GALILEO
and the
Universality of Free Fall:
the facts behind the legend

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Erice May 5, 2004

NEWTON's VIEW OF FREE FALL

In the gravitational field of a source body (e.g. Earth):

$$m_i \ddot{\vec{g}} = -\frac{GM_\oplus m_g}{r^3}\vec{r}$$

PRINCIPIA opening paragraph: "This quantity that I mean hereafter under the name of ... **mass** ... is known by the **weight** ... for it is proportional to the weight as I have found by experiments on pendulums, very accurately made..."

(Equivalence of inertial-to-gravitational mass: Equivalence Principle)

$$m_g = m_i \Rightarrow \eta \equiv \frac{\Delta g}{g} = 0$$

Universality of Free Fall (UFF)

GR IS BASED ON THE EQUIVALENCE "PRINCIPLE" (I)

- **Einstein (1907): "hypothesis of complete physical equivalence" between a gravitational field and an accelerated reference frame: In a freely falling system all masses fall equally fast, hence gravitational acceleration has no local dynamical effects ("Weak Equivalence Principle-WEP")**

An observer inside Einstein elevator will not be able to tell, before hitting the ground, whether he is moving with an acceleration g in empty space, far away from all masses, or else he is falling in the vicinity (height of fall<<Earth radius) of a body (the Earth) whose local gravitational acceleration is also g (and in the same direction).

The experimental consequence is the Universality of Free Fall (UFF):

$$\eta \equiv \frac{\Delta g}{g} = 0$$

… unique to gravity!

GR IS BASED ON THE EQUIVALENCE "PRINCIPLE" (II)

- "Einstein Equivalence Principle (EEP)" (assumes WEP): all gravitational effects are (globally) replaced by the metric of a curved, 4-dimension space-time…

General Relativity is based on WEP

WEP ⇒ UFF: should bodies of different composition fall with different accelerations, the elevator and the test mass inside it would generally fall with different accelerations and the observer would be able to tell that he is close to the surface of the Earth and not in an accelerated frame in empty space.

a deviation from UFF would require a modification of GR

STATE OF THE ART

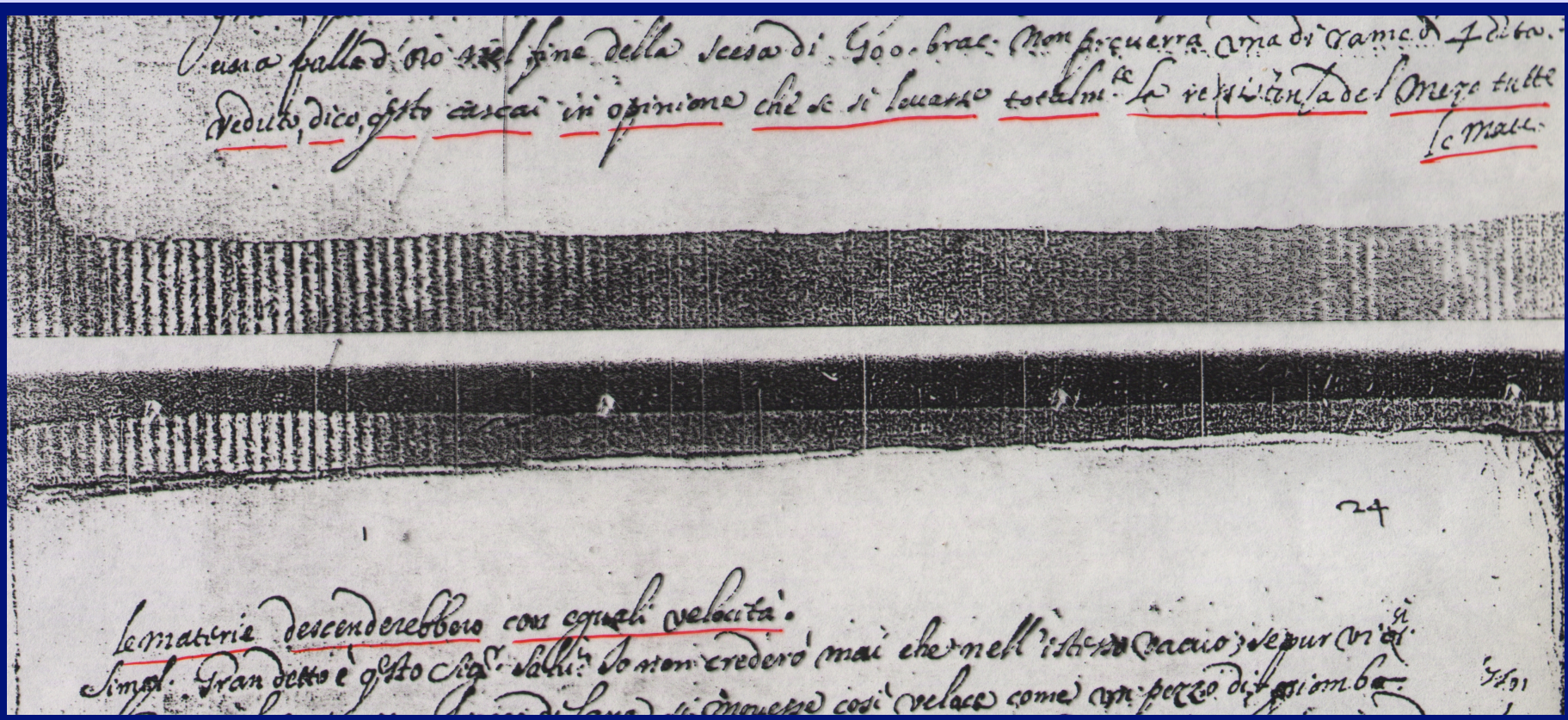
From experiments by the "Eöt-Wash" group (slowly rotating torsion balance PRD 50, 3614 1994)

$$\eta(Be,Cu) = (-1.9 \pm 2.5)\cdot 10^{-12}$$

From recent predictions based on string theory (PRD 66 046007; PRL 89 081601, 2002) deviations from UFF might appear already at a level just below these experimental result:

$$\eta(Be,Cu) \leq 10^{-12}$$

GALILEO's Handwritten Statement of the UNIVERSALITY of FREE FALL



Veduto, dico, questo cascai in opinione che se si levasse totalmente la resistenza del mezzo tutte le materie descenderebbero con uguali velocità.

