S2 - Lezioni 23 e 25 Febbraio 2016:

- Effetto della accelerazione centrifuga sulla superficie della Terra.
- Effetto della accelerazione di Coriolis su corpi in caduta (deviazione verso Est)
- Uso della conservazione dell'energia per il calcolo della veloci
- Esempio di equazione del moto in un riferimento rotante: applicazione ad un satellite soggetto alla attrazione gravitazionale della Terra e in orbita circolare attorno ad essa. Relazione tra raggio dell'orbita e periodo. Velocità e relazione con il raggio. Calcolo dell'energia totale. Uso della conservazione dell'energia per calcolare la velocità di fuga dall'attrazione gravitazionale della Terra

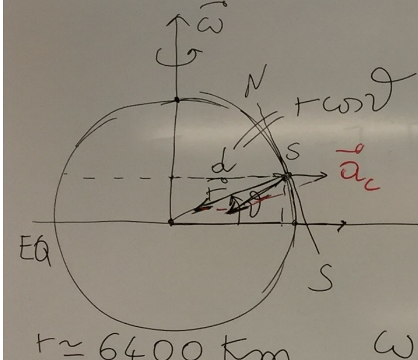
$$\vec{a}_{Rot} = \boxed{\vec{a}_{RI} = \frac{\vec{F}}{m}} \underbrace{- \vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\text{acc. centrifuga}} \underbrace{- 2\vec{\omega}\times\vec{v}_{ROT}}_{\text{acc. di Coriolis}}$$

$\vec{\omega}$

$P = \frac{2\pi}{\omega}$

$\omega^2 \cdot d$

distanza (minima) dall'asse di rotazione



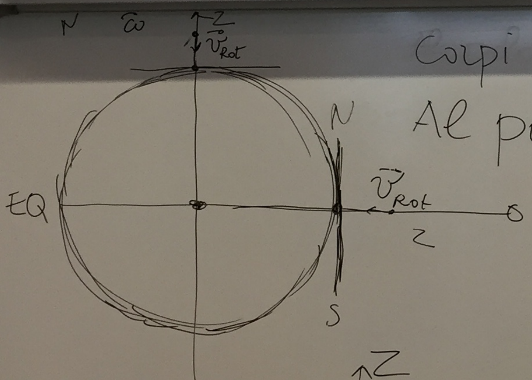
$$\frac{a_{c\,max}}{g} = \frac{\omega^2 r}{g} = \frac{(7.3\times10^{-5})^2\cdot(6.4\times10^{6})}{9.8} \simeq 3.47\times10^{-3}$$

$r \simeq 6400$ Km

$$\omega = \frac{2\pi}{86164}\,\frac{rad}{s} \simeq 7.3\times10^{-5}\ rad\ s^{-1}$$

Corpi in caduta libera

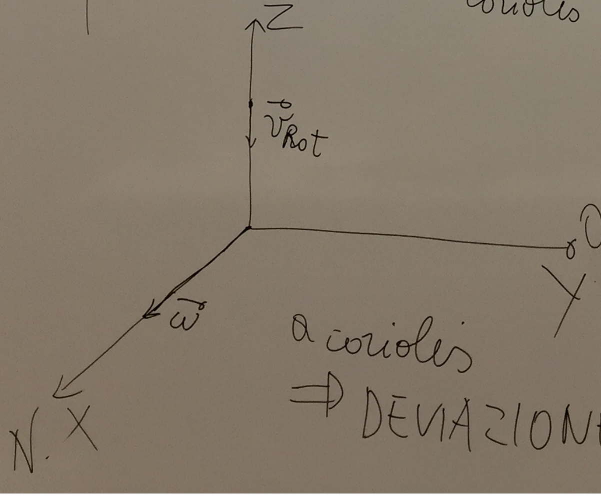
Al polo: $\vec{\omega} \,//\, \vec{v}_{Rot} \Rightarrow \vec{a}_{coriolis} = 0$



All'equatore:

