No precise target accuracy at which a violation should occur has been predicted by theories predicting new, composition dependent interactions. A violation is expected, but only below the level reached so far, probably well below it; whether this is really so, only high accuracy experiments can tell.

2 Experiment principle and the expected signal

An experiment to test the Universality of Free Fall requires two test bodies of different composition falling in the field of a source mass, and a read–out system to detect their motions relative to one another searching for a *differential* effect –pointing in the direction of the source mass and with a frequency determined by the relative motion of the test bodies with respect to it– which cannot be explained on the basis of known, classical phenomena. This requires that differential gravitational effects of classical origin (e.g. tidal effects or differential coupling due to different multipole moments of the test bodies as bodies of finite dimension), as well as non gravitational effects (e.g. due to residual air, radiation pressure, electric forces, magnetic forces), must be smaller than the signal expected in case of a deviation from the UFF (hence, of a violation of Equivalence). Which amounts to saying that, in order to be interpreted as a violation of Equivalence, the effect detected should go to zero for test bodies made of the same material.

In ground experiments the test bodies can be either free–falling (the so called *mass dropping* experiments) or suspended against the local acceleration of gravity; the source mass can be either the Earth or the Sun.

In *mass dropping* experiments the test bodies are released from a height and the *driving* acceleration acting upon them is the local acceleration of gravity $g=GM_\oplus/R_\oplus{}^2 \cong 980\ cm\ s^{-2}$ ($M_\oplus$, $R_\oplus$ being the mass and radius of the Earth). The differential acceleration expected because of a deviation from the UFF quantified by a given value η of the Eötvös parameter is the fraction η of g: $\Delta g=\eta\cdot g$. The smaller is η ($\eta << 1$), the better is the accuracy of the test, the smaller is the differential acceleration Δg that the apparatus must be able to detect. Δg is in the direction of free fall and its frequency depends on the rotation state of the free falling apparatus (it is a DC signal for not rotating falling bodies, while it is modulated at the frequency of rotation if the free falling apparatus rotates in the reference frame of the laboratory). Mass dropping experiments have the advantage of a large driving acceleration (the largest possible for an observer confined to the surface of the Earth or in orbit around it), but the disadvantage of a short duration of fall (half a second only for a dropping height of 10 *m*).

If a test body is suspended on the surface of the Earth against the local acceleration of gravity it is subject to the centrifugal force due to the diurnal rotation of the Earth at angular velocity $\omega_\oplus$, which acts in the meridian plane of the suspended body and is proportional to its inertial mass. The motion of the body is limited to the plane of the horizon; the component of the centrifugal force in this plane is directed in the North-South direction towards South and depends on the latitude ϑ :

$$f_c = m_i \omega_\oplus^2 \cdot R_\oplus \cdot cos\vartheta \cdot sin\vartheta \tag{4}$$

Equilibrium is reached at a position where this force is balanced by a component, in the same direction, of the local acceleration of gravity, which is proportional to the gravitational mass m_g of the body. This is the well known fact that a plumb line does not point to the center of the Earth but instead is displaced towards South by an angle which is maximum at 45° of latitude (where the deviation is of about 2 *milliradians*), is zero at the poles and along the equator. No deviation from the UFF (and no violation of Equivalence) means that all plumb lines at any given latitude are displaced by the same angle regardless of the material of the suspended body. Which is to say that two test masses of different composition suspended from two wires of the same length, when released from the local vertical should return to their maximum elongation angle at exactly the same time, always keeping in step. The experiment should monitor how accurately the pendula keep in step. In such pendulum experiments the driving acceleration is only 1.69 $cm \cdot s^{-2}$ –the maximum value (at ϑ=45°) of the centrifugal force per unit mass given by (4)– and it is a DC effect. However, the advantage is that the experiment is not limited by short duration.

