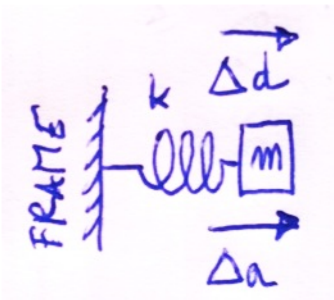
An accelerometer for spaceborne applications with interferometric readout: heterodyne interferometry and realization of LIG

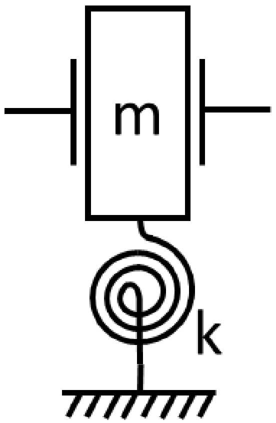
Marco Pisani and Massimo Zucco, INRIM, Torino, Italy
Anna Nobili, University of Pisa

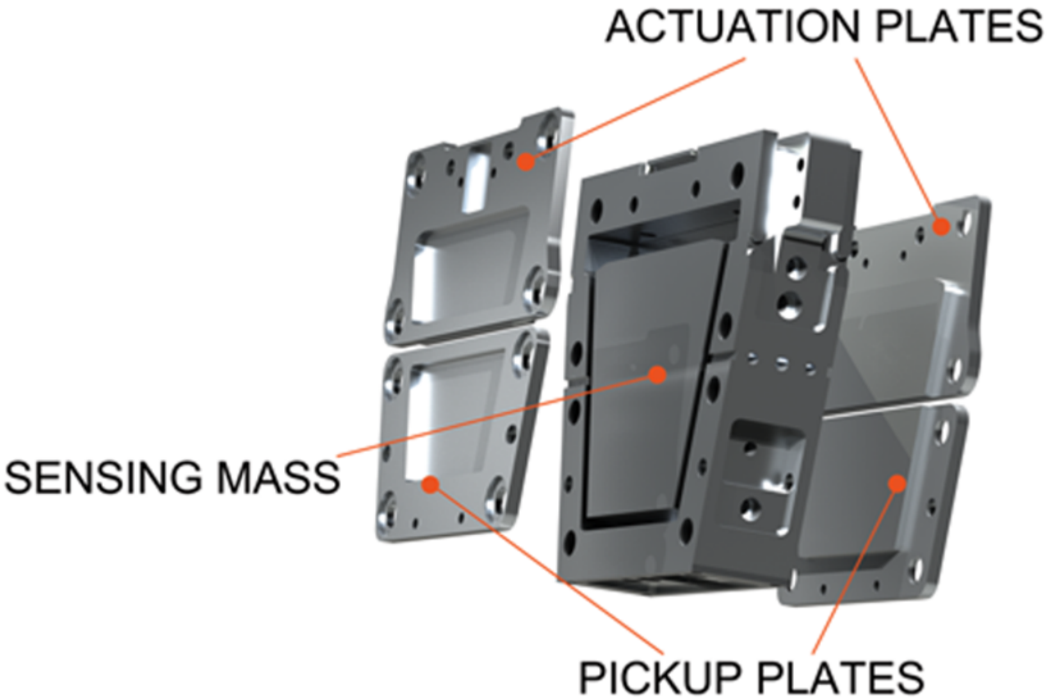
How does an accelerometer work?

- An accelerometer measures the acceleration of a weakly coupled mass relative to its enclosure by measuring the relative displacement it gives rise to.
- Mass m, stiffness of the suspension k, natural coupling frequency: $\omega_0 = \sqrt{(k/m)}$ (small k can suspend the test mass in absence of weight, hence low ω_0)
- Low ω_0 means high sensitivity: an acceleration Δa between the test mass and its enclosure at frequency $\omega < \omega_0$ yields a relative displacement $\Delta d \approx \Delta a/\omega_0^2$ (e. g. $\omega_0 = 1\ rad\ s^{-1}$, $\Delta a = 10^{-12}$ m/s^2 → Δd = 1 pm)



Capacitive reading





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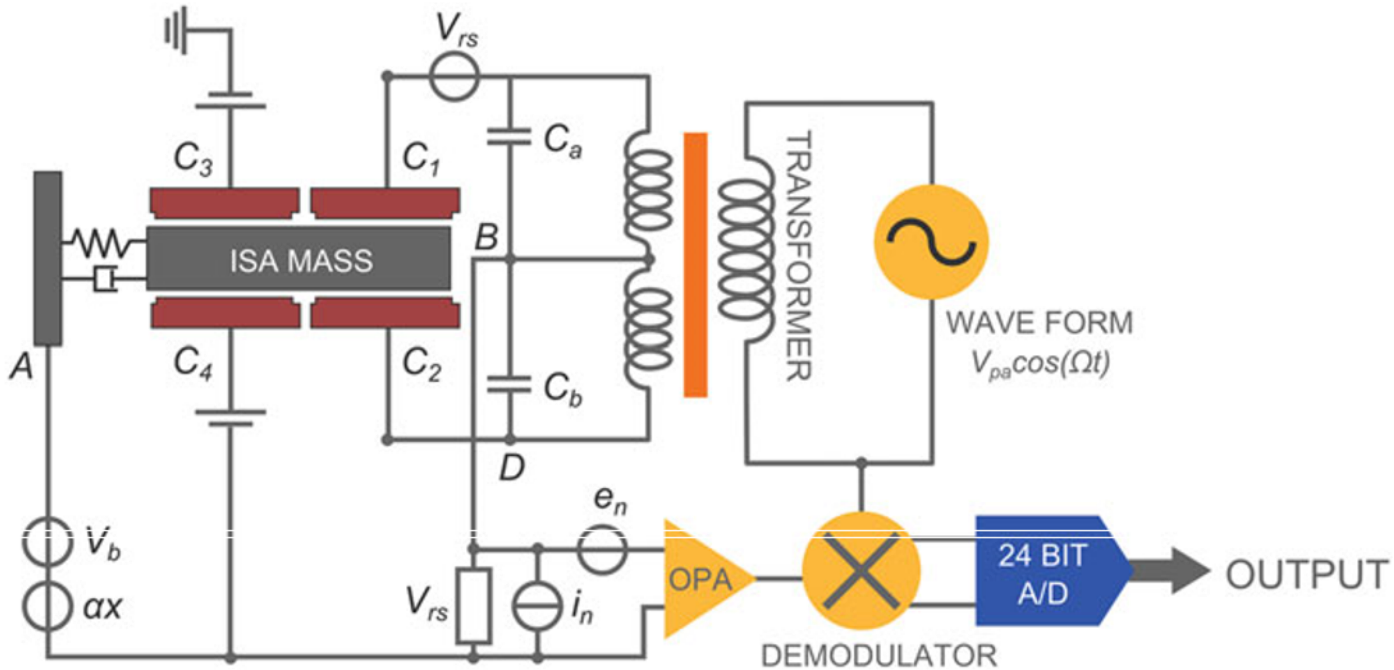
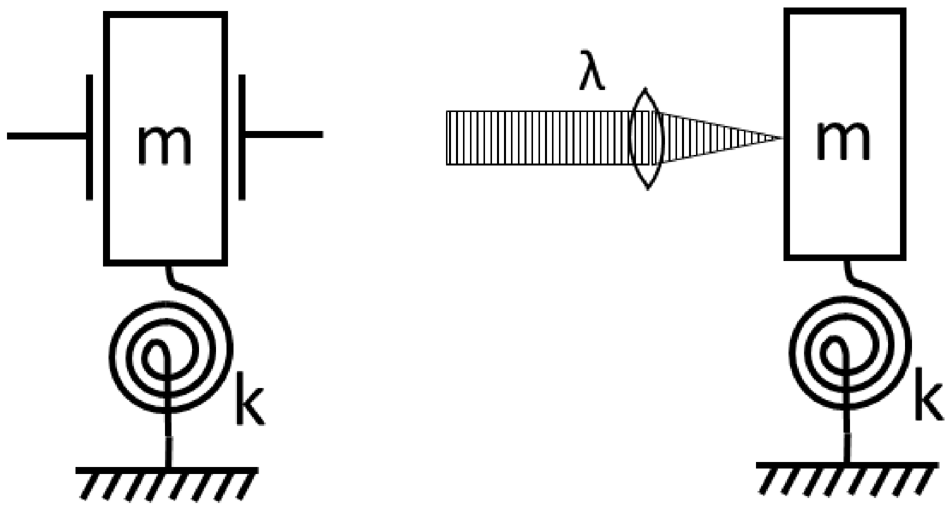


