active dampers themselves (in GG the dampers are electrostatic). Since losses in the dampers depend in turn on the magnitude of the control forces they are required to provide, we must first evaluate the mechanical losses in the springs. All the other parts are rigid and no losses are expected in them.

It is very important to establish at which frequency the mechanical suspensions do dissipate energy. If we consider the two body model of Fig. 5, the only frequencies involved are the spin frequency ω_s (of each body around its own centre of mass) and the frequency of whirl (of the two bodies around their common centre of mass with a radius of whirl r_w), which is equal to the natural frequency of oscillation ω_n. In GG the ratio ω_s/ω_n is very high ($\simeq 10^3$); thus, if we neglect ω_n with respect to ω_s, it is apparent from Fig. 5 that the suspension springs are dragged at the spin frequency through a "corridor" of uneven width. As a consequence, in their own reference system, the springs contract and expand with amplitude $2r_w$ at the spin frequency and therefore dissipate energy at this frequency [5]. It can be easily shown that, in the general case in which ω_n is not negligible compared to ω_s, energy is in fact dissipated at frequency $\omega_s - \omega_n$.

Mechanical losses in the GG suspension springs can therefore be measured experimentally by setting them in oscillation at the frequency at which the GG spacecraft will spin, with a clamping similar to the one to be used in the space mission, in realistic conditions of temperature and vacuum. The (adimensional) quantity to be measured is the *quality factor* Q of the suspensions *at the relevant frequency* [5]; this is routinely done by monitoring the decrease in time of the amplitude of oscillation, just recalling that the timescale for its exponential decay is $Q/(\pi\nu)$, ν being the relevant frequency at which losses occur (see Eq. (2) below).

Helicoidal springs very similar to the ones that can be used to suspend the GG test bodies during the space mission have been manufactured in $CuBe$ and their Q at oscillation frequencies close to $5\,Hz$ has been measured, at room temperature and in vacuum, in the laboratory of LABEN in Florence. Care has been taken in considering transversal oscillations in a horizontal plane, so that they are not affected by the local gravity, like in space. The Q values obtained so far are between $16,000$ and $19,000$.

Since energy losses in the mechanical suspensions are responsible for the onset of the whirl motion (in order to conserve the total angular momentum of the system) (see Fig. 5), the whirl motion (at the natural frequency ω_n w.r.t. the fixed frame) is nothing but an oscillatory motion of growing amplitude, i.e. with a *negative* quality factor equal and opposite to the measured quality factor Q of the suspensions. Hence, the radius of whirl will grow, from a given value $r_w(0)$, according to the equation:

$$r_w(t) = r_w(0)e^{\omega_n t/(2Q)} \tag{2}$$

That is, the larger the Q, the slower the growth of the whirl instability. For large Q, as in this case, we can write:

$$\frac{(\Delta r_w)_{T_n}}{r} \simeq \frac{\pi}{Q} \tag{3}$$

where $(\Delta r_w)_{T_n}$ is the increase in the amplitude of whirl in one natural period of oscillation T_n. For the GG test bodies, taking the measured value $Q \simeq 16,000$ and having $T_n \simeq 2^m$, the amplitude of the whirl motion will need about one week to double: such a very slow growth of the whirl instability is obviously very important for making the control forces required to damp it very small.

The increase in amplitude $(\Delta r_w)_{T_n}$ (during one natural period) is due to an increase of the along-track velocity of the bodies, which in turn is caused by an average *destabilizing* acceleration a_d, also along-track, such that:

$$\frac{1}{2}a_d T_n^2 \simeq 2\pi(\Delta r_w)_{T_n} \quad , \quad a_d \simeq \frac{1}{Q}\omega_n^2 r_w \tag{4}$$

which means, an average *destabilizing* force (along-track) of magnitude

$$|F_d| \simeq \frac{1}{Q} m\omega_n^2 r_w \tag{5}$$

Since $F_c = m\omega_n^2 r_w$ is the centrifugal force, equal and opposite to the elastic force of the spring $F_{spring} = -kr_w$, the destabilizing force which generates the unstable whirl motion depicted in Fig. 5 turns out to be

$$|F_d| \simeq \frac{1}{Q}|F_{spring}| \tag{6}$$

i.e., only a fraction of the elastic spring force [5]: the higher is the Q of the suspensions (at the spin frequency) the smaller is the destabilizing force which needs to be damped in order to stabilize the system.

