I. APPENDIX: THE GG SIMULATOR

A. Motivation and Background

Space missions in Fundamental Physics like GG require high precision experiments to be performed in space with no direct access to the apparatus once in orbit. Though this is commonplace for all space missions, space missions at large do not rely on weightlessness as a key feature for the experiment performance. To the contrary, GG as well as other missions in this field (e.g. GP-B, Goce, μScope, Lisa-PF, Step), are designed to perform experiments in absence of weight. Therefore, the experimental apparatus is designed and built to work at zero-g, not at 1-g, and the issue arises as to how effectively such an apparatus can be tested in the lab before launch, and what are the chances for the space mission to perform as expected. Since flight opportunities are scarce - especially for basic science - and flying a mission, even in Low Earth Orbit, is typically as expensive as large ground projects, it is a must to provide firm evidence before launch that chances to succeed are high. Considerable effort has been devoted during several years in order to provide strong evidence for the success of the GG mission. This has been done by proceeding along the following three lines:

1. Build up a numerical Space Experiment Simulator of the GG mission in space based on the best know-how available to the most advanced space industries, particularly those with direct expertise in missions which fly zero-g designed payloads

2. Build up a prototype in the lab which is demonstrated to have the key physical features of the payload to fly, and provide experimental evidence that the major requirements of the space mission are met

3. Feed the values of the physical parameters measured in the lab into the Space Experiment Simulator to assess the overall performance of the GG mission.

It is apparent that this was a very challenging plan. Space industry should have the capability required; the payload should have been designed so that a 1-g version of it can be built which maintains its key features; the scientific and industrial teams should be able to work in very close collaboration. All such conditions are met in GG, in terms of space industry specific know-how, of physical design of the experimental apparatus for it to be significantly tested in the lab, as well as for what concerns a well established tradition of close collaboration between scientists and space industry. Thales Alenia Space Italia (To) is well equipped to perform the task of building a GG Space Experiment Simulator. In its capacity as prime contractor of the Goce mission of ESA, TAS-I designed, built, tested and demonstrated the performance of a complete Space Experiment Simulator for this mission, which has considerable commonalties with GG.

Goce has been a very challenging high-tech mission devoted to measuring the gravitational field of the Earth to very high degree and order. It was equipped with very sensitive accelerometers arranged in diamond configuration to accurately measure gravity gradient effects. The accelerometers - designed and built by Onera (Chatillon, France)- were based on a "free floating" test mass with electrostatics bearing and pick up. Though a tiny mechanical connection was added in order to provide passive electrostatic grounding, the accelerometers as such could work only at zero-g and no full test was possible in the lab. Onera - with the support of CNES - had indeed already flown similar accelerometers (onboard the Space Shuttle, as well as in previous satellite geodesy missions such as Champ and Grace). However, being the tasks of Goce more challenging, a Space Experiment Simulator was built by Thales Alenia Space in Torino to design and test the calibration procedure (actually 2 different types of calibrations were carried out during the mission, both implemented and tested only by the E2E GOCE Simulator), to check the consistency of spacecraft and payload specifications with the overall system requirements, to support trade-off, sensitivity and worst-case analyses, to support design and pre-validation testing of the Drag-Free and Attitude Control (DFAC) laws, to prepare and test the on-ground and in-flight gradiometer calibration concepts, to prototype the post-processing algorithms, and to validate the performance of the mission. The GOCE simulator was extensively used during the design and construction of the spacecraft and payload and was the core of the flight commissioning and calibration activities carried out at that time. The GOCE mission results are well known all over the world and the E2E Simulator represents a Thales Alenia Space Italia flagship.

The GG Space Experiment Simulator was initiated - with ASI support - since the first study of the mission (1998), [1] precisely because it was immediately rated as a crucial validating tool. This preliminary simulator allowed the basic physical features of the GG system to be identified and checked; however, it was still too simplified (e.g., it was mostly a 2-dimensional model). Building up on the expertise acquired with the Goce Simulator, the GG Space Experiment Simulator could be raised to a very advanced level in the very short time of the GG Phase A2 study (2009), [2], for the case of equatorial orbit and FEEP thrusters. In the framework of the GG Phase A2 Study 31 documents were produced [3], in order to describe with high detail all the mission aspects and to be ready for the implementative B and C phases. Furthermore, by incorporating in it the physical parameters as experimentally measured in the lab with the Payload Prototype, the accuracy of the simulation was correspondingly enhanced. The combination of (a) flight validation of the orbit and spacecraft environment simulation and (b) lab validation of the experiment parameters, makes the GG simulator an extremely reliable performance validation tool, the like of which was seen in no other similar mission. The last significant update of the simulator has been carried out during 2011 and 2012 in order to exploit the sun-synchronous orbit and to introduce the laser gauge devices.

B. GG Simulator Architecture

As for the GOCE E2E simulator, the GG Simulator core relies on the DCAP (Dynamics and Control Analysis Package) software package, developed by Thales Alenia Space Ialia under ESA contract. DCAP provides the user with capability to model, simulate and analyze the dynamics and control performances of coupled rigid and flexible (NASTRAN interfaced) structural systems subjected to possibly time varying structural characteristics and space environment loads. It uses the formulation for the dynamics of multi-rigid/flexible-body systems based on Order(n) Kane's method, an elegant mean to develop the dynamics equations for multibody systems in an automated numerical computation. This avoids the explicit computation of a system mass matrix and its inversion, and it results in a minimum-dimension formulation exhibiting close to Order(n) behavior, n being the number of system degrees of freedom. A dedicated symbolic manipulation pre-processor is further used in the coding optimization, which is particularly oriented towards the simulation of the non-linear dynamics of multi-body systems. The modeling capability is completed with the possibility of user-defined software, allowing for modeling of specific control feature not directly included in the dynamic package's library. For the latter control modeling, it has been developed an interface allowing to describe the user control directly by Matlab/Simulink blocks.

A dedicated quadruple precision version of the DCAP software package has been devised for the GG simulator due to the huge dynamic range required by this scientific mission: from half a picometer differential displacement of the test masses with some significant decimal digits ($< 10^{-14}$ m), to the orbital radius of the satellite around the Earth (order of 10^7 m). Such range of order 10^{21} requires the science simulations to be run with quadruple machine precision.

The GG simulator solves for the satellite dynamics along an orbit resulting from the application of the Earth's gravity field, the non-conservative environmental disturbances (atmospheric drag, wind, solar radiation pressure, coupling with Earth's magnetic field, etc.) and the DFAC control forces and torques.

