



The GG and GGG experiments

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(on behalf of the GG collaboration)

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What is special about WEP/UFF tests?



WEP/UFF is at the basis of Newtonian gravity and of GR (I)

Newtonian gravity rests on the experimental fact that inertial and gravitational mass are the same. This is the Equivalence Principle (EP) – later to become the Weak Equivalence Principle (WEP) – whereby in a gravitational field all bodies fall with the same acceleration (known as the Universality of Free Fall – UFF)

Newton regarded the experimental proof of this “Principle” so important that he tested it personally, and reported the results in the opening paragraph of the *Principia* (1687):

*“This quantity that I mean hereafter under the name of ... mass ... is known by the weight ... for it is proportional to the weight **as I have found by experiments on pendulums, very accurately made...**”*



WEP/UFF is at the basis of Newtonian gravity and of GR (II)

Einstein put UFF at the basis of General Relativity

.. but rumors are that he did not care about experimental tests and knew nothing about torsion balance tests performed in the same years by Eötvös and collaborators, which improved Bessel's pendulum experiments by at least 3 orders of magnitude!!

In the Editorial of CQG 2012 Focus Issue devoted to WEP, by Will & Speake, we read:

“Einstein took WEP for granted in his construction of general relativity, never once referring to the epochal experiments by Baron Eötvös”.

But it is not so! In *“The foundation of the General Theory of relativity”* (1916) §2 *The need for an extension of the postulate of relativity*, Einstein wrote:

*... This view is made possible for us by the teaching of experience as to the existence of a field of force, namely the gravitational field, which possesses the remarkable property of imparting the same acceleration to all bodies. Footnote: **Eötvös has proved experimentally that the gravitational field has this property in great accuracy.***

This footnote was not added in the English translation; it is there in the original paper in German!

WEP/UFF tests are null experiments

- Physical quantity to be measured: Δa
differential acceleration between test masses made of different material falling in the gravitational field of a source body with average acceleration a .
Dimensionless Eötvös parameter:

$$\eta = \frac{\Delta a}{a}$$

- Not an absolute measurement, like measuring G or the gravitational redshift (they require measurement to be compared with theoretical prediction of the effect measured, with knowledge/measurement of all physical parameters involved in the model) \implies
... can reach very high precision & accuracy
... **especially if performed as a differential experiment**: avoids recovering a very small physical quantity from the difference of two much larger ones



Why UFF/WEP tests can be more accurate than measurements of gravitational redshift by many orders of magnitude?

A measurement of gravitational redshift is an *absolute measurement*. GP-A result (PRL 1980) is:

$$\eta = \frac{\Delta a}{a}$$

$$\left(\frac{\Delta \nu}{\nu} \right)_{GP-A} = [1 + (2.5 \pm 70) \cdot 10^{-6}] \cdot \left(\frac{\varphi_s - \varphi_e}{c^2} - \frac{|\vec{v}_s - \vec{v}_e|^2}{c^2} - \frac{\vec{r}_{se} \cdot \vec{a}_e}{c^2} \right)$$

If TMs are coupled the experiment measures Δa directly, hence η : no experiment signal, no violation (to the level of noise); the smaller the signal (or the noise), the better the test.

No prediction must be made to which the measured signal should be compared in order to obtain the physical quantity of interest!

... you must “only” beat random errors and carefully check systematics...

The measured frequency shift had to be compared with the sum of the 3 terms (gravitational potential difference, second order Doppler shift, residual of first order Doppler), whose values depend on various physical quantities, some of which to be measured during the experiment itself. It is only by comparing the theoretical prediction and the measured shift that the authors could establish the ratio $[1 + (2.5 \pm 70) \cdot 10^{-6}]$ for a measurement of gravitational redshift to 1st order.

It took 4 years to publish the results of an experiment that lasted only about 2 hours!

... more difficult as clocks improve; **measurement to 2nd order still out of reach**; experimental result very hard to interpret (especially for space measurements). What if a discrepancy is found? Would it question GR or call for a better physical model?