$a_{coriolis} = 2\,\omega\, v_{Rot}$

velocità di caduta



piano dell'orizzonte all'equatore

a coriolis

$\Rightarrow$ DEVIAZIONE GRAVI VERSO EST

$$m g h = \frac{1}{2} m v_{fin}^2 \qquad \text{CONS. ENERGIA}$$

$$v_{fin} = \sqrt{2gh} \qquad \sqrt{m^2 s^{-2}}$$

g

$$v(t) = g t \qquad v(t_{fin}) = g t_{fin} = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$$

$$v(t_{fin}) \equiv v_{fin}$$

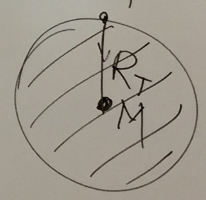
$$z(t) = \int_0^t g t \, dt = g \int_0^t t' dt' = g\left[\frac{t'^2}{2}\right]_0^t = \frac{1}{2} g t^2$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

$$h = z(t_{fin}) = \frac{1}{2} g t_{fin}^2 \quad \Rightarrow \quad t_{fin} = \sqrt{\frac{2h}{g}}$$

$h = 40\,m \qquad t_{fin} = \sqrt{\frac{2 \cdot 40}{9.8}} = 2.86\,s$

$$g = \frac{G M_T}{R_T^2} = 9.8\, m s^{-2}$$

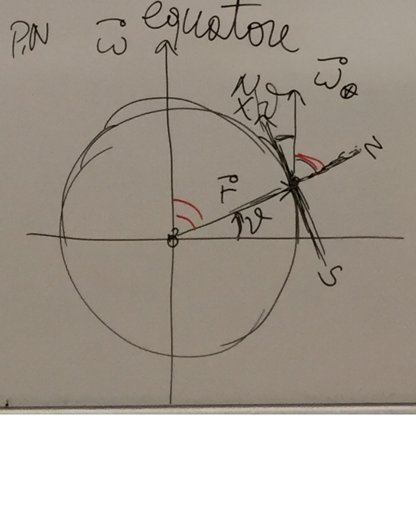


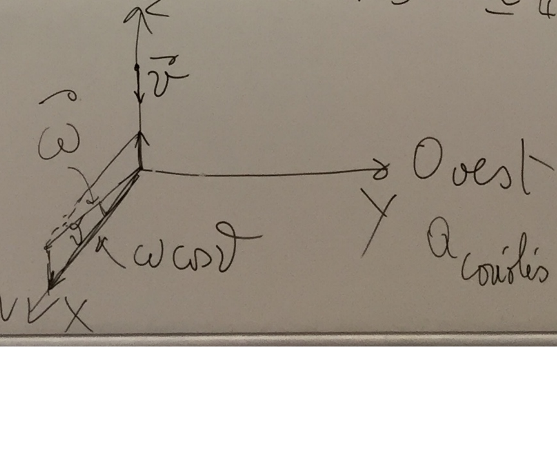
$$v_{fin} = \sqrt{2gh} = \sqrt{2 \cdot 40 \cdot 9.8} = 28\, m s^{-1} =$$

$$= 28 \cdot \frac{3.6\cancel{00}}{1\cancel{000}}\, km/h \simeq 101\, km/h$$

$$a_{coriolis} \simeq 2 \omega v_{fin} \simeq 2 \times 7.3 \times 10^{-5} \times 28\, m s^{-2}$$

$$\simeq 4.1 \times 10^{-3}\, m s^{-2} \simeq 4.2 \times 10^{-4} g$$



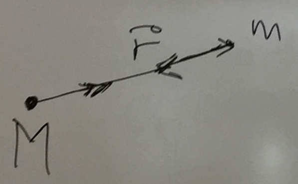


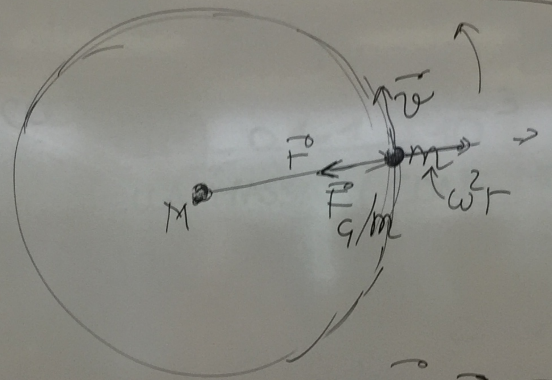
$$a_{coriolis} \simeq 2 \omega \cos\vartheta_{lat}\, v$$

