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COMMENT

Evaluation of a proposed test of the weak equivalence principle using Earth-orbiting bodies in high-speed co-rotation: re-establishing the physical bases

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Abstract. Test masses coupled by weak mechanical suspensions are sensitive to differential forces such as the force due to a possible violation of the equivalence principle (EP). If in addition they are put in rapid rotation, the differential signal is modulated at high frequency, which is beneficial for noise reduction. Galileo Galilei (GG) is a proposed space experiment for testing the equivalence principle to 1 part in 10^{17} based on these concepts. A recent paper by Jafry and Weinberger (1998 *Class. Quantum Grav.* **15** 481–500) claims that GG can only reach 10^{-14}. We show that the analysis of this paper is flawed (by several orders of magnitude) because of two misconceptions: one on the physical nature of mechanical damping and the other on active control methods for the stabilization of spinning bodies.

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1. Introduction

Paper [1] has been devoted to the Galileo Galilei (GG) space experiment [2–4] addressing the issue of the stabilization of whirl motions that weakly coupled rotors are known to develop because of non-zero dissipation between rotating parts of the system. The conclusion of [1] is that the required stabilizing forces overcome by far the weak passive forces of the mechanical suspensions (springs) on which the GG experiment relies, thus making it inadequate for a very high accuracy equivalence principle (EP) test. We show that [1] is affected by two serious misconceptions which invalidate in full its conclusions: a *misconception on the physical nature of mechanical damping* (section 2) and a *misconception on the active control of spinning bodies* (section 3).

GG is a small satellite project aiming at testing the equivalence principle to 10^{-17} with concentric hollow test cylinders in rapid rotation around their symmetry axes. The test bodies are suspended and coupled by very weak mechanical suspensions; the corresponding

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frequencies of natural oscillations are much smaller than the spin frequency. The experiment is run at room temperature; the spacecraft is spin-axis stabilized and no active attitude control is needed. In the current version, non-gravitational forces acting on the spacecraft surface are largely compensated by field emission electric propulsion (FEEP) ion thrusters needing only a few grams of propellant for the entire mission duration. If the test bodies fall differently in the field of the Earth because of an EP violation their centres of mass will show a relative displacement of pointing to the centre of the Earth, whose amplitude depends on the stiffness of the differential mechanical coupling. Such a mechanical displacement is transformed into an electric potential signal via a capacitance read-out system whose plates are located halfway in the gap between the coaxial test cylinders. Since the plates spin with the system (at $\cong 5$ Hz), the signal is modulated at this frequency. In the original torsion balance experiments by Eötvös the signal was DC. Subsequent experiments with better results (finding no violation to the level of $\cong 10^{-12}$) were based on modulation frequencies at least four to five orders of magnitude smaller than that proposed in GG: 24 h in [5, 6] and 2 h in [7]. At the time of writing, GG is one of six projects selected and funded for a Phase A study by the Italian Space Agency (ASI) [8]. Information on GG is available on the web [9].

2. Misconception on the physical nature of mechanical damping.

GG is constructed of rigid bodies coupled by weak suspensions of high mechanical quality (particularly those of the test bodies) which moreover undergo only minute deformations (a few μm at most). The suspensions are carefully clamped so as to avoid parts sliding against one another, which is the main cause of mechanical losses in the clamps and in the whole system. There are no bearings, since, after spin up is completed, there is no need of a motor. There are no viscous materials: no fluids, no oils, no greases. Therefore, the main loss factors (inverse of the quality factor Q) are those due to the very small internal dissipation of the mechanical suspensions as they undergo minute deformations at the spin frequency. The only remaining cause of dissipation is the electrostatic sensors/actuators used to damp the whirl motions, since all other parts are rigid and have no losses. Calculation of thermal noise in the active dampers shows that the corresponding losses are negligible by far compared to those achievable with mechanical suspensions [4, 10] (assuming all parameters as for the GG experiment and a very conservative value of 10 for the electric quality factor). Crandall [11] has calculated (using [12]) the back-reaction force on the plates of the capacitors from the high-frequency measurement voltage, finding that the electrical contributions to the mechanical stiffness and damping are negligible. Losses in the dampers will be measured with the GG prototype on the ground after active rotating control, similar to the GG active control, has been implemented ([13], chapter 3).