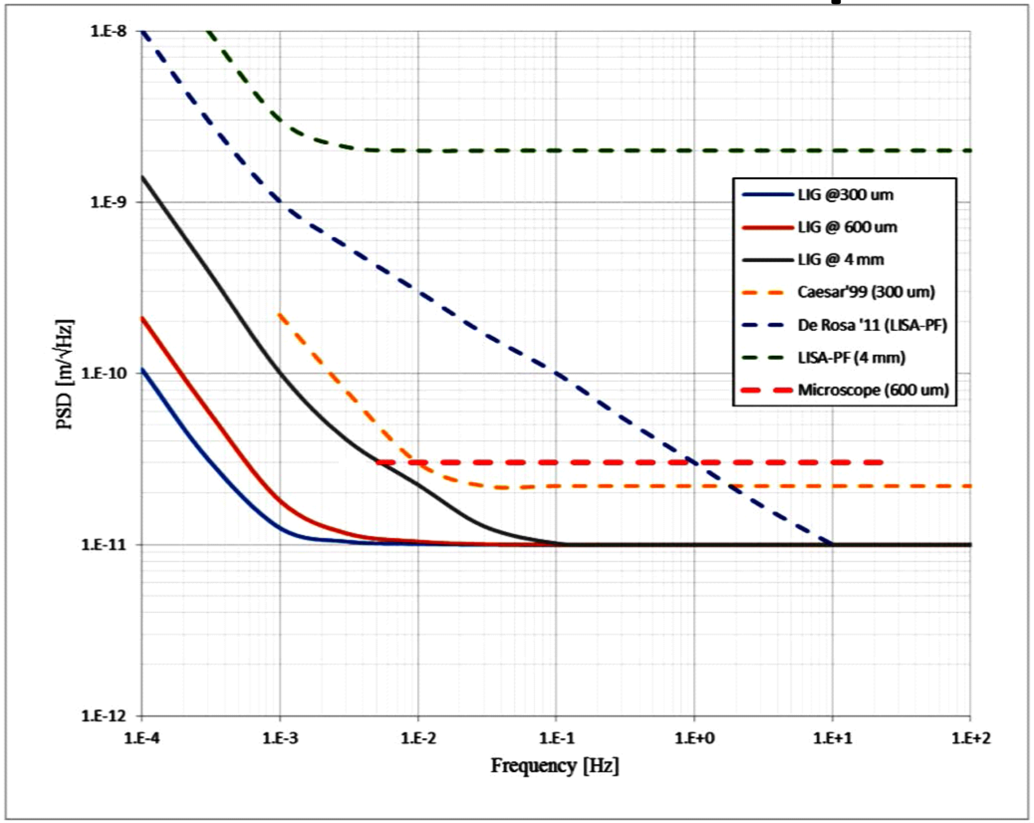
Interferometric readout



INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

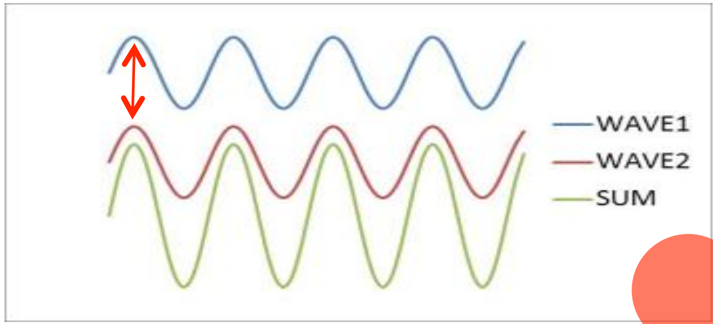
UNIVERSITÀ DI PISA

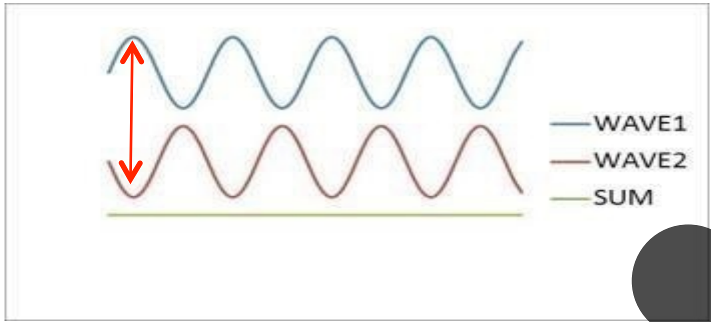
Interferometer vs capacitive



The principle of Interference

- An interferometer compares a displacement D with the wavelength λ of a monochromatic light source (a laser). The wavelength λ is our measurement unit.
- The interferometer is based on constructive and distruptive interference between two (or more) laser beams
- An intensity signal is converted in electrical signal S by a detector





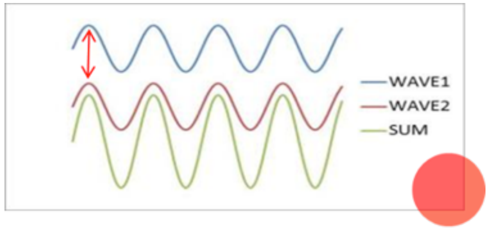
Interferometer types and measurement needs

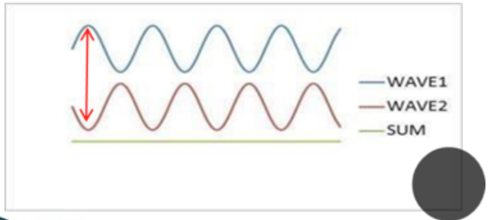
- Michelson
  - Homodyne
  - Heterodyne
- Synthetic wavelength (or dual wavelength)
- Fabry-Perot
- Femto-second laser
- …