Rather than being suspended independently, the test bodies can be attached at the opposite ends of a horizontal beam which is then suspended with a wire from its midpoint. Any force acting differently on the test bodies and perpendicular to the plane formed by the beam and the wire, gives rise to a non zero net torque which will twist the wire by an angle proportional to the differential force itself. This is the *torsion balance* originally designed by John Mitchell and used by Henry Cavendish at the end of the 18th century to determine the universal constant of gravity G from the measurement, and the theoretical prediction, of the gravitational effect produced on the balance by two large source masses suspended outside the balance on opposite sides of the test masses. In experiments to test the UFF the test masses at the opposite ends of the beam have different composition and equal mass (although having equal mass is not strictly needed) and there are no artificial source masses outside the balance. If the beam is oriented in the East–West direction the force (4) gives a non zero torque on each test mass. The UFF implies a zero twist angle for any composition of the test bodies and any direction of the beam; a deviation from the UFF results in a non zero net torque and consequent twist of the balance by a constant angle (maximum for the beam in the East–West direction) if the balance is stationary in the laboratory (DC effect). The driving acceleration is the same as in pendulum experiments; the differential acceleration whose effect on the torsion balance (twist angle) should be detected in order to test the Equivalence Principle to the level η is

$$a_{EP}^{\oplus} = \eta \cdot \omega_{\oplus}^2 \cdot R_{\oplus} \cdot cos\vartheta \cdot sin\vartheta \qquad (a_{EP}^{\oplus} \cong \eta \cdot 1.69\ cm \cdot s^{-2} \quad at \quad \vartheta = 45^o) \tag{5}$$

A constant angle of twist means that there is no zero–check on the result of the experiment, unless the test bodies themselves are swapped on the balance (to prove a violation of Equivalence the balance should twist by the same angle –but in the opposite direction– when the masses are swapped). If the balance is located on a rotating tray the twist angle is modulated at the rotation frequency of the tray. The torsion constant of the wire should be as weak as possible (given the need to withstand the weight of the bodies) in order to result in a detectable twist angle even for a very small differential acceleration acting on the bodies of the balance, i.e. for $\eta << 1$ in (5).

Taking the Sun as the source mass instead of the Earth, the signal expected from a violation of Equivalence is derived by writing the equations of motion of the test bodies (of different composition A and B) in the (non inertial) reference frame centered at the center of the Earth and orbiting around the Sun at the annual angular velocity $\Omega_{\oplus}$ (the gravitational attraction from the Sun is proportional to the *gravitational mass* of the test bodies, while the centrifugal force due to the rotation of the Earth around the Sun –or the rotation of the Sun around the Earth in a geocentric frame of reference– is proportional to their inertial mass):

$$
\begin{aligned}
m_i^A \vec{a}^{\,A} &= -m_g^A \cdot \frac{GM_{sun}}{r_A^{\,3}} \vec{r}_A + m_i^A \cdot (\vec{\Omega}_\oplus \times (\vec{\Omega}_\oplus \times \vec{R})) \\
m_i^B \vec{a}^{\,B} &= -m_g^B \cdot \frac{GM_{sun}}{r_B^{\,3}} \vec{r}_B + m_i^B \cdot (\vec{\Omega}_\oplus \times (\vec{\Omega}_\oplus \times \vec{R}))
\end{aligned} \tag{6}
$$

where $\vec{r}_A$, $\vec{r}_B$ are the relative position vectors of the test bodies with respect to the Sun, $\vec{R}$ is the Earth–to–Sun vector, and M_{sun} is the mass of the Sun. If the test bodies are concentric: $\vec{r}_A = \vec{r}_B \equiv \vec{r}$. Otherwise, $\vec{r}_A \cong \vec{r}_B \equiv \vec{r}$ where the relative distance of the test bodies has been neglected with respect to their distance from the Sun. Any classical tidal effect due to the centers of mass of the bodies not being exactly coincident, and depending on the specific experimental set–up, must be smaller than the signal expected for the level of violation the experiment is aiming to detect. From this, and the centrifugal acceleration of the Earth at its distance form the Sun, we have:

$$
\begin{aligned}
m_i^A \vec{a}^{\,A} &= -m_g^A \cdot \frac{GM_{sun}}{r^3} \vec{r} + m_i^A \cdot \frac{GM_{sun}}{R^3} \vec{R} \\
m_i^B \vec{a}^{\,B} &= -m_g^B \cdot \frac{GM_{sun}}{r^3} \vec{r} + m_i^B \cdot \frac{GM_{sun}}{R^3} \vec{R}
\end{aligned} \tag{7}
$$