A firm estimate of the losses in the GG mechanical suspensions requires them to be measured experimentally, by setting the springs in oscillation under realistic operating conditions (oscillation frequency, vacuum, temperature, clamping); note that there is no need to perform this measurement with the system rotating [14].

In order to measure, for a given mechanical system, the quality factor Q (defined as the ratio of the total energy stored in the system to the amount of energy dissipated in one cycle) the system is made to oscillate and then the oscillation amplitude $A(t)$ is recorded as it decays with time. Q can also be defined as follows:

$$A(t) = A(0)\,\mathrm{e}^{-\omega t/2Q} \tag{1}$$

where ω is the frequency of the oscillation and $A(0)$ is its amplitude at the initial time. Hence,

$$Q = \frac{\omega(t_2 - t_1)}{2\ln(A_1/A_2)} \tag{2}$$

which yields the value of Q from measurements of A_1, A_2 at times t_1, t_2. Consider a helical spring with its (unavoidable) clamping and the attached mass necessary to obtain the oscillation frequency of interest. Horizontal oscillations avoid pendulum-like motion due to local gravity which would yield a higher Q because gravity contributes to the total energy but not to the dissipation. In vacuum ($\cong 10^{-5}$ torr) at room temperature and for oscillation frequencies from 2 to 10 Hz, the measured Q values of the prototype springs manufactured for the suspension of the GG test masses were between 16 000 and 19 000. Oscillations were excited with a small electromagnet and their amplitudes were measured optically [9, 15]. Although further improvement is possible, these values are quite good because of how the suspensions are made: they are helical springs carved out of a single piece of material (Cu–Be) by electroerosion in three dimensions, followed by an appropriate thermal treatment. The Q measurement procedure (by recording the decaying oscillation amplitude) is a standard one, which obviously does not require the system to be taken into space, even though in this case it is designed for use in space; and since measurements are made for the springs as designed for space, no scaling is necessary either.

Energy is dissipated because of different types of losses (structural or viscous, in the spring material as it undergoes deformations, because of imperfect clamping or because of resistance of residual air) and the oscillation amplitude decay is due to *all* of them. Consequently, the measured Q is the Q of the whole system and gives a quantitative measurement of *all* losses in it: *whatever their physical nature*. Once dissipation has been measured experimentally, model-dependent estimates of it are no longer needed and, in any case, should be consistent with experimental results. In contrast, speculations in [1] (appendix) that dissipation in the GG system should be amplified by a factor ω_s/ω_n (the ratio of the spin-to-natural frequency; $\cong 10^3$ in GG) over the measured value are proven to be wrong by experimental measurements.

The dissipation discussed above—in the springs and their clamping as they are deformed at the frequency of spin, referred to as 'rotating damping'—is known to give rise to unstable whirl motion at the natural frequency ω_n with respect to the non-rotating frame. If Q quantifies all losses at the spin frequency, the fractional variation of the radius of whirl r_w in one natural period of oscillation $T_n = 2\pi/\omega_n$ is

$$\frac{(\Delta r_w)_{T_n}}{r_w} \cong \frac{\pi}{Q}. \tag{3}$$

In GG the ratio ω_s/ω_n is 630 for the test masses and 1600 for the pico-gravity box (PGB) suspended laboratory inside which the test masses are suspended in turn. The time scales for doubling the whirl radius are 2.5 weeks for the test masses (with $Q = 16\,000$) and 2.5 h for the PGB (with $Q = 90$), that is the whirl motions grow very slowly, which makes it easier to keep them under control and to damp them.

