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Radiometer effect in space missions to test the equivalence principle

A. M. Nobili, D. Bramanti, G. Comandi, and R. Toncelli
Space Mechanics Group, Department of Mathematics, University of Pisa, I-56127 Pisa, Italy

E. Polacco
Department of Physics, University of Pisa, and INFN, I-56127 Pisa, Italy

G. Catastini
Alenia Spazio, Department of Advanced Studies, I-10146 Turin, Italy

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Experiments to test the equivalence principle in space by testing the universality of free fall in the gravitational field of the Earth have to take into account the radiometer effect, caused by temperature differences in the residual gas inside the spacecraft as it is exposed to the infrared radiation from Earth itself. We report the results of our evaluation of this effect for the three proposed experiments currently under investigation by space agencies: μSCOPE, STEP, and GG. It is found that in μSCOPE, which operates at room temperature, and even in STEP, where the effect is greatly reduced by means of very low temperatures, the radiometer effect is a serious limitation to the achievable sensitivity. Instead, by axially spinning the whole spacecraft and with an appropriate choice of the sensitivity axes—as proposed in GG—the radiometer effect averages out and becomes unimportant even at room temperature.

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The equivalence principle (EP) stated by Galileo, reformulated by Newton and reexamined by Einstein to become the founding principle of general relativity, has attracted the attention of space agencies around the world. NASA, ASI, and CNES (in the US, Italy, and France) are considering space missions for EP testing; the European Space Agency (ESA) is contributing to the American and the French projects. All missions will fly experiments to test the most direct consequence of the EP: the universality of free fall whereby all bodies fall with the same acceleration regardless of their mass and composition. The most accurate EP experiments on the ground have been carried out with test bodies suspended on a torsion balance [1–3] finding no violation to about 10^{-13}. Test bodies in low Earth orbit are subject to a driving acceleration much stronger than on torsion balances on the ground, by about 3 orders of magnitude. Moreover, the absence of weight is ideal in small force experiments. As a consequence, space missions can potentially improve by several orders of magnitude the current sensitivity in EP tests. The goals are 10^{-15} for the French μSCOPE [4], 10^{-17} for the Italian ‘‘GALILEO GALILEI’’ (GG) [5–8], 10^{-18} for the American STEP [9–12]. All disturbances which have the same frequency and phase as the signal of an EP violation must be smaller than the expected signal for the experiment to achieve its goal.

In all the proposed space experiments the test bodies are concentric, hollow test cylinders especially sensitive to differential accelerations acting between them. In the case of STEP and μSCOPE the system is sensitive along the symmetry axis of the test cylinders, and their orientation is fixed with respect to inertial space while orbiting around the Earth as shown in Fig. 1. This must be achieved by active control of the spacecraft which carries the experimental apparatus. An EP violation in the field of the Earth would generate a signal whose amplitude is modulated at the orbital frequency of the satellite. During some selected time intervals the spacecraft can also be slowly rotated (with a rotation period of about 10^3 s) so as to rotate the sensitivity axis of the test cylinders with respect to the Earth, thus modulating an EP violation signal at the rotation frequency of the spacecraft.

In the case of GG, the concentric test cylinders spin around the symmetry axis at a rather high frequency (2 Hz with respect to the center of the Earth) and are sensitive to differential effects in the plane perpendicular to the spin/symmetry axis. A cylindrical spacecraft encloses, in a nested configuration, a cylindrical cage and the test bodies inside it;

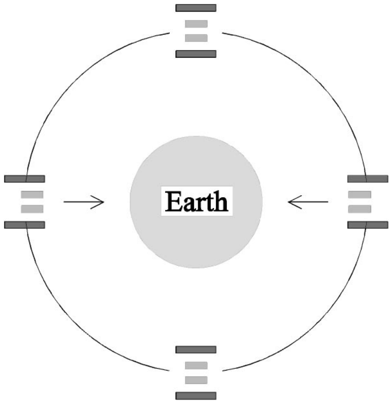

FIG. 1. Section through the symmetry/sensitivity axis, of the STEP and μSCOPE outer and inner test cylinders (of different composition) as they orbit around the Earth inside a space-fixed, actively controlled spacecraft (not shown). The arrows indicate how the system is sensitive at the orbital frequency to a violation of equivalence in the field of the Earth. During the time intervals in which the spacecraft undergoes slow, controlled rotation (at 10^{-3} Hz) in the plane of the figure the signal is modulated at the rotation rate, to help distinguish it from spurious effects at the orbit frequency (figure not to scale).