The GG simulator is based on three different main modules: the Environment, the Dynamics and the Post-Processing ones. Figure 1 shows a description of the GG simulator logical breakdown, highlighting the main data sets exchanged.

The *Environment Module* is in charge of computing the gravity field, the gravity gradient and the non-gravitational forces/torques acting on the spacecraft. These forces and torques are added to the forces of the DFAC and AOCS actuators, in order to realize the GG orbit.

The *Dynamics Module* is in charge of computing the GG orbit and the relative dynamics of PGB w.r.t. the spacecraft, and of test masses w.r.t. PGB. This module has to take into account the gravity gradient acting on each body inside the GG spacecraft, and of the WEP violating signal acting on the test masses. The control laws for the damping of the PGB and test masses whirling motion, for the drag free and the AOCS are also dedicated blocks of the dynamics module. It is also in charge of computing:

- the capacitance measurements used to feed the Whirl control of the PGB and of the test masses (simulation of the capacitance sensors, which feed the whirl controller)
- the forces necessary to damp the PGB and test masses whirl motions (simulation of the actuators, which realise the whirl controller commanded forces)
- the effects due to temperature variations on the inertia properties of spacecraft, PGB and test masses
- the effects due to temperature variations on the mechanical suspension (degrade of the $CMRR_{xy}$ and $CMRR_z$) and on the mechanical balancing of the read-out capacitance bridge
- the effects of the temperature gradients on the mechanical suspension (degrade of the $CMRR_{xy}$ and $CMRR_z$) and on the mechanical balancing of the read-out capacitance bridge
- the DFAC and AOCS sensors' measurements
- the DFAC and AOCS actuators' forces and torques (simulation of the cold gas thrusters)

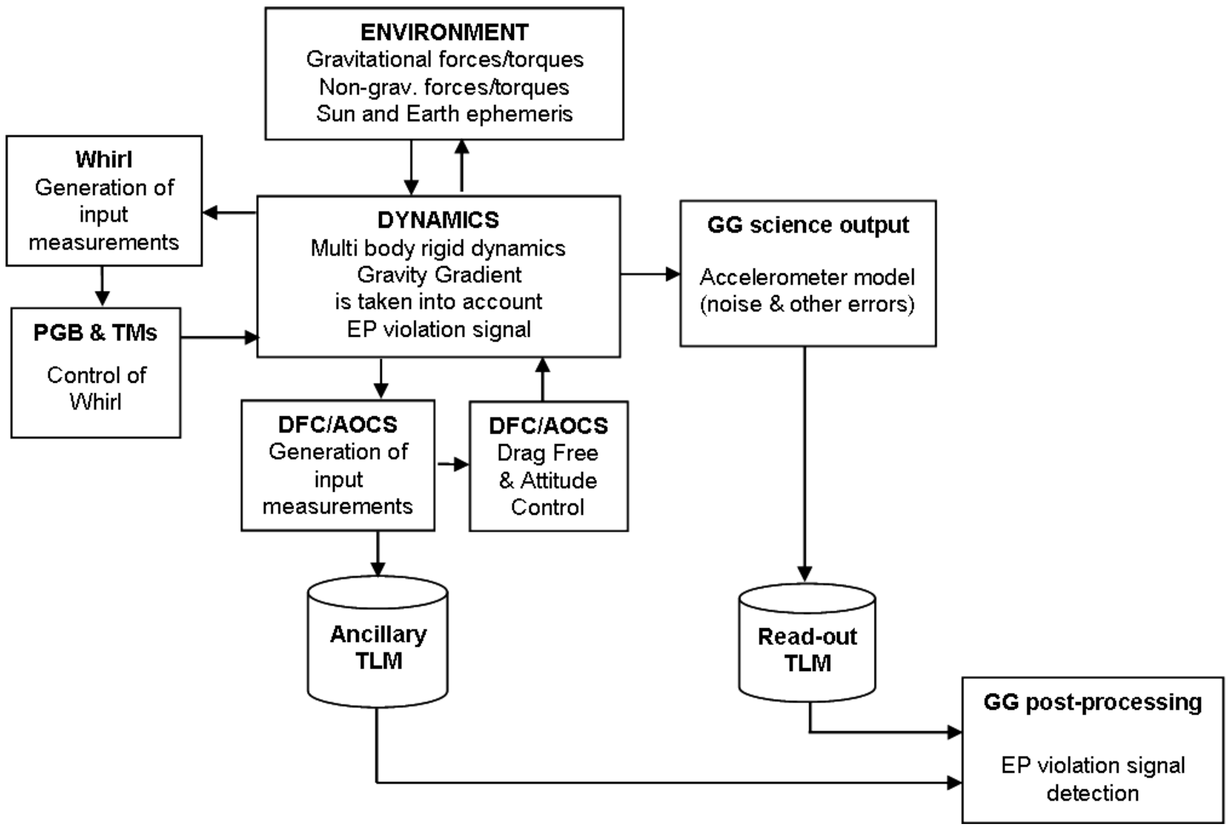

FIG. 1. The three main modules of the simulator are the Environment, the Dynamics and the Post-Processing modules. The blocks containing the controllers for the damping of the whirling motion of PGB and of the test masses, the Drag Free and the Attitude and Orientation of the satellite are components of the Dynamics module.

- the ancillary telemetry data and the other spacecraft data
- the GG science output (simulation of the science capacitance sensors measurements, which feed the post-processing module)

The *Post-Processing Module* is a self standing off line post-processor, which is in charge of detecting the WEP violating signal in terms of differential test mass displacement starting from the science output (and ancillary telemetry if needed). It is also used to compute accelerations, displacements and other useful vectors in the hereafter defined different reference frames.