“On the universality of free fall, the equivalence principle, and the gravitational redshift”, AJP 2013

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2 and the gravitational redshift**

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21 (Received 10 August 2012; accepted 15 March 2013)

22 Through the contributions of Galileo, Newton, and Einstein, we recall the universality of free fall
23 (UFF), the weak equivalence principle (WEP), and the strong equivalence principle (SEP), in order
24 to stress that general relativity requires all test masses to be equally accelerated in a gravitational
25 field; that is, it requires UFF and WEP to hold. The possibility of testing this crucial fact with null,
26 highly sensitive experiments makes these the most powerful tests of the theory. Following Schiff,
27 we derive the gravitational redshift from the WEP and special relativity and show that, as long as
28 clocks are affected by a gravitating body like normal matter, measurement of the redshift is a test
29 of UFF/WEP but cannot compete with direct null tests. A new measurement of the gravitational
30 redshift based on free-falling cold atoms and an absolute gravimeter is not competitive either.
31 Finally, we compare UFF/WEP experiments using macroscopic masses as test bodies in one case
32 and cold atoms in the other. We conclude that there is no difference in the nature of the test and
33 that the merit of any such experiment rests on the accuracy it can achieve and on the physical
34 differences between the elements it can test, macroscopic proof masses being superior in both
35 respects. © 2013 American Association of Physics Teachers.
36 [http://dx.doi.org/10.1119/1.4798583]
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Why have torsion balances defeated Galileo-like mass dropping tests?



Release errors in mass dropping tests

Any position difference (error) at initial time in the distance of TMs to the source body perfectly mimics a violation (velocity errors also matter..):

$$\eta_{class} = 3 \frac{\Delta h}{d}$$

True on ground as well as in space, whatever the test masses (macroscopic as well as cold atoms), whatever the time of fall...

Blaser CQG 2001; Nobili et al. GRG 2008

So far have wiped out the advantage of a very strong driving acceleration:

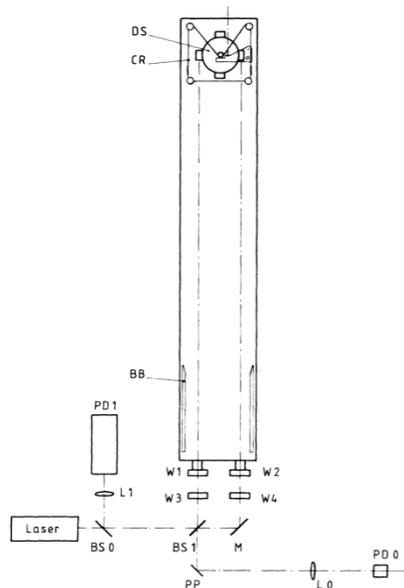
$$\eta = \Delta a/a, a = 9.8 \text{ m/s}^2 \text{ (slightly less in low Earth orbit)}$$

- 500 times larger than for torsion balances in the field of the Earth
- 1600 times larger in the field of the Sun

Release errors in mass dropping tests: the GAL experiment (I)

GAL a modern differential mass dropping experiment (Carusotto, Polacco et al. PRL, 1992)

- Clever idea + laser interferometer read-out to try compete with torsion balance.
- If WEP/UFF holds a disk made of two halves of different material should not rotate.
- Rotation angle measured with modified Michelson interferometer
- The fringe frequency shift, proportional to disk angular acceleration is the effect to be measured.





Release errors in mass dropping tests: the GAL experiment (II)

Experimental results show that the torsion balance was far from being defeated!!

Homogeneous *Al* disk, 70 runs:

$$\frac{\Delta g}{g} = (3.2 \pm 9.5) \cdot 10^{-10}$$

Al-Cu disk, 63+65 drops (disk reversed):

$$\left(\frac{\Delta g}{g}\right)_{Al-Cu} = (2.9 \pm 7.2) \cdot 10^{-10}$$

Carusotto, Polacco et al, PRL 1992



What's magic about the torsion balance?



What's magic about the torsion balance (I)

Signal much weaker than in mass dropping, but..

- If fiber is thin, it has very low natural frequency (Eöt-Wash group balance 798 s period). TMs very weakly coupled \Rightarrow highly sensitive to differential effects
- **On ground (not in space!)** suspension fiber aligns itself with local gravity (towards the center of mass of the Earth) \Rightarrow common mode forces are rejected (almost) perfectly by physics (no torque from common mode forces, no deflection of the wire, no residual differential signal.. (almost))



What's magic about the torsion balance (II)

Violation signal from Earth DC, but..