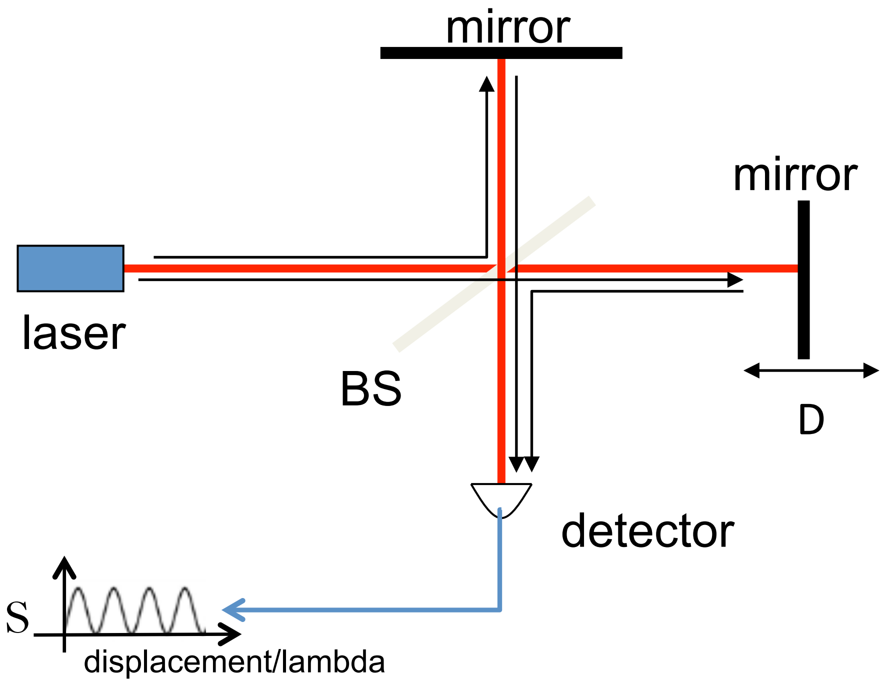
- Long distance
- Absolute
- Incremental
- High resolution/accuracy
- High speed
- Nano metrology
- Vacuum/air
- …

Homodyne Michelson interferometer

The signal S is proportional to the ratio D/λ







$S = 2(1 + \cos\varphi)$, $\varphi = (4\pi D/\lambda)$

Michelson 1881

THE

AMERICAN

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JULY TO DECEMBER, 1881.

WITH SIX PLATES.

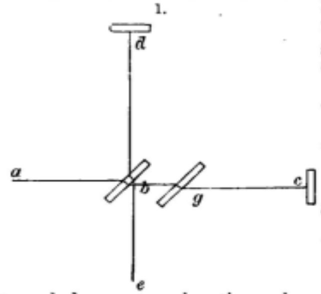
122 *A. A. Michelson—The relative motion of the Earth*

If, now, the apparatus be revolved through 90° so that the second pencil is brought into the direction of the earth's motion, its path will have lengthened $\frac{4}{100}$ wave-lengths. The total change in the position of the interference bands would be $\frac{8}{100}$ of the distance between the bands, a quantity easily measurable.

The conditions for producing interference of two pencils of light which had traversed paths at right angles to each other were realized in the following simple manner.

Light from a lamp *a*, fig. 1, passed through the plane parallel glass plate *b*, part going to the mirror *c*, and part being reflected to the mirror *d*. The mirrors *c* and *d* were of plane glass, and silvered on the front surface. From these the light was reflected to *b*, where the one was reflected and the other refracted, the two coinciding along *be*.

1.



The distance *bc* being made equal to *bd*, and a plate of glass *g* being interposed in the path of the ray *bc*, to compensate for the thickness of the glass *b*, which is traversed by the ray *bd*, the two rays will have traveled over equal paths and are in condition to interfere.

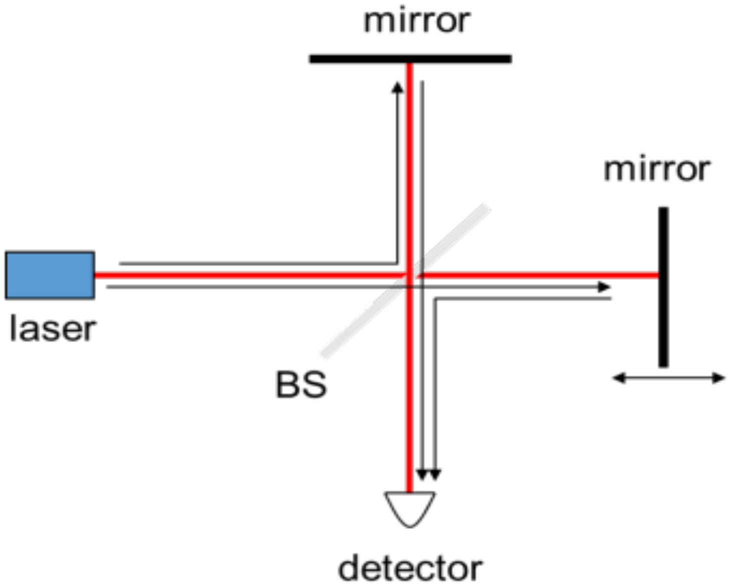
The instrument is represented in plan by fig. 2, and in perspective by fig. 3. The same letters refer to the same parts in the two figures.

The source of light, a small lantern provided with a lens, the flame being in the focus, is represented at *a*. *b* and *g* are the two plane glasses, both being cut from the same piece; *d* and *c* are the silvered glass mirrors; *m* is a micrometer screw which moves the plate *b* in the direction *bc*. The telescope *e*, for observing the interference bands, is provided with a micrometer eyepiece. *w* is a counterpoise.

In the experiments the arms, *bd*, *bc*, were covered by long paper boxes, not represented in the figures, to guard against changes in temperature. They were supported at the outer ends by the pins *k*, *l*, and at the other by the circular plate *o*. The adjustments were effected as follows:

The mirrors *c* and *d* were moved up as close as possible to the plate *b*, and by means of the screw *m* the distances between a point on the surface of *b* and the two mirrors were made approximately equal by a pair of compasses. The lamp being

Michelson 1881



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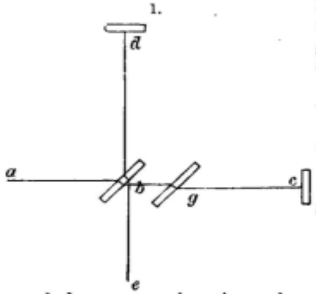