C. Simulator Environment Module

This block is dedicated to the computation of the forces and torques acting on the spacecraft and resulting from the interaction with the orbital environment. As such, this module computes the gravity force, the gravity gradient torque, the aerodynamic force and torques, the magnetic torque, the solar radiation pressure force and torque. It includes:

- the Earth gravity field taking into account for J2 effect, with gravitational constant $GM = 3.986004418\Delta 1014 m^3/s^2$, according to the EGM96 Earth Gravity field solution, and $EarthMeanRadius = 6378144m$. The gravity gradient torque (taking into account for J2 effect, too) is also applied on each body
- the MSIS86 atmospheric model for the computation of the air density, temperature and chemical composition along the satellite orbit. The F10 and F10.B indexes related to the Solar activity and the Geomagnetic indexes Ap and Kp are used to feed the MSIS model

- a model of the Earth's magnetic field derived from the Oersted satellite measurements
- the celestial bodies ephemeredes computation

The solar radiation pressure and Earth albedo are computed modeling the satellite surface as a set of one cylinder and two simple flat surfaces. The pressures due to the solar and Earth albedo, which depend mainly on the distance from the Sun, on the altitude of the satellite during its orbit, and on the angle between the Sun direction and the local-vertical direction, is computed for each elementary surface. Once the pressures have been computed, the corresponding forces for each surface are obtained considering the normal and tangent components depending on the surface extension and on the aspect angle of the surface with respect to fluxes direction. The force is computed for each surface and applied to the surface's centre of pressure. The resulting force and torque on the centre of mass of each body is then computed.

D. Simulator Dynamics Module

The complete GG system, which takes into account the spacecraft, the Pico Gravity Box, the inner and outer test masses, has been simulated. An additional dummy body has been introduced in order to solve for the orbit without introducing numerical errors due to the high spinning frequency of the spacecraft itself (DCAP solves the orbit in the reference frame of the first body of the kinematics chain). The dummy body is a massive point coincident with the spacecraft centre of mass: its motion w.r.t. the Inertial Reference Frame is defined by the degrees of freedom (3) of the Hinge 1, which connects it (Node 1) to the origin of the IRF. The bodies are characterized by the up-to-date values of the mass and inertia properties, while the dummy body - Body 1 - is a unitary massive point coincident with the spacecraft centre of mass (Node 10). The bodies of the GG dynamical model (schematically represented in Figure 2) and the degrees of freedom (summarised in Table I D) are defined as follows:

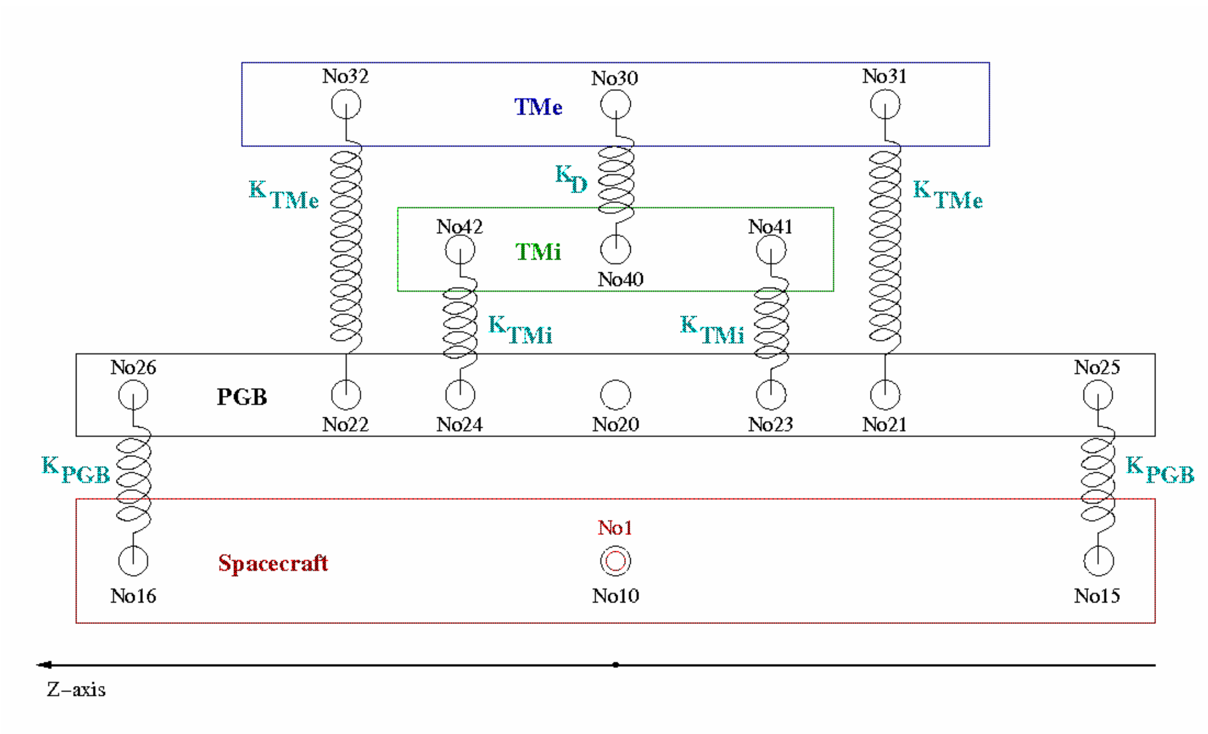

FIG. 2. Logical scheme of the dynamical model implemented within DCAP software code for finite element simulation of the space experiment. The z axis is the spin/symmetry axis of the system; all elastic connections along z are very stiff; the plane of sensitivity is perpendicular to z. The model encompasses all bodies (spacecraft, PGB and 2 test masses), each one with its 6 degrees of freedom in 3D (3 for translation and 3 for rotation), mass and moments of inertia. All non rigid components of the system (sketched as springs) are implemented with their designed stiffness (in the sensitive plane as well as in the z direction) and mechanical quality factors Q for simulation.

Hinge Id.	Translational DoFs (x,y,z)	Rotational DoFs (x,y,z)
1	F, F, F	L, L, L
2	L, L, L	F, F, F
3-4-5	F, F, F	F, F, F

TABLE I. Translation and rotational degrees of freedom of the hinges defining the kinematics chain of the GG system. Hinge 1 connects the IRF to the dummy body, and the null-length Hinge 2 connects the dummy body to the spacecraft centre of mass. The meaning of L and F are: L = Hinge DoF Locked, F = Hinge DoF Free

- Body 1 is the dummy body. Its representative node (Node 1) is coincident with the spacecraft centre of mass. The Hinge 1, which connects the IRF to Node 1, has 3 degrees of freedom: its translation completely describes the orbit motion of the spacecraft
- Body 2 is the spacecraft. The satellite has its centre of mass (Node 10) coincident with the dummy body (Node 1). The null-length Hinge 2 permits satellite rotations only. In particular, the rotation about the Z axis defines the spin direction
- Body 3 is the Pico-Gravity Box (PGB). The Hinge 3, which connects the s/c centre of mass (Node 10) to the PGB centre of mass (Node 20), provides the 6 degrees of freedom (3 rotations and 3 translations) of the PGB-s/c relative motion
- Body 4 is the outer (external) test mass (TMe). The Hinge 4, which connects the PGB centre of mass (Node 20) to the TMe centre of mass (Node 30), provides the 6 degrees of freedom (3 rotations and 3 translations) of the TMe-PGB relative motion
- Body 5 is the inner test mass (TMi). The Hinge 5, which connects the PGB centre of mass (Node 20) to the TMi centre of mass (Node 40), provides the 6 degrees of freedom (3 rotations and 3 translations) of the TMi-PGB relative motion

This type of multi-body connection grants an open-loop kinematics topology, with no need for cut-joint hinges. All hinges are described by a Euler sequence Type 1, x-y-z, or 3, z-x-y. The active degrees of freedom (DoF) defined by hinges can be differently set depending on the required type of simulation.

