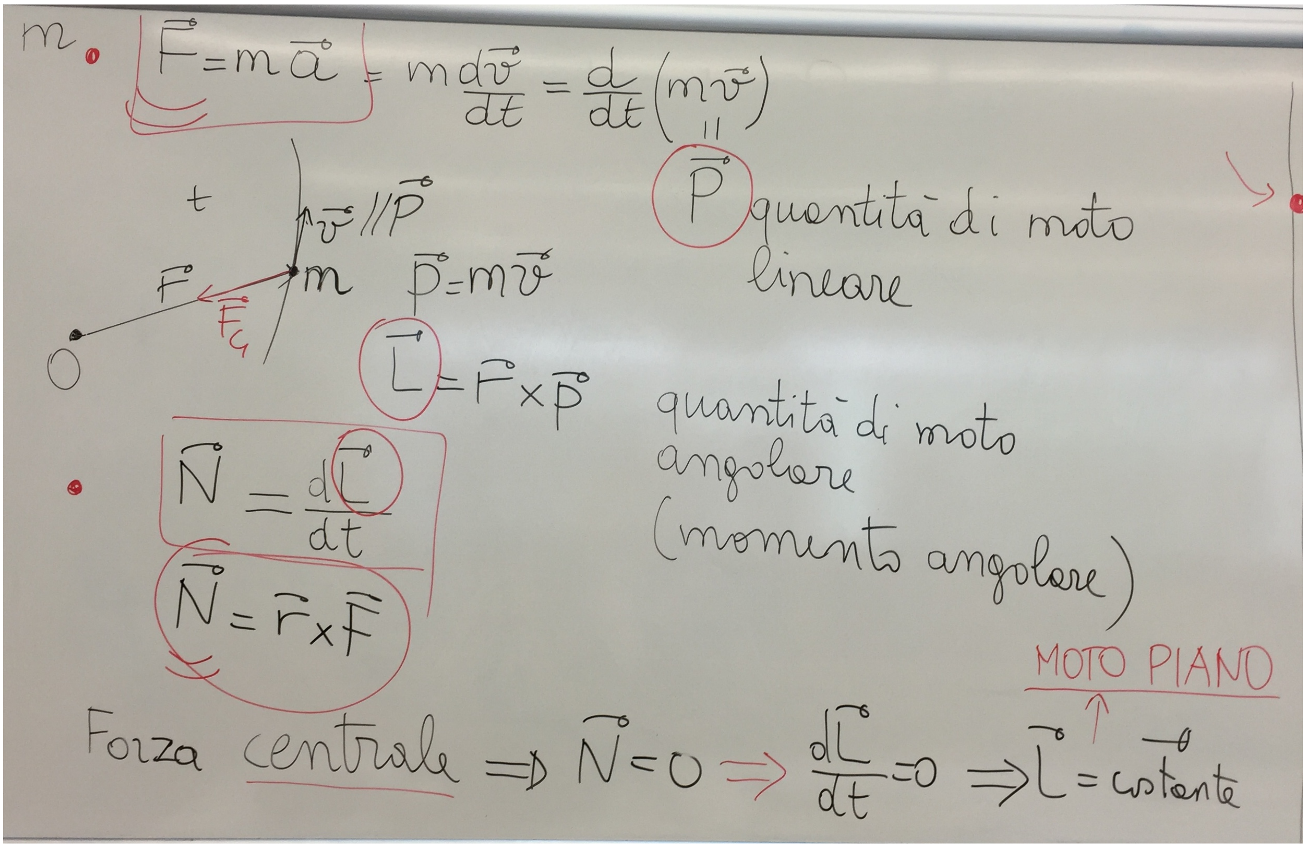
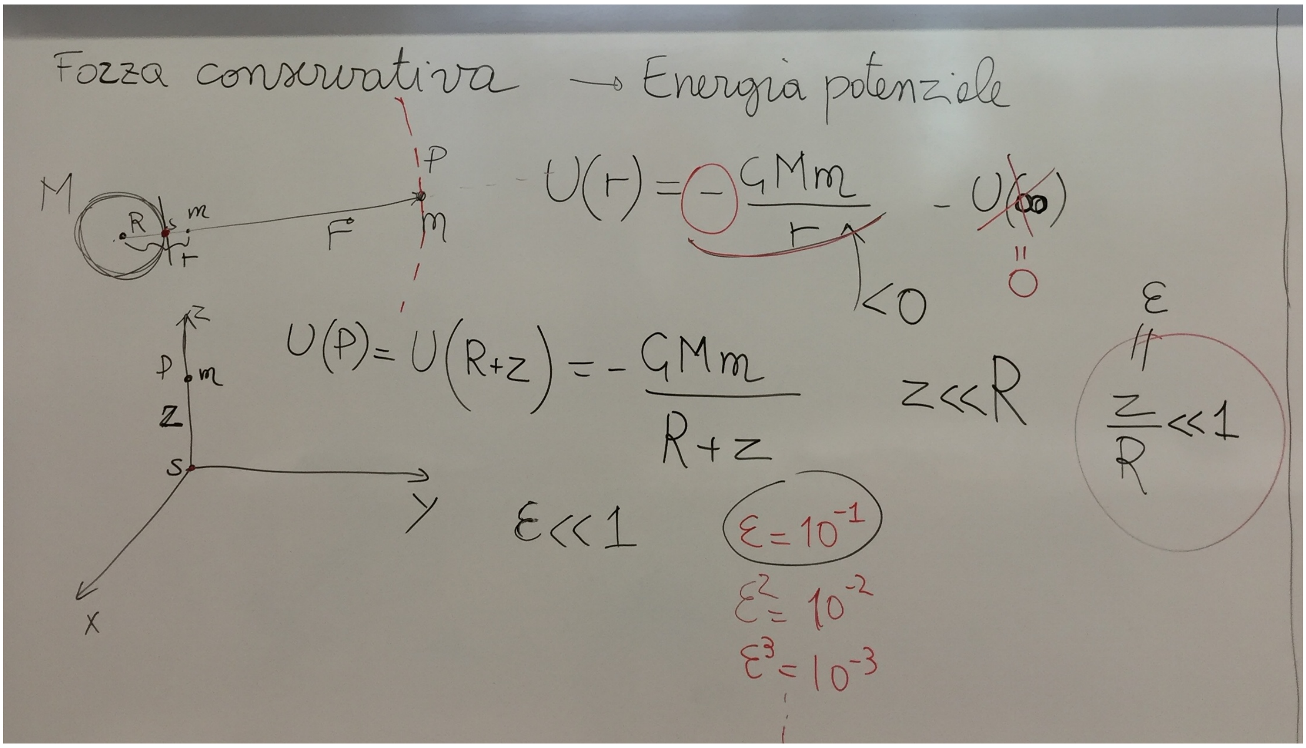
S3 - Lezioni 1 e 3 Marzo 2016:

