1 Scientific relevance of the Equivalence Principle

It is possible to ascribe two conceptually different kinds of masses to a body: an *inertial mass* and a *gravitational mass*. The inertial mass is the proportionality factor between a force (any kind of force) applied to the body and the acceleration it acquires in response to it in an inertial laboratory. The gravitational mass is a measurement of the property of the body to attract gravitationally any other body (gravitational *active* mass), or to be gravitationally attracted by any other body (gravitational *passive* mass). Assuming the validity of the action–reaction principle (which leads to conclude that the center of mass of an isolated system must move with constant velocity in an inertial frame of reference) also implies that the gravitational passive and active mass of a body must be the same. Since both concepts refer to the same physical interaction, this result appears to be quite natural. The gravitational mass is the analog in a gravitational field, of the electric charge in an electric field –it can be viewed as a *gravitational charge*– while it has no apparent relation (in spite of the name) with the concept of inertial mass.

Using Newton's law of gravitation to write the equation of motion of a body of inertial mass m_i and gravitational mass (both active and passive) m_g in the field of a source body of gravitational mass M_g (for instance, the Earth), we have:

$$m_i \ddot{\vec{r}} = -\frac{GM_g m_g}{r^3} \cdot \vec{r} \qquad (1)$$

where $\ddot{\vec{r}}$ is the relative position vector between the centers of mass of the bodies and G the universal constant of gravity. If $m_i \propto m_g$ *(Equivalence of inertial and gravitational mass),* then the acceleration $\ddot{\vec{r}}$ is the same for all bodies. With the measured value of G and a proportionality factor of 1 $(m_i = m_g)$, the local acceleration of gravity on the surface of the Earth –the same for all bodies regardless of their mass and composition– amounts to about 980 $cm\ s^{-2}$. This is the so called *Universality of Free Fall* (UFF*).* No such thing holds for all other fundamental forces of Nature. For instance, a proton and an electron do not have –in the same electric field– the same (in modulus) acceleration, because the inertial mass of the proton is much bigger than the inertial mass of the electron and no proportionality holds between the inertial mass of a body and its *electric charge,* as it does between the inertial mass and the *gravitational charge*

Galileo was most probably the first one to provide experimental evidence for the UFF (see Sec. 3). However, he was not aware of the law which rules the gravitational interaction and which leads to writing Eq. (1). Therefore, he had no awareness of the equivalence between inertial and gravitational mass, and of the link between this concept and his own experimental results on the UFF.

The fact that the two concepts of inertial and gravitational mass refer in fact to the same physical quantity was first stated by Newton in the opening paragraph of the *Principia* [1]:

> "*This quantity that I mean hereafter under the name of ... mass ... is known by the weight ... for it is proportional to the weight as I have found by experiments on pendulums, very accurately made...* "

At the beginning of the 20th century, almost 300 years since Galileo's work, Einstein realized that because of the proportionality between the gravitational mass m_g and the inertial mass m_i, the effect of gravitation is locally equivalent to the effect of an accelerated frame and can be locally canceled. This is known as the *Weak Equivalence Principle* which Einstein introduced in 1907 [2] as the *hypothesis of complete physical equivalence* between a gravitational field and an accelerated reference frame: in a freely falling system all masses fall equally fast, hence gravitational acceleration has no *local* dynamical effects. Any test mass located inside the famous Einstein elevator –falling with the local acceleration of gravity *g*– and zero initial velocity with respect to it, remains motionless for the time of fall. An observer inside Einstein elevator will not be able to tell, before hitting the ground, whether he is moving with an acceleration *g* in empty space, far away from all masses, or else he is falling in the vicinity of a body (the Earth) whose local gravitational acceleration is also *g* (and in the same direction).