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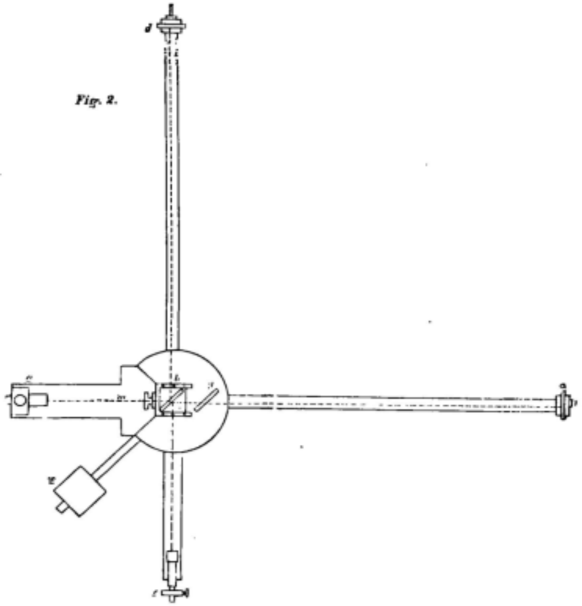
The mirrors *c* and *d* were moved up as close as possible to the plate *b*, and by means of the screw *m* the distances between a point on the surface of *b* and the two mirrors were made approximately equal by a pair of compasses. The lamp being lit, a small hole made in a screen placed before it served as a point of light; and the plate *b*, which was adjustable in two planes, was moved about till the two images of the point of light, which were reflected by the mirrors, coincided. Then a sodium flame placed at *a* produced at once the interference bands. These could then be altered in width, position, or direction, by a slight movement of the plate *b*, and when they were of convenient width and of maximum sharpness, the

Fig. 2.



sodium flame was removed and the lamp again substituted. The screw *m* was then slowly turned till the bands reappeared. They were then of course colored, except the central band, which was nearly black. The observing telescope had to be focussed on the surface of the mirror *d*, where the fringes were most distinct. The whole apparatus, including the lamp and the telescope, was movable about a vertical axis.

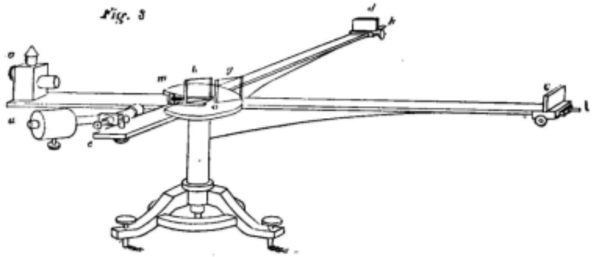
It will be observed that this apparatus can very easily be

made to serve as an "interferential refractor," and has the two important advantages of small cost, and wide separation of the two pencils.

The apparatus as above described was constructed by Schmidt and Hænsch of Berlin. It was placed on a stone pier in the Physical Institute, Berlin. The first observation showed, however, that owing to the extreme sensitiveness of the instrument to vibrations, the work could not be carried on during the day. The experiment was next tried at night. When the mirrors were placed half-way on the arms the fringes were visible, but their position could not be measured till after twelve o'clock, and then only at intervals. When the mirrors were moved out to the ends of the arms, the fringes were only occasionally visible.

It thus appeared that the experiments could not be performed in Berlin, and the apparatus was accordingly removed

Fig. 3



to the *Astrophysicalisches Observatorium* in Potsdam. Even here the ordinary stone piers did not suffice, and the apparatus was again transferred, this time to a cellar whose circular walls formed the foundation for the pier of the equatorial.

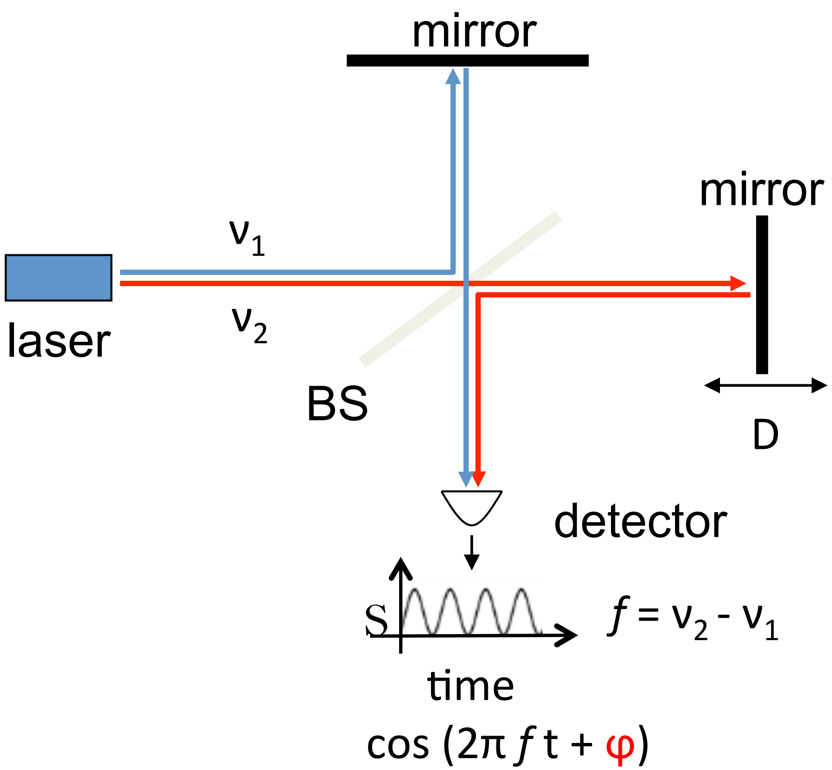
Here, the fringes under ordinary circumstances were sufficiently quiet to measure, but so extraordinarily sensitive was the instrument that the stamping of the pavement, about 100 meters from the observatory, made the fringes disappear entirely!

If this was the case with the instrument constructed with a view to avoid sensitiveness, what may we not expect from one made as sensitive as possible!

At this time of the year, early in April, the earth's motion in its orbit coincides roughly in longitude with the estimated direction of the motion of the solar system—namely, toward the constellation Hercules. The direction of this motion is inclined at an angle of about +26° to the plane of the equator,