- Choosing Sun as source (signal a factor 3 weaker than from Earth): Earth's rotation up-converts DC signal to diurnal frequency... “passive” rotation of the balance. **First exploited by Dicke: 3 orders of magnitude improvement w.r.t Eötvös; 1 more gained by Braginsky & Panov**
- If balance rotates on a turntable (20' reached by Eöt-Wash group) signal from Earth modulated to higher frequency (+ effects of daily disturbances reduced) and signal from Sun modulated too. Almost 1 order of magnitude improvement in the field of the Sun; 4 orders of magnitude improvement in the field of the Earth



*What can space (low Earth orbit) provide
which is not available on ground??*



The advantages of space for testing WEP/UFF

- Signal (from Earth) only slightly smaller than in Galileo dropping tests on ground ($\simeq 8 \text{ m/s}^2$): $\simeq 500$ times stronger than in ground balances with Earth as source and $\simeq 1400$ with Sun as source. Note: does not apply to mass-dropping tests
- Absence of weight: on ground the balance is suspended against $1 g$, in space against $a_{\text{iner-drag}} \simeq 10^{-8} g$ (the largest acceleration on TMs is the inertial acceleration in response to air drag of the s/c – with GG numbers) \Rightarrow suspending 100 kg mass in GG is like suspending 1 mg on ground! \Rightarrow low stiffness, low natural frequency, high sensitivity..
- “lab” (the spacecraft) isolated in space: local disturbances (from terrain tilts, nearby masses...) much reduced provided that a dedicated and well designed s/c is used..
- If s/c attitude is kept fixed in space (actively) violation signal is at the orbital frequency (100' period). s/c rotation would up-convert it to higher frequency. GG is stabilized by 1-axis rotation at 1 Hz provided once for all at mission start, angular momentum conservation, no motor, no bearings, whole “lab” co-rotating. “Passive” rotation as in Dicke experiment...

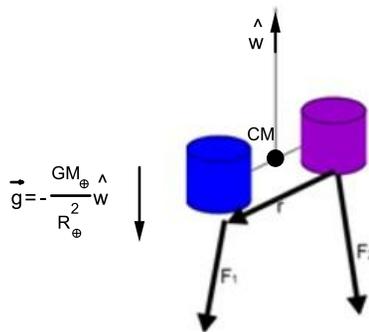


Why not flying a torsion balance?

A torsion balance in space

Perfect common mode rejection needs 1g and is lost in weightlessness conditions

\vec{F}_1 and \vec{F}_2 are the forces acting on each mass. Their vectorial sum applied to the center of mass CM is balanced, on the ground, by the suspension fiber directed along \hat{w} (to the center of mass of the Earth). Only the component of the total torque along \hat{w} does twist the wire. It is found to be:



$$T_w = \frac{\vec{r} \cdot \vec{F}_1 \times \vec{F}_2}{|\vec{F}_1 + \vec{F}_2|}, \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

- only forces not parallel to each other do twist the wire
- forces parallel to each other (of equal as well as different size) do not twist the wire

In space the largest common mode effect comes from residual air drag on s/c:

$$a_{iner-drag} \simeq 10^{-8} g \simeq 10^7 \Delta a_{EP} \quad (\eta_{GG} = 10^{-17})$$

..even if partially compensated by drag free control, common mode rejection is needed...



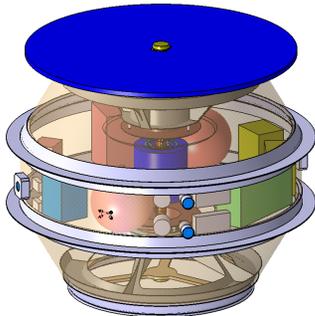
*GG: a “balance” and its spacecraft
for testing WEP to 10^{-17} in the field of the Earth*



The reasons behind every choice..