- Confronto tra gli effetti della forza e del momento della forza; dimostrazione che se la forza è centrale il moto deve svolgersi in un piano
- Calcolo della energia potenziale gravitazionale a grande distanza e vicino alla superficie della Terra. Importanza del segno nei due casi.
- Richiami sullo sviluppo in serie di potenze
- Forze dissipative e loro effetto
- Forza di resistenza dell'aria su un satellite in orbita e su un corpo in caduta
- Forza di Archimede





$f(x)$ $\quad x \ll 1$ parametro di piccolezza

SVILUPPI IN SERIE DI POTENZE $\quad f=(1+\varepsilon)^{-1}$

$$f(x) \simeq f(0) + x \left.\frac{df}{dx}\right|_{x=0} + \frac{x^2}{2}\left.\frac{d^2f}{dx^2}\right|_{x=0} + \frac{x^3}{3!}\left.\frac{d^3f}{dx^3}\right|_{x=0} \cdots + \frac{x^n}{n!}\left.\frac{d^nf}{dx^n}\right|_{x=0} + O(x^{n+1})$$

$x=0$

$$n! = n(n-1)(n-2)\cdots(n-(n-1))$$

energia potenziale di m sulla sup. della Terra

$$U(P) = -\frac{GMm}{(R+z)} = -\frac{GMm}{R\left(1+\frac{z}{R}\right)} = -\left(\frac{GMm}{R}\right)\frac{1}{1+\varepsilon} = -\left(\frac{GMm}{R}\right)(1+\varepsilon)^{-1}$$