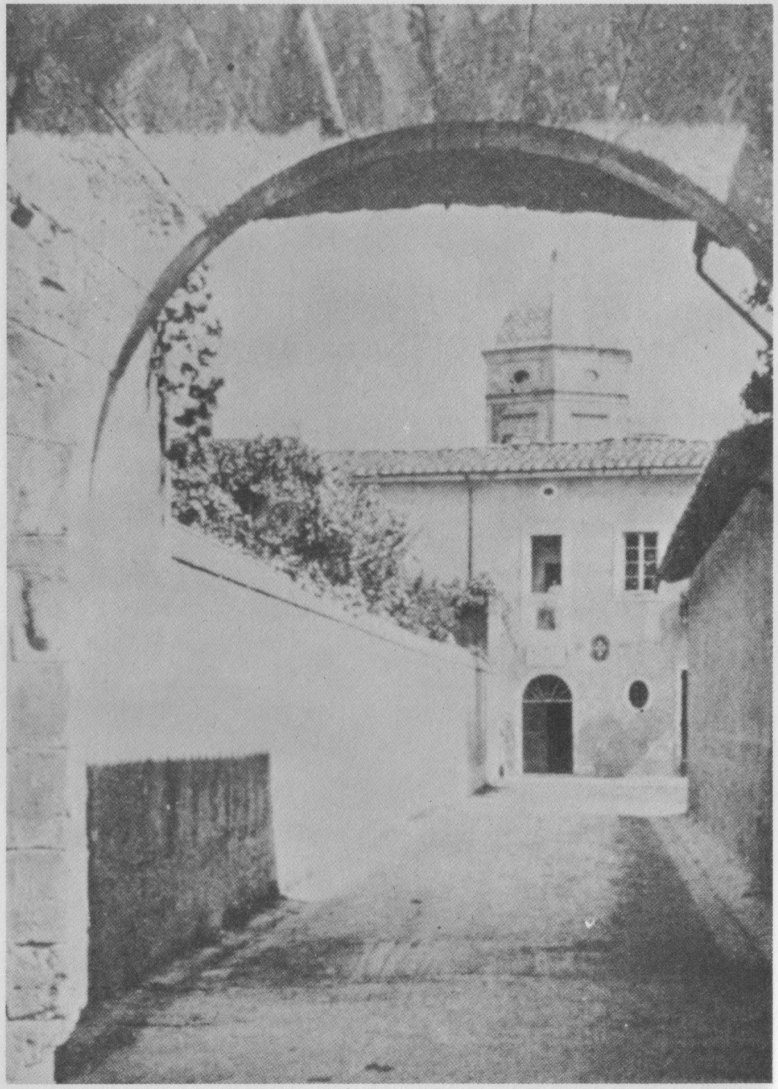
Having observed this I came to the conlcusion that in a medium totally devoid of resistance all bodies [substances] would fall with the same speed.

GALILEO's "DISCORSI" (I)

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI;
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

GALILEO's "DISCORSI" in the English Translation



Galileo's birthplace in Pisa

Dialogues Concerning

TWO NEW SCIENCES

GALILEO GALILEI

TRANSLATED BY
Henry Crew & Alfonso de Salvio

WITH AN INTRODUCTION BY
Antonio Favaro

DOVER PUBLICATIONS, INC., NEW YORK

GALILEO's DISCORSI (II)

"I Discorsi": Completed in 1634, published in 1638 in Leiden (because at that time Galileo was not allowed by the Church of Rome to publish in Italy)

Galileo was under house arrest (Arcetri, Florence) from 1636 till his death in 1642 (his troubles with the Church started in 1632)

Galileo was blind since 1637

GALILEO's FULL STATEMENT (I)

SALV. Or sia questa, in grazia del Sig. Simplicio, la soluzione del nostro dubbio: e lasciato il digredire, torniamo al nostro proposito. Veduto come la differenza di velocità, ne i mobili di gravità diverse, si trova esser sommamente maggiore ne i mezzi più e più resistenti; ma che più? nel mezzo dell'argento vivo l'oro non solamente va in fondo più velocemente del piombo, ma esso solo vi descende, e gli altri metalli e pietre tutti vi si muovono in su e vi galleggiano, dove che tra palle d'oro, di piombo, di rame, di porfido, o di altre materie gravi, quasi del tutto insensibile sarà la disegualità del moto per aria, chè sicuramente una palla d'oro nel fine della scesa di cento braccia non preverrà una di rame di quattro dita; veduto, dico, questo, cascai in opinione che se si levasse totalmente la resistenza del mezzo, tutte le materie descenderebbero con eguali velocità.

SIMP. Gran detto è questo, Sig. Salviati. Io non crederò mai che nell'istesso vacuo, se pur vi si desse il moto, un´ fiocco di lana si movesse così veloce come un pezzo di piombo.

GALILEO's FULL STATEMENT (II)

..the difference of speed between bodies of different specific gravity is most marked in those media which are the most resistant….even more, in quick silver not only gold falls faster than lead, but in fact gold only does descend while all other metals and stones float..

On the other hand the variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so light that in a fall of 100 cubits a ball of gold would surely not outstrip one of copper by as much as four fingers. Having observed this I came to the conclusion that in a medium totally devoid of reistance all bodies [substnaces] would fall with the same speed

1cubit (Tuscany)=58.3 cm

"I DISCORSI" ARE BASED ON MUCH EARLIER WORK... (I)

88.

GALILEO a GUIDOBALDO DEL MONTE [in Montebaroccio].

Padova, 29 novembre 1602.

Bibl. Naz. Fir. Mss. Gal., P. VI, T. VI, car. 10. — Copia di mano del secolo XIX, trascritta quando fu messa insieme la raccolta Palatina dei Mss. Galileiani, e derivata da copia che dall'originale aveva tratto di sua mano VINCENZIO VIVIANI. Alla copia moderna è premessa la seguente indicazione, che certamente fu riprodotta dalla copia di pugno del VIVIANI: « Copia di lettera del Sig.r Galileo, da Padova li 29 Novembre 1602, al Sig.r Marchese Guid'Ubaldo dal Monte, a Monte Baroccio, cavata da me dall'originale mandatomi da Pesaro dal Sig.r Dottor Costanzo Pompei con sua lettera del primo Gennaio 1667 *ab Inc.ne* e da esso trovata in un sacco di varie scritture attenenti all'eredità di detto Sig.r Guid'Ubaldo, esistente oggi in Pesaro appresso . . . ».