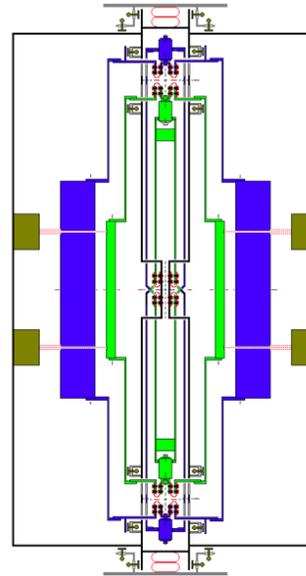
- TMs are cylinders; they should be weakly coupled to form a balance very sensitive to differential accelerations (low natural coupling frequency) with possibility to reduce common mode effects as much as possible
- TMs should be concentric to reduce classical tidal (differential) effects \Rightarrow concentric co-axial cylinders
- Each TM orbiting the Earth is a 2-body problem, with 2DOF (orbital plane) \Rightarrow the balance should be sensitive in 2D too \Rightarrow the plane perpendicular to the symmetry axis of the cylinders is the sensitive plane and lies, nominally, in the plane of the orbit \Rightarrow violation signal is a vector pointing to the CM of the Earth as the balance orbits around it (constant size if orbit circular) - it is at the orbital frequency
- Rotation around the symmetry axis of the cylinders will up-convert the signal to the rotation frequency. If the s/c has the same cylindrical symmetry, stabilizing it by 1-axis rotation around it will provide, after initial spin-up, “passive” rotation of the whole system. Note: since entire “lab” rotates, local mass anomalies give DC effects \Rightarrow no terrific requirements on mass test manufacture \Rightarrow ample choice of materials, also H rich like polyethylene can be considered...
- Since the test needs low coupling frequency and high spin rate, this is by definition a rotor in supercritical regime. Theory & long record of such rotors tell us that while it is highly unstable in 1D, in 2D it provides self centering (by physics). There is a known weak instability (whirl motion) at known frequency (natural, away from signal frequency) which does not interfere with the measurement

GG: how it looks like (I)



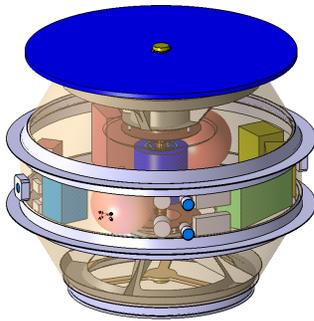
The GG satellite. 400 kg total mass, 1.4 m width, 1.2 m height to be flown in a standard near circular, low altitude (≈ 600 km) sun-synchronous orbit for a 9-month mission duration.

The GG differential acceleration sensor is located at the center of mass (outer test cylinder visible in blu). \rightarrow see section along spin axis to the right..

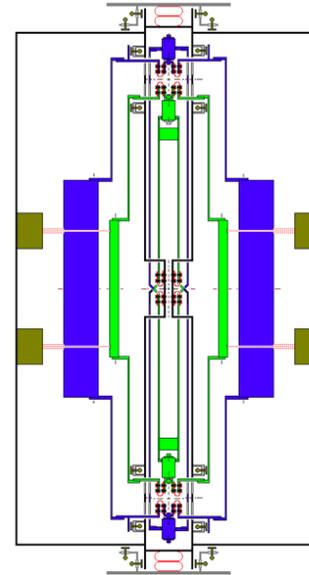


Section of GG balance along symmetry/rotation axis. Inner and outer test cylinder (green and blue) are suspended at their centers of mass from the PGB (an intermediate suspension stage sketched with a rectangular section) which encloses the sensor, screens it from various disturbances and mounts instrumentation (here the read-out laser gauge is shown)

GG: how it looks like (II)



Note the cylindrical symmetry. Attitude stabilization is by rotation around symmetry axis



Relative displacements of the test cylinders are read by a low noise laser interferometry gauge developed at JPL.

In essence the sensor is a beam balance with concentric test masses rotating around its beam.