The forces required to damp the slow whirl motions are (in modulus) slightly larger than the destabilizing forces which give rise to the whirl, whose value is known to be smaller than the passive spring forces by a factor of $1/Q$ (see, for example, [16], equation (35)), where Q quantifies *all* losses at the frequency of spin and must be measured as discussed above. Hence, the required stabilizing force, anti-parallel to the slow velocity of whirl $\vec{v}$, is (slightly larger than)

$$\vec{F}_{stab} = -\frac{1}{Q} m\omega_n \vec{v} \tag{4}$$

where m is the reduced mass of the system. Because of the misconception on the nature of damping, [1] erroneously gives the time scales of whirl motion to be a factor of ω_s/ω_n shorter (5.6 s for the PGB and 0.7 h for the test masses) and the forces (4) a factor of ω_s/ω_n larger; and hence also the effects of imperfections and errors in these forces are amplified by the same factor. In the GG prototype experiment the required stabilizing force (for test bodies of 10 kg each at spin frequencies from 2 to 10 Hz) is provided by a very light disc (0.5 g only) on a Teflon surface ([13], figure 3.8).

3. Misconception on active control methods for the stabilization of spinning bodies

The GG bodies are stabilized actively, by means of small capacitance sensors/actuators rotating with the system at a velocity $\cong 10^3$ times higher than the velocity of whirl they are required to damp. In order to recover and damp this slow (and slowly growing) velocity with much more rapidly rotating sensors/actuators it is necessary to develop a control strategy [13, 15, 17] in which:

(a) the relative velocity of the bodies is computed from differences of measurements taken by the rotating displacement sensors one spin period apart;
(b) the relative velocity is averaged over several spin periods ($\cong$10);
(c) the relative velocity data are best-fitted to a vector rotating at the known angular frequency of whirl.

A reference signal at the spin frequency ($\cong$ 5Hz) is constructed continuously (so as to avoid accumulation of errors; averaging over a few minutes) from the output of commercial Earth elevation sensors which measure the angular phase (and hence also the spin rate) of the spacecraft. Note that for a time interval as short as the whirl period the rotation of the system (whose spin energy is very large) can be regarded as constant. Instead, in [1] the relative velocity is computed by taking differences of successive measurements from the sensors and without making use of the reference signal. In this way they fail to recover the correct value of this slow relative velocity, since it is overwhelmed by the much larger velocity of spin of the sensors themselves (by a factor of $\omega_s/\omega_n \cong 10^3$). Consequently, their control forces are also a factor of ω_s/ω_n larger than in (4). This is strikingly apparent already in the simple case of the two-body system made of the GG outer spacecraft and the PGB when the two control strategies are compared (figure 1).

Since the control laws used in [1] fail so completely already in the simpler two-body model, they are certainly useless for the scope claimed in the paper, that is to evaluate the sensitivity achievable in EP testing by the full six-body GG system.

How the full GG system (four bodies plus two small coupling arms) is stabilized by controlling all whirl motions at the same time is shown in figure 2. The resulting relative distance between the test bodies is shown in figure 3, while figure 4 gives the intensity of both the passive elastic force and the control force.

Figures 2–4 refer to planar simulations; simulations in three dimensions have been carried out ([13], chapter 6) showing that the dynamical behaviour is not affected by the increased number of degrees of freedom; however, the required computing time increases significantly.

As for the effect of drag (and of solar radiation pressure), it is huge compared to the expected signal; however, it is transferred to the test masses as an inertial acceleration in common mode by its nature, while an EP violation signal would be differential. This is why the GG test cylinders are arranged in a coupled suspension similarly to an ordinary beam balance (except for the fact that the beam is vertical rather than horizontal): by adjusting the length of the arms with piezoelectric actuators common mode forces can be rejected, leaving only a much smaller differential effect to compete with the signal. In physics experiments this is known as *common mode rejection*; the attainable level of rejection depends on the specific system and mechanism for rejection. With the prototype of GG in the laboratory we have achieved a rejection level of 1 part in 200 000, which is better than the current requirement for the GG experiment in space [13], where we assume that drag is partially compensated (by drag-free control with FEEP mini-thrusters) and partially rejected. Drag could also be totally rejected (no compensation) [4] provided the rejection level is improved accordingly. The drag-free control of GG is based on a notch filter at the orbital frequency ([13], chapter 6); it has also been tested in combination with whirl control for the full six-body GG system in three dimensions. No additional difficulties are encountered in the six-body case as compared to the two-body model, but the computing time required by the simulations is much greater.

