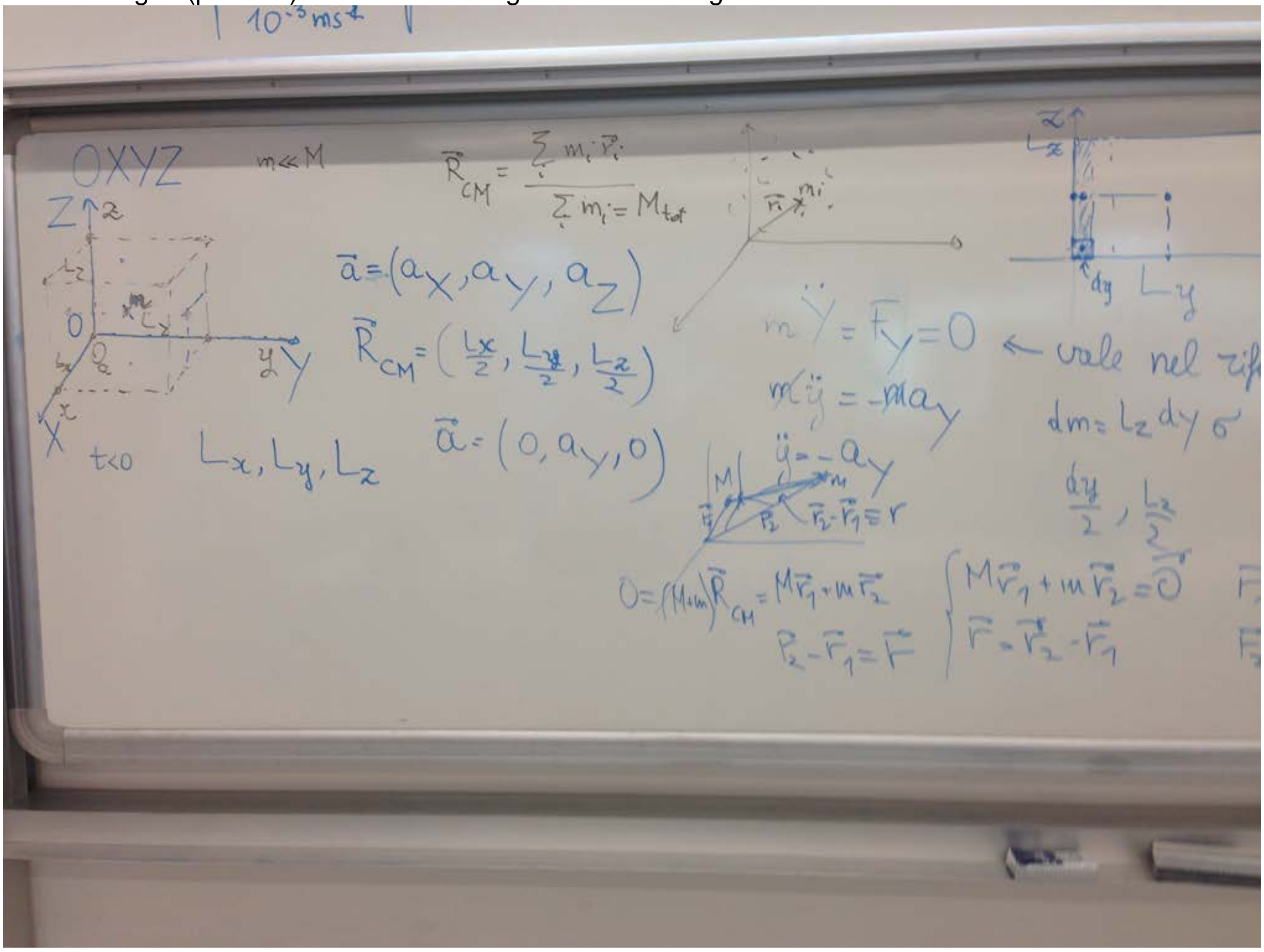
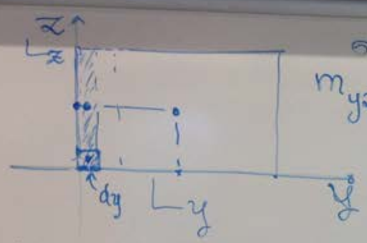
Fisica 1 per studenti di chimica - Anno accademico 2014-2015 - Secondo Semestre - Anna Nobili

17 Febbraio 2015

- Correzione del compitino del 16 Dicembre 2014
- Consegna (parziale) e discussione degli errori con i singoli studenti



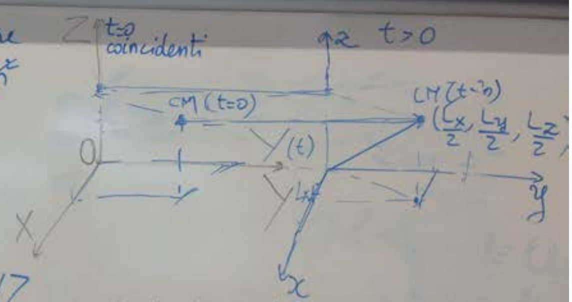
$$\frac{g}{a_y}\,y(t_1') + \frac{L_z}{2} - \frac{g}{a_y}\frac{L_y}{2} > 0 \qquad a_y \geq g\,\frac{L_y}{L_z}$$



σ kg/m² $\quad \sigma$ uniforme

$m_{yz} = \sigma L_y L_z \qquad$ kg $\cancel{m^{-2}}\,\cancel{m^2}$

$\vec{R}_{CM} = \left(\frac{L_y}{2}, \frac{L_z}{2}\right)$



← vale nel riferimento inerziale OXYZ

$dm = L_z\, dy\, \sigma$

$\frac{dy}{2}, \frac{L_z}{2}$

$Y(t) = \frac{1}{2} a_y t^2 + y(t)$

$\dot{Y}(t) = a_y t + \dot{y}(t)$

$\ddot{Y}(t) = a_y + \ddot{y}(t) \implies \ddot{y}(t) = \ldots$

0

$M\vec{r}_1 + m\vec{r}_2 = \vec{0} \qquad \vec{F}_1 = -\frac{m}{M}\vec{r} \qquad \frac{|\vec{r}_1|}{|\vec{r}_2|} = \frac{m}{M}$

$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{F}_2 = +\frac{M}{m}\vec{r}$

$t<0 \qquad L_x, L_y, L_z$

$M \qquad \vec{g} = -a_y \qquad m \qquad \vec{r}_1 \qquad \vec{r}_2 \qquad \vec{r}_2 - \vec{r}_1 = r$

$\frac{dy}{2}, \frac{L_z}{2}$

$\ddot{y}(t) = -a_y$

$\dot{y}(t) = -a_y \int_0^t dt' = -a_y t + \cancel{\dot{y}(0)}$

$y(t) = -\frac{1}{2} a_y t^2 + y(0) = -\frac{1}{2} a_y t^2 + \frac{L_y}{2}$

$$\boxed{y(t) = -\frac{1}{2} a_y t^2 + \frac{L_y}{2}}$$

$y(t_{fin}) = 0 = -\frac{1}{2} a_y t_{fin}^2 + \frac{L_y}{2}$

$t_{fin} = \sqrt{\frac{L_y}{a_y}} = \sqrt{\frac{10\,m}{10^{-3}\,m s^{-2}}} = \sqrt{10^4 s^2} = 10^2 s$

$$\begin{cases} \ddot{y}(t) = -a_y \\ \ddot{z}(t) = -g \end{cases}$$

$$\begin{cases} \dot{y}(t) = -a_y t \\ \dot{z}(t) = -g t \end{cases}$$

$$\begin{cases} y(t) = -\frac{1}{2} a_y t^2 + \frac{L_y}{2} \\ z(t) = -\frac{1}{2} g t^2 + \frac{L_z}{2} \end{cases}$$

$\vec r_2 - \vec r_1 = \vec r$

$\dot y(t) = a_y t + \cancel{\dot y}(t)$

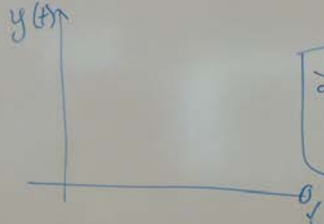
$$\begin{cases} \ddot y(t) = -a_y \\ \ddot z(t) = -g \end{cases}$$

$R_\oplus, M_\oplus$

$\cancel{m}\ddot z = -\dfrac{G M_\oplus \cancel{m}}{R_\oplus^2}$

$\ddot z = -\dfrac{G M_\oplus}{R_\oplus^2} = -g \simeq$

$$\begin{cases} \dot y(t) = -a_y t \\ \dot z(t) = -g t \end{cases}$$



$\boxed{z(t) = \dfrac{g}{a_y} y(t) + \left(\dfrac{L_z}{2} - \dfrac{g}{a_y} \dots\right.}$

$$\begin{cases} y(t) = -\frac{1}{2} a_y t^2 + \frac{L_y}{2} \\ z(t) = -\frac{1}{2} g t^2 + \frac{L_z}{2} \end{cases}$$

t_{fi}: $y(t_{fi}) = 0$ $\quad L_z >$

$t'_{fi} = \sqrt{\dfrac{L_z}{g}}$ $\quad t_{fi} = \sqrt{\dfrac{L_y}{a_y}}$

$a_y \geq$

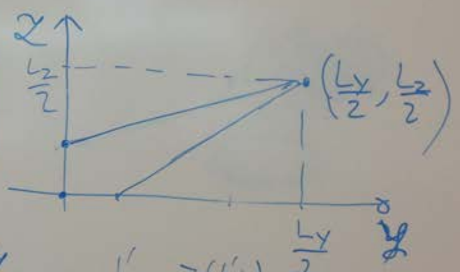
$\dfrac{g}{a_y} y(t'_{fi}) + \dfrac{L_z}{2} - \dfrac{g}{a_y}\dfrac{L_y}{2} \geq$