$G = 6.67\times10^{-11}\ m^3 s^{-2} kg^{-1}$

Costante di gravitazione universale

$$\vec{F}_{M,m} = -\frac{GMm}{r^3}\vec{r}$$





$m << M$

$\omega = \frac{2\pi}{P}$

$\vec{\omega} \perp$ piano orbita

$$a^r_{Rot} = -\frac{GM}{r^2} + \omega^2 r + 0 = 0$$

$\vec{v} = \vec{\omega}\times\vec{r}$

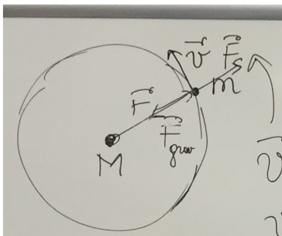
$v = \omega r$

$$\boxed{\omega^2 r^3 = \underbrace{GM}}$$

$$\frac{4\pi^2}{P^2} r^3 = GM$$

$$v = \sqrt{\frac{GM}{r^3}}\cdot r = \sqrt{\frac{GMr^2}{r^3}} = \sqrt{\frac{GM}{r}}$$

$$v \propto \frac{1}{r^{1/2}}$$



$\vec{\omega}$ $\qquad P = \frac{2\pi}{\omega}$

$$\omega_\oplus = \frac{2\pi}{\underbrace{1\cdot365.25\cdot86400\,s}_{3.15\times10^7}} = 1.99\times10^{-7}\ \frac{rad}{s}$$

$$\times\frac{180}{\pi} = 1.14\times10^{-5}\ \frac{°}{s}$$

$\vec{v} = \vec{\omega}\times\vec{r}$

$v = \omega r$

$$v_\oplus = \omega_\oplus r = 1.99\times10^{-7}\times1.49\times10^{11}\ m s^{-1} \simeq 0.986\ \frac{°}{1d}$$

$r = 1.49\times10^{11}\ m$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}\omega^2 r$$

$$\boxed{\omega^2 r^3 = (GM)}$$

$[m^2 s^{-2}]$

$$\simeq (1.99\times1.49)\cdot10^4\ m s^{-1} \simeq 30\ Km/s$$

$30\times3600 =$

$3\cdot3.6\times10^3\ Km/h$

$10^4\ Km/h$

$$\omega = \left(\frac{GM}{r^3}\right)^{1/2}$$

$$v = \omega r = \left(\frac{GM}{r}\right)^{1/2}$$

$$\frac{E}{m} = \underbrace{\frac{1}{2}v^2}_{T} \underbrace{- \frac{GM}{r}}_{U} = \frac{1}{2}\omega^2 r^2 - \frac{GM}{r}$$

$$= \frac{1}{2}\cancel{r^2}\frac{GM}{\cancel{r^3}} - \frac{GM}{r} = \boxed{-\frac{GM}{2r} = E}$$

$$dL = \vec{F}\cdot d\vec{s}$$

$$[Kg\, m^2 s^{-2}] = [J]$$

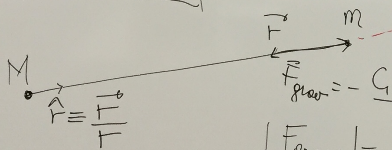
$$\frac{dL}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v}$$

$$\left[\frac{J}{s}\right] = W$$

Se $\vec{F}$ è conservativa $\Rightarrow$

$$U(P) - U(P_0) = -\int_{P_0}^{P} \vec{F}\cdot d\vec{s}$$

$$U(P) = -\int_{P_0}^{P} \vec{F}\cdot d\vec{s} + U(P_0)$$



$$\vec{F}_{grav} = -\frac{GMm}{r^3}\vec{r}$$

$$|F_{grav}| = \frac{GMm}{r^2}$$

$$U(r) = -\int^{r}\left(-\frac{GMm}{r^3}\vec{r}\cdot d\vec{r}\right) + \underbrace{U(\infty)}_{=0}$$

$$= GMm\int_{\infty}^{r} r'^{-2}\, dr' + \cancel{0}$$

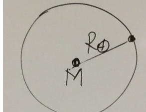
$$U(r) = GMm\left[\frac{r'^{-2+1}}{-2+1}\right]_{\infty}^{r} = -GMm\left[\frac{1}{r'}\right]_{\infty}^{r} = -\frac{GMm}{r}$$