Ill.mo Sig.e e P.ron Col.mo

V. S. Ill.ma scusi la mia importunità, se persisto in voler persuaderle vera la proposizione de i moti fatti in tempi uguali nella medesima quarta del cerchio [1]; perchè, essendomi parsa sempre mirabile, hora viepiù mi pare, che da V. S. Ill.ma vien reputata come impossibile: onde io stimerei grand' errore e mancamento il mio, s'io permettessi che essa venisse repudiata dalla di lei speculazione, come quella che fusse falsa, non meritando lei questa nota, nè tampoco d'esser bandita dall'intelletto di V. S. Ill.ma, che più d'ogn'altro la

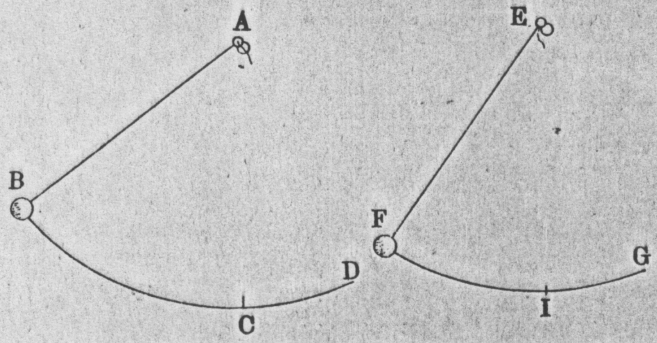
19. Prima aveva scritto *alla terza*, e poi corresse *a sesta*. —

[1] Cfr. Vol. II, pag. 259.

"I DISCORSI" ARE BASED ON MUCH EARLIER WORK... (II)

potrà più presto ritrarre dall' esilio delle nostre menti. E perchè l' esperienza, con che mi sono principalmente chiarito di tal verità, è tanto certa, quanto da me confusamente stata esplicata nell' altra mia, la replicherò più apertamente, onde ancora lei, facendola, possa accertarsi di questa verità.

Piglio dunque due fili sottili, lunghi ugualmente due o tre braccia l' uno, e siano AB, EF, e gli appicco a due chiodetti A, E, e nell' altre estremità B, F lego due palle di piombo uguali (se ben niente importa se fussero disuguali), rimuovendo poi ciascuno de' detti fili dal suo perpendicolo, ma uno assai, come saria per l' arco CB, e l' altro pochissimo, come saria secondo l' arco IF; gli lascio poi nell' istesso momento di tempo andar liberamente, e l' uno comincia a descrivere archi grandi, simili al BCD, e l' altro ne descrive de' piccoli, simili all' FIG; ma non però consuma più tempo il mobile B a passare tutto l' arco BCD, che si faccia l' altro mobile F a passare l' arco FIG. Di che mi rendo sicurissimo così:



Il mobile B passa per il grand' arco BCD, e ritorna per lo medesimo DCB, e poi ritorna verso D, e va per 500 e 1000 volte reiterando le sue reciprocazioni; l' altro parimente va da F in G, e di qui torna in F, e parimente farà molte reciprocazioni; e nel tempo ch' io numero, verbi grazia, le prime cento grandi reciprocazioni BCD, DCB etc., un altro osservatore numera cento altre reciprocazioni per FIG piccolissime, e non ne numera pure una sola di più: segno evidentissimo che ciascheduna particolare di esse grandissime BCD consuma tanto tempo, quanto ogni una delle minime particolari FIG.

LE OPERE (GALILEO's COLLECTED WORKS)

LE OPERE

DI

GALILEO GALILEI

NUOVA RISTAMPA DELLA EDIZIONE NAZIONALE

SOTTO L'ALTO PATRONATO

DEL

PRESIDENTE DELLA REPUBBLICA ITALIANA

GIUSEPPE SARAGAT

VOLUME VIII.

FIRENZE

G. BARBÈRA - EDITORE

1968

HOW DID GALILEO COME TO FORMULATE THE UFF?

Aristoteles view cannot be right!

Experimental proof is needed!

Guess: the difference in time of fall may be due to air resistance (… test guess by amplifying resistance of medium)

Difficulties with experiments based on dropping masses from a height …

Inclined plane experiments

Pendulum experiments

ARITOTELES CANNOT BE RIGHT!

Aristoteles view: the free fall velocity of bodies is proportional to their weight

Galileo's simple logical argument…

" If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower and the slower will be somewhat hastened by the swifter… hence the heavier body (made by the two tied together) moves with less speed than the lighter; an effect which is contrary to your supposition."

Ex: two bricks tied together to form a single brick do obviously fall at the same speed as each brick separately, while according to Aristoteles should fall faster because they are twice as heavy…

EXPERIMENTAL PROOF IS NEEDED!

“ The facts set forth by me up to this point and, in particular, the one which shows that difference of weight, even when very great, is without effect in changing the speed of falling bodies, so that as far as weight is concerned they all fall with equal speed: this idea is, I say, so new, and at first glance so remote from fact that if we do not have the means of making it just as clear as sunlight, it had better not be mentioned; but having once allowed it to pass my lips I must neglect no experiment or argument to establish it”

This is what makes the difference w.r.t Philopponus (6th century) and Benedetti (1553) who had questioned Aristoteles view before Galileo…

MAKE A GUESS AND TEST IT!

..the difference of speed between bodies of different specific gravity is most marked in those media which are the most resistant….

….even more, in quick silver not only gold falls faster than lead, but in fact gold only does descend while all other metals and stones float..

It is typical of Galileo's scientific method to make a guess on a possible physical effect and put it to test by amplifying it…

DIFFICULTIES IN MASS DROPPING FROM A HEIGHT (I)

"The experiments made to ascertain whether two bodies, differing greatly in weight will fall from a given height with the same speed offers some difficulty; because, if the height is considerable, the retarding effect of the medium, which must be penetrated and thrust aside by the falling body, will be greater in the case of the small momentum of the very light body than in the case of the great violenza of the heavy body;

so that, in a long distance, the light body will be left behind; if the height be small, one may well doubt whether there is any difference; and if there be a difference it will be inappreciable."

DIFFICULTIES IN MASS DROPPING FROM A HEIGHT (II)

"It occurred to me therefore to repeat many times the fall through a small height in such away that I might accumulate all those small intervals of time that elapse between the arrival of the heavy and the light bodies respectively at their common terminus, so that this sum makes an interval of time which is not only observable, but easily observable."

GALILEO's INCLINED PLANE EXPERIMENTS

"In order to employ the slowest speeds possible and thus reduce the change which the resisting medium produces upon the simple effect of gravity it occurred to me to allow the bodies to fall along a plane slightly inclined to the horizontal. For in such a plane, just as well as in a vertical plane, one may discover how bodies of different weight behave:…"

Galileo's tools: an inclined plane to show the fall + a **water clock** to measure the duration of fall

He measured time by the weight of water flowing from a large vessel through a narrow pipe

+ he knew that: $s \propto t^2$ (letter to Paolo Sarpi, 1604; original at University of Pisa library)

⇒ controlled experiment, much better than the tower experiment.