$\dot y(t) = a_y t + \cancel{\dot y}(t)$

$\cancel{m}\ddot z = -\dfrac{G M_\oplus \cancel{m}}{R_\oplus^2}$

$\ddot z = -\dfrac{G M_\oplus}{R_\oplus^2} = -g \simeq -9.8\,\mathrm{m\,s^{-2}}$

$$\boxed{z(t) = \frac{g}{a_y} y(t) + \left(\frac{L_z}{2} - \frac{g}{a_y}\frac{L_y}{2}\right)}$$



t_{fi}: $y(t_{fi}) = 0$ $\quad L_z > \dfrac{g}{a_y} L_y$

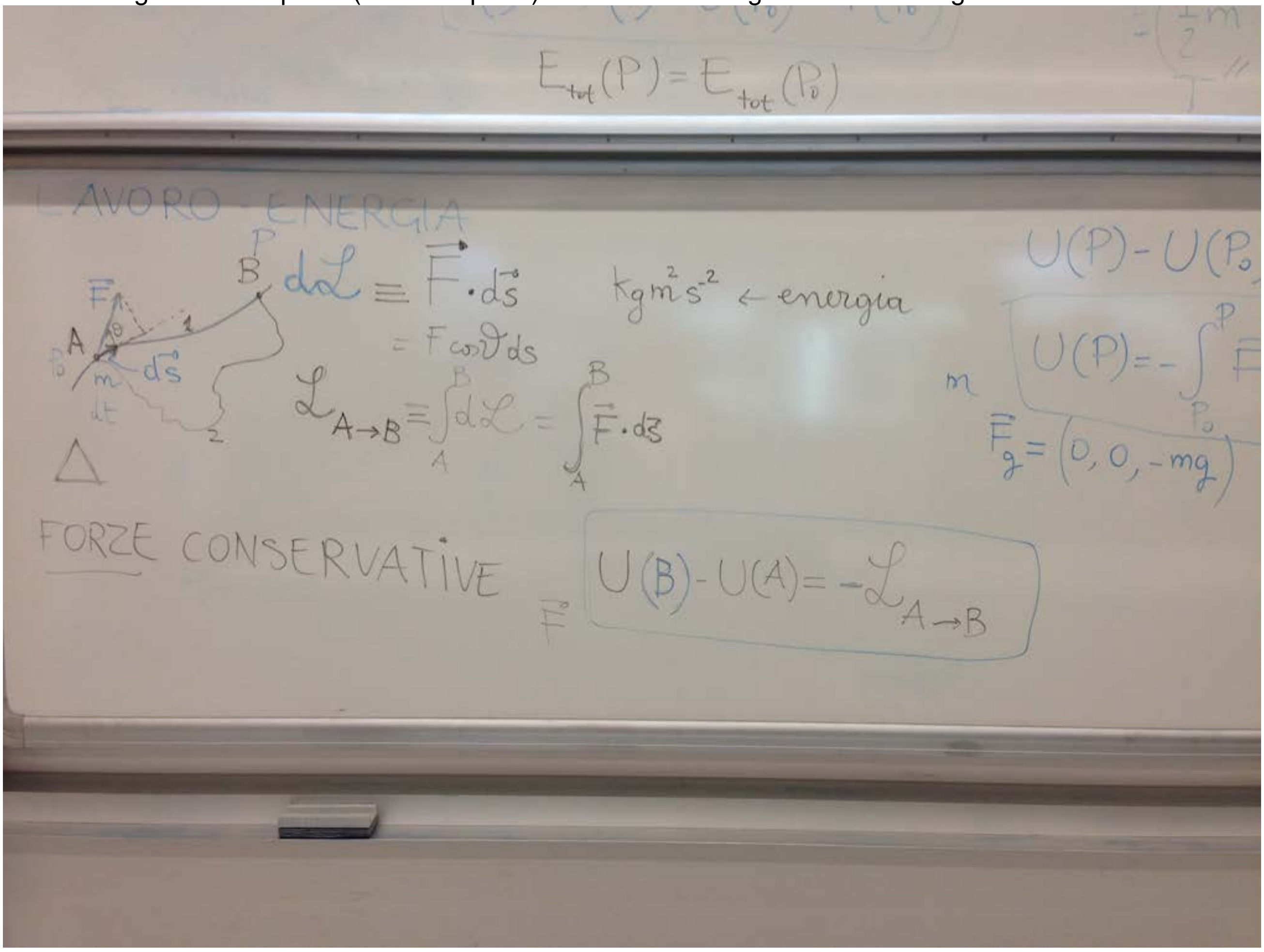
t'_{fi}: $z(t'_{fi}) = 0$

$t'_{fi} = \sqrt{\dfrac{L_z}{g}}$ $\quad t_{fi} = \sqrt{\dfrac{L_y}{a_y}}$

$\dfrac{g}{a_y} y(t'_{fi}) + \dfrac{L_z}{2} - \dfrac{g}{a_y}\dfrac{L_y}{2} \geq 0$ $\quad a_y \geq g\dfrac{L_y}{L_z}$ $\rightarrow a \leq g\dfrac{L_y}{L_z}$

19 Febbraio 2015

- Concetto di lavoro, forze conservative, definizione di energia potenziale e conservazione dell'energia totale
- Consegna del compitino (seconda parte) e discussione degli errori con i singoli studenti



$$U(P) + T(P) = U(P_0) + T(P_0)$$

$$E_{tot}(P) = E_{tot}(P_0)$$

LAVORO - ENERGIA

$$d\mathcal{L} \equiv \vec{F} \cdot d\vec{s} \qquad kg\,m^2 s^{-2} \leftarrow \text{energia}$$

$$= F \cos\vartheta \, ds$$

$$\mathcal{L}_{A \to B} \equiv \int_A^B d\mathcal{L} = \int_A^B \vec{F} \cdot d\vec{s}$$

$$U(P) - U(P_0) = -\mathcal{L}_{P_0 \ldots}$$

$$\boxed{U(P) = -\int_{P_0}^{P} \vec{F} \cdot d\vec{s} + \ldots}$$

m

$$\vec{F}_g = (0, 0, -mg)$$

FORZE CONSERVATIVE

$$\vec{F} \qquad \boxed{U(B) - U(A) = -\mathcal{L}_{A \to B}}$$

$$d\mathcal{L} \equiv \vec{F} \cdot d\vec{s} \qquad kg\,m^2 s^{-2} \leftarrow \text{energia}$$

$$= F \cos\vartheta \, ds$$

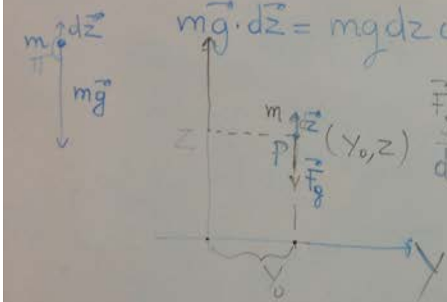
$$\mathcal{L}_{A \to B} \equiv \int_A^B d\mathcal{L} = \int_A^B \vec{F} \cdot d\vec{s}$$

$$U(P) = -\int_{P_0}^{P} \vec{F} \cdot d\ldots$$

m

$$\vec{F}_g = (0, 0, -mg)$$

$$m\vec{g} \cdot d\vec{z} = mg\,dz \cos(\widehat{\vec{g}, d\vec{z}}) = mg\,dz \cos\pi = -mg\,dz$$



$$\vec{F}_g = (0, -mg)$$

$$d\vec{z} = (0, dz)$$

$$\vec{r} = (y_0, z)$$

$$\vec{F}_g \cdot d\vec{z} = 0 - mg\,dz$$

$$U(P) = -\ldots$$

$$U(P_0) = 0$$

m

$\vec{F}$ conser…

$$U(P) - U(P_0) = -\frac{m}{2}\int_{P_0}^{P} d\ldots$$

$$0$$

$$= \left(\frac{1}{2} m v^2\right) \ldots$$

$$T$$

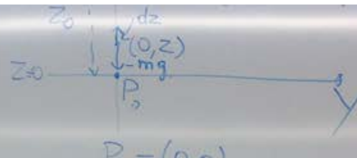
$$U(P) + T(P) = U(P_0) + T(P_0)$$

$$E_{tot}(P) = E_{tot}(P_0)$$

$P_0 \to P$

$$U(P) = -\int_{P_0}^{P} \vec F \cdot d\vec s + U(P_0)$$

$\vec F_g = (0, 0, -mg)$



$P_0 = (0,0)$

$= +mg \int_0^{z_0} dz$

π $d\vec z$ $m\vec g$

$$U(P) = -\int_{P_0}^{P} \vec F \cdot d\vec s = -\int_{P_0}^{P} m\vec a \cdot d\vec s = -m\int_{P_0}^{P} \frac{d\vec v}{dt} \cdot d\vec s = -m\int_{P_0}^{P} \frac{d\vec v}{\cancel{dt}} \cdot (\vec v\, \cancel{dt}) = -m\int_{P_0}^{P} \vec v \cdot d\vec v$$