The Hinge 1 defines also the initial condition for the GG orbit (w.r.t. the IRF), with the convention that at t = 0 the satellite position is $(R_\oplus + h, 0, 0)$ and the satellite velocity is $(0, v_y, v_z)$, with $R_\oplus$ the Earth equatorial radius and h the GG orbit altitude. The Hinge 2 defines instead the satellite spin w.r.t. the IRF: initially it has been assumed to be 1 Hz, according to the last analyses, which have been performed in order to verify the capability of cold gas thrusters (originally FEEP thrusters) for the drag free compensation.

The mass and inertia properties used in the simulator have been fixed at the beginning according to the last Phase A Report description (2000); then, they have been updated according to the 2012 March review mass budget. At the end they shall be updated according to the last mass budget, provided after test mass procurement. The mass and inertia properties were not continuously updated in order to make more efficient the "growth" of the simulator (the main job was adding all the possible spurious effect masking or competing with the WEP signal).

More than 50 nodes were required to implement suspension connections, sensors and actuators munting points, optical markers for laser gauges and reference points to have a complete information and additional checks of the system dynamics (e.g. range variation of laser gauge measurement due to the small variation of the test masses and PGB relative attitude).

The mechanical suspensions connecting the PGB to the spacecraft and the test masses to the PGB, which are schematically represented by springs in Figure 2, are implemented in order to provide realistic suspension modes (according to the GG on the Ground measured values) and transfer function (see Table I D). The PGB-s/c suspension is also characterised by its realistic dissipative term, corresponding to a mechanical quality factor $Q_{PGB} \cong 90$, in order to provide the most realistic representation of the PGB-s/c dynamics behaviour: the PGB-s/c relative motion not only must be measured in order to feed its control of the whirl motion, but provides also the input of the DFAC control, which is a key point for the GG science performance.

The dissipation of the PGB-test masses suspensions implemented within the GG simulator is instead by far much greater than the true one, which is measured on the GG Ground prototype: this is necessary in order to reduce the simulation time required to observe the increase of the whirling motion (the GG simulator performs at almost real time speed also if it runs on a workstation with multicore Xeon @3.4 GHz) by adopting a mechanical quality factor $Q_{TM} = 500$ (vs. a true value of $Q_{TM} = 20000$). With this choice the whirl-radius doubling time trw2 is of the

GG sub-system	Planar oscillation period [s]	Axial oscillation period [s]
s/c-PGB	360	30
PGB-TMe	$30 < TCM_{xy} < 120$	30
PGB-TMi	$30 < TCM_{xy} < 120$	30
TMe-TMi	540	0

TABLE II. Oscillation periods of the various GG subsystems along the satellite spin axis and in the plane perpendicular to it

order of $trw2 \simeq 10000s$ (vs. a true value of $trw2 \simeq 500000s$). This shorter trw2 permits to carry out simulations covering a mission time duration not longer than $200000 \div 300000s$, but which cover all the relevant aspects in terms of disturbing effects and science performance (the WEP signal is in fact detected). The adoption of the true value for the mechanical quality factor of the PGB-test masses suspension would force the length of one science performance simulation to several millions of seconds, just to verify the growth of the whirling motion! It is apparent that there is nor loss of generality, neither a "favourable" assumption in this choice. It has to be noted here that the absence of motor and bearings, which is a key point of the GG satellite, makes the structural damping the main contributor to the whirling motion.

The WEP violating signal is simulated with a force with amplitude $F_{WEP} = m_{TM} \cdot g(h) \cdot \eta N$, which is directed from the Earth centre of mass to the centre of mass of the outer test body only (it is a pure differential force for the test masses). The mass of the proof body is 10 kg, the value of the local gravity sensed from the test mass depends on the GG altitude, and it is about $8 \div 8.4 m/s^2$. The Eötvös parameter η is an input value for the GG simulator, with η in the range of $10^{-17} \div 10^{-13}$.

High-fidelity models represent sensors feeding the control algorithms and the actuators (cold gas thrusters and spinning-up gas thrusters), which are in charge of generating the DFAC and AOCS commanded forces and torques. The high-fidelity models for sensor and actuators have to account for intrinsic noises, transfer functions, non linearity, mounting errors, sensor/actuators inner geometrical imperfections (see Figure 3), temperature fluctuations, quantization and all the other effects which can degrade the scientific performance of the sensors/actuators, since the goal of the simulation is predicting the realistic mission scientific performance.