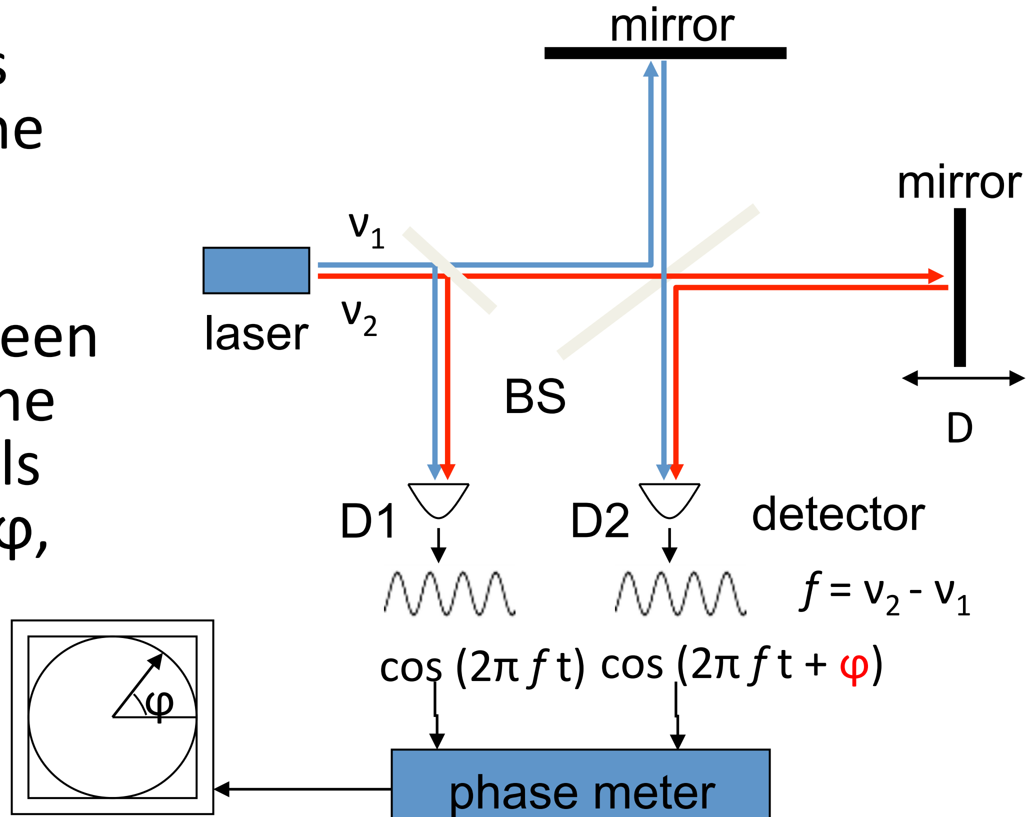
Heterodyne Michelson Interferometer

- The laser source generates two beams with slightly different frequencies ν_1 and ν_2
- The two beams are addressed to the two mirrors and than recombined on the detector
- A beat signal having frequency $f = \nu_2 - \nu_1$ is generated
- $S = 1 + \cos(2\pi f t + \varphi)$
- The phase φ is proportional (again) to D/λ

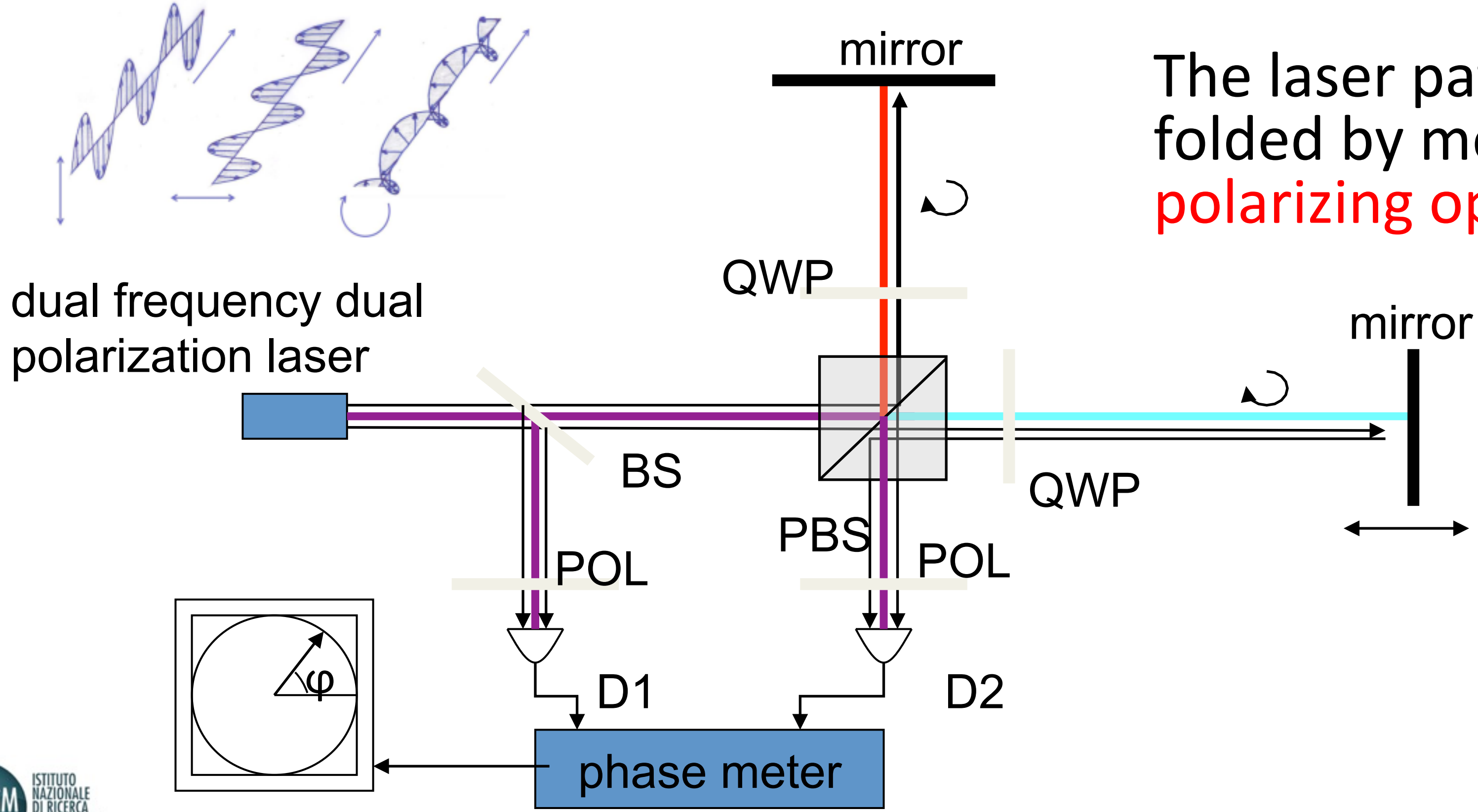


Heterodyne Michelson Interferometer

- A reference signal is generated before the beam separation
- A dynamic phase measurement between the reference and the measurement signals allows us to obtain φ, hence D/λ



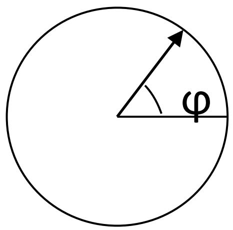
Practical realization of a heterodyne interferometer