$U(P_0) = 0$

m

$\vec F$ conservativa

RI

$\vec F = m\vec a$

$\vec a = \frac{d\vec v}{dt}$

$\vec v = \frac{d\vec s}{dt}$

$\vec v \cdot d\vec v = \frac{1}{2} d(\vec v \cdot \vec v)$

$d(\vec v \cdot \vec v) = \vec v \cdot d\vec v + d\vec v \cdot \vec v =$

$= 2\,\vec v \cdot d\vec v$

$\vec v$

$v^2 \equiv \vec v \cdot \vec v = (v_x, v_y, v_z) \cdot (v_x, v_y, v_z)$

$= v_x^2 + v_y^2 + v_z^2$

$(P_0) = -\frac{m}{2}\int_{P_0}^{P} d(v^2) =$

$= -\left(\frac{1}{2} m v^2\right)_P + \left(\frac{1}{2} m v^2\right)_{P_0}$

T

$mg\, dz \cos\pi = -mg\, dz$

$\vec F_g \cdot d\vec z = 0 - mg\, dz$

$$U(P) = -\int_{P_0}^{P} \vec F \cdot d\vec s = -\int_{P_0}^{P} m\vec a \cdot d\vec s$$

$U(P_0) = 0$

m

RI

$\vec F = m\vec a$

$\vec F$ conservativa

$\vec a = d$

$\vec v =$

$$U(P) - \underset{0}{U(P_0)} = -\frac{m}{2}\int_{P_0}^{P} d(v^2) =$$

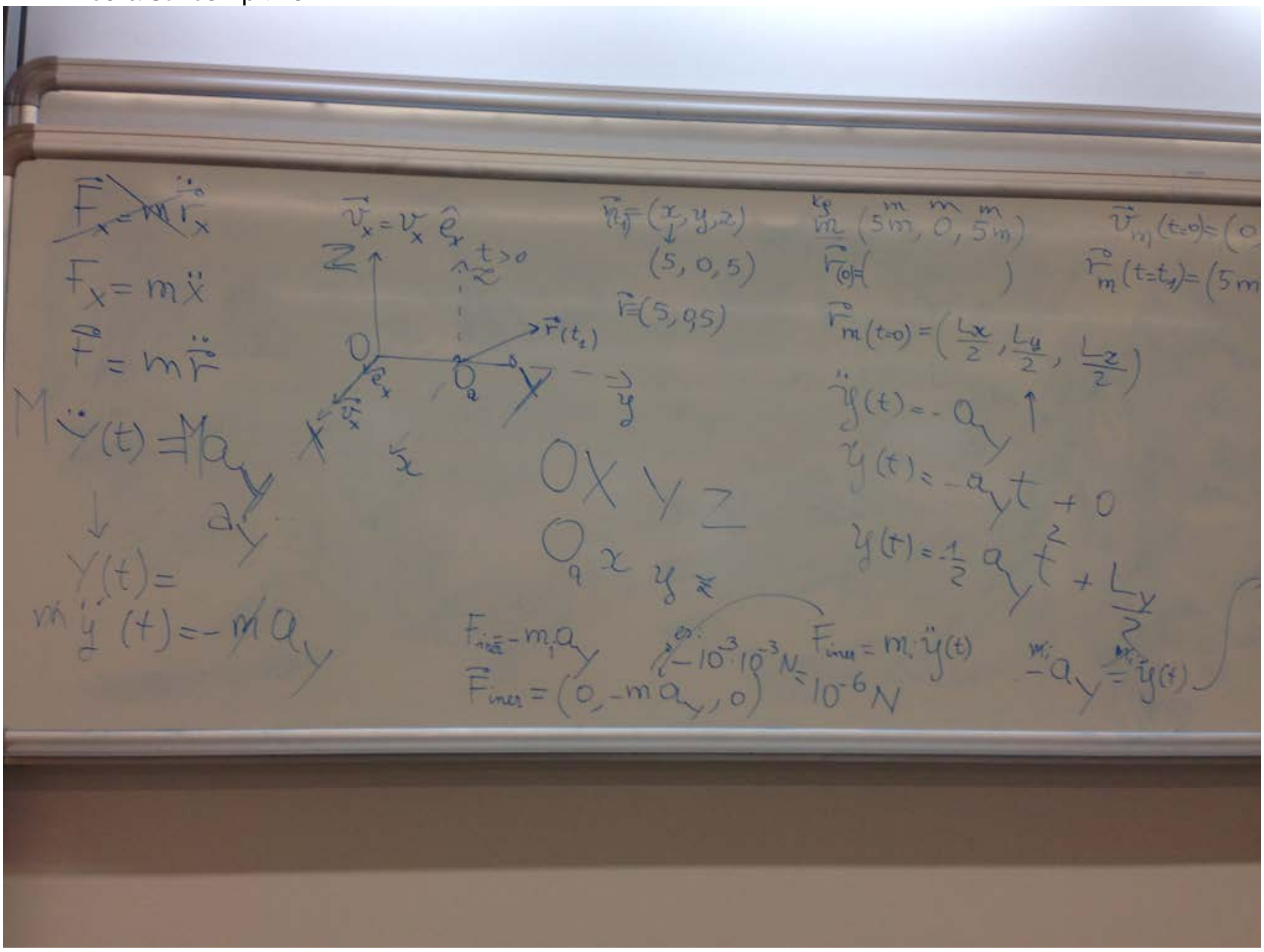
$$= -\underbrace{\left(\frac{1}{2} m v^2\right)_P}_{T} + \left(\frac{1}{2} m v^2\right)_{P_0}$$