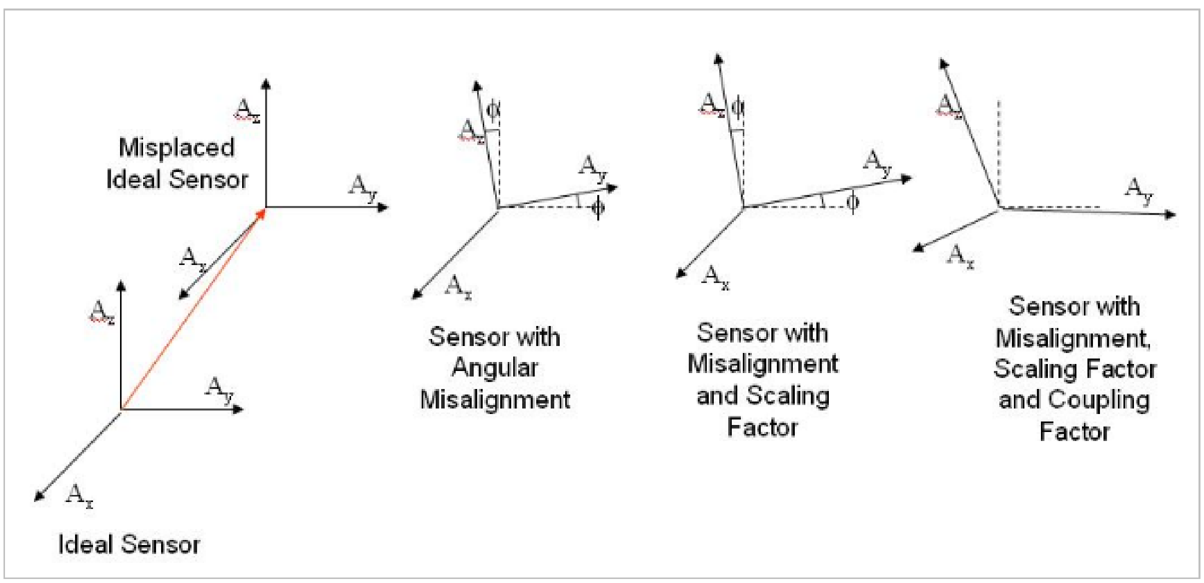

FIG. 3. Sensor/actuator geometrical imperfection: mispositioning, misalignment, scale factor error and coupling error

The simulation of sensor/actuator noise and of temperature fluctuation is based on the superposition of deterministic noise (implemented by sinusoidal terms) and of stochastic noise (implemented by noise shaping technique). With the noise shaping technique, a properly devised (in the frequency domain) noise shaping filter is used to provide the desired shape to its feeding unitary band limited white noise. The block-diagram of the white noise shaping technique is reported in Figure 4: a unitary (one-sided) white noise is passed through an analog/digital filter that builds up the requested noise Spectral Density and which integrates also an anti-aliasing filter (if necessary). The output of the filter is then decimated (if requested) and saved into a dedicated variable.

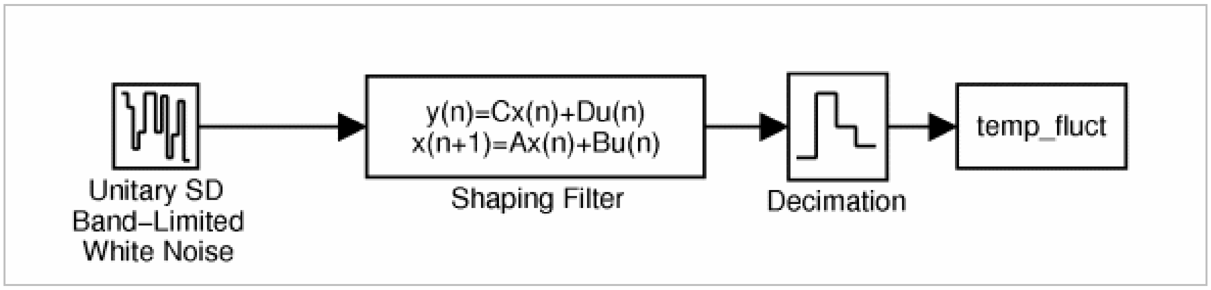

FIG. 4. Block diagram of the white noise shaping technique. A white noise with unitary (one-sided) Spectral Density is passed through a shaping filter (applying also the anti-aliasing, if necessary) and decimated (if required) before saving the output into a dedicated variable.

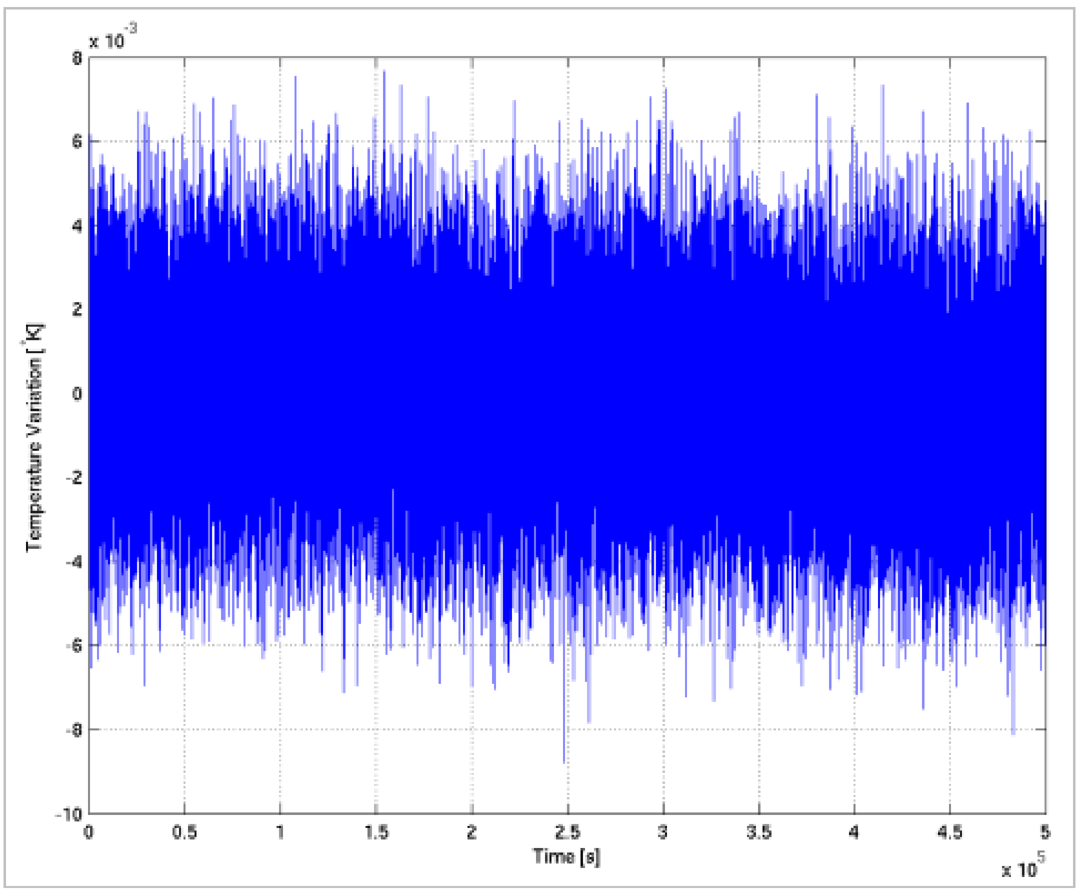

FIG. 5. Time history of the fluctuating component of temperature obtained with the white noise shaping technique.