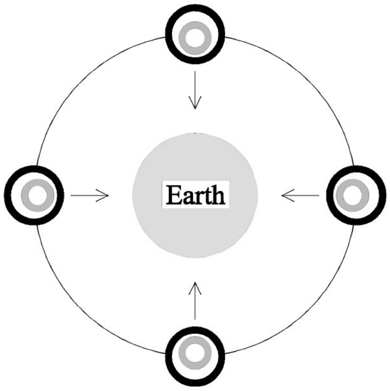

FIG. 2. Section across the spin/symmetry axis of the GG outer and inner test cylinders (of different composition) as they orbit around the Earth inside a co-rotating, passively stabilized spacecraft (not shown). The centers of mass of the test cylinders are shown to be displaced towards the center of the Earth as in the case of a violation of equivalence in the field of the Earth (indicated by the arrows). The signal is modulated at the spin frequency of the system (2 Hz with respect to the center of the Earth) (figure not to scale).

the whole system has a dominant moment of inertia with respect to the symmetry axis and is (passively) stabilized by 1-axis rotation around it. As shown in Fig. 2, an EP violation in the field of the Earth would generate a signal of constant amplitude (for zero orbital eccentricity) whose direction is always pointing to the center of the Earth, hence changing orientation with the orbital period of the satellite. The readout, also rotating with the system, will therefore modulate an EP violation signal at its spin frequency.

In all cases there will be a residual gas around the test cylinders and, were the satellite exposed to a *radiation source* from a given direction, there will also be nonzero temperature gradients inside the satellite and the experimental chamber. These two factors together give rise to the *radiometer effect*.

In order to evaluate how relevant is the radiometer effect for the STEP and μSCOPE experiments we must compute it along the symmetry axis of the test cylinders. The radiometer acceleration along the symmetry axis s of a cylinder of density ρ is given by

$$a_s=\frac{p}{2\rho}\frac{1}{T}\frac{dT}{ds} \tag{1}$$

where p is the pressure of the residual gas and T its temperature. In EP space experiments the test cylinders have a different composition (and typically also different dimensions); they are therefore subject to a different radiometer acceleration. If in addition the radiation source that the satellite is exposed to is the Earth, the resulting radiometer acceleration on the test cylinders will have the same frequency and phase of a possible violation of equivalence having the Earth as the source mass, thus making it indistinguishable from the expected signal. The additional rotation of the STEP and μSCOPE spacecraft, whose purpose is to modulate an EP violation signal at a frequency different from (and larger than) the orbital frequency around the Earth, will modulate the radiometer differential acceleration as well. Equation (1) is derived (by differentiation) from the equation of stationary equilibrium for the gas surrounding the test cylinders [13]:

$$\frac{p}{\sqrt{T}}=\text{const.}$$

In the STEP experiment the residual pressure is extremely low: $p\cong10^{-13}$ torr (1.33×10^{-10} dyn/cm^2) because the apparatus is maintained at 1.8 K inside a Dewar filled with superfluid He (enclosed by another He Dewar at 4.2 K). An EP violation to the level of 1 part in 10^{18} (the target of this mission) would give a differential acceleration $(a_{\rm EP})_{\rm STEP}\cong8.4\times10^{-16}$ cm/s^2; the radiometer effect resulting from (1) must be below this value. The test body of lower density (Be, with $\rho_{Be}=1.85$ g/cm^3) is subject to a larger radiometer effect and will dominate in the differential one because the other body is much denser. For this to be smaller than the target signal of EP violation, the condition on temperature gradients across the Be test mass of the STEP experiment (10–15 cm size) is

$$\left(\frac{dT}{ds}\right)_{\rm STEP}<4.2\times10^{-5}\ \text{K/cm} \tag{2}$$

which must be ensured over the period of the signal (i.e., the orbital period around the Earth) or—in case of signal modulation—over the rotation period of the spacecraft. The superfluid He Dewar, besides ensuring extremely low pressure, is pivotal in reducing temperature gradients so as to meet the requirement given by (2). However, this concerns the test bodies only. It does not prevent the spacecraft outer shell from expanding and contracting in response to the infrared radiation from the Earth while orbiting around it; the spacecraft mass so displaced will act on the test bodies giving rise to a differential acceleration (because the test cylinders cannot be exact monopoles) with the same frequency and phase as an EP violation. In addition, the He mass itself will respond with zero friction to the varying tidal potential of the Earth, with the result of moving around the test bodies at exactly the orbital frequency [10,11]. It is not an easy task to stop completely this motion inside a large Dewar which contains a few hundred liters of He.