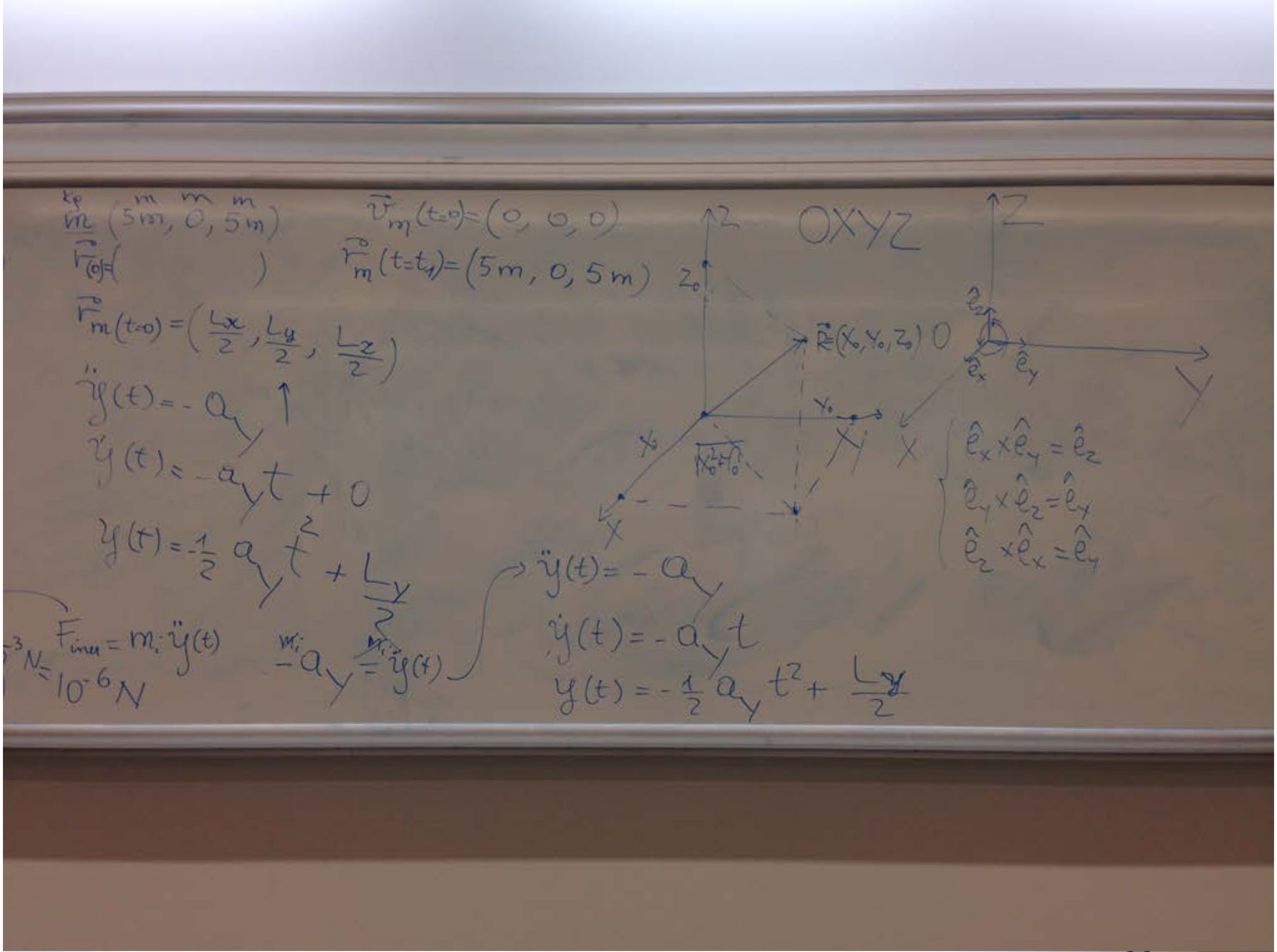
$) = U(P_0) + T(P_0)$

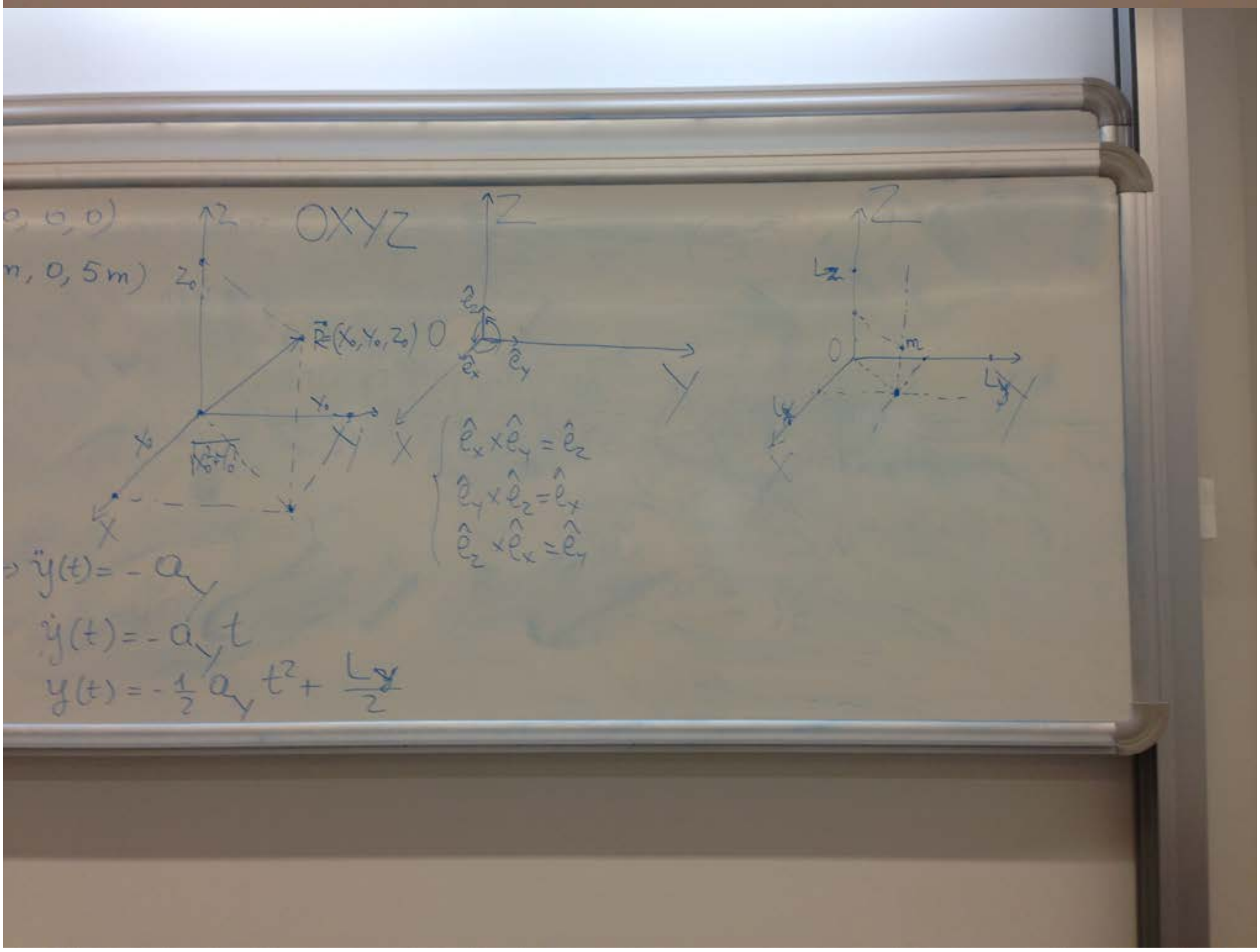
$) = E_{tot}(P_0)$

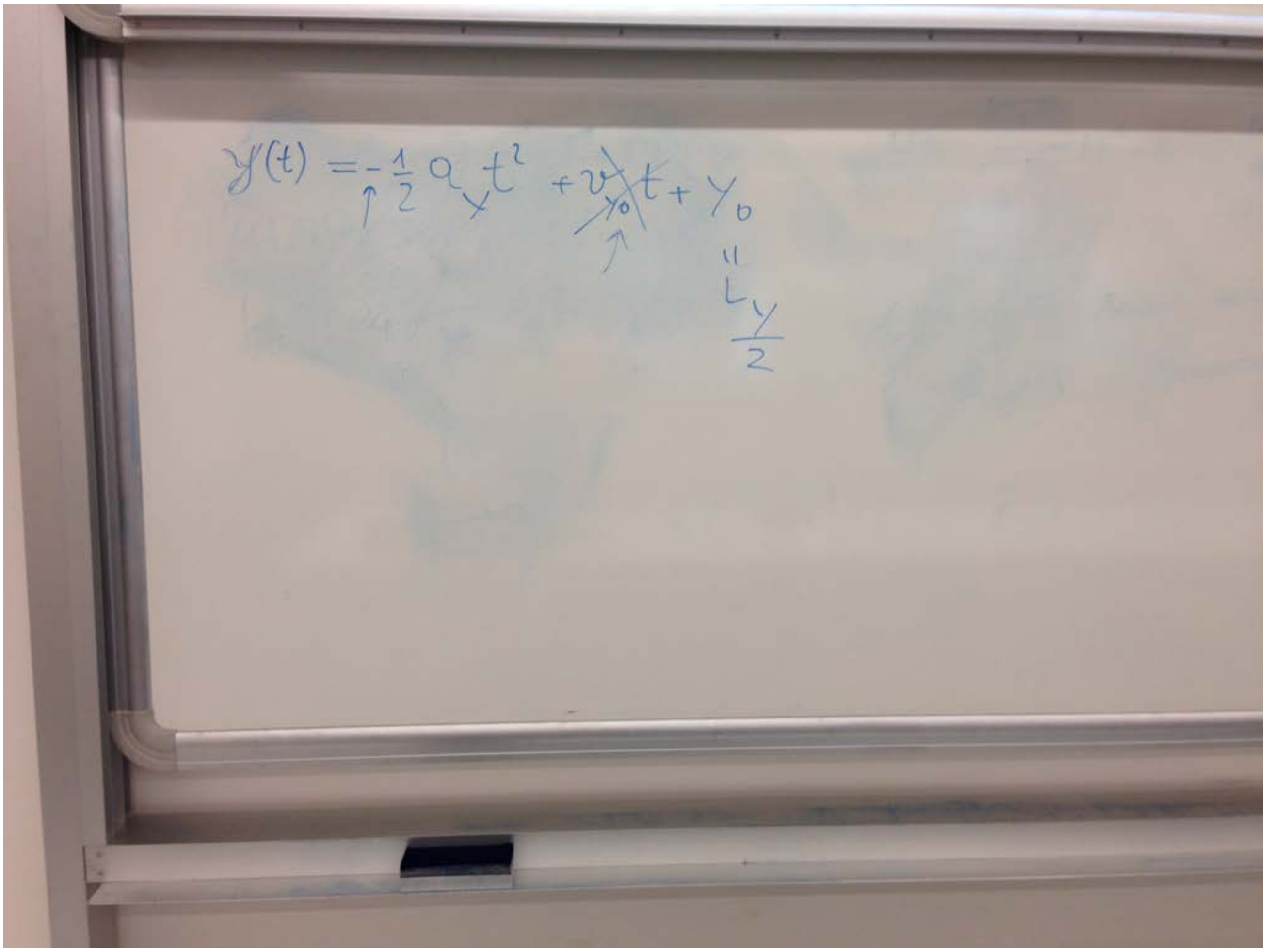
24 Febbraio 2015

– Ancora sul compitino




m (5m, 0, 5m)
$\vec{v}_m(t=0)=(0,0,0)$
$\vec{r}_m(t=t_1)=(5m, 0, 5m)$
OXYZ
$\vec{r}_m(t=0)=\left(\frac{L_x}{2}, \frac{L_y}{2}, \frac{L_z}{2}\right)$
$\ddot{y}(t)=-a_y$
$\dot{y}(t)=-a_y t+0$
$y(t)=-\frac{1}{2}a_y t^2+\frac{L_y}{2}$
$F_{iner}=m_i\,\ddot{y}(t)$
$10^{-6}N$
$-a_y=\ddot{y}(t)$
$\vec{R}(X_0,Y_0,Z_0)$ O
$\hat{e}_x\times\hat{e}_y=\hat{e}_z$
$\hat{e}_y\times\hat{e}_z=\hat{e}_x$
$\hat{e}_z\times\hat{e}_x=\hat{e}_y$
$\ddot{y}(t)=-a_y$
$\dot{y}(t)=-a_y t$
$y(t)=-\frac{1}{2}a_y t^2+\frac{L_y}{2}$