AIR RESISTANCE

Galileo did not know that air drag increases as V^2 :

$$a_{drag} \propto \rho_{atm} \left(\frac{A}{M} \right) V^2$$

$$\left(\frac{A}{M} \right) \propto \frac{1}{\rho r}$$

area-to-mass ratio of the falling body

.. had he known, he might have compensated for drag by a proper choice of density and radius of the falling bodies, so as to make their product (hence drag) the same!

However he knew that slowing down the falling speed would also reduce the resistance of the medium, so inclined plane experiments were the right choice

FROM INCLINED PLANE TO PENDULUM EXPERIMENTS (I)

How to get rid of friction of inclined plane experiments?

Galileo's answer was: pendulum experiments…and it was a very subtle idea indeed
What has pendulum to do with falling bodies (either vertically or on inclined planes)?

Cord theorem: given a circle in a vertical plane, the times of fall along the vertical diameter and whatever cord through the lowest exterme of the same diameter, are the same

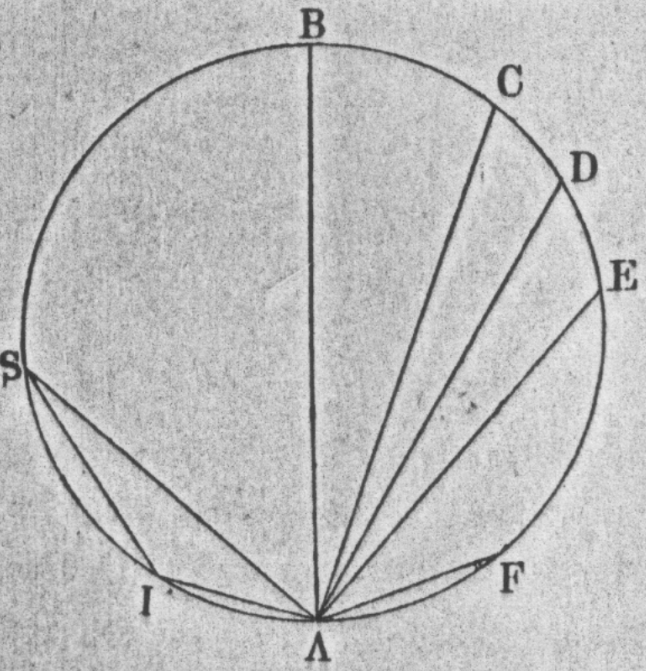
Galileo, letter to Guidobaldo dal Monte, 1604

FROM INCLINED PLANE TO PENDULUM EXPERIMENTS (II)

Galileo, letter to Guidobaldo dal Monte, 1604

basi uguali siano sempre uguali, potendone fare uno brevissimo e l' altro lungo mille miglia. Ma restando nella medesima materia, io credo haver dimostrato questa conclusione, non meno dell' altra inopinabile.

Sia del cerchio BDA il diametro BA eretto all' orizzonte, e dal punto A sino alla circonferenza tirate linee *utcumque* AF, AE, AD, AC: dimostro, mobili uguali cadere in tempi uguali e per la perpendicolare BA e per piani inclinati secondo le linee CA, DA, EA, FA; sicchè, partendosi nell' istesso momento dalli punti B, C, D, E, F, arriveranno in uno stesso momento al termine A, e sia la linea FA piccola quant' esser si voglia.



59. *l' altra* — 75. *linee utrumque AF* —

FROM INCLINED PLANE TO PENDULUM EXPERIMENTS (III)

In addition to the cord theorem, Galileo showed that the ratio of the times taken by a pendulum to reach the vertical along a small arc and the time of fall of a body through a distance equal to the length of the pendulm is a fixed adimensional number which he determined quite accurately (to $2 \cdot 10^{-3}$):

$$\frac{P/4}{t_{fall}} = \frac{\pi\sqrt{2}\sqrt{R/g}}{4\sqrt{R/g}} = \frac{\pi\sqrt{2}}{4}$$

".. E passando più avanti, ho anco voluto liberarmi da qualche impedimento che potesse nascer dal contatto di essi mobili su' l detto piano declive: e finalmente ho preso due palle, una di piombo e un di sughero…."

GALILEO's PENDULUM TEST OF THE EQUIVALENCE PRINCIPLE

Accordingly I took two balls, one of lead and one of cork, the former more than a hundred times heavier than the latter, and suspended them by means of two equal fine threads, each four or five cubits long. Pulling each ball aside from the perpendicular, I let them go at the same instant, and they, falling along the circumferences of circles having these equal strings for semi-diameters, passed beyond the perpendicular and returned along the same path. This free vibration [per lor medesime le andate e le tornate] repeated a hundred times showed clearly that the heavy body maintains so

[129]

nearly the period of the light body that neither in a hundred swings nor even in a thousand will the former anticipate the latter by as much as a single moment [minimo momento], so perfectly do they keep step. We can also observe the effect of the medium which it offers to motion, diminishes the vibration of the cork more than that of the lead, but without altering the frequency of either; even when the arc traversed by the cork did not exceed five or six degrees while that of the lead was fifty or sixty, the swings were performed in equal times.

Most probably the first null experiment in Physics!

ACCURACY OF THE TEST

Same test performed by Newton much later, who explicitly quotes an accuracy of 1 part in 10^3

Phase lag after N oscillation only determined by an error in relative length of the pendulums: 0.1 % reasonable, agrees with Galileo's statement

Experiment was repeated by Fuligni & Iafolla in Rome, showing that it is in fact difficult to be less accurate than that…

Note: Newton could use more precise clocks (based on the novel work of Huygens on cycloidal pendulum), but the point is that no clock was needed in this null experiment!

PISA TOWER EXPERIMENT BY RENIERI

Renieri, letter to Galileo (march 1641): “ Habbiamo qui avuto occasione di far un’ esperienza di due gravi cadenti da alto, di diversa materia, ma dell’ istessa grandezza; perché un tal Gesuita scrive che scendono nello stesso tempo, e con pari velocità arrivano a terra.. Ma finalmente habbiamo trovato il fatto in contrario, poiché dalla cima del campanile del Duomo tra la palla di piombo e quella di legno vi corrono tre braccia almeno di differenza. Si fecero anche esperienze di due palle di piombo, una della grandezza eguale a un’ ordinaria d’ artiglieria e l’ altra da moschetto, e si vedeva tra la più grossa e la più piccola, dal’ altezza dello stesso campanile, esservi un buon palmo di differenza, del quale la più grossa anticipava la più piccola. “

In another letter to Galileo 1 week later, Renieri promises to read Galileo’ s “Discorsi” and also to repeat the tower experiments to better check

But he was right, even though he did not know why….