Note that the *quality factor* Q in Eq. (6) expresses the *total losses* in the system, regardless of their physical nature, e.g. structural (also known as *hysteretic*) or viscous, and also regardless of *where* in the system they occur (e.g., in GG, inside the suspension springs and at their clampings). Once the Q has been measured experimentally, at the relevant frequency and in realistic conditions, this gives the total dissipation in the system and is therefore the value to be entered in Eq. (6) in order to get the intensity of the destabilizing force. Were the measured Q of the system to be *interpreted* as if *entirely* due to *viscous dissipation*, the corresponding value of the *viscous quality factor* would have to be, according to current models which give a ratio of ω_s/ω_n for the structural-to-viscous Q, correspondingly larger. In GG this would be 10^3 times larger than the measured values, meaning a very small viscous dissipation. Indeed, this result had to be expected because GG has no bearings, no fluids, no oils, no greases; it has only carefully clamped suspensions of high mechanical quality (particularly for the test bodies) which, moreover, undergo only minute deformations. Therefore, viscous losses must be very very small. Nonetheless, they are included in the value of the quality factor measured experimentally. Losses in the electrostatic dampers themselves have not been measured yet, but theoretical estimates show that they are negligible [1, 4, 6].

It must be pointed out that in the analysis of GG performed at ESTEC [7] the destabilizing force is taken to be a factor $\omega_s/\omega_n \simeq 10^3$ larger than in Eq. (6); this amounts to assuming the GG system as dominated by viscous damping, which is ruled out by the values of Q measured in the laboratory. Q values of $16,000$ to $19,000$, as obtained for the *total losses* in the helicoidal suspensions of the test masses, clearly show that viscous losses –if any– must be very very small, and therefore the assumption made in [7] is incorrect.

The following question becomes relevant at this point: how much energy (per unit time) is gained by the (destabilizing) whirling motion as fraction of the energy lost by the spinning rotor? Let us consider the rotor in the two body model of Fig. 5. The spin energy of the rotor is:

$$E_{rotor} = \frac{1}{2} I \omega_s^2 \qquad (7)$$

with I the total moment of inertia of the two bodies with respect to the spin axis (perpendicular to the plane of the Figure). The energy (kinetic plus elastic) of the whirl motion, at the constant natural frequency ω_n with respect to the fixed frame and with radius of whirl r_w is:

$$E_w = 2m\omega_n^2 r_w^2 \qquad (8)$$

The time derivatives of E_w and E_{rotor} are:

$$\dot{E}_w = 4mr_w\omega_n^2\dot{r}_w \quad , \quad \dot{E}_{rotor} = I\omega_s\dot{\omega}_s \qquad (9)$$

From the conservation of angular monetum it is possible to relate $\dot{\omega}_s$ to $\dot{r}_w$. The total spin angular momentum of the rotor is:

$$L_{rotor} = I\omega_s \qquad (10)$$

The angular momentum of the whirl motion is:

$$L_w = 2mr_w^2\omega_n \qquad (11)$$

Since the total angular momentum has to be conserved it must be:

$$\dot{L}_{rotor} + \dot{L}_w = 0 \qquad (12)$$

from which, since ω_n is constant, it follows:

$$\dot{\omega}_s = -\frac{4mr_w\omega_n}{I} \cdot \dot{r}_w \qquad (13)$$

Using Eq. (13) for $\dot{\omega}_s$, we get from (9):