OXYZ
$\vec{R}(X_0,Y_0,Z_0)$ O
$\hat{e}_x\times\hat{e}_y=\hat{e}_z$
$\hat{e}_y\times\hat{e}_z=\hat{e}_x$
$\hat{e}_z\times\hat{e}_x=\hat{e}_y$
$\ddot{y}(t)=-a_y$
$\dot{y}(t)=-a_y t$
$y(t)=-\frac{1}{2}a_y t^2+\frac{L_y}{2}$
Lz
m
Ly
Lx



26 Febbraio 2015

- Calcolo dell’energia potenziale di un campo gravitazionale uniforme
- Studio del moto del pendolo semplice a partire dall’energia; come si usa la legge di conservazione dell’energia

$E = T + U = \text{cost}$

(forze conservative)

$g = 9.81 \ m\,s^{-2}$

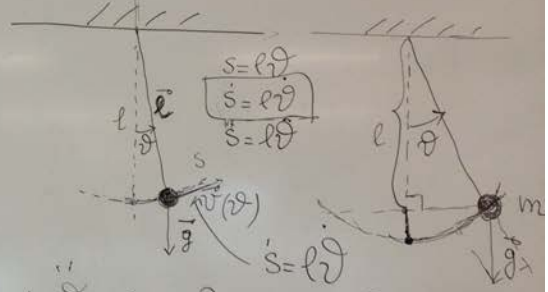
$g = \frac{G M_{\oplus}}{R_{\oplus}^2}$

$R_{\oplus} = 6.378 \times 10^6 \ m$

$g(h) = \frac{G M_{\oplus}}{(R_{\oplus} + h)^2}$

W

$m, z \qquad U(z) = +mgz$



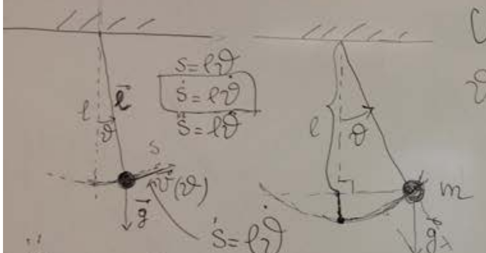
$$\boxed{\ddot{\vartheta} + \frac{g}{\ell}\sin\vartheta = 0}$$

$\vartheta(t=0) = \vartheta_0$

$\dot{\vartheta}(t=0) = \dot{\vartheta}_0$

$E_1(\vartheta_0, 0) = mg\ell(1 - \cos\vartheta_0)$

$m, z \qquad U(z) = +mgz$



$$\boxed{\ddot{\vartheta} + \frac{g}{\ell}\sin\vartheta = 0}$$

$\vartheta(t=0) = \vartheta_0$ rad

$\dot{\vartheta}(t=0) = \dot{\vartheta}_0$ rad/s

$U(\vartheta) = mg\ell(1 - \cos\vartheta)$

$U(0) = 0$

$\vartheta = \frac{\pi}{2} \qquad U\left(\frac{\pi}{2}\right) = mg\ell$

$E = T(\vartheta) + U(\vartheta)$

$T(\vartheta) = \frac{1}{2} m (v(\vartheta))^2 = \frac{1}{2} m \ell^2 \dot{\vartheta}^2$

$\frac{d}{dt}\dot{\vartheta}^2_{(t)} = \frac{d}{d(\dot{\vartheta})}(\dot{\vartheta}^2) \cdot \frac{d\dot{\vartheta}}{dt} = 2\dot{\vartheta}\ddot{\vartheta}$

$\frac{dx^2}{dx}$

$\underbrace{E(\vartheta, \dot{\vartheta}) = \frac{1}{2} m \ell^2 \dot{\vartheta}^2_{(t)} + mg\ell(1 - \cos\vartheta_{(t)})}$

$\vartheta(t), \dot{\vartheta}(t)$

$\underset{0}{\dot{E}}(\vartheta_{(t)}, \dot{\vartheta}_{(t)}) = \frac{1}{2} m\ell^2 \frac{d}{dt}\dot{\vartheta}^2_{(t)} + \frac{d}{dt}(m$

$= \cancel{m}\ell^2 \dot{\vartheta}\ddot{\vartheta} + \cancel{m}g\ell$

$\frac{d}{dt}\cos\vartheta_{(t)} = \frac{d(\cos\vartheta)}{d\vartheta} \cdot \frac{d\vartheta}{dt}$

$E_1(\vartheta_0, 0) = mgl(1-\cos\vartheta_0)$ $\quad E_1 = E_2$

$\Downarrow$

$\dots gl(1-\cos\vartheta)$

$U\left(\frac{\pi}{2}\right) = mgl$

$T(\vartheta) + U(\vartheta)$

$\dots) = \frac{1}{2} m (v(\vartheta))^2 = \frac{1}{2} m l^2 \dot\vartheta^2$

$\frac{d}{dt}\dot\vartheta^2_{(t)} = \frac{d}{d(\dot\vartheta)}(\dot\vartheta^2) \cdot \frac{d\dot\vartheta}{dt} = 2\dot\vartheta\ddot\vartheta$

$\uparrow$ $\frac{dx^2}{dx}$

$$\underbrace{E(\vartheta,\dot\vartheta) = \frac{1}{2} m l^2 \dot\vartheta^2_{(t)} + mgl(1-\cos\vartheta_{(t)})}_{\vartheta(t),\ \dot\vartheta(t)} = \text{costante} \quad \text{(al variare di } t\text{)} \qquad E(\vartheta(0), \dot\vartheta(0)) = E^* \quad kg\, m^2 s^{-2}$$

$$\dot E(\vartheta_{(t)}, \dot\vartheta_{(t)}) = \frac{1}{2} m l^2 \frac{d}{dt}\dot\vartheta^2_{(t)} + \frac{d}{dt}(mgl) - mgl\frac{d}{dt}(\cos\vartheta_{(t)}) =$$

$$\overset{\displaystyle \dot E}{\underset{0}{\parallel}} \qquad = \cancel{m}l^{\cancel{2}}\dot\vartheta\ddot\vartheta + \cancel{m}gl\,\cancel{\dot\vartheta}\sin\vartheta = 0 \Rightarrow l\ddot\vartheta + g\sin\vartheta = 0$$

$$\boxed{\ddot\vartheta + \frac{g}{l}\sin\vartheta = 0}$$

$$\frac{d}{dt}\cos\vartheta_{(t)} = \frac{d(\cos\vartheta)}{d\vartheta}\cdot\frac{d\vartheta}{dt} = -\dot\vartheta\sin\vartheta$$

$E_1(\vartheta_0, 0) = mgl(1-\cos\vartheta_0)$ $\qquad E_1 = E_2$

$\Downarrow$

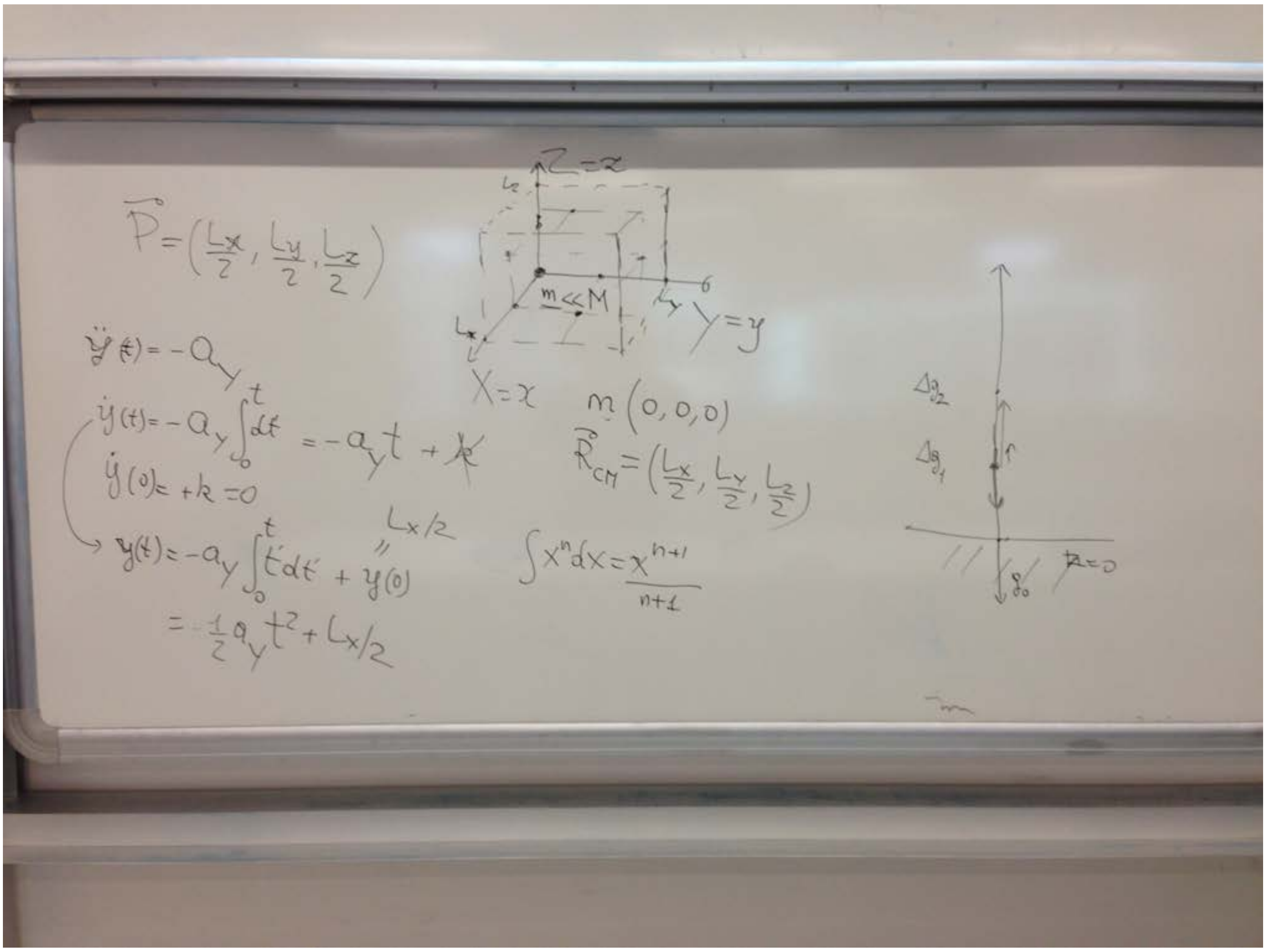
$E_2(0, \dot\vartheta_0) = \frac{1}{2} m l^2 \dot\vartheta_0^2$