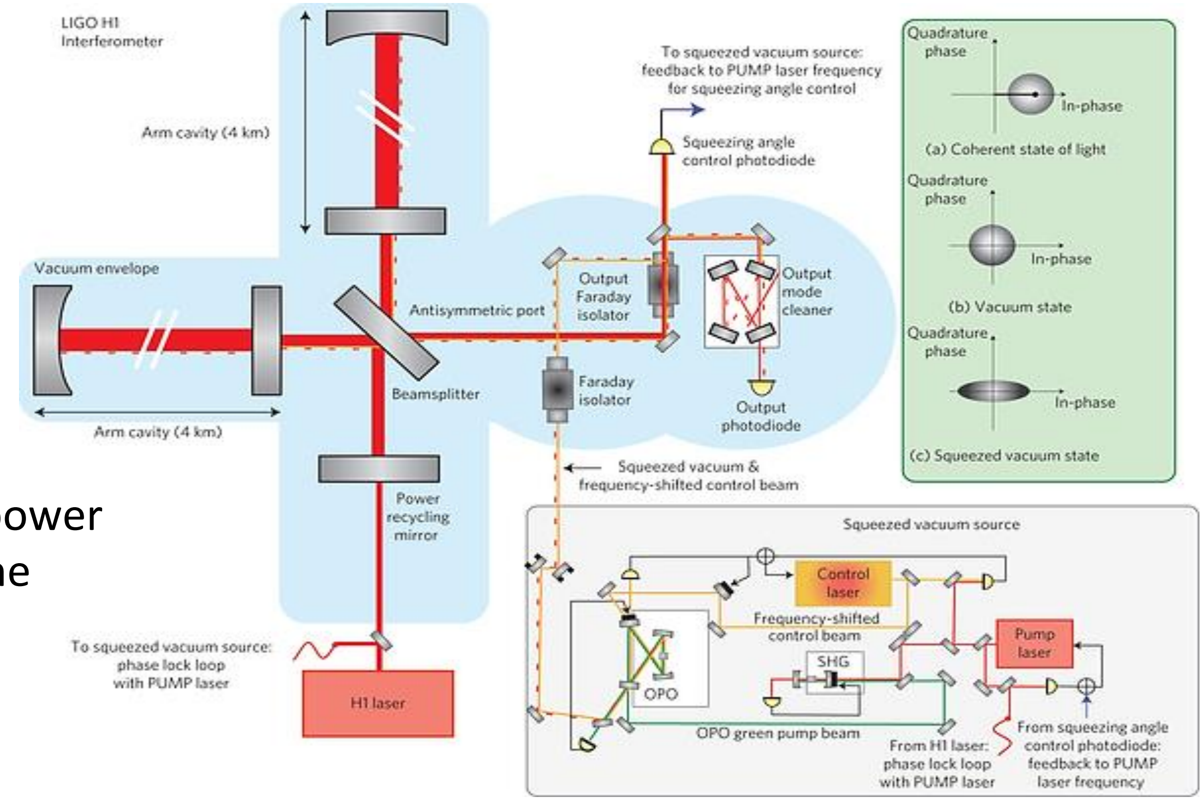
The laser path is folded by means of polarizing optics

Can we achieve infinite resolution?

- The resolution is given by the S/N ratio
- The noise limit is the shot noise: N = √#photons
- With a sufficiently high number of photons the S/N can be very high
- A S/N ≈ 10^6 can be «easily» achieved: with λ ≈ 1 μm → picometer resolution



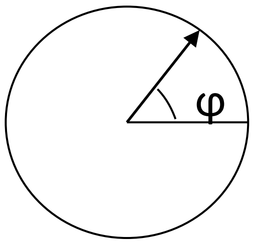
Ligo and Virgo interferometers: 10^{-18} m!



KW of optical power circulating in the interferometer

But, what about accuracy?

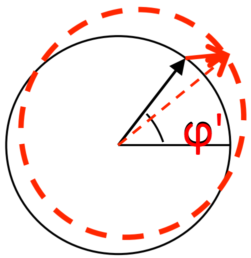
- The 2π phase must be divided into equal parts

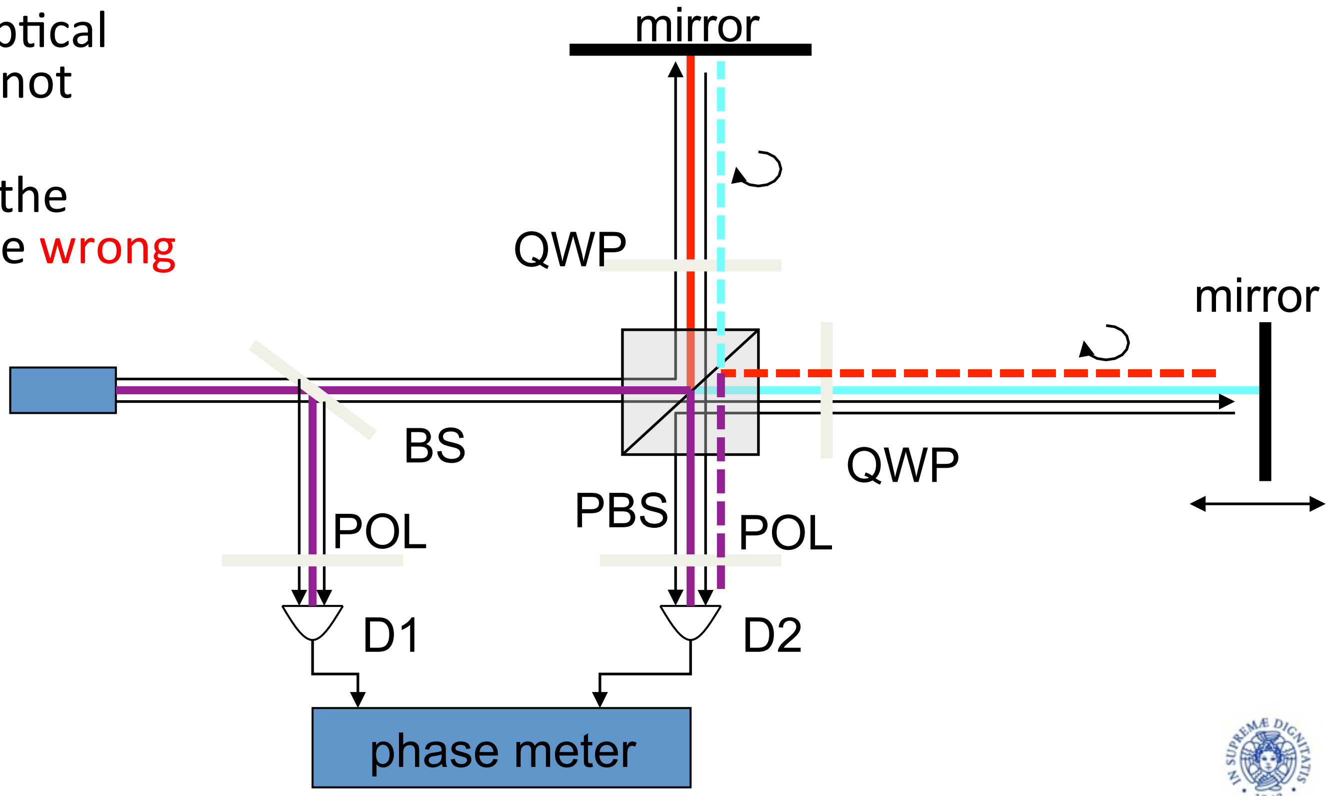


$$\varphi = (4\pi\, D/\lambda))$$

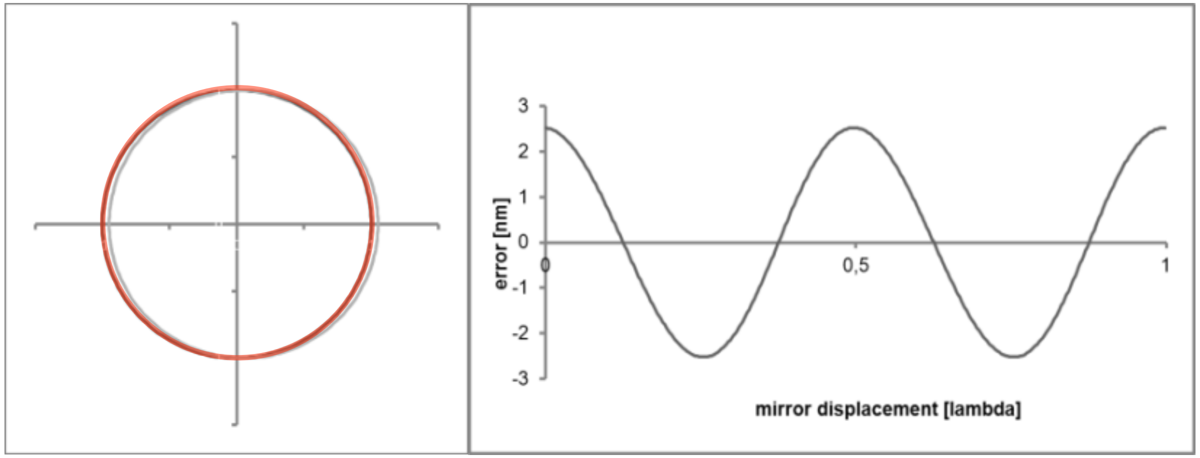
The polarization mixing
(or cross-talk, or cyclic error) problem