$\varepsilon = \frac{z}{R} \ll 1$

$$f(\varepsilon) = (1+\varepsilon)^{-1} \simeq f(0) + \varepsilon\left.\frac{df}{d\varepsilon}\right|_{\varepsilon=0} + \frac{\varepsilon^2}{2}\left.\frac{d^2f}{d\varepsilon^2}\right|_{\varepsilon=0} + O(\varepsilon^3) \simeq 1 - \varepsilon + \frac{\varepsilon^2}{2} + O(\varepsilon^3)$$

$f(0) = 1$, $\varepsilon = 0$

$$\frac{d(1+\varepsilon)^{-1}}{d\varepsilon} = -1(1+\varepsilon)^{-2}\cdot\frac{d(1+\varepsilon)}{d\varepsilon} = -(1+\varepsilon)^{-2}\cdot 1 = -(1+\varepsilon)^{-2}$$

$$\frac{d\left[-(1+\varepsilon)^{-2}\right]}{d\varepsilon} = +(1+\varepsilon)^{-3}\cdot\frac{d(1+\varepsilon)}{d\varepsilon} = (1+\varepsilon)^{-3}$$

$$\frac{d^2(1+\varepsilon)^{-1}}{d\varepsilon^2}$$

$$U(P) = -\frac{GMm}{R}\left(1 - \frac{z}{R} + \frac{1}{2}\frac{z^2}{R^2} + O\left(\frac{z^3}{R^3}\right)\right) \simeq -\frac{GMm}{R}\left(1 - \frac{z}{R} + \frac{1}{2}\frac{z^2}{R^2}\right) - \cancel{U(\infty)}$$

$U(\infty) = 0$

$U(z) = mgz > 0$

$U(R+z)$ $\quad = -(U(R)) = -\left(-\frac{GMm}{R}\right)$

$$U(R+z) + \frac{GMm}{R} \simeq +\frac{GMm}{R^2}z - \frac{GMm}{2R^3}z^2$$

g

$$g = \frac{GM}{R^2} \simeq \frac{3.99\times10^{14}}{(6.4)^2\times10^{12}} \simeq \frac{3.99}{(6.4)^2}\times10^2\ ms^{-2} \simeq 9.8\ ms^{-2}$$

$(GM)_\oplus = 3.99\times10^{14}\ m^3 s^{-2} \cancel{kg^{-1}} \cancel{kg}$

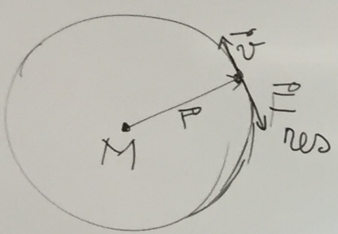
Es: $\frac{40\,m}{2\cdot6.4\times10^6} \simeq \frac{2}{6.4}\times10^{-5} \simeq 3\times10^{-6}$

0.000003

$$U(R+z) - U(R) \simeq mgz - mgz\left(\frac{z}{2R}\right) \simeq mgz\left(1 - \frac{z}{2R}\right)$$

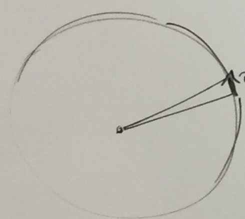
$U(R) = 0$, $\to 0$

$$\simeq mgz \qquad U(z) = mgz$$



$$v = \omega r = \sqrt{\frac{GM}{r}}$$

$$\omega^2 r^3 = GM \qquad P = \frac{2\pi}{\omega} \qquad E_{grav} = -\frac{GMm}{2r}$$



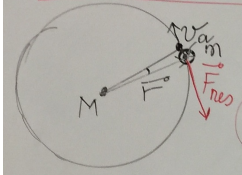
$d\vec{s} = \vec{v}\,dt$

$$dL = \vec{F}_{res}\cdot d\vec{s} = \vec{F}_{res}\cdot\vec{v}\,dt \qquad \text{energia}$$

$$\frac{dL}{dt} = \vec{F}_{res}\cdot\vec{v} = -F_{res}\,v \qquad \text{potenza}$$

$$\frac{d}{dt}\left(-\frac{GMm}{2r}\right) \leftarrow \frac{dE_{grav}}{dt} = \frac{dL}{dt} = -F_{res}\,v$$