Figure 5 shows as example the time histories of the fluctuating temperature, temp_fluct, realized according to the white noise shaping technique. Figure 6 shows the Spectral Density (SD) as computed from the simulated time history, vs the desired analytic SD: the desired analytic SD can be superimposed to the computed one.

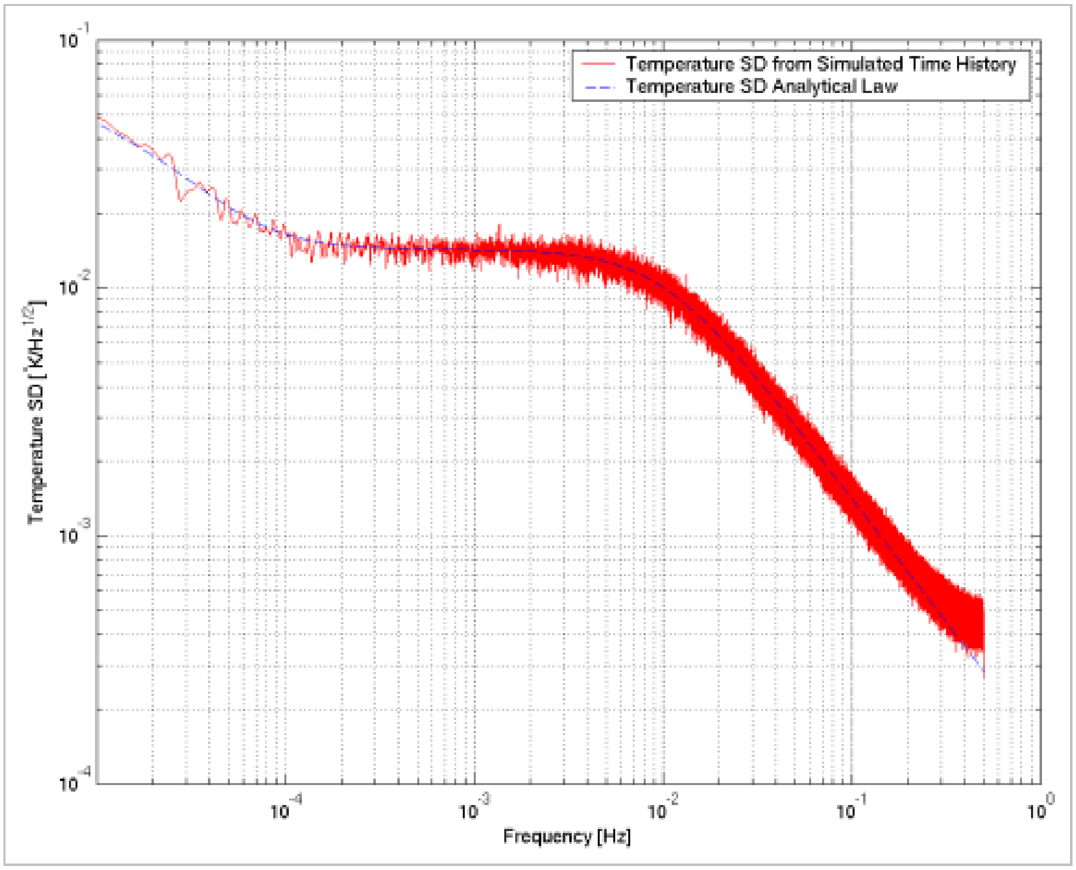

FIG. 6. Spectral density of the temperature fluctuation computed from the simulated time history vs. the Spectral density desired analitic law.

The above cited technique provides the capability of adding the temperature effects sensed at the level of the test masses (thermal noise, temperature fluctuation), of the mechanical balancing of the science read-out, and of the $CMRR_{xy}$ and $CMRR_z$ variation due to temperature.

An analytic function for the temperature variation sensed from spacecraft and PGB has been instead modelled in order to introduce their inertia variations.

E. Post-Processing Module

The GG sensor and actuators are fixed w.r.t. the spinning satellite: this means that the detection of the WEP violation signal and of the other interesting information (non-gravitational acceleration, whirling motion, temperature effects, gravity gradient contributions, etc.) is completely masked by the GG spinning frequency.

The Post-Processing module consists of a Matlab macros package, which has been implemented in order to allow the transformation of all the sensor measurements from the BF to the IRF and to the WEP reference frames. Also the ancillary information about the satellite, PGB and test masses dynamics is provided in all the reference frames, in order to allow the science performance check by using the time histories generated by the simulator. The implemented macro package requires the Signal Processing and Statistics Matlab Toolboxes. The coding of this module is such that it should be easy a full porting to the GNU Octave environment, which is a free Matlab clone providing also packages equivalent to the Signal Processing and Statistics Toolboxes.

The Post-Processing module allows the analysis on both the time and frequency domain, providing results in terms of Amplitude Spectrum and Spectral Density of the interesting signals (e.g. Fourier analysis of the non-gravitational accelerations w.r.t to IRF and Spectral Density accelerations and displacements).

F. Whirling Stabilization

The stabilization of whirl motion can be implemented (see for example the historical references [4] and [5]) by building in a rotating frame a damping command proportional to the relative velocity between bodies in the inertial (non-rotating) reference frame. A simple and didactic way to show this capability is by examining the problem of 2 bodies connected by a dissipative spring. Let us consider r_s the vector pointing to the Spacecraft center of mass, r_p the vector pointing to the PGB center of mass, ϵ the vector locating the suspension point of the spring with respect to the Spacecraft center of mass, S the Suspension point of the spring (see 7). The spacecraft is rotating around the z-axis, which is perpendicular to the xy plane. The rotation is counter-clockwise. In the inertial reference frame the equations of motion read: being $\epsilon = \epsilon(t)$ a rotating vector in the inertial frame (fixed in the reference frame corotating

$$\begin{cases} m_s \ddot{\underline{r}}_s = -k(\underline{r}_s - \underline{r}_p + \underline{\varepsilon}(t)) - c_r(\dot{\underline{r}}_s - \dot{\underline{r}}_p - \underline{\omega}_s \times (\underline{r}_s - \underline{r}_p)) - \underline{F}_{ext} \\ m_p \ddot{\underline{r}}_p = -k(\underline{r}_p - \underline{r}_s + \underline{\varepsilon}(t)) - c_r(\dot{\underline{r}}_p - \dot{\underline{r}}_s - \underline{\omega}_s \times (\underline{r}_p - \underline{r}_s)) \end{cases}$$

with the spacecraft) and ω_s the spin angular velocity. These equations show clearly the nature of rotating damping, namely damping between the rotating bodies. Using: $x = r_p - r_s$ (relative position vector); $m_r = m_s m_p/(m_s + m_p)$ (reduced mass), $\omega_0^2 = k/m_r$ (natural frequency of oscillation), $c_r = k/(Q\omega_s)$ (coefficient of rotating damping, with Q the mechanical quality factor of the spring), we can write the equation for the relative motion of the two bodies:

$$\ddot{\underline{x}} = -\omega_0^2(\underline{x} - \underline{\varepsilon}(t)) - \frac{\omega_0^2}{\omega_s Q}(\dot{\underline{x}} - \underline{\omega}_s \times \underline{x}) - \frac{\underline{F}_{ext}}{m_s}$$