$$\frac{\dot{E}_w}{\dot{E}_{rotor}} = -\frac{\omega_n}{\omega_s} \qquad (14)$$

which is a very important result. In the GG case, where the frequency of the whirl motion is very small compared to the spin frequency ($\omega_n \simeq 10^{-3}\omega_s$), Eq. (14) tells us that the energy gained by the whirling motion is one thousand times smaller than the energy lost by the rotor. All the rest, that is $1-(\omega_n/\omega_s) \simeq 99.9\%$!!, is dissipated as heat inside the springs; which means that it is *not transferred* to the (destabilizing) whirl motion. In simple terms one can say that in "supercritical" rotation the energy balance is essentially between the rotor and the springs; the rotor looses spin energy and the springs dissipate almost all of it as heat: the faster the spin, the larger is the energy dissipated inside the springs, as it can be seen from Eq. (9). On the other hand, the springs do not enter at all in the balance of the angular momentum; the onset of the whirl motion is inevitable for the total angular momentum to be conserved, but the energy it gains from the rotor is only the small fraction given by Eq. (14). So, the idea one might have that the faster the spin (as compared to the natural frequency) the higher the energy gained also by the whirl motion (by which argument the GG system would be highly unstable), is proved to be incorrect. As a consequence, also the objection raised in [7] that the GG system should be much more unstable than ground based rotating machines due to the smaller ratio ω_n/ω_s (by one order of magnitude or more) does not hold, because the fraction of destabiling energy is also correspondingly smaller, as shown by Eq. (14)

3.2 Active Stabilization and Control with Rotating Dampers

All the GG bodies spin at the same rate. There are no non-rotating parts, and therefore there can be no *non-rotating friction* to damp the whirl motions. They must be damped actively, with dampers necessarily fixed to the rotating bodies. We have shown (Eq. (6)) that the destabilizing forces are only a small fraction of the passive spring forces, which in turn are very small because the suspensions are designed to be extremely weak ($k \simeq 10\,dyn/cm$), as it is in fact possible in space in spite of the fact that the test bodies have masses of $10\,kg$ each. Small capacitance plates (see Fig. 7), with surfaces of $1\,cm^2$, turn out to be sufficient to provide the required active forces.

Let the whirling motion be damped using electrostatic sensors/actuators fixed to the rotating bodies. The capacitors are required to provide a

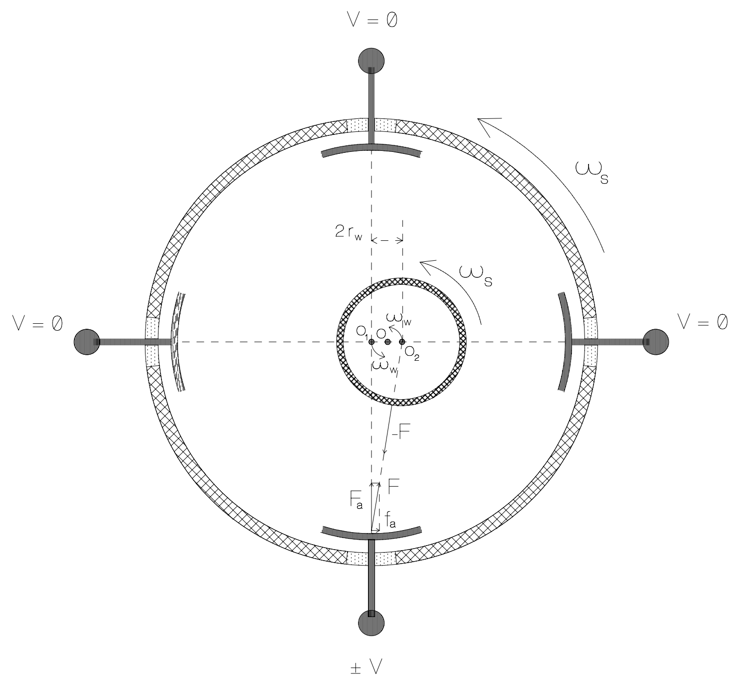

Figure 7: Four capacitance plates, at 90° from one another, rotate with the system at angular frequency ω_s. They provide an electrostatic force F in order to prevent the growth of the whirl motion of the bodies (at a frequency ω_w equal to the natural one ω_n). The reaction of the active force on the plate has a small tangential component f_a which spins up the rotor by transferring to it the angular momentum of whirl, which would otherwise grow. The figure is not to scale; the distance $2r_w$ between the centres of mass O_1 and O_2 of the two bodies is in reality many orders of magnitude smaller than R.