In the case of μSCOPE the experiment is run at room temperature, the satellite is designed to fly at higher altitude than STEP (600–700 km) and the target for EP violation is 10^{-15}, corresponding to $(a_{\rm EP})_{\mu\rm SCOPE}\cong8.2\times10^{-13}$ cm/s^2. The less dense test cylinder is made of Ti ($\rho_{\rm Ti}=4.5$ g/cm^3), about 8 cm long. The residual gas pressure as reported for a predecessor of the current instrument (STAR [14]) is of 3.7×10^{-6} torr. A pressure of 7.5×10^{-9} torr is foreseen for the Laser Interferometer Space Antenna (LISA) mission [15,16], far ahead in the future. Assuming that a value of 10^{-8} torr is achieved with μSCOPE, the condition on temperature gradients analog to (2) (but referring to an EP test 3 orders of magnitude less sensitive) is

$$\left(\frac{dT}{ds}\right)_{\mu\mathrm{SCOPE}} < 1.7\times 10^{-4}\ \ \mathrm{K/cm}. \tag{3}$$

According to [14], temperature differences in the STAR accelerometer reach 0.5 K, thus exceeding the limit (3) by several orders of magnitude. An instrument of the same family, but much more demanding, has been proposed for the LISA gravity wave mission. It is named CAESAR and aims to achieve a noise level not exceeding 3×10^{-13} cm s$^{-2}/\sqrt{\mathrm{Hz}}$ between 10^{-4} Hz and a few 10^{-3} Hz (see [15], Chap. 6). At 10^{-3} Hz this is about two orders of magnitude smaller than the target signal of μSCOPE. Although CAESAR would benefit from the very good thermal stability expected in LISA ($10^{-6}\mathrm{K}/\sqrt{\mathrm{Hz}}$), the authors quote a spatial temperature difference of 0.01 K (with a very dense proof mass of Au-Pt) [17]. It therefore seems quite difficult for a room temperature mission such as μSCOPE to reach its goal of testing the equivalence principle to one part in 10^{15}.

In GG the radiometer effect must be computed in the plane perpendicular to the symmetry axis of the test cylinders (Fig. 2). Any temperature gradient in this plane in the direction of the Earth will result in a radiometer effect indistinguishable from the signal. However, azimuthal temperature asymmetries on the surface of the spacecraft itself are reduced by its 1-axis rotation according to the formula

$$\Delta T_{sc} = \frac{\alpha\Phi\ \mathrm{hr}\, P}{\mathrm{cm}/2},$$

yielding a temperature difference ΔT_{sc} of a few mK for a spinning spacecraft exposed to the infrared radiation flux of the Earth ($\Phi \cong 2.4\times 10^{5}\ \ \mathrm{erg\,cm^{-2}\ \ s^{-1}}$) and resembling the outermost shell of GG [radius $r=0.5$ m, height $h=1.3$ m, mass $m=33$ kg, specific heat $c=0.9\times 10^{7}\ \ \mathrm{erg\,g^{-1}\,K^{-1}}$ ($0.2\ \ \mathrm{cal\,g^{-1}\,K^{-1}}$), absorption coefficient $\alpha=0.73$, spin period $P=0.5$ s]. A thin (0.2 cm) insulating layer (Mylar or Kapton) inside the spacecraft shell, whose timescale of thermal inertia is $\tau=40$ s, will further reduce the temperature difference to $\Delta T_{\mathrm{Mylar}}$ of the order of 10 μK:

$$\Delta T_{\mathrm{Mylar}} = \frac{\Delta T_{sc} P}{2\tau}.$$

Finally, vacuum between this layer and the cage enclosing the test cylinders (made in Cu and coated with Mylar) does ensure radiative transfer of heat and consequent very effective reduction of temperature differences in the direction of the Earth, to ΔT_{cage} of the order of 10^{-10} K (comparable to quantum fluctuations over half the spin period) according to

$$\Delta T_{\mathrm{cage}} = \frac{\sigma S_{\mathrm{cage}} T^{3} \epsilon_{\mathrm{Mylar}} P}{c_{\mathrm{cage}} M_{\mathrm{cage}}} \Delta T_{\mathrm{Mylar}}$$

where σ is the Stefan-Boltzmann constant, $S_{\mathrm{cage}}=1\ \ \mathrm{m}^{2}$, $M_{\mathrm{cage}}=40$ kg, $c_{\mathrm{cage}}=3.9\times 10^{6}\ \ \mathrm{erg\,g^{-1}\,K^{-1}}$ are the mass, surface, and specific heat of the cage and $\epsilon_{\mathrm{Mylar}}=0.05$ is the emissivity of Mylar.

It is apparent that temperature differences as small as these make the radiometer effect totally negligible in the GG mission. The result holds at room temperature and shows no need for a cryogenic experiment. The radiometer effect is not a limiting factor in EP testing for this mission. Thermal noise is higher at higher temperature, but since it is proportional to $\sqrt{(T/M)}$ (with M the mass of a test body), more massive test cylinders (10 kg in GG, about 100 g in STEP) compensate for the higher temperature.

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