$\cancel{m}g\cancel{l}(1-\cos\vartheta_0) = \frac{1}{2}\cancel{m}l^{\cancel{2}}\dot\vartheta_0^2$

$2\frac{g}{l}(1-\cos\vartheta_0) = \dot\vartheta_0^2$

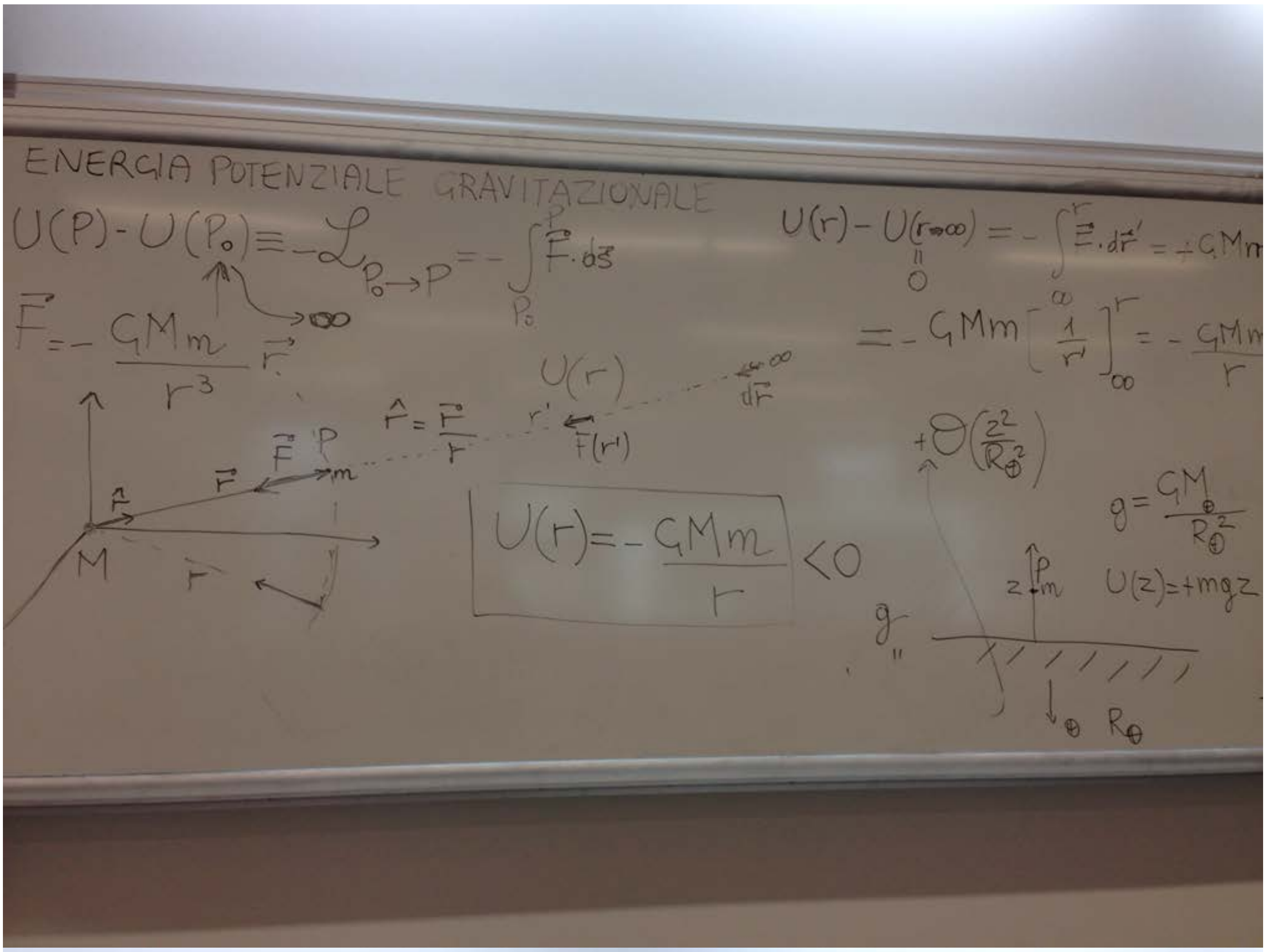
$\dot\vartheta_0 = \sqrt{2\frac{g}{l}(1-\cos\vartheta_0)}$

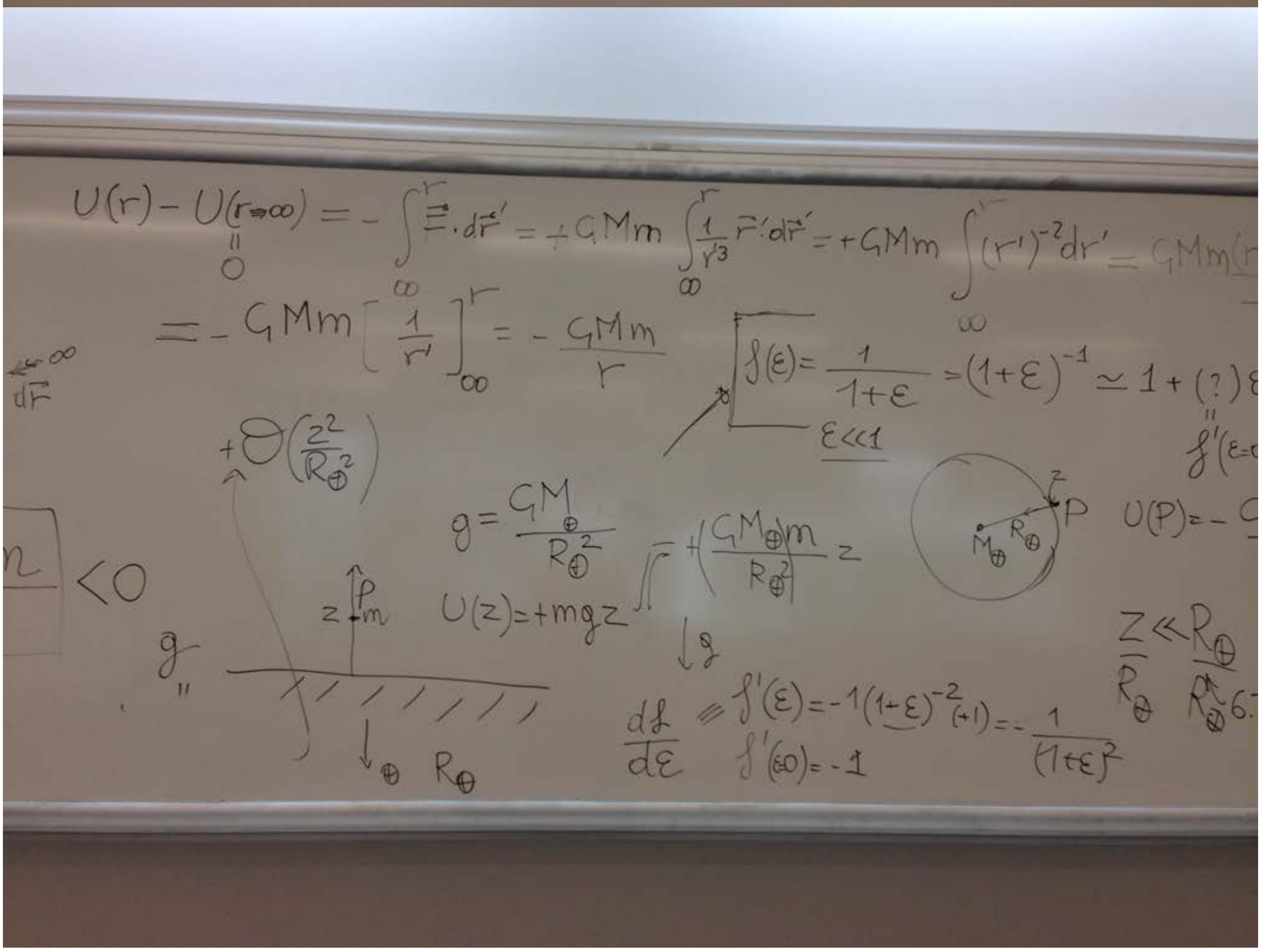
$\uparrow$ max



3 Marzo 2015

- Calcolo dell'energia potenziale della forza gravitazionale
- Caso generale, caso del campo gravitazionale uniforme e coerenza dei due risultati
- Sviluppi in serie di Taylor e loro rilevanza in Fisica (caso particolare della energia potenziale gravitazionale della Terra)