FORZE DISSIPATIVE E LORO EFFETTO



$$P = \frac{2\pi}{\omega} \qquad \omega^2 r^3 = GM$$

$$v = \omega r = \sqrt{\frac{GM}{r}}$$

$$E_{grav}(r) = -\frac{GMm}{2r}$$

$$\frac{dE_{grav}}{dt}\,?$$

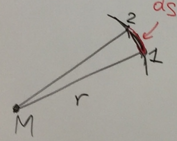
$$E_{grav}(r) = -\frac{GMm}{2r} = \left(-\frac{GMm}{2}\right) r^{-1}$$

$$\frac{dE_{grav}}{dt} = \left(-\frac{GMm}{2}\right)\left(-r^{-2}\frac{dr}{dt}\right)$$

$$\frac{dE_{grav}}{dt} = \frac{GMm}{2r^2}\left(\frac{dr}{dt}\right) \quad \dot r$$

$$\dot E_{grav} = -F_{res}\,\omega r$$

$$d\mathcal{L} = \vec F_{res}\cdot d\vec s = \vec F_{res}\cdot \vec v\, dt$$

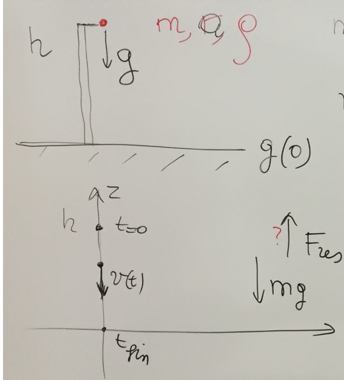


$$\vec v = \frac{d\vec s}{dt}$$

$$\frac{d\mathcal{L}}{dt} = -F_{res} v = -F_{res}\,\omega r$$

$$\dot r = -2F_{res}\frac{\omega r^3}{GMm} = -\frac{2F_{res}\,\omega r^3}{\omega^2 r^3 m}$$

$$\boxed{\dot r = -\frac{2}{\omega}\left(\frac{F_{res}}{m}\right)} < 0$$



$$m = \left(\frac{4}{3}\pi a^3\right)\rho \qquad v(0) = 0$$

$$F_{res} = \left(\frac{1}{4}\right)\rho_{aria}(\pi a^2)\, v^2$$

$$\frac{kg}{m^3}\cdot m^2\cdot m^2 s^{-2} = kg\, m\, s^{-2}$$

$$E(t=0) = E(t_{fin})$$

$$K + U = mgh = \frac{1}{2} m v_{fin}^2$$

$$v(t_{fin}) = \sqrt{2gh}$$

$$F_{res} \ll mg$$

$$1 \gg \frac{(F_{res})_{max}}{mg} = \frac{1}{2}\frac{\rho_{aria}\pi a^2 g h}{\frac{4}{3}\pi a^3 g \rho} = \frac{3}{8}\frac{\rho_{aria}}{\rho a} h \approx 0.1$$

$$(F_{res})_{max} = \frac{1}{4}\rho_{aria}\pi a^2 v^2(t_{fin}) = \frac{1}{4}\rho_{aria}\pi a^2 g h$$

$h = 15\ m$

$\rho_{aria} \simeq 1.2\ kg/m^3$

$\rho = 2.7\ g\ cm^{-3}$

$a = 25\ cm$

$$F_A = g\left(\frac{4}{3}\pi a^3\right)\rho_{aria}$$

$$\frac{F_A}{mg} = \frac{g\,\frac{4}{3}\pi a^3\,\rho_{aria}}{g\,\frac{4}{3}\pi a^3\,\rho} = \frac{\rho_{aria}}{\rho} \simeq 4.4\times 10^{-4}$$

$$g(0) = \frac{GM}{R_T^2} \qquad g(h) < g(0) \qquad h \ll R_T \qquad \underbrace{\frac{h}{R_T}}_{=\varepsilon} \ll 1$$

$$g(h) = \frac{GM}{(R_T+h)^2} = \frac{GM}{R_T^2\left(1+\frac{h}{R_T}\right)^2} = \underbrace{\frac{GM}{R_T^2}}_{g(0)}(1+\varepsilon)^{-2} \simeq g(0)\left(1 - 2\frac{h}{R_T}\right) \simeq g(0)\left(1 - 4.7\times 10^{-6}\right)$$

$h = 15\ m$, $R_T = 6400\ km$

$$f(\varepsilon) = (1+\varepsilon)^{-2} \simeq 1 - 2\varepsilon$$

$$-2(1+\varepsilon)^{-3}$$