However, Einstein's formulation of the *Weak Equivalence Principle* whereby the effect of gravity disappears in a freely falling reference frame, holds only *locally.* The elevator is free falling *in the vicinity* of the Earth, which amounts to saying that $h<<R_\oplus$, namely, that the height of fall is much smaller than the radius of the Earth. The cancellation of gravity in a freely falling frame holds *locally* for each frame, but the direction of free fall is not the same in all of them. Which is the direct consequence of the fact that the gravitational field of a body (like Earth) is not uniform, giving rise to the so called *tidal forces* between test particles whose centers of mass are not coincident (and also inside a body of non zero dimensions). With the *Weak Equivalence Principle* Einstein has moved from Newton's concept of one global reference frame with gravitational forces and the UFF, to many free falling *local* frames *without* gravitational forces.

A step further in the development of General Relativity leads Einstein to a generalization of the *Weak Equivalence Principle,* known as the *Strong Equivalence Principle,* stating that in an electromagnetically shielded laboratory, freely falling and non rotating, the laws of physics –including their numerical content– are independent of the location of the laboratory. In such a laboratory all particles free of non gravitational forces have no relative accelerations. According to General Relativity and the *Strong Equivalence Principle* (which assumes the *Weak* one), all gravitational effects are replaced by the metric of a curved, 4-dimension space-time. In this sense the Equivalence Principle expresses the very essence of General Relativity and as such it deserves to be tested as accurately as possible.

The *Weak Equivalence Principle*, although obviously not known to Galileo, also leads –as in the case of Newton's *Equivalence of inertial and gravitational mass*– to the UFF*:* should bodies of different composition fall with different accelerations, the elevator and the test mass inside it would generally fall with different accelerations and the observer would be able to tell that he is close to the surface of the Earth and not in an accelerated frame in empty space.

In the last 30 years since the advent of the space age General Relativity has been subject to extensive experimental testing as never before in its first 50 years of existence, and so far it has come out having no real competitors; the crucial area where experimental gravitation is likely to play an important rôle is in the verification of the universality of free fall as a test of the weak equivalence principle itself, since it is tantamount to testing whether gravitation can be ascribed to a metric structure of space-time.

The adimensional parameter which quantifies a deviation from the UFF (hence, also a violation of Equivalence) for test bodies of different composition, *A* and *B*, inertial mass m_i and gravitational mass m_g , is the so-called Eötvös parameter η:

$$\eta = \frac{2[(m_g / m_i)_A - (m_g / m_i)_B]}{[(m_g / m_i)_A + (m_g / m_i)_B]} \tag{2}$$

The finding of a value $\eta \neq 0$ would disprove the UFF and indicate a violation of the Weak Equivalence Principle on which General Relativity ultimately relies. Instead, $\eta = 0$ –as reported by all experiments so far– confirms the basic assumption of General Relativity and has additional profound significance.

The total mass–energy of a body can be expressed as the sum of many terms, corresponding to the energy of all the conceivable interactions and components: $m = \Sigma_k m_k$. For instance, at the atomic level, the rest mass contributes (as a fraction of the total) for ≅ 1; the nuclear binding energy for $8{\cdot}10^{-3}$ (for light elements), the mass difference between neutron and proton for $1.4{\cdot}10^{-3}$ $(A\text{-}Z)/A$ (*A* being the number of protons plus neutrons and *Z* the number of protons in the nucleus), the electrostatic energy of repulsion in the nuclei for $6{\cdot}10^{-4} Z^2 A^{-4/3}$, the mass of electrons for $5{\cdot}10^{-4} Z/A$, the antiparticles for $\cong 10^{-7}$, the weak interactions responsible of β decay for 10^{-9} or less. For an extended spherical body of radius *R* and (homogeneous) density ρ, the gravitational self–energy contributes by $-4/5\pi\rho G R^2/c^2$. The conventional Eötvös parameter (2) can therefore be generalized into:

$$\eta_k = \frac{2[(m_g / m_i)_{A_k} - (m_g / m_i)_{B_k}]}{[(m_g / m_i)_{A_k} + (m_g / m_i)_{B_k}]} \tag{3}$$

such that a non-zero value of η_k would define the violation of equivalence between the inertial and gravitational mass-energy of the *k-th* type.