ENERGIA POTENZIALE GRAVITAZIONALE
U(P) - U(P₀) ≡ -L P₀→P = -∫ F·ds
F = - GMm/r³ r
U(r)
r̂ = r/r
F(r')
dr
U(r) = - GMm/r < 0
U(r) - U(r→∞) = - ∫ F·dr' = + GMm
= - GMm [1/r'] = - GMm/r
+ O(z²/R⊕²)
g = GM⊕/R⊕²
U(z) = + mgz
M
P
m
z
R⊕


U(r) - U(r→∞) = - ∫ F·dr' = + GMm ∫ 1/r'³ r'·dr' = + GMm ∫ (r')⁻² dr' = GMm
= - GMm [1/r'] = - GMm/r
f(ε) = 1/(1+ε) = (1+ε)⁻¹ ≃ 1 + (?) ε
ε≪1
f'(ε=0)
+ O(z²/R⊕²)
g = GM⊕/R⊕²
U(z) = + mgz
+ (GM⊕m/R⊕²) z
U(P) = -
z ≪ R⊕
z/R⊕
df/dε = f'(ε) = -1(1+ε)⁻²(+1) = - 1/(1+ε)²
f'(ε=0) = -1
M⊕
R⊕
P
< 0

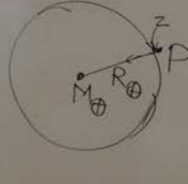
$$\ldots Mm \int_{\infty}^{r} \frac{1}{r'^3}\,\vec{r}'\cdot d\vec{r}' = +GMm \int_{\infty}^{r} (r')^{-2}\,dr' = GMm\,\frac{(r')^{-1}}{-1} = \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} \quad n \neq -1$$

$$\ldots \frac{Mm}{r}$$

$$\boxed{f(\varepsilon) = \frac{1}{1+\varepsilon}} = (1+\varepsilon)^{-1} \simeq 1 + \underset{\substack{\| \\ f'(\varepsilon=0)}}{(?)}\,\varepsilon + \underset{\substack{\uparrow \\ \frac{f''(\varepsilon=0)}{2}}}{(?)}\,\varepsilon^2 + \underset{\substack{\uparrow \\ \frac{f'''(\varepsilon=0)}{3!}}}{(?)}\,\varepsilon^3 \cdots \simeq 1-\varepsilon + \mathcal{O}(\varepsilon^2)$$

$$\varepsilon \ll 1$$

SVILUPPO ...
SERIE DI ...
DI POTE...



$$U(P) = -\frac{GMm}{(R_\oplus + z)} = -\frac{GMm}{R_\oplus\left(\frac{R_\oplus}{R_\oplus} + \frac{z}{R_\oplus}\right)} = -\frac{GMm}{R_\oplus\left(1 + \frac{z}{R_\oplus} \ldots\right.}$$

$$+\left(\frac{GM_\oplus m}{R_\oplus^2}\right) z \qquad \downarrow g$$

$$z \ll R_\oplus \qquad \frac{z}{R_\oplus} \qquad \varepsilon = \frac{z}{R_\oplus} \ll 1$$

$$R_\oplus \simeq 6.378 \times 10^6\ \text{m}$$

$$\varepsilon^2 \qquad \varepsilon^3 \qquad \varepsilon^4$$

$$\frac{df}{d\varepsilon} = f'(\varepsilon) = -1(1+\varepsilon)^{-2}(+1) = -\frac{1}{(1+\varepsilon)^2}$$

$$f'(\varepsilon=0) = -1$$

$$U(R_\oplus + z) = -\frac{GMm}{R_\oplus}\Big(1 + \underbrace{\frac{z}{R_\oplus}}_{\varepsilon}\Big)^{-1} \simeq -\frac{GMm}{R_\oplus}\left(1 - \varepsilon + \mathcal{O}(\varepsilon^2)\right) = -\frac{GMm}{R_\oplus}\left(1 - \frac{z}{R_\oplus} \ldots\right.$$

$$U(R_\oplus + z) - \left(-\frac{GMm}{R_\oplus}\right) = mgz + \left(\frac{GMm}{R_\oplus}\right)\mathcal{O}\!\left(\frac{z^2}{R^2}\right)$$

$$U(P) - U(P_0) \simeq mgz + U(P_0)\,\mathcal{O}\!\left(\frac{z^2}{R_\oplus^2}\right) \qquad = \frac{GMm}{R_\oplus}\,\frac{z^2}{R_\oplus^2}$$

$$f(\varepsilon) = (1+\varepsilon)^{-1} \ldots$$

$$F(z) = - \ldots$$

$$g(\varepsilon) = \frac{1}{(1+\varepsilon \ldots}$$

$$g'(\varepsilon) = \ldots$$

$$g'(\ldots$$

$$\downarrow_{\oplus} \quad R_{\oplus}$$

$$\frac{df}{d\varepsilon} = f'(\varepsilon) = -1(1+\varepsilon)^{-2}(+1) = -\frac{1}{(1+\varepsilon)^2} \qquad R_{\oplus} \; 6.37$$

$$f'(\varepsilon 0) = -1$$

$$+\mathcal{O}(\varepsilon^2)\Big) = -\frac{GMm}{R_{\oplus}}\left(1 - \frac{z}{R_{\oplus}} + \mathcal{O}\left(\frac{z^2}{R_{\oplus}^2}\right)\right) = -\frac{GMm}{R_{\oplus}} + \left(\frac{GM}{R_{\oplus}^2}\right)mz + \frac{GMm}{R_{\oplus}}\mathcal{O}\Big(\frac{z}{R}$$

$$f(\varepsilon) = (1+\varepsilon)^{-1} \simeq 1 - \varepsilon + \varepsilon^2$$

$$+\frac{GMm}{R_{\oplus}}\frac{z^2}{R_{\oplus}^2}$$

$$F(z) = -\frac{GMm}{(R_{\oplus}+z)^2} = -\frac{GMm}{R_{\oplus}^2\left(1+\underbrace{\frac{z}{R_{\oplus}}}_{=\varepsilon}\right)^2} = -\frac{GMm}{R_{\oplus}^2}g(\varepsilon) \qquad f''(\varepsilon = 0) = 2$$

$$g(\varepsilon) = \frac{1}{(1+\varepsilon)^2} = (1+\varepsilon)^{-2} \simeq 1 - 2\varepsilon$$

$$g'(\varepsilon) = -2(1+\varepsilon)^{-3}$$

$$g'(\varepsilon=0) = -2$$

$=0$

$$F(z) = -\left(\frac{GMm}{R_{\oplus}^2}\right)\left(1 - \frac{2z}{R_{\oplus}}\right) =$$

$$g(R_{\oplus}+z) = -g(R_{\oplus}) + g\frac{2}{R_{\oplus}}z$$

$$(+1) = -\frac{1}{(1+\varepsilon)^2} \qquad R_{\oplus}\; 6.378 \times 10^6\ \mathrm{m} \qquad \varepsilon^2 \qquad \varepsilon^3 \qquad \varepsilon^4 \qquad \varepsilon$$

$$f'(\varepsilon 0) = -1$$

$$= -\frac{GMm}{R_{\oplus}} + \left(\frac{GM}{R_{\oplus}^2}\right)mz + \cancel{\frac{GMm}{R_{\oplus}}\mathcal{O}\left(\frac{z^2}{R_{\oplus}^2}\right)}$$

$$f''(\varepsilon) = \underbrace{f'}_{=\frac{d}{d\varepsilon}}\left(-(1+\varepsilon)^{-2}\right) = +2(1+\varepsilon)^{-3}\underbrace{\frac{d}{d\varepsilon}(1+\varepsilon)}_{=1} = 2(1+\varepsilon)^{-3}$$

$$-\frac{GMm}{R_{\oplus}^2\left(1+\frac{z}{R_{\oplus}}\right)^2} = -\frac{GMm}{R_{\oplus}^2}g(\varepsilon) \qquad f''(\varepsilon=0) = 2$$