The solution of the equation of relative motion is:

$$\underline{x}(t) = \frac{\omega_0^2 \underline{\varepsilon}(\omega_s t)}{\omega_0^2 - \omega_s^2} + \underline{x}_{drag}(t) + \underline{x}_{whirl}(\omega_w t)$$

with x_{drag} the vector describing the displacement of the equilibrium position due to the action of the external drag and $x_{whirl}(\omega_w t)$ the exponential term responsible of the motion divergence (spiral growing with time).

The above 2-body system is unstable. The simplest way to stabilize it is adding some Non Rotating Damping (i.e. damping between non rotating parts of the system), which is mathematically expressed by the terms containing C_{nr} (the equations refer again to the inertial reference frame):

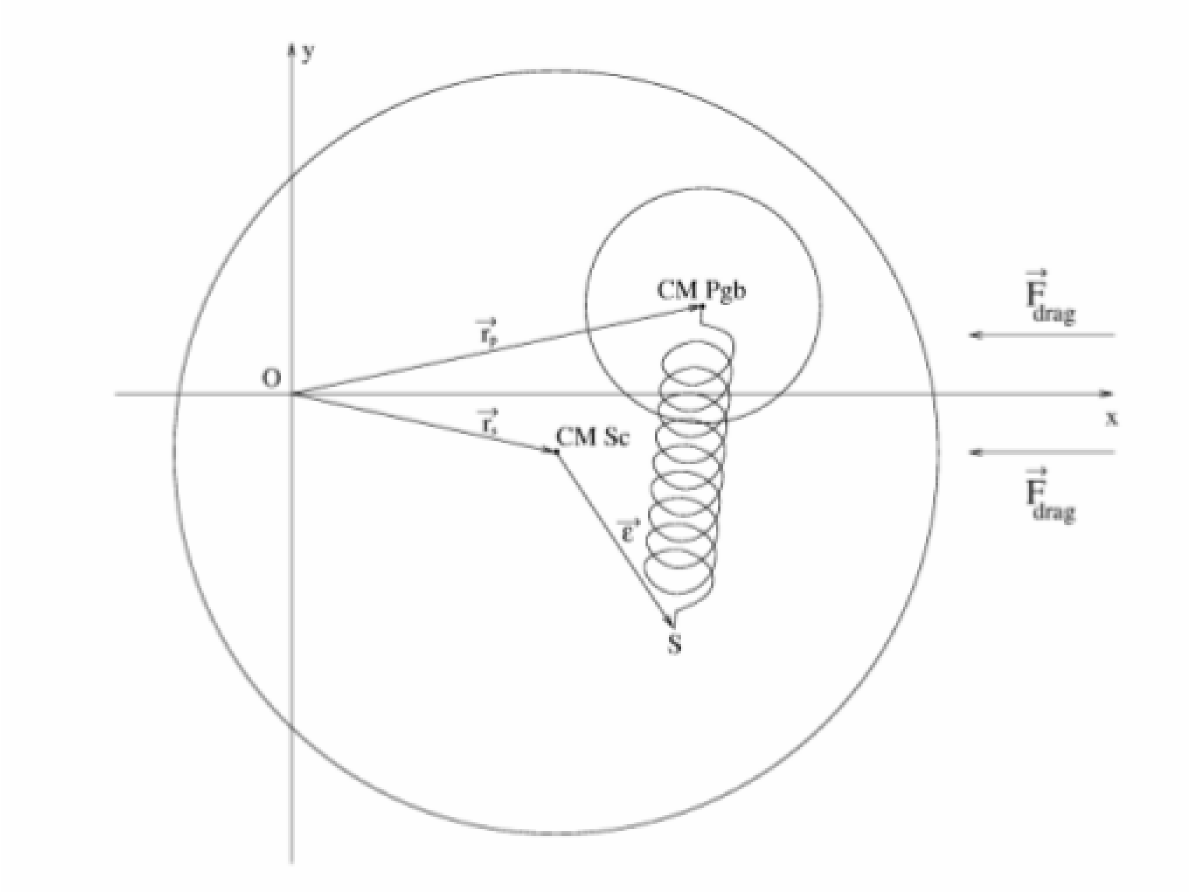

FIG. 7. Schematization of the two-body model problem.

$$\underline{x}_{drag} = -\frac{F_{ext}}{m_s}\frac{1}{\omega_0^2 + \omega_0^2/Q^2}\hat{i} - \frac{F_{ext}}{m_s}\frac{1}{\omega_0^2 + \omega_0^2/Q^2}\frac{1}{Q}\hat{j}$$

$$\begin{cases} m_s \ddot{\underline{r}}_s = -k(\underline{r}_s - \underline{r}_p + \underline{\varepsilon}(t)) - c_r(\dot{\underline{r}}_s - \dot{\underline{r}}_p - \underline{\omega}_s \times (\underline{r}_s - \underline{r}_p)) - C_{nr}(\dot{\underline{r}}_s - \dot{\underline{r}}_p) - \underline{F}_{ext} \\ m_p \ddot{\underline{r}}_p = -k(\underline{r}_p - \underline{r}_s + \underline{\varepsilon}(t)) - c_r(\dot{\underline{r}}_p - \dot{\underline{r}}_s - \underline{\omega}_s \times (\underline{r}_p - \underline{r}_s)) - C_{nr}(\dot{\underline{r}}_p - \dot{\underline{r}}_s) \end{cases}$$

$$\ddot{\underline{x}} = -\omega_0^2(\underline{x} - \underline{\varepsilon}(t)) - \frac{\omega_0^2}{\omega_s Q}(\dot{\underline{x}} - \underline{\omega}_s \times \underline{x}) - \frac{C_{nr}}{m_r}\dot{\underline{x}} - \frac{\underline{F}_{ext}}{m_s}$$