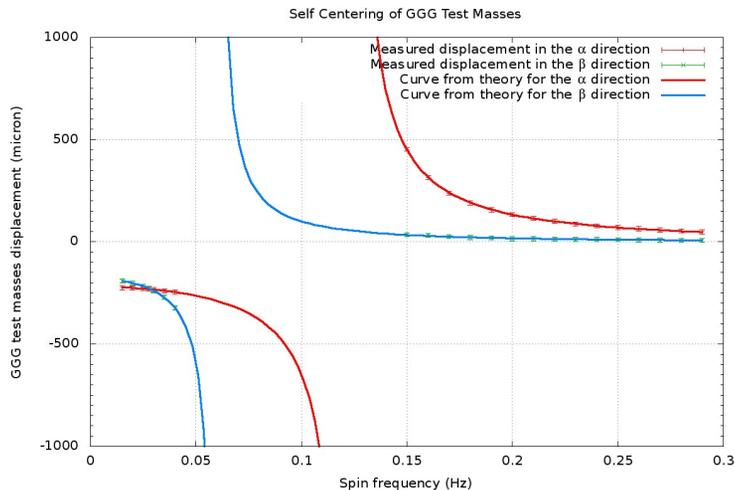
Each arm has 2 parts and they are all adjustable (with capacitors as sensors and PZT as actuators) for common mode rejection.

How physics allows rapid rotation in 2D

The centers of mass of the test bodies cannot be perfectly concentric. The offset vector $\vec{\varepsilon}$ (fixed with the rotating masses) is not zero but in 2D it is reduced by the factor $\omega_{diff}^2/\omega_{spin}^2$. The solution (in the non rotating frame) is:

$$\vec{r}(t) \simeq -\varepsilon \left(\frac{\omega_{diff}^2}{\omega_{spin}^2 - \omega_{diff}^2} \right) \begin{pmatrix} \cos(\omega_{spin}t + \varphi) \\ \sin(\omega_{spin}t + \varphi) \end{pmatrix} \simeq -\varepsilon \left(\frac{\omega_{diff}^2}{\omega_{spin}^2} \right) \begin{pmatrix} \cos(\omega_{spin}t + \varphi) \\ \sin(\omega_{spin}t + \varphi) \end{pmatrix}$$

Proof masses are centered on one another by physics.



Experimental data from the GGG accelerometer agree with the theoretical curves in both directions α, β of the rotating plane:

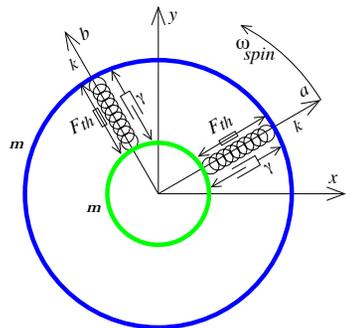
$$r_{\alpha,\beta}(\nu_{spin}) = \varepsilon_{\alpha,\beta} \cdot \frac{\nu_{\alpha,\beta}^2}{\nu_{\alpha,\beta}^2 - \nu_{spin}^2}$$



“Abatement” of thermal noise by rotation



The ultimate limitation to UFF tests comes from thermal noise due to internal dissipation in the suspensions: shown experimentally by slowly rotating torsion balances (Adelberger et al., PPNP 2009)



The integration time required to reduce thermal noise random force below the force signal $F(\omega_{signal})$ with a given SNR is:

$$t_{int} = SNR^2 \frac{\langle |\hat{F}_{th}(\omega_{signal})|^2 \rangle}{F(\omega_{signal})^2}$$

In the 2D rotating oscillator the signal is up-converted to $\omega_{spin} \gg \omega_{signal}$, its effect is not attenuated while the thermal noise force due to internal dissipation with loss angle $\phi = 1/Q$ is reduced to:

$$\langle |\hat{F}_{th}(\omega_{spin})|^2 \rangle \simeq 4K_B T \frac{\mu \omega_{diff}^2 \phi(\omega_{spin})}{\omega_{spin}} \quad \downarrow$$

*Integration time is reduced by the (very large) factor $\omega_{signal}/\omega_{spin} \gg 1$
More effective than cryogenics. (Pegna et al., PRL 2011)*



Integration time for GG to reach 10^{-17}

By up-converting the signal to 1 Hz GG thermal noise from internal damping is reduced to slightly below the level of gas damping and eddy currents thermal noise, yielding a total integration time of 3 to 4 *hr* for SNR=2 (0.6 pm displacement violation signal expected)

Pegna et al. 2013, submitted

⇒ readout with such low noise needed. JPL laser gauge has demonstrated 1 pm/ $\sqrt{\text{Hz}}$ noise @ 1 Hz, and less at higher frequencies. Purely differential and linear (no dynamic range issue). It works also at 2-3 cm separation as in GG test masses.