force at the whirling frequency ω_n while spinning at angular frequency ω_s; therefore, they must actuate at frequency $\omega_s - \omega_n$. By providing forces internal to the system they cannot possibly change its total angular momentum: they can only transfer the angular momentum of whirl to the rotation angular momentum of the rotor by spinning it up. This is what happens if they are made to provide a stabilizing force of the same intensity as the destabilizing one given by Eq. (6) (in fact a slightly larger one). This force must always act along the vector of relative velocity of the centres of mass of the bodies in their whirling motion, as seen in the fixed frame of reference. Since the centres of mass of the bodies are displaced from the centre of mass of the system by an amount r_w (see Fig. 7), the electrostatic plates will necessarily apply also a small force f_a tangent to the surface of the rotor amounting to a fraction r_w/R ($R \simeq 10\,cm$ is the linear dimension of the rotor) of the the active force F, of intensity $F \simeq (2/Q)|F_{spring}|$ (for both bodies), which will damp the relative velocity of whirl (see Fig. 7). Of the corresponding reaction components on the electrostatic actuators only the reaction to the small tangential component $f_a \simeq (1/Q)|F_{spring}|(2r_w/R)$ will produce a non zero angular momentum by spinning up the rotor at the expense of exactly the angular momentum of whirl:

$$f_a R \simeq \frac{1}{Q}|F_{spring}|\cdot 2r_w \simeq \frac{1}{Q}2m\omega_n^2 r_w^2 = \dot{L}_w \quad (15)$$

which will therefore increase the spin angular momentum of the rotor $L_{rotor} = I\omega_s$ in such a way that the total angular momentum of the system is conserved. That is:

$$I\dot{\omega}_s \simeq \frac{1}{Q}2m\omega_n^2 r_w^2 \quad (16)$$

thus producing a spin up of the rotor at the rate $\dot{\omega}_s$ given by Eq. (16). By integrating $\dot{\omega}_s$ for the entire duration of the mission $T_{mission} = t_f - t_i$, from intial to final epoch, the ratio ω_f/ω_i of final–to–initial spin angular velocity of the rotor is obtained:

$$\frac{\omega_f}{\omega_i} = 1 + \frac{\omega_i}{Q}\frac{2mr_w^2}{I}\frac{\omega_n^2}{\omega_i^2}T_{mission} \quad (17)$$

In the case of the GG test bodies, taking the (smallest) measured value of $16,000$ for the quality factor Q at the spin frequency of $5\,Hz$, $r_w \simeq 10^{-6}\,cm$ and $\omega_n^2/\omega_{spin}^2 \simeq 2.5\cdot 10^{-6}$ we get, for a

6-month duration of the mission:

$$\frac{\omega_f}{\omega_i} - 1 \simeq 10^{-15} \tag{18}$$

The corresponding angular advance is:

$$\Delta\theta \simeq \frac{1}{2Q}\frac{2mr_w^2}{I}\omega_n^2 T_{mission}^2 \simeq 4 \cdot 10^{-2}\, arcsec \tag{19}$$

which is clearly negligible. The amount of spin energy gained by the rotor at the end of the mission is vanishingly small:

$$\Delta E_{rotor} \simeq 2 \cdot 10^{-15} E_{rotor} \simeq 2 \cdot 10^{-6}\, erg \tag{20}$$

The corresponding (negative) quality factor Q_{rotor} for the spin up of the rotor (in absence of any external disturbances to its spin rate) is obviously huge (in modulus):

$$Q_{rotor} = -2\pi \frac{T_{mission}}{T_s}\frac{E_{rotor}}{\Delta E_{rotor}} \simeq -2.5 \cdot 10^{23} \tag{21}$$

(T_s is the spin period). The amount of spin angular momentum gained by the rotor in 6 months is, similarly to the spin energy, vanishingly small:

$$\Delta L_{rotor} \simeq 10^{-15} L_{rotor} \simeq 6 \cdot 10^{-8}\, g\, cm^2\, s^{-1} \tag{22}$$

The contribution to the energy of the system by the electrostatic dampers is obtained by computing the work done by both components (F_a and f_a, shown in Fig. 7) of the active force they provide. While only f_a will transfer angular momentum from the whirl motion to the rotor, both components of the active force will provide energy. It can be easily demonstrated that f_a supplies the rotor with the energy necessary to increase its spin rate, while the energy supplied by F_a balances exactly the energy dissipated by the springs as heat, which would otherwise spin down the rotor.