These equations are useful to see the difference between Rotating Damping (term with c_r), and Non Rotating Damping (term with C_{nr}). In the space experiment everything is corotating with the spacecraft, so we need to recast these equations in the rotating reference frame:

$$\begin{cases} m_s \ddot{\underline{r}}_s = -k(\underline{r}_s - \underline{r}_p + \underline{\varepsilon}) - c_r(\dot{\underline{r}}_s - \dot{\underline{r}}_p) - C_{nr}(\dot{\underline{r}}_s - \dot{\underline{r}}_p + \underline{\omega}_s \times (\underline{r}_s - \underline{r}_p)) + m_s \omega_s^2 \underline{r}_s - 2 m_s \underline{\omega}_s \times \dot{\underline{r}}_s - \underline{F}_{ext}(t) \\ m_p \ddot{\underline{r}}_p = -k(\underline{r}_p - \underline{r}_s + \underline{\varepsilon}) - c_r(\dot{\underline{r}}_p - \dot{\underline{r}}_s) - C_{nr}(\dot{\underline{r}}_p - \dot{\underline{r}}_s + \underline{\omega}_s \times (\underline{r}_p - \underline{r}_s)) + m_p \omega_s^2 \underline{r}_p - 2 m_p \underline{\omega}_s \times \dot{\underline{r}}_p \end{cases}$$

$$\ddot{\underline{\zeta}} = -\omega_0^2(\underline{\zeta} - \underline{\varepsilon}) - \frac{\omega_0^2}{\omega_s Q} \dot{\underline{\zeta}} - \frac{C_{nr}}{m_r} \dot{\underline{\zeta}} - \frac{C_{nr}}{m_r} \underline{\omega}_s \times \underline{\zeta} + \omega^2 \underline{\zeta} - 2\underline{\omega}_s \times \dot{\underline{\zeta}} - \frac{\underline{F}_{ext}(t)}{m_s}$$

with ζ and $\dot{\zeta}$ the relative displacement and its rate of change in the rotating reference frame. The minimum C_{nr} for stabilizing the whirling motion is $C_{nr} = c_r \cdot \frac{\omega_s}{\omega_0}$, with, for $\omega_s \gg\gg \omega_0$ exploits $c_r \simeq k\frac{\eta}{\omega_s}$ and $\eta = 1/Q_{spin}$, i.e. depending on the internal loss of the material at the frequency at which the material goes through the elastic hysteresis cycle, the spin frequency.

The control implementation adopted for the GG satellite does not rely on a PD or PID implementation, but on ad hoc devised double Fourier reconstruction of the modulating signal (see [1] for details).

G. GG Simulator and Error Budget

The GG simulator provides for each run a binary output file containing the time histories of all the science performance relevant variables (bodies position, velocities, accelerations, attitude - e.g. for checking the test masses axes relative inclination -, angular velocity and acceleration, sensor input and output, actuators output, control commanded forces/torques, etc). Useful information provided from the simulator are also the time histories of the non gravitational acceleration acting on the spacecraft all along the orbit, which feeds the computation of experiment error budget. The post processing modules provides the performance of the simulation by computing the demodulation of the measurements and providing the test mass differential displacement at the signal frequency. We have to remember here that the structural dissipation has been increased of little less than two orders of magnitude in order to make the whirling timescale shorter, thus limiting the simulation to one or two days. Moreover, a change of the test mass material requires a new simulation, a change in one single requires a new simulation, and the time cost to perform a different simulation for each parameter change is huge. For this reason the configuration of the simulator has been frozen with a fixed choice of test mass material, a fixed sun-synchronous orbit, and a fixed set of parameters defining suspension, sensors and actuators. The science mission error budget, unfortunately, requires to verify that changing test mass materials (test mass quadruples) or something in the suspension, or some other parameter does not affect the science performance. For this reason a set of Matlab macros collecting all the space science mission parameters (verified also on the GGG prototype) and some output of the GG simulator has been implemented, in order to provide a tool that is in charge of modeling all the spurious effects and that has been validated vs the simulator when the parameters are those ones defined in the GG simulator. Of course, the non gravitational accelerations acting on the spacecraft cannot be modelled: the time history of the non gravitational accelerations provided by the GG simulator is used as input (the generation of a long time history of the non gravitational acceleration acting on the spacecraft can be generated by using a double precision version of the GG simulator, when the science performance, i.e. picometer scale is not required). The non gravitational acceleration acting on the spacecraft, which are provided in the Inertial Reference Frame, are transformed on the required reference frame and the Fourier amplitude spectrum is computed in order to have the component of the acceleration at the same frequency of the WEP signal: this value, multiplied by the common mode rejection ratio in the sensitive plane, provides the measure of the most important competing effect. In order to properly take into account the variation of the angle between the spin axis and the orbit normal, a time history of 90 days of orbit has been used to compute the tidal perturbing effects and the coupling between test masses quadruple moments and Earth gravity field. At the same way, time series of test mass magnetic couplings and of other perturbing effects are computed. The Fourier amplitude spectrum provides at the end the frequency and amplitude of the effects competing with the WEP one. The most important perturbing effect are those ones with the same WEP frequency (there is only the coupling of the residual non gravitational acceleration multiplied by the overall common mode rejection) and its harmonics.

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[1] *"Galileo Galilei" (GG) Phase A Report*, ASI (Agenzia Spaziale Italiana), 1st edn. November 1998, 2nd edn. (January 2000) http://eotvos.dm.unipi.it/ggweb/phaseA/ (January 2000)
[2] *"Galileo Galilei" (GG) Phase A-2 Study Report,* ASI (Agenzia Spaziale Italiana) (2009) http://eotvos.dm.unipi.it/PA2/GGPA2.pdf
[3] *"GG Preliminary Requirements Review Data Package (GG-PRR-Data-Pack)"*, ASI-Thales Alenia Space Contract No. I/039/08/0 "Studio di Fase A2 della Missione Galileo Galilei-GG", 2008-2009
[4] J. P. Den Hartog, *Mechanical Vibrations*, Dover Publ. Inc N.Y. 1985 (first published in 1934)
[5] S. H. Crandall, J. Sound Vib. 11(1), 3-18, (1970)