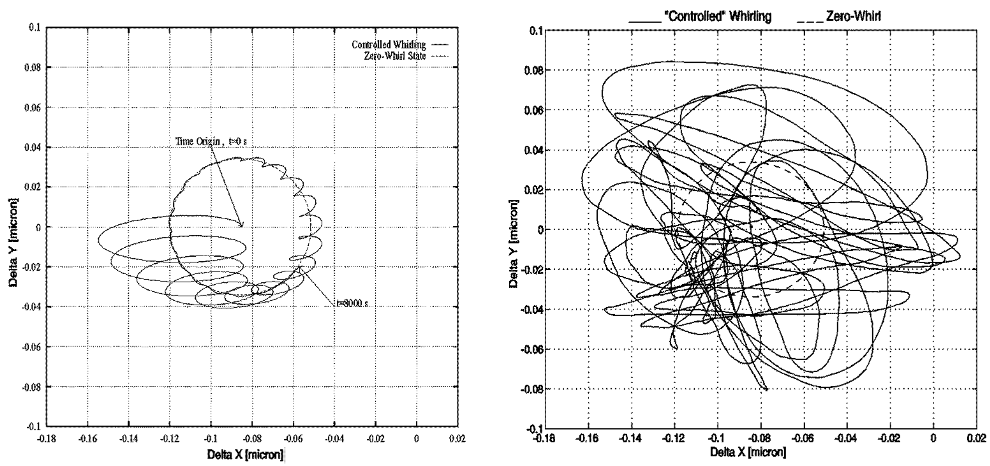

Figure 1. Trajectory of the relative motion of the centres of mass of the GG outer spacecraft and the PGB in the plane perpendicular to the spin axis in a two-body model (coupling constant 0.02 N m^{-1}, $Q = 90$). The Y-axis is pointed to the centre of the Earth, hence the largest effect of the residual atmospheric drag, assumed to be 5×10^{-9} N, is a constant displacement along the X-axis (of $\cong 0.08$ μm); its second harmonic (assumed to be 40% of it) appears in this system as a variation at the orbital period (5700 s). This is the broken circle, showing (in both plots) the stationary state that the system would reach if the whirl motion were perfectly damped. The plot on the left is obtained with the control laws of the GG Team assuming the following errors: initial bias of 10 μm linear and 1° angular; fractional error in spin rate measurements $\Delta\omega_s/\omega_s = 10^{-4}$; offset (by construction and mounting) of 10 μm; errors in the capacitors of 0.1 μm RMS. Whirl oscillations with the natural period of 314 s (around the points of the broken circle) and of decreasing amplitude are apparent as the system is brought to its stationary state in 8000 s only. Note that at this point the relative distance of the two centres of mass is below 5 Å. These results have been obtained independently using a DCAP software package (of Alenia Spazio) and Matlab. The plot on the right shows, for the same system, but under much more ideal assumptions (perfect knowledge of spin rate; perfect centring of the rotor; an initial linear bias of 1 μm and no angular bias; an error in the sensors/actuators 10 times smaller, i.e. of 10^{-2} μm) the results obtained by applying the control laws proposed in paper [1]. It is apparent that even in a much more favourable situation the same system has been unwittingly transformed into one dominated by very large active forces for which there is in fact no need, as the plot on the left demonstrates. Note that the dissipation has been assumed to be the same in both cases ($Q = 90$), hence failure to stabilize the whirl motion (right-hand plot) has to be ascribed only to the control laws implemented in that case. Regarding the plot on the left, note that the assumptions for the various error sources are conservative. For instance, small capacitors like those designed for GG can be shown in the laboratory to be sensitive to relative displacements of 10^{-2} μm.