FREE FALL IN AIR (I)

4 forces act on a free falling spherical body (ρ, r) in air:

$$F_g = (4/3)\rho\pi r^3 g$$ gravitational force

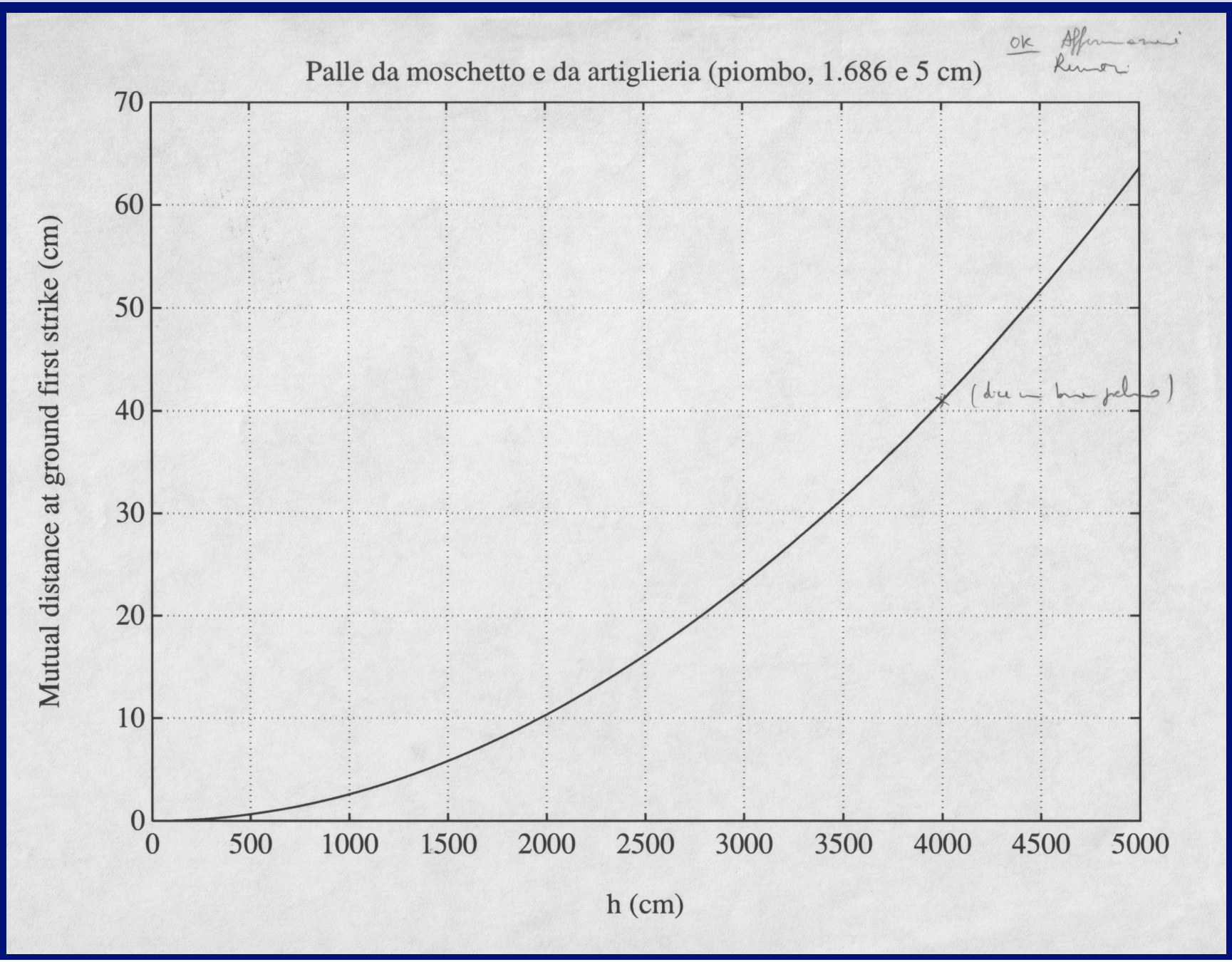
$$F_a = (4/3)\rho_{atm}\pi r^3 g$$ Archimedes force

$$F_{drag} = (1/2)C_D\pi r^2 \rho_{atm} V^2$$ drag force, C_D=0.5 at low h

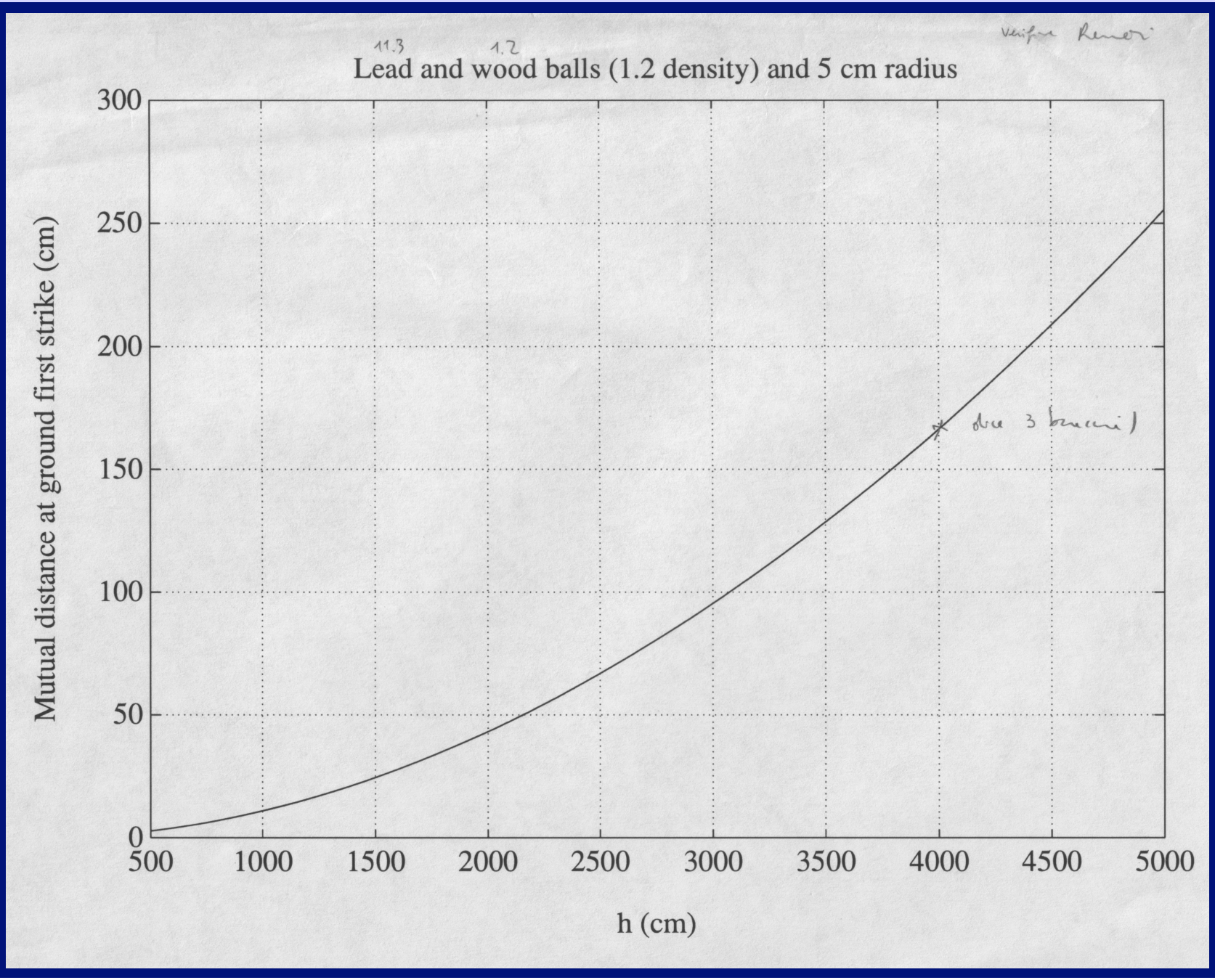
$$F_f = (1/2)(4/3)\pi r^3 \rho_{atm} dV/dt$$ friction force