The capacitors must also act as sensors in order to recover, from relative position measurements, the linear relative velocity of the whirling motion which needs to be damped to stabilize the system; or, for example (see Fig. 7), two of the capacitors can be used as sensors and the other two as actuators. In GG the capacitors spin (together with the bodies) at a frequency about 10^3 times larger than the frequency of whirl. Is it possible, by means of rotating sensors, to recover the slow relative velocity of whirl as with non-rotating sensors? This question can be answered in a totally general way. The answer would be *"No"*, if the rotating observer had no other means of *"looking outside"* to measure independently (i.e. by means of some other instrument) its own phase and rotation rate; the answer is *"Yes"*, if the rotating observer can do that, which is the case in GG using the *Earth Elevation Sensors*. An independent knowledge of the phase and the rotation rate of the sensors makes it possible to subtract from the measurements of the rotating sensors their own velocity of rotation, hence to recover the (much slower) relative velocity which would be obtained with fixed sensors. The problem amounts to that of computing numerically a time derivative, because the capacitors measure relative displacements while the electrostatic active dampers require to know the relative velocity of the bodies *in the fixed frame*. If the velocity were computed by taking successive measuremets of the sensors and computing their difference, it would be dominated by the rotation velocity, i.e. it would be larger than the velocity to be damped by a factor about 10^3 (i.e. the ratio spin–to–natural velocity). If then such a velocity output were used to drive the electrostatic actuators which must damp it, the required force would be (correspondingly) about 10^3 times larger than necessary. Clearly this procedure would not be appropriate for a system like GG where the spin–to–natural velocity ratio is very large. The analysis of GG performed at ESTEC [7] has been carried out following this procedure; as a result, their final value of the active control forces has been oversized by another factor 10^3. In addition to the 10^3 factor by which the same authors have overestimated the magnitude of the destabilizing forces to be damped, this further amplification of the active control forces brings their total oversizing to a factor of 10^6, thus making their analysis inapplicable to the GG system.

Instead, the problem of reducing the magnitude of the active control forces close to that of the destabilizing ones (given by Eq. (6)), is solved as follows. A reference signal is built from a 5 Hz oscillator (clock) synchronized by the EES output averaged over many spin periods of the spacecraft. Since the spin period is 0.2 s, it can certainly be considered constant over time intervals of the order of many times the spin period itself. Note that the reference signal is continoulsy synchronized to the output of the *Earth Elevation Sensors* so that any error in their measurement of the spin rate will not build up. The reference signal is needed in order to perform the transformation from the rotating to the fixed frame (and back) as accurately as possible. A relative displacement of two bodies due to the growing unstable mode is measured by the rotating capacitance sensors. Instead of taking successive measurements of the sensors and computing their difference, the difference is computed between measurements which are 1 spin period apart, using the reference signal. The sensor signal is sampled a certain number of times per spin period (e.g. 10 to 20), at regular intervals, and for each sampled point the difference is computed with respect to the sampled point 1 spin period later. In this way the rotation velocity of the sensors is subtracted from their measurements; a best fit to the sampled data points (each one is a relative position difference after 1 spin period) gives the relative velocity vector $\vec{v}$ between the two bodies in the fixed frame. To this velocity corresponds a whirling motion of amplitude $r_w \simeq v/\omega_n$, and then, taking into account Eq. (5), we can obtain the required stabilizing force, antiparallel to the velocity $\vec{v}$,

$$\vec{F}_{stab} = -\frac{1}{Q} m\omega_n \vec{v} \tag{23}$$

This force will prevent the whirl motion from growing, i.e. it will maintain the system in a steady condition. To actually damp a whirling motion one has to apply a slightly larger force: for instance, a force twice as large will damp the whirling motion at the same rate at which it would grow in the absence of $\vec{F}_{stab}$.