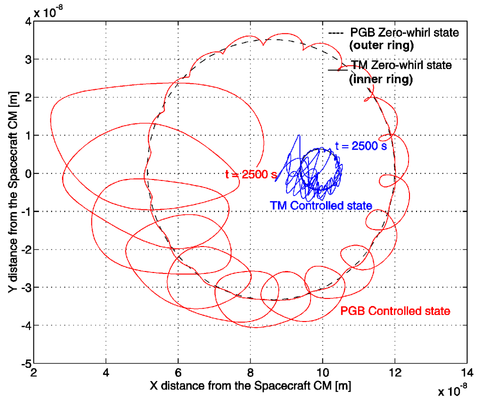

Figure 2. Evolution of the full six-body GG system: outer spacecraft, PGB, two test masses with two gimballed coupling arms. These arms are pencil-like in shape and have no rings. As individual bodies they would be unstable; in GG they couple two much more massive test bodies which are individually stabilized by whirl control, hence, the arms are also found to be automatically stabilized with no need to add rings around their midpoints. Only the trajectories of the PGB and of one test mass are plotted (for 9500 s after the first 2.500 s) showing their distance from the centre of mass of the spacecraft $(0, 0)$. The plane of the figure is perpendicular to the spin axis and the Y-axis is pointed to the centre of the Earth. The residual drag acting on the spacecraft has a DC component equal to 5×10^{-9} N (giving rise to a constant X displacement in this plot) plus an orbital frequency term which is 40% of the DC component (giving rise to the broken circles) and a 10% noise on both components. Whirl motions appear as oscillations at the natural periods around the points of the corresponding broken circles: if active control is effective their amplitude must decrease. This is indeed what happens. Here we have assumed Q values of 90 for the PGB and 500 for the test masses (a very conservative assumption for the test masses, since the measured Q for their suspensions is of 16 000–19 000). The errors included were: 10^{-2} μm RMS (tested in the laboratory for capacitance plates of $\cong 2$ cm^2 as in the GG active dampers), 10 μm linear bias, 1° angular bias for the capacitance sensors; $\Delta\omega_s/\omega_s = 10^{-4}$ RMS for the Earth elevation sensors (this is possible with EES by 'Officine Galileo', Firenze); 1 μm initial offset of the suspension springs.

(This figure can be viewed in colour in the electronic version of the article; see `http://www.iop.org`)

4. Conclusions

We have shown that paper [1] overestimates the required stabilizing forces of the GG system by a factor $\omega_s/\omega_n \cong 10^3$ because of a misconception about the physical nature of mechanical damping. In addition, it overestimates the active control forces to be applied by rotating sensors/actuators by another factor of ω_s/ω_n because of a misconception on the control laws of spinning bodies. Overall this amounts to an error by a factor $\simeq 10^6$. This invalidates in full the evaluation of GG as carried out in [1], according to which GG could only reach a sensitivity in EP testing of 1 part in 10^{14}. Paper [1] is the final version of a precursor technical report [18] prepared by the same authors in support of the Fundamental Physics Advisory Group (FPAG) of ESA for its evaluation of GG [19]. Therefore, we can also answer a few questions raised in [19]. In particular, (a) [19] states that 'The high spin rate is not an advantage for the experiment. The advantages conveyed by spin (suppressing the effects of low-frequency noise) are outweighed by the disadvantages of having unstable modes around the signal frequency.' Instead, unstable modes can be stabilized and they are so slow that scientific data acquisition can take place while whirl control is off, hence the advantages of high spin rate can be fully exploited; (b) [19] states that 'The servo forces will dominate the