- Real polarizing optical components are not ideal
- Some portion of the beams goes in the wrong direction



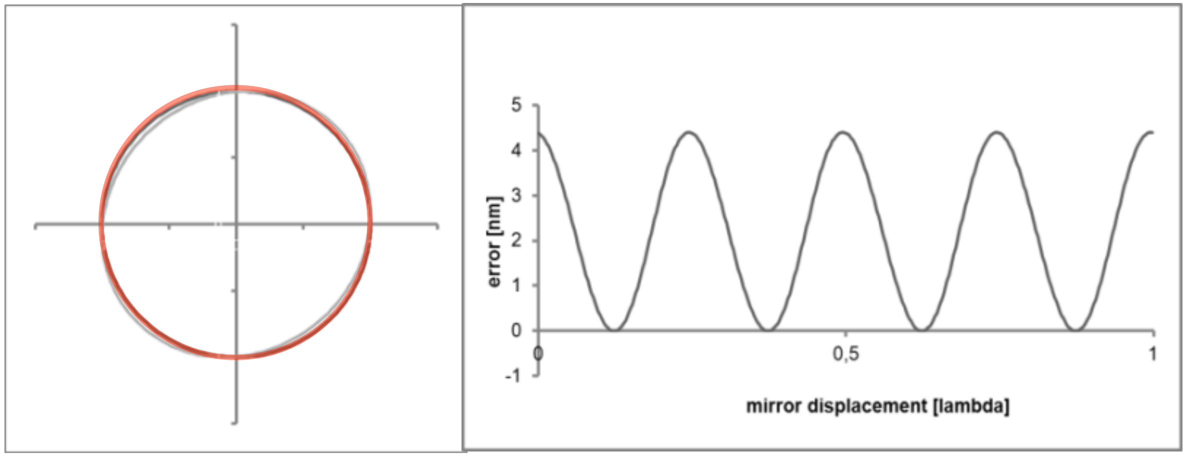


Cyclic error simulation



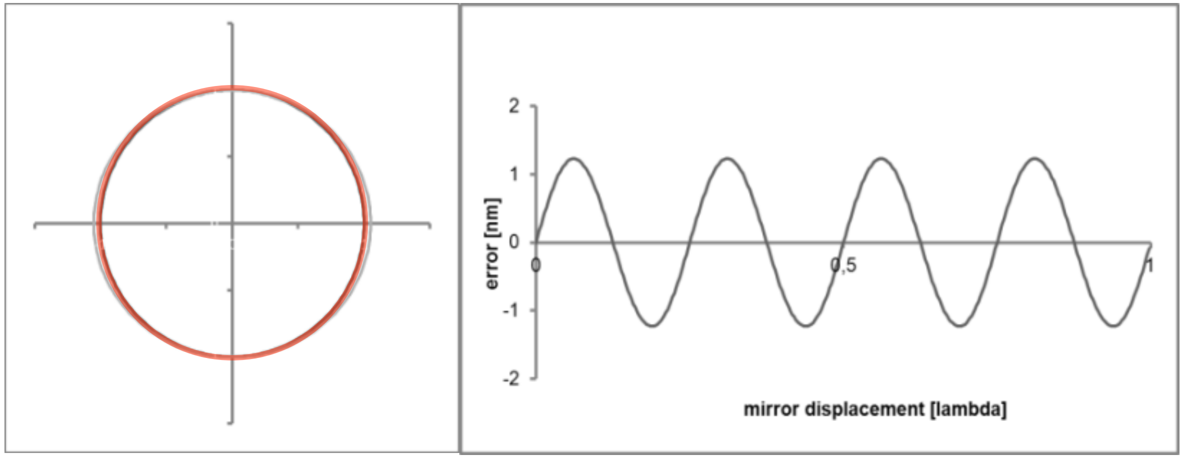
effect of 5 % offset unbalance

Cyclic error simulation



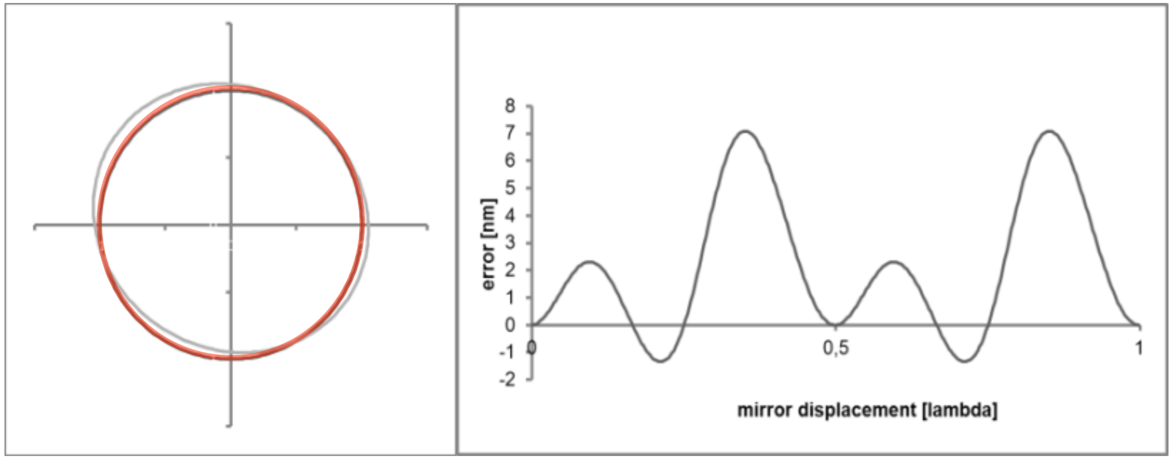
effect of 5 % gain unbalance

Cyclic error simulation



effect of 5° error with respect to perfect quadrature

Cyclic error simulation