By defining:

$$
\begin{aligned}
m_g^A &= m_i^A \cdot \left(1 + \frac{\eta}{2}\right) \\
m_g^B &= m_i^B \cdot \left(1 - \frac{\eta}{2}\right)
\end{aligned} \tag{8}
$$

we also get:

$$
\begin{aligned}
m_i^A \vec{a}^{\,A} &= -GM_{sun} m_i^A \cdot \left(\frac{\vec{r}}{r^3} + \frac{\eta}{2} r \frac{\vec{r}}{r^3} - \frac{\vec{R}}{R^3} \right) \\
m_i^B \vec{a}^{\,B} &= -GM_{sun} m_i^B \cdot \left(\frac{\vec{r}}{r^3} - \frac{\eta}{2} \frac{\vec{r}}{r^3} - \frac{\vec{R}}{R^3} \right)
\end{aligned} \tag{9}
$$

and therefore, for the differential acceleration between the test bodies resulting from a violation of Equivalence η, we get:

$$
\vec{a}_{EP}^{\,sun} = \vec{a}^{\,B} - \vec{a}^{\,A} = \eta \cdot \frac{GM_{sun} \vec{r}}{r^3} \qquad a_{EP}^{sun} \cong \eta \cdot 0.6 cm \cdot s^{-2} \tag{10}
$$

In a geocentric reference frame the position vector of the Sun, can be written as:

$$\vec{R}_{sun} = R\ (cos\delta\, sin H,\, cos\delta\, cos H,\, sin\delta\,) \tag{11}$$

where δ and H are its declination and hour angle giving its angular position on the celestial sphere at any time of the year and the day (e.g. $\delta = 0$ at the equinoxes, $H=0$ when the Sun is at the local meridian). In the same frame, the position vector of the laboratory, at latitude ϑ, where the test bodies are located is:

$$\vec{R}_{lab} = R_{\oplus}\,(0,\, cos\vartheta,\, sin\vartheta\,) \tag{12}$$

and the position vector $\vec{r}$ of Eq. (10), giving the relative position of the Sun with respect to the laboratory, is:

$$\begin{aligned}\vec{r} = \vec{R}_{sun} - \vec{R}_{lab} &= \left(R\ cos\delta\, sin H,\, R\,cos\delta\, cos H - R_{\oplus}\, cos\vartheta,\, R\,sin\delta - R_{\oplus}\, sin\vartheta\right)\\ &\equiv (\,r_x, r_y, r_z\,)\end{aligned} \tag{13}$$

We now rewrite it in the reference frame of the laboratory itself in which the first coordinate is the East–West direction, the second coordinate is the North–South direction (the resulting plane is the plane of the horizon) and the third coordinate in the direction of the Zenith (the vertical plane is the meridian plane):

$$\vec{r} = (\,R_{sun} - R_{lab}\,) = (\,r_x\,, r_y sin\vartheta - r_z\, cos\vartheta, r_z\, sin\vartheta + r_y\, cos\vartheta\,) \tag{14}$$

which immediately gives the North–South and East–West components (in the horizontal plane of the laboratory) of the differential acceleration (10) which would result from a violation of Equivalence η having the Sun as the source mass:

$$a^{sun}_{EP,NS} = \eta \cdot G\frac{M_{sun}}{R^3} \cdot (\,r_y sin\vartheta - r_z\, cos\vartheta\,) \tag{15}$$

$$a^{sun}_{EP,EW} = \eta \cdot G\frac{M_{sun}}{R^3} \cdot r_x \tag{16}$$

showing a dependence from the *daily* and annual motion of the Sun (contained in r_x, r_y, r_z as defined by (13)), as well as from the latitude ϑ of the laboratory. It is therefore apparent that using the Sun as the source mass ensures a frequency modulation of the signal (with a 24-*hr* period) even for a stationary apparatus. This fact was successfully exploited in the 1960s and 1970s (see Sec. 5) yielding a considerable improvement over past torsion balance experiments having the Earth as the source mass, despite a weaker driving acceleration (0.6 $cm\ s^{-2}$ instead of 1.69 $cm\ s^{-2}$).

It is worth noticing that a different source mass (the Sun instead of the Earth) changes the distance range of the test (from the radius of the Earth to the Astronomical unit); a fact that should be taken into account when using the result of the tests to place limits on the existence of new composition-dependent interactions.