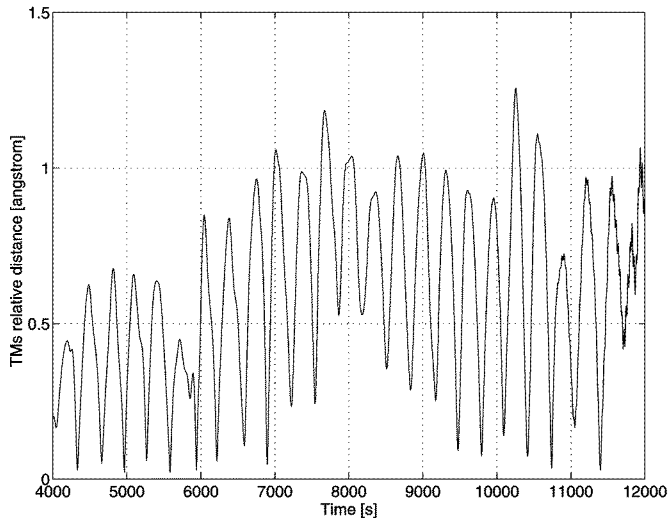

Figure 3. Numerical simulation of the same six-body GG system as in figure 2, with the same residual drag and the same error sources. Here we plot the relative displacement between the inner and the outer test mass (differential displacement) as obtained after applying active control of their whirl motions. Note that in this simulation whirl control is always on, i.e. this is a worst-case simulation because whirl control can, in fact, be switched off during scientific data acquisition. This result is impressive in that it shows how active control by means of electrostatic sensors/actuators can be so accurate as to make the GG macroscopic test bodies self-centre on one another as expected in supercritical rotation in the absence of dissipation (infinite mechanical quality factor, zero whirl).

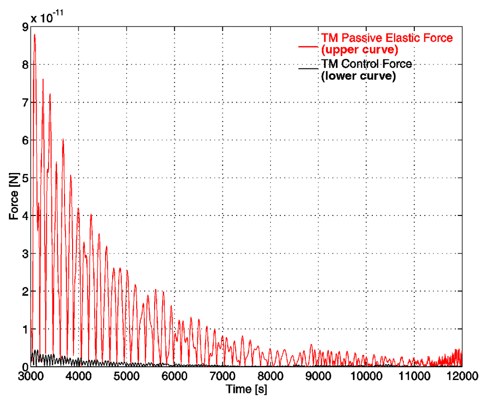

Figure 4. Numerical simulation of the six-body GG system with the same error sources as in figures 2 and 3. Here we plot the passive elastic force of the suspension springs (upper curve) for comparison with the control force acting on the outer test mass (lower curve). The control force is clearly much smaller than the elastic force. We recall that, in order to speed up the simulation, the quality factor of the test bodies suspension springs was taken to be 500 (four times worse than measured); in addition, the system was controlled with a force 11 times larger than the minimum required theoretically by (4). In point of fact, we have also run experiments in which a control force only 2.5 times larger than the minimum could stabilize the system.

(This figure can be viewed in colour in the electronic version of the article; see `http://www.iop.org`) passive spring forces.' Instead, we have implemented control forces which are smaller by far than the elastic forces and yet can stabilize the whirl motions (figures 2–4); (c) [19] states that the gimballed rods (the coupling arms) '. . . appear to be highly unstable in high-speed rotation and are a source of significant perturbations.' Instead, numerical simulations of the full GG system show that this is not the case (see further details in [13], chapter 6), confirming the physical guess made by the GG Team before a full simulation could be carried out; (d) [19] states that 'The control forces have to mimic damping forces in the non-rotating frame but must be synthesized from measurement in the rotating frame. Imperfections in the sensors and actuators will cause significant disturbances in the differential mode.' The first statement is true, but the second one has been found to be incorrect if control forces are properly computed and applied; which is not the case in [1, 18]. Another issue raised in [19], that of the usefulness of the PGB laboratory, has not been touched on here; it has been answered in [15] and [13], figure 2.6.

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