From the point of view of conventional field theory, the verification of all these separate *Equivalence Principles* corresponds to a very peculiar coupling of each field to gravity; whether and why it should be so in all cases is a mystery.

Nearly all attempts to extend the present framework of physics predict the existence of new interactions which are composition dependent and therefore violate the Equivalence Principle. Equivalence Principle tests are by far the most sensitive low energy probes of such new physics beyond the present framework. Any deviation from the UFF –expressed as a fractional differential acceleration $\Delta a/a$ between falling bodies of different composition– is proportional to the post-Newtonian deviations from General Relativity measured, for instance, by the adimensional parameter $\gamma^* \equiv \gamma - 1$ (γ the Eddington parameter). The estimated value of the proportionality factor linking $\Delta a/a$ to γ^* changes depending on scalar (10^{-5}) or vector models, and in the latter case, on the kind of coupling expected for a new interaction $(10^{-2} \div 10^{-3})$. Since experimental tests of the UFF have shown that $\Delta a/a \leq 10^{-13}$ [3, 4, 5], they also constrain γ^* to much smaller limits than it has been obtained from post-Newtonian or pulsar tests, which provide only $|\gamma^*| \leq 10^{-3}$, clearly showing the superior probing power of *Equivalence Principle* tests. [see e.g. 6].

No precise target accuracy at which a violation should occur has been predicted by theories predicting new, composition dependent interactions. A violation is expected, but only below the level reached so far, probably well below it; whether this is really so, only high accuracy experiments can tell.

2 Experiment principle and the expected signal

An experiment to test the Universality of Free Fall requires two test bodies of different composition falling in the field of a source mass, and a read–out system to detect their motions relative to one another searching for a *differential* effect –pointing in the direction of the source mass and with a frequency determined by the relative motion of the test bodies with respect to it– which cannot be explained on the basis of known, classical phenomena. This requires that differential gravitational effects of classical origin (e.g. tidal effects or differential coupling due to different multipole moments of the test bodies as bodies of finite dimension), as well as non gravitational effects (e.g. due to residual air, radiation pressure, electric forces, magnetic forces), must be smaller than the signal expected in case of a deviation from the UFF (hence, of a violation of Equivalence). Which amounts to saying that, in order to be interpreted as a violation of Equivalence, the effect detected should go to zero for test bodies made of the same material.

In ground experiments the test bodies can be either free–falling (the so called *mass dropping* experiments) or suspended against the local acceleration of gravity; the source mass can be either the Earth or the Sun.

In *mass dropping* experiments the test bodies are released from a height and the *driving* acceleration acting upon them is the local acceleration of gravity $g=GM_\oplus/R_\oplus^{\,2} \cong 980\ cm\ s^{-2}$ ($M_\oplus$, $R_\oplus$ being the mass and radius of the Earth). The differential acceleration expected because of a deviation from the UFF quantified by a given value η of the Eötvös parameter is the fraction η of g: $\Delta g=\eta\cdot g$. The smaller is η ($\eta \ll 1$), the better is the accuracy of the test, the smaller is the differential acceleration Δg that the apparatus must be able to detect. Δg is in the direction of free fall and its frequency depends on the rotation state of the free falling apparatus (it is a DC signal for not rotating falling bodies, while it is modulated at the frequency of rotation if the free falling apparatus rotates in the reference frame of the laboratory). Mass dropping experiments have the advantage of a large driving acceleration (the largest possible for an observer confined to the surface of the Earth or in orbit around it), but the disadvantage of a short duration of fall (half a second only for a dropping height of 10 *m*).

If a test body is suspended on the surface of the Earth against the local acceleration of gravity it is subject to the centrifugal force due to the diurnal rotation of the Earth at angular velocity $\omega_\oplus$, which acts in the meridian plane of the suspended body and is proportional to its inertial mass. The motion of the body is limited to the plane of the horizon; the component of the centrifugal force in this plane is directed in the North-South direction towards South and depends on the latitude ϑ:

$$f_c = m_i \omega_\oplus^2 \cdot R_\oplus \cdot cos\vartheta \cdot sin\vartheta \qquad (4)$$