In 1 day (almost 15 GG orbits, 6-8 times the integration time) a reliable test to 10^{-17} is expected

1.4 d integration time is estimated for μSCOPE to reach 10^{-15} . It means that current thermal noise should be reduced by a factor 10^4 to reach 2 orders of magnitude better with the same integration time (should thermal noise be the limiting factor).



What if a “violation-like” effect is measured?



GG null checks (I)

Violation signal: we know the frequency, the phase, that it must not change sign
 \Rightarrow *lock-in detection*

We have a 2D sensor and can make a complex Fourier analysis of data:
 $z = x + iy$ in the non spinning frame; $\zeta = a + ib$ in the rotating frame;
If the s/c spins at $+\omega_{spin}$ a violation signal $z = \rho e^{\pm\omega_{orb}t}$ from the Earth is measured by the rotating read-out as

$$\zeta = \rho e^{i(-\omega_{spin}t \pm \omega_{orb})t}$$

i.e. in the complex FFT the line of the signal must be found close to $-\omega_{spin}$ only (noise close to ω_{spin} is eliminated (2D sensor + complex FFT allows a factor 2 to be gained, not $\sqrt{2}$, and at one specific side of it depending on the sign of ω_{orb}

...not enough for systematics at the same frequency as the violation signal (the Earth's monopole coupling differently with TMs quadrupole moment) or close to it (for the rotating sensor) like Earth tides (at twice the orbital frequency)...



GG null checks (II)

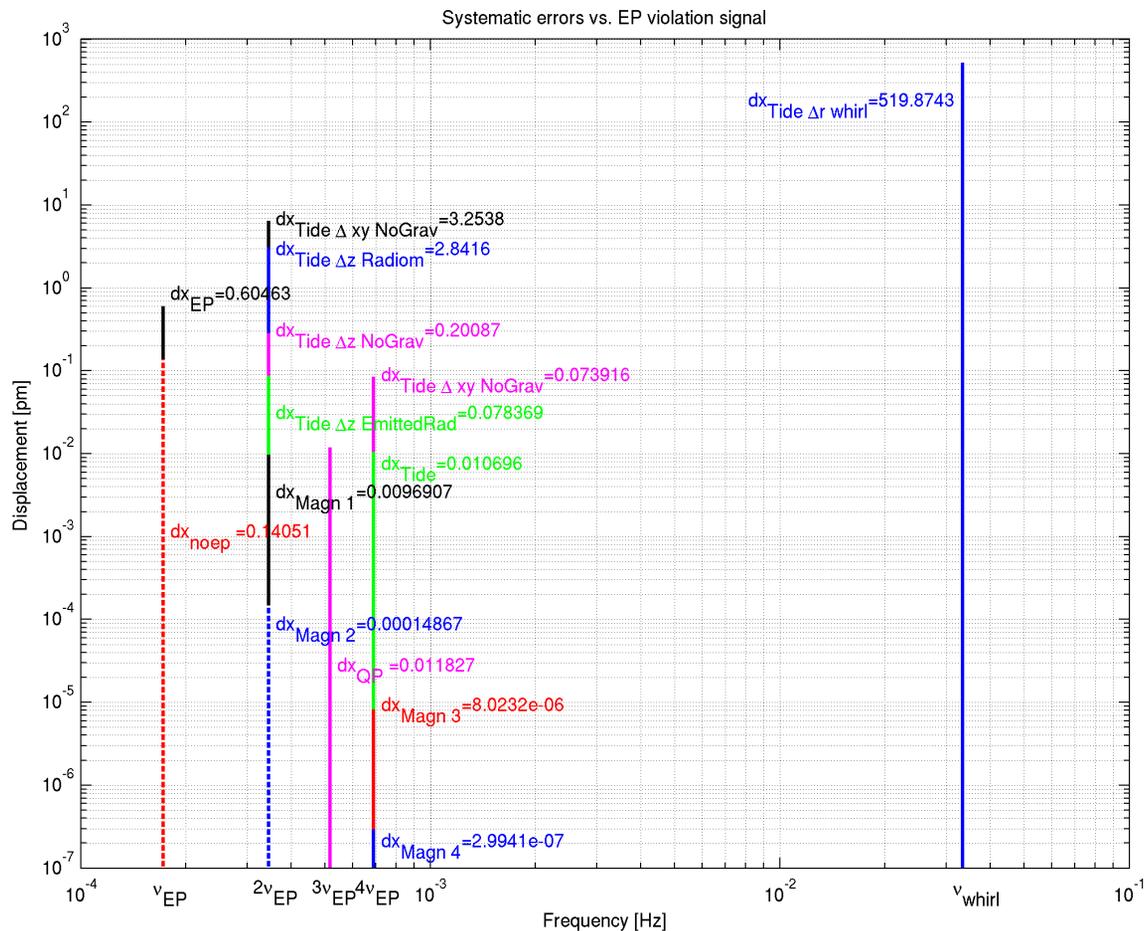
In 9-month mission and 1 WEP test to 10^{-17} per day, passive stability of GG spin axis under while the regression of the node of the orbit due to the quadrupole mass moment of the Earth (J_2) changes the inclination of the spin/symmetry axis to the orbit plane by $\simeq 1$ deg/day allows the most dangerous known sources of systematics to be identified and discriminated from the signal due to their different physical signature (known from celestial mechanics).