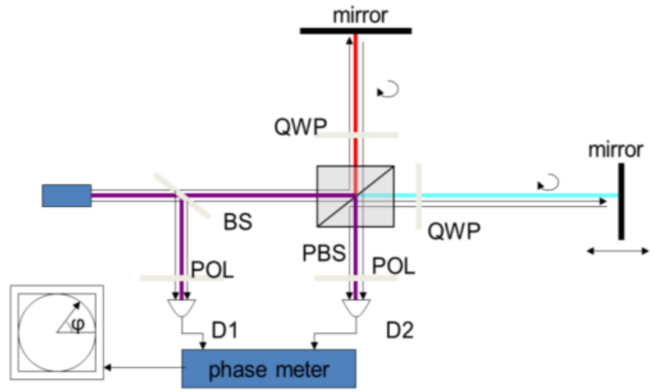
combination of the three

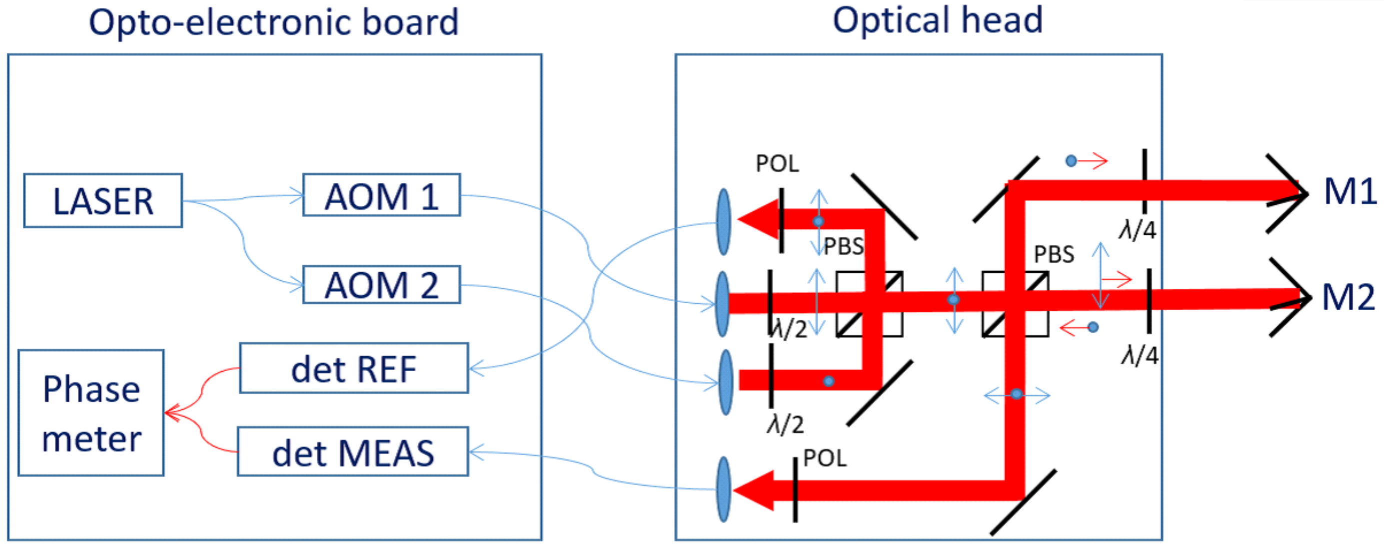
Ways to reduce cyclic errors

- Reduce lambda or multipy the optical path
- Use a Fabry-Perot interferometer
- Reduce mixing by implementing spatial separation

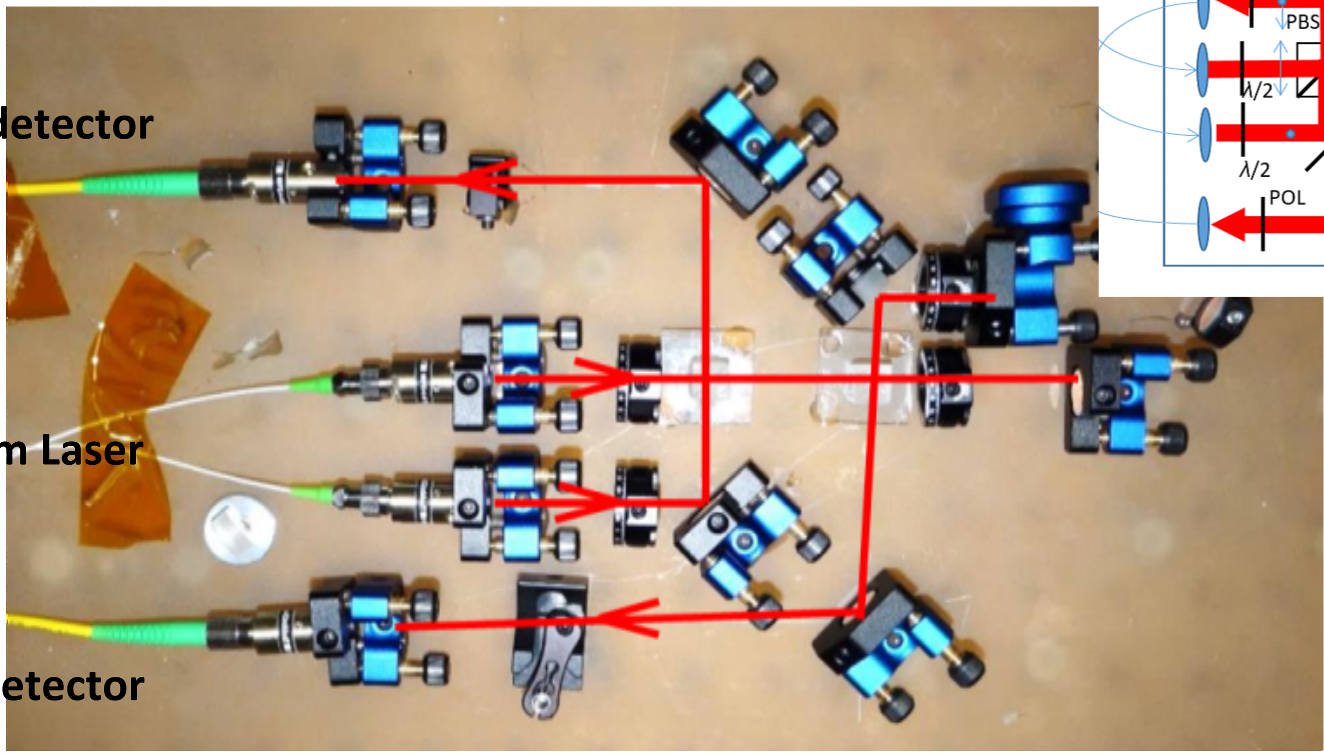
... cyclic errors small enough for LIG

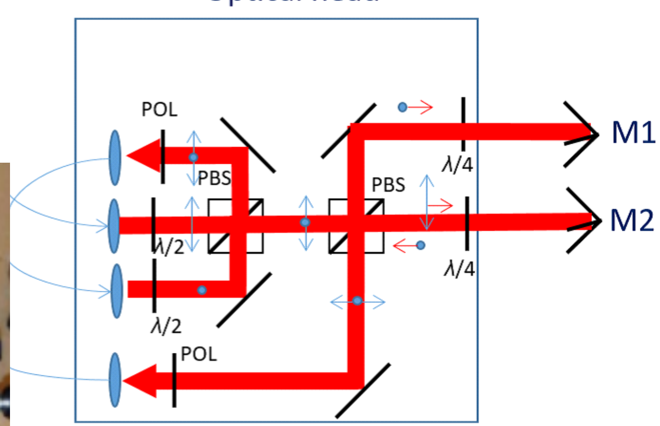
Prototype implementation of LIG





Prototype of LIG optical head





30 x 20 cm
Zerodur board