Let us check Renieri's results…

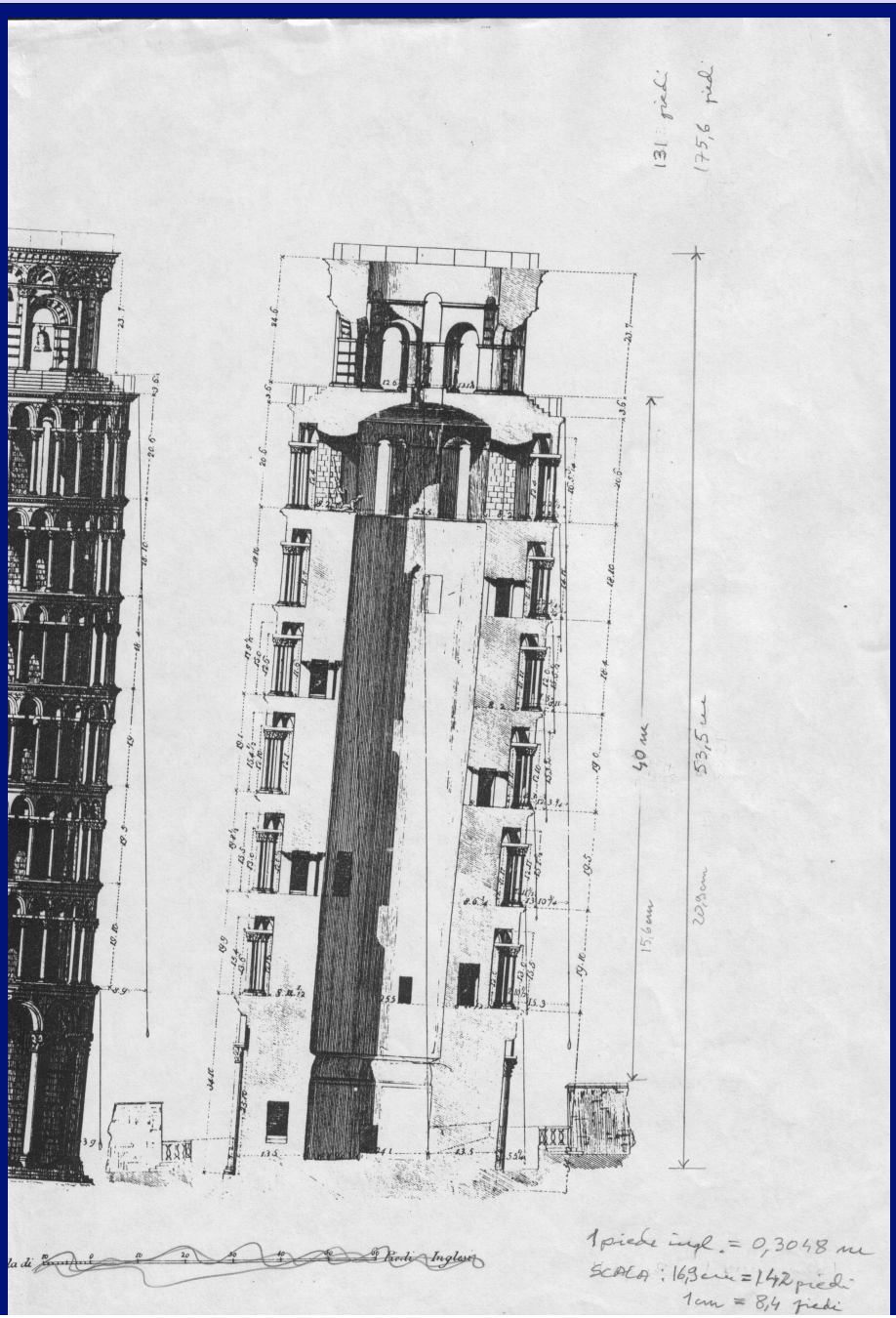
FREE FALL IN AIR (II)