We have demonstrated that in such long term run analysis the quadrupole effect will appear at $3\nu_{orb}$ in addition to its dominant term at ν_{orb} (also ν_{WEP}) while tides appear at $4\nu_{orb}$ in addition to $2\nu_{orb}$

This analysis is done offline; is extremely rigorous (based on celestial mechanics...); requires no sensor in addition to the one at the center of mass of the whole satellite.



GG null checks (III)





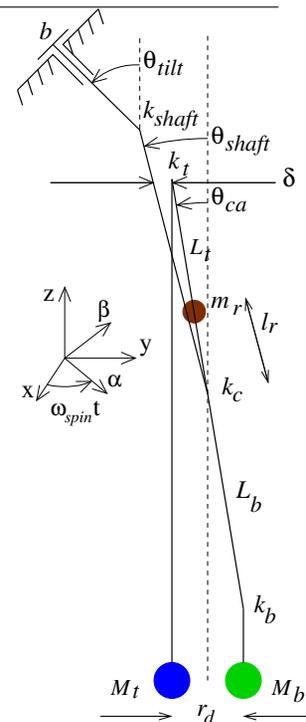
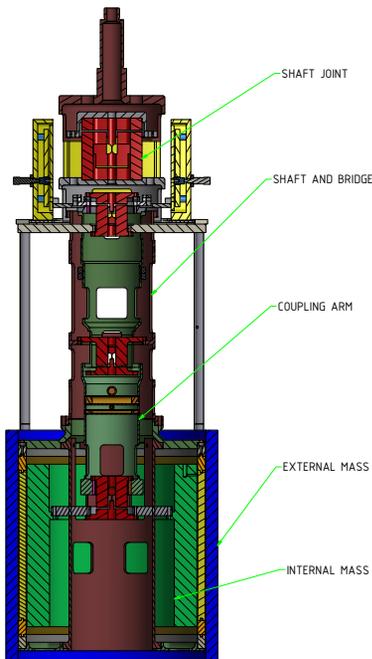
GG on Ground (GGG)

Possible because the GG sensor has 2 DOF: use spin/symmetry axis to suspend it, sensitive in the horizontal plane of lab (same number of DOF as in space), full scale, rotation in supercritical regime...