→ Sulla

$U(R_\oplus + z) =$

$z \ll R_\oplus$

$\frac{z}{R_\oplus} \ll 1$



m

$$E_{lancio} = E_\infty = \frac{1}{2}m\underbrace{v_\infty^2}_{=0} - \frac{GMm}{r_\infty} = 0$$

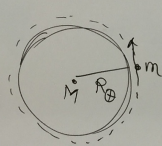
$$\frac{1}{2}\cancel{m}v_{lancio}^2 - \frac{GM\cancel{m}}{R_T} = 0$$

$$v_{lancio\ fuga} = \sqrt{\frac{2GM_\oplus}{R_\oplus}} = \sqrt{2}\cdot\sqrt{\frac{3.99\times10^{14\ 8}}{6.4\times10^{6}}} = \sqrt{2}\sqrt{\frac{3.99}{6.4}}\ 10^4\ ms^{-1}$$

$$\simeq \sqrt{2}\cdot 8\cdot 10^3\, ms^{-1}$$

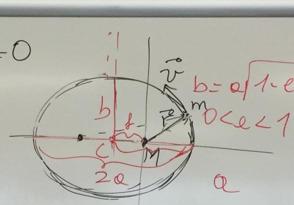
$$\simeq 11\ Km s^{-1}$$

$$GM_\oplus = 6.67\times10^{-11}\cdot 5.98\times10^{24}\ m^3 s^{-2} = 3.99\times10^{+14}\, m^3 s^{-2}$$



$$v_{sputnik} \simeq \sqrt{\frac{GM_\oplus}{R_\oplus}} \simeq 8\ Kms^{-1}$$

$$\omega = \sqrt{\frac{GM_\oplus}{R_\oplus^3}} \simeq 1.2\times10^{-3}\, rad\, s^{-1} \qquad P = \frac{2\pi}{\omega} \simeq 5.000\ s \simeq 1.4\ h$$



$b = a\sqrt{1-e^2}$

$0 < e < 1$

a

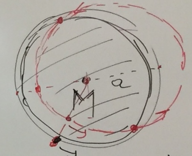
$e = 0\quad b = a$

circonferenza

$$\frac{4\pi^2}{P^2}a^3 = GM$$

$$\omega^2_{media}$$

$$E = -\frac{GM}{2a}$$



$$E = -\frac{GM}{2a}$$

$$E = -\frac{GM}{2a}$$

NEWTON (R1)

$$\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

$\vec{p}$ quantità di moto lineare

$$\vec{r}\times\vec{F} = \frac{d}{dt}(\vec{J})$$

$$\vec{N}$$

$$\vec{J} = \vec{r}\times\vec{p}$$

$$F_{grav} \Rightarrow \vec{N} = 0 \Rightarrow \vec{J} = \text{costante}$$

$-\frac{GMm}{r}$

$+U(\infty)$, $U(\infty) = 0$

→ Sulla sup. della Terra

$$U(R_\oplus + z) = -\frac{GMm}{(R_\oplus + z)} = -\frac{GMm}{R_\oplus\left(1+\frac{z}{R_\oplus}\right)} = -\frac{GMm}{R_\oplus}\left(1 - \frac{z}{R_\oplus}\right) = -\frac{GMm}{R_\oplus} + \frac{GMm}{R_\oplus^2}z$$

$z \ll R_\oplus$

$\frac{z}{R_\oplus} \ll 1$

$$U(R_\oplus + z) - U(R_\oplus) = -\cancel{\frac{GMm}{R_\oplus}} + \underbrace{\frac{GM}{R_\oplus^2}}_{=g}m\, z + \cancel{\frac{GMm}{R_\oplus}} = +mgz$$