Were the capacitance sensors perfect, this would be enough to stabilize the GG bodies, as it can be also checked with numerical simulations. However, the sensitivity of the sensors in realistic flight conditions has to be taken into account because, being the growth rate of the whirl instabilities very slow, the relative displacement during one spin period is so small to be dominated by the noise of the sensors. As a result, the recovered velocity vector is also very noisy, and much larger than its actual value in the fixed frame. If the actuators were programmed to damp the velocity vector recovered in this way, the resulting active force would be driven by the noise, i.e. it would be much larger than the force which destabilizes the system. Such a controlled system would be dominated by the active control forces rather than by the very low stiffness passive mechanical suspensions. This is in contrast with the basic physical design of the GG experimental test of the universality of free fall whereby the test bodies must be very weakly coupled so as to be sensitive to tiny differential forces.

A better active control can be devised in which active damping is not applied until the (slow) relative velocity of the whirl motion of the bodies has been recovered from the noise after appropriate averaging. To do this, the relative velocity vector is averaged over a few spin periods (typically a fraction 1/100 of one period of whirl), and then fitted to the whirl period (the natural period of oscillation) in order to reconstruct its rotation in the fixed frame. Since the growth rate of the instability is slow, this averaging procedure can be carried on over a large number of whirl periods (particularly for the test bodies), thus making the determination of the velocity to be damped more and more accurate. Active control starts only once the small relative velocity of the bodies w.r.t. the fixed frame has been reconstructed and separated from the noise. Indeed we have found that a velocity vector obtained from averaging over 10 spin periods only (which amounts to a small fraction of one whirl period), if used to drive the dampers with no further averaging/fitting, already reduces the intensity of the control force by one order of magnitude. The control forces provided by the electrostatic dampers can be further tuned to adjust for remaining phase errors. Again, such adjustments are successful because the instabilities grow very slowly: the dampers spin very fast (1 turn every $0.2\,s$), but the instabilities take several days to grow (for the test bodies), so plenty of time is available for the active control forces to achieve stabilization by counteracting the physical destabilizing forces very accurately, without *overshooting*. Which is very important, because it means that active forces only slightly larger than the destabilizing ones can stabilize the system, no effort being wasted in attempting to damp ill determined relative motions.

Although the GG bodies can be stabilized by small active control forces, it is very important to ensure that the application of these forces does not affect the expected sensitivity of the EP testing experiment. If the stabilizing forces were applied perfectly antiparallel to the actual relative velocity vector of the bodies, they would not change their relative position of equilibrium, hence they would not affect the EP experiment at all. However, there will be a phase error, and therefore also a component of the active control force which will perturb the equilibrium position thus affecting the EP experiment. In our estimates of these perturbations we take $1.5\,deg$ for the phase error and $10^{-6}\,cm$ for the minimal displacement (r.m.s) that can be sensed by the capacitance sensors; note that this figure refers to the small capacitors used for active control, not to those of the read out system, which are located in between the test bodies, are much more sensitive. The largest stabilizing forces are those applied to the PGB laboratory inside which the test bodies for the EP experiment are located. This is due to the lower Q value of the PGB suspension springs (as compared to the springs which suspend the test bodies), as discussed in Sec. 2.2. Although an alternative solution is possible to get a higher value of Q, with helicoidal springs made of separate wires insulated at the clamping, this is not crucial as far as the perturbations from the PGB control forces on the test bodies are concerned. Since the test bodies are suspended inside the PGB laboratory, any force applied to the PGB is the same for both the test bodies, that is, it is a *common mode* force. And since the test bodies are arranged in a balanced suspension in order to reject common mode forces (to 1 part in $50,000$ in the current baseline [1]), this means that only a fraction $1/50,000$ of the PGB control forces will affect the test bodies *differentially* and therefore compete with the expected signal. It can be checked easily that such a perturbation is well below the expected signal. As for the perturbation due to the active forces which stabilize the test bodies themselves, since each one is stabilized with respect to the PGB laboratory (as envisaged above), the perturbation is differential (rather than common mode) and therefore competes directly with the expected signal. With $Q = 16,000$ (the lower value measured for the suspension springs of the test masses), and the errors given above for the capacitance sensors and the phase, this perturbation too is below the expected signal. It is therefore concluded that not only all the GG bodies can be stabilized, but also that such stabilization is compatible with the scientific goal of the mission to test the Equivalence Principle to 1 part in 10^{17}.