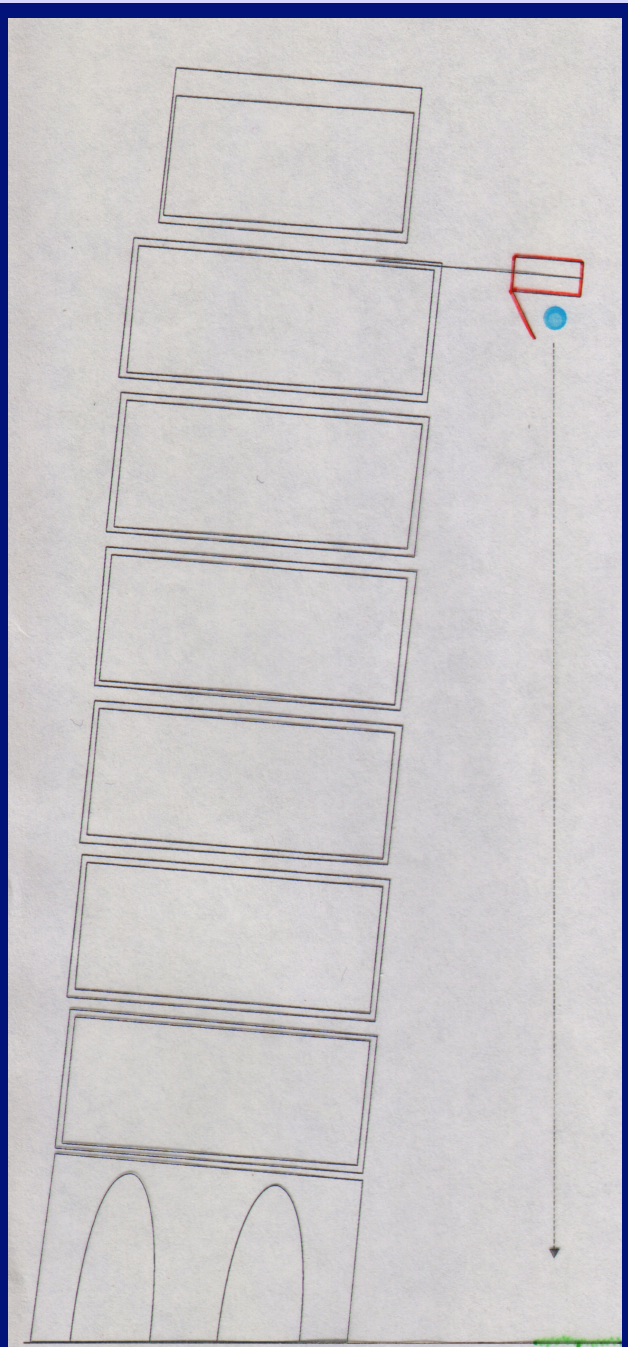
FREE FALL IN AIR (II)



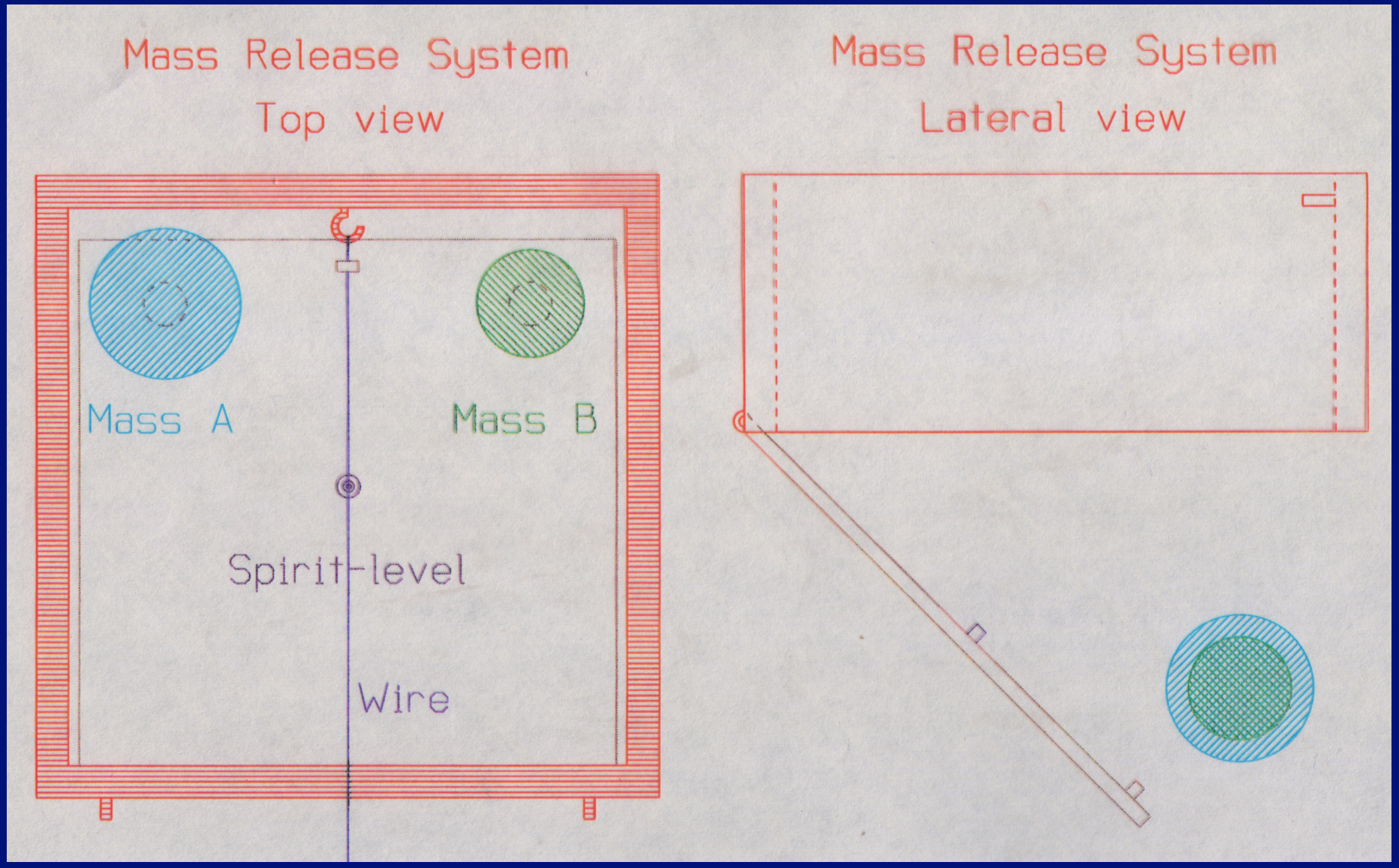
OUR EXPERIMENTS from the LEANING TOWER of PISA