... remember: **it is the prototype of a sensor designed and optimized for space. At 1 g torsion balances are better (much higher sensitivity..)**



The GGG prototype



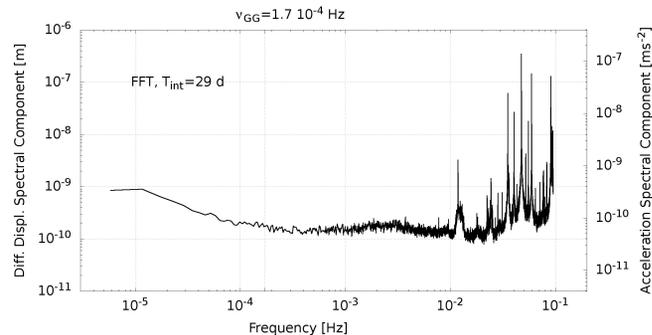
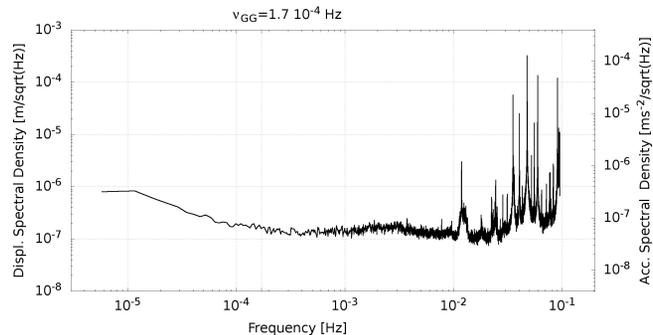
GG in space needs no motor no bearings, has no “terrain” tilts, has weaker coupling and higher sensitivity by 3 orders of magnitude; the driving signal from Earth is 500 times stronger ...yet the key features are the same as in GGG

Monolithic rotating 2D joint provides attenuation of low frequency terrain microseismicity (much better than active control in closed loop on conventional tiltmeter...)

Discovered way of rejecting tilts. But currently dominated by ball bearings rotation noise close to spin frequency on the rotor (ball bearings degradation with time..)



GGG low frequency sensitivity



At $\nu_{GG} \simeq 1.7 \cdot 10^{-4}$ Hz the measured relative displacement SD is $\simeq 1.8 \cdot 10^{-7} \text{m}/\sqrt{\text{Hz}}$ and the measured relative acceleration SD is $\simeq 6 \cdot 10^{-8} \text{ms}^{-2}/\sqrt{\text{Hz}}$

Over 29 d the integrated differential displacement noise @ ν_{GG} is $\simeq 180$ pm (GG in space must detect 0.6 pm) and the differential acceleration noise is $\simeq 7 \cdot 10^{-11} \text{m/s}^2$

Sensitivity to WEP violation in the field of the Sun (diurnal frequency, $a_{\odot-PI} \simeq 0.0057 \text{ms}^{-2}$) is:

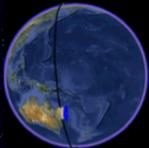
$$\eta_{GGG-\odot} \simeq \frac{3.5 \cdot 10^{-10} \text{ms}^{-2}}{a_{\odot-PI}} \simeq 6 \cdot 10^{-8}$$

Best GGG sensitivity if used as prototype of GG experiment in space:

$$\eta_{1.7 \cdot 10^{-4} \text{Hz}} \simeq \frac{7 \cdot 10^{-11} \text{m/s}^2}{8 \text{m/s}^2} \simeq 8.9 \cdot 10^{-12}$$

You are welcome to visit the GG website
<http://eotvos.dm.unipi.it>

"GALILEO GALILEI" GG



A small satellite to test the Equivalence Principle of Galileo, Newton and Einstein to 1 part in 10^{17} (4 orders of magnitude improvement)



ESA debriefing on GG proposal for a small mission opportunity in 2017

- On the universality of free fall, the equivalence principle and the gravitational redshift, Am. J. Phys., paper in press & talk presented at Microscope Colloquium II, Paris January 2013
- Sensitivity and error budget of the GGG laboratory prototype, Q2C5 October 2012
- GG: space test of the WEP to 10^{-17} and laboratory demonstrations, Class. Quantum Grav. focus issue on WEP, August 2012
- Abatement of thermal noise due to internal damping in 2D oscillators with rapidly rotating test masses, Phys. Rev. Lett, 2011
- Report of GG space mission Study (Phase A2 level), as of April 2009
- Testing the Weak Equivalence Principle: a review in 2009
- The GGG ("GG on the Ground") lab prototype

Documents and papers