In essence, the driving principle of active damping in GG is *not to activate* the dampers until the (small) relative velocity of the bodies has been reconstructed from the noisy measurements of the rotating sensors through appropriate averaging. Otherwise, active forces driven by a poor analysis of the sensors measurements do themselves generate instabilities, instabilities which do *not exist in the real system* and which in turn would require large forces to be damped. The active control of the GG bodies as described in [7] does just that, thus transforming the system into a different one entirely dominated by the active control forces. In contrast, our numerical simulations show that it is possible to implement control laws in which the magnitude of the control forces is of the order of the value derived theoretically (Eq. (6)) for the forces which destabilize the system. This analysis is carried out with the DCAP [8] software package developed at Alenia Spazio and now running also at the University of Pisa.

In addition, numerical simulations confirm the expected linear behaviour of the GG system. This is in agreement with what is reported in the literature on Rotordynamics. From [9] we can quote: *"Most of the components that comprise a rotordynamic system can be quite accurately modeled as linear. The strongly non linear components of the system, if present, are usually localized in the sense that they are directly coupled to only a small number of the system coordinates. Such components include fluid-film bearings, squeeze-film dampers, piecewise linear supports (e.g. deadbands, rubs) working fluid interaction.."*. We can also quote from [10]: *".. The system as a whole may however be strongly non linear because of the behavior of the bearings, in the case of fluid-handling machines, because of the non linear fluid mechanics in the blade passages and in the seals. Both fluid-film bearings and rolling elements bearings severely limit the total relative radial displacement of the rotating journal with respect to the stationary bearing. Nevertheless, the dynamic characteristics of the bearing reactions change dramatically over this small range of permitted displacement so that even very small radial displacements of the rotor introduce nonlinearities"*. GG has neither bearings nor any of the recognized sources of non linearities mentiones above. This means, in particular, that no jumps phenomena or bifurcation of the solutions occur as the system parameters are varied, and that the method of superposition is valid, whereby all excitation sources need not to be considered simultaneously.

4 Conclusions

Attempts at modulating the EP signal at high frequency are almost as old as EP experiments themselves. The GG modulation frequency is by far the highest attempted so far (by several orders of magnitude). The underlying feature is that of weakly (mechanically) coupled test bodies rotating much faster compared to their natural periods of oscillations. The azimuthal symmetry and fast spin reduce thermal effects so that the experiment can be performed at room temperature.

The specific issue of active stabilization of such weakly coupled rotors has been brought up in the ESTEC analysis [7] of GG as a very critical issue: having derived the need for control forces oversized by a factor of about *1 million* this analysis couldn't but conclude that the GG proposed experiment was totally inadequate for testing the Equivalence Principle. This conclusion has been endorsed by the *Fundamental Physics Advisory Group* (FPAG) of ESA (chaired by M. Jacob), which made it the key element of its resolution about GG [11] (points 2,3 and 5 of the resolution). The scientific analysis of this controversial issue carried out by members of the GG Science Team, whereby the GG system can be stabilized with control forces 1 *million* times smaller than in [7], was made available to the FPAG [12, 13]; the opinion expressed by a leading scientist in the field of Rotordynamics was also made available [5].

Theoretical analysis, numerical simulations and experimental measurements have confirmed so far the validity of the GG concept for high accuracy EP testing (1 part in 10^{17} being the target); all efforts are now being focused on the GGG ground test at the laboratory of LABEN in Florence. Once accounted for the smaller driving signal on Earth, and for the different coupling of the test bodies, the target signals to be detected –on the ground and in space– are comparable. As a result, key features of the space experiment can be validated on the ground. Among them, the stabilization of the whirl motions with rotating electrostatic dampers and the rejection of common mode forces by mechanical balancing of the test bodies. We have no doubt that a validation of the GG concept on the ground is going to make the case for a small satellite mission much stronger.

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