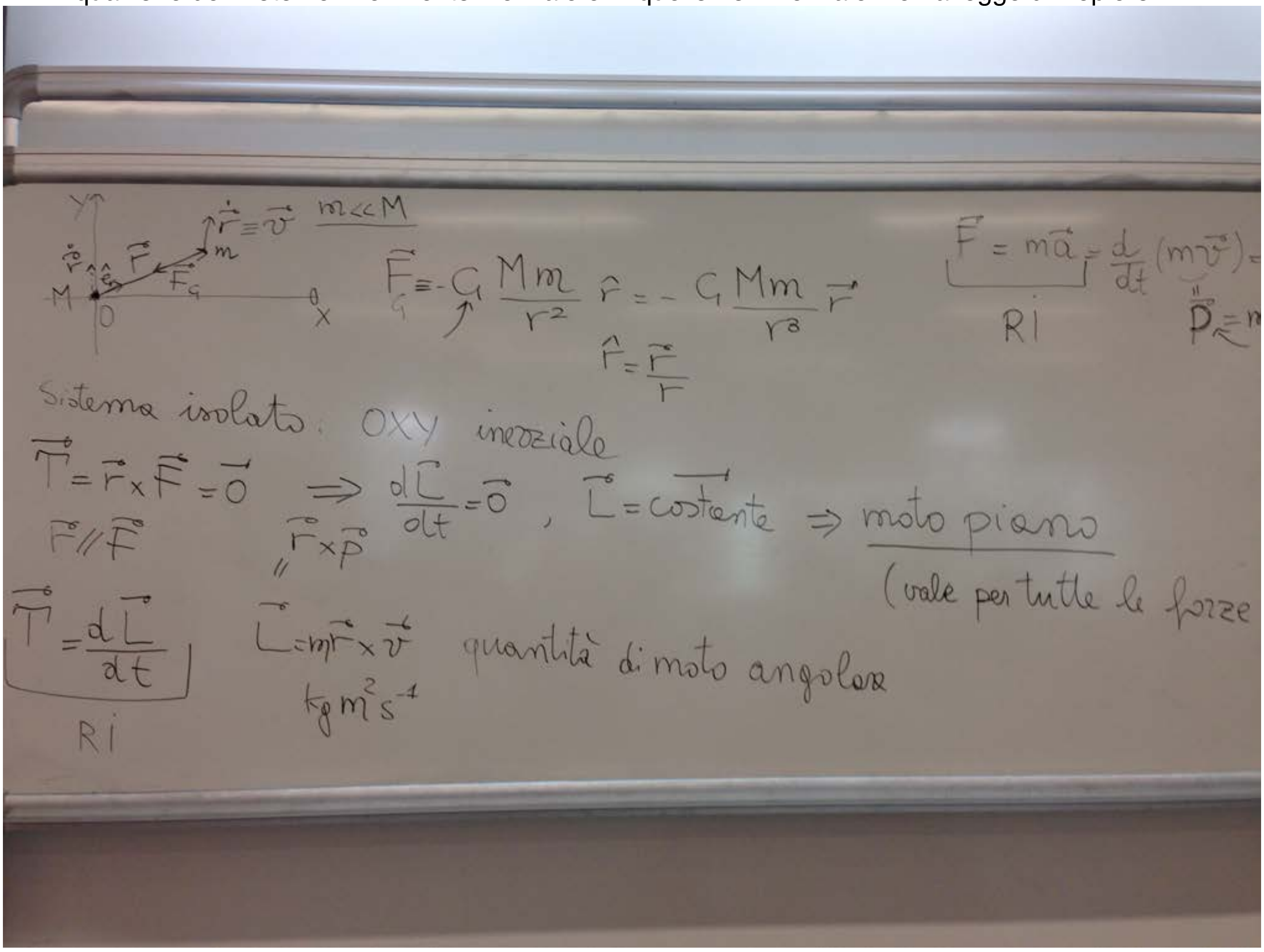
$$\simeq 1 - 2\varepsilon$$

$$F(z) = -\left(\frac{GMm}{R_{\oplus}^2}\right)\left(1 - \frac{2z}{R_{\oplus}}\right) = -\underbrace{\frac{GMm}{R_{\oplus}^2}}_{\mathrm{Kg\,m\,s^{-2}}} + 2\underbrace{\left(\frac{GMm}{R_{\oplus}^3}\right)}_{\mathrm{Kg\,m\,s^{-2}/m}} \cdot z$$

$$g(R_{\oplus}+z) = -g(R_{\oplus}) + g\frac{2}{R_{\oplus}}z$$

5 Marzo 2015

- Moto di satelliti e pianeti, caso particolare di due soli corpi di cui il secondo con massa trascurabile rispetto al primo, e di orbita circolare
- Forze centrali e moto piano
- Equazione del moto nel riferimento inerziale e in quello non inerziale. Terza legge di Keplero



$$\underbrace{\vec{F} = m\vec{a}}_{RI} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$\vec{p} = m\vec{v}$ quantità di moto lineare

$kg\, m\, s^{-1}$

$\hat{e}_\vartheta = (-\sin\vartheta, \cos\vartheta) = -\sin\vartheta\,\hat{e}_x + \cos\vartheta\,\hat{e}_y$ RI

$$\dot{\hat{e}}_\vartheta = (-\dot\vartheta\cos\vartheta, -\dot\vartheta\sin\vartheta) = -\dot\vartheta\underbrace{(\cos\vartheta, \sin\vartheta)} = -\dot\vartheta\,\hat{e}_r \qquad \dot{\hat{e}}_x = \dot{\hat{e}}_y = 0$$

moto piano

(vale per tutte le forze centrali)

$$m\ddot{\vec{r}} = \vec{F}_G$$

$$\cancel{m}\ddot{\vec{r}} = -G\frac{M\cancel{m}}{r^3}\vec{r}$$

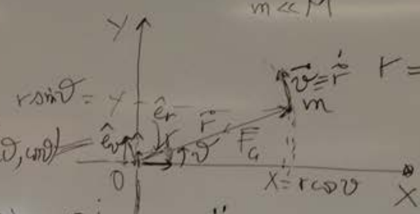
$$\vec{r} = r\hat{e}_r$$

$$r\dot\vartheta\,\hat{e}_\vartheta = \dot{\vec{r}} = \underset{0}{\dot{r}}\,\hat{e}_r + r\dot{\hat{e}}_r$$

$$\ddot\vartheta = 0 \Leftarrow \text{costante} = \frac{d\vartheta}{dt} = \dot\vartheta \text{ rad/s}$$

velocità circolare

PRESENTI 26



$r = \text{cost.} \Rightarrow \dot{r} = 0$ $\qquad \vec{r} = r\hat{e}_r$

$\vec{r} = (x, y)$

$\vec{r} = (r\cos\vartheta, r\sin\vartheta)$

$r = \sqrt{x^2 + y^2} > 0$

$$\boxed{\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{e}_r}$$

$\hat{e}_r = (\cos\vartheta, \sin\vartheta)$

$\dot{\hat{e}}_r = (-\dot\vartheta\sin\vartheta, \dot\vartheta\cos\vartheta) = \dot\vartheta(-\sin\vartheta, \cos\vartheta) = \dot\vartheta\,\hat{e}_\vartheta$

centrali)

$$r\dot{\vartheta}\,\hat{e}_\vartheta = \dot{\vec{r}} = \underset{0}{\underset{\parallel}{\dot{r}}}\,\hat{e}_r + r\,\dot{\hat{e}}_r$$

$\hat{e}_r = (\cos\vartheta, \sin\vartheta)$

$\dot{\hat{e}}_r = (-\dot{\vartheta}\sin\vartheta, \dot{\vartheta}\cos\vartheta) = \dot{\vartheta}(-\sin\vartheta, \cos\vartheta) = \dot{\vartheta}\,\hat{e}_\vartheta$

$$\ddot{\vartheta} = 0 \Leftarrow \text{costante} = \frac{d\vartheta}{dt} = \dot{\vartheta} \quad \text{rad/s}$$

orbita circolare

$$\ddot{\vec{r}} = \frac{d}{dt}\left(r\dot{\vartheta}\hat{e}_\vartheta\right) = \underset{0}{\underset{\parallel}{\dot{r}}}\,\dot{\vartheta}\hat{e}_\vartheta + r\underset{0}{\underset{\parallel}{\ddot{\vartheta}}}\,\hat{e}_\vartheta + r\dot{\vartheta}\,\dot{\hat{e}}_\vartheta = -r\dot{\vartheta}^2\hat{e}_r$$

$$\boxed{\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{e}_r} \Longrightarrow -r\dot{\vartheta}^2\hat{e}_r = -\frac{GM}{r^2}\hat{e}_r$$

$$r\dot{\vartheta}^2 = \frac{GM}{r^2} \Rightarrow \boxed{\dot{\vartheta}^2 r^3 = GM}$$

3ª legge di Keplero

$$P = \frac{2\pi}{\dot{\vartheta}}$$

$\vec{v} \equiv \dot{\vec{r}} = r\dot{\vartheta}\,\hat{e}_\vartheta$

$v = r\dot{\vartheta} = \omega r$ = costante

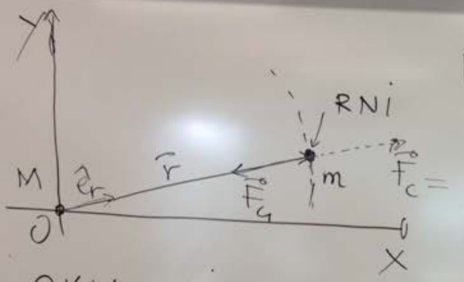
$v = r\sqrt{\frac{GM}{r^3}} = \sqrt{\frac{GM}{r}}$ $\quad ms^{-1}$

$v \propto \frac{1}{\sqrt{r}}$

$\dot{\vartheta}$ rad/s

$\dot{\vartheta} \equiv \omega$ velocità angolare (costante)

$\vec{\omega} = (0, 0, \omega)$



OXY $\quad$ Ri

$|\vec{F}_G| = |\vec{F}_c|$

$$\vec{F}_c = \frac{GM\cancel{m}}{r^2} = \cancel{m}\dot{\vartheta}^2 r$$

$$\dot{\vartheta}^2 r^3 = GM$$