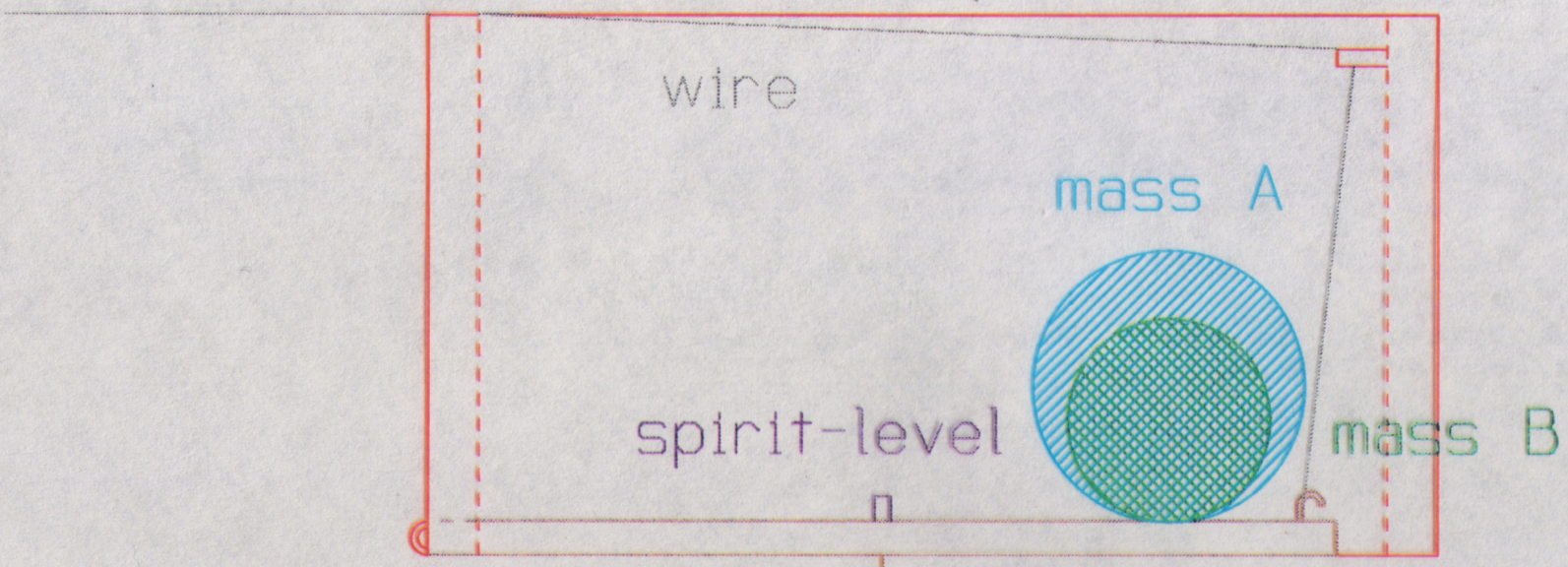
HOW to RELEASE the TEST MASSES at the SAME TIME?



OUR RELEASE MECHANISM (I)



OUR RELEASE MECHANISM (II)



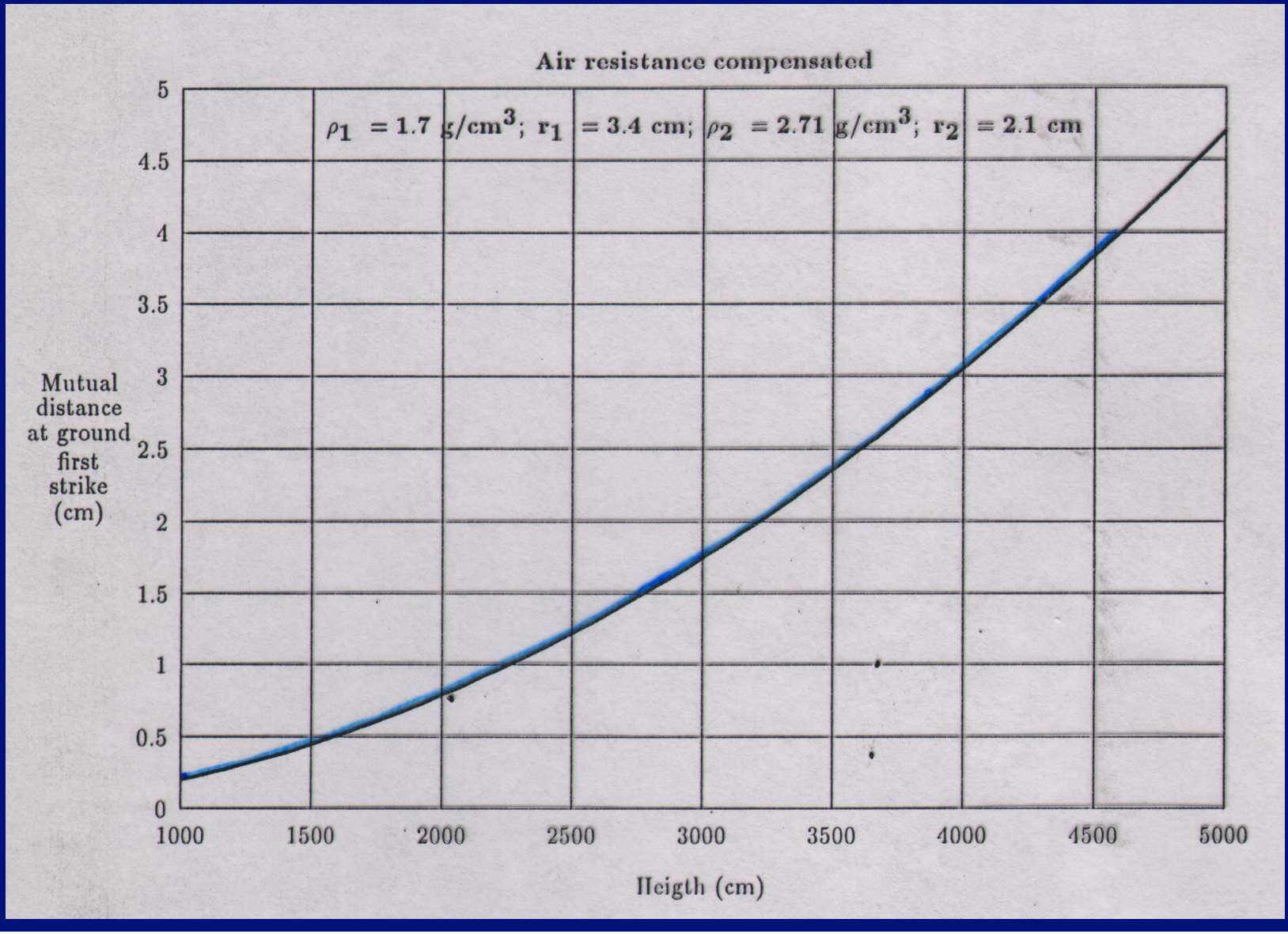
$$\vec{N} = \vec{x}_g \wedge m\vec{g} = I_o\vec{\omega}$$

$$I_o = (4/3)m x_g^2 \qquad \omega = \frac{3g}{4x_g}$$

$$OP = 1.7\,x_g \qquad a(P) = OP\,\omega = (5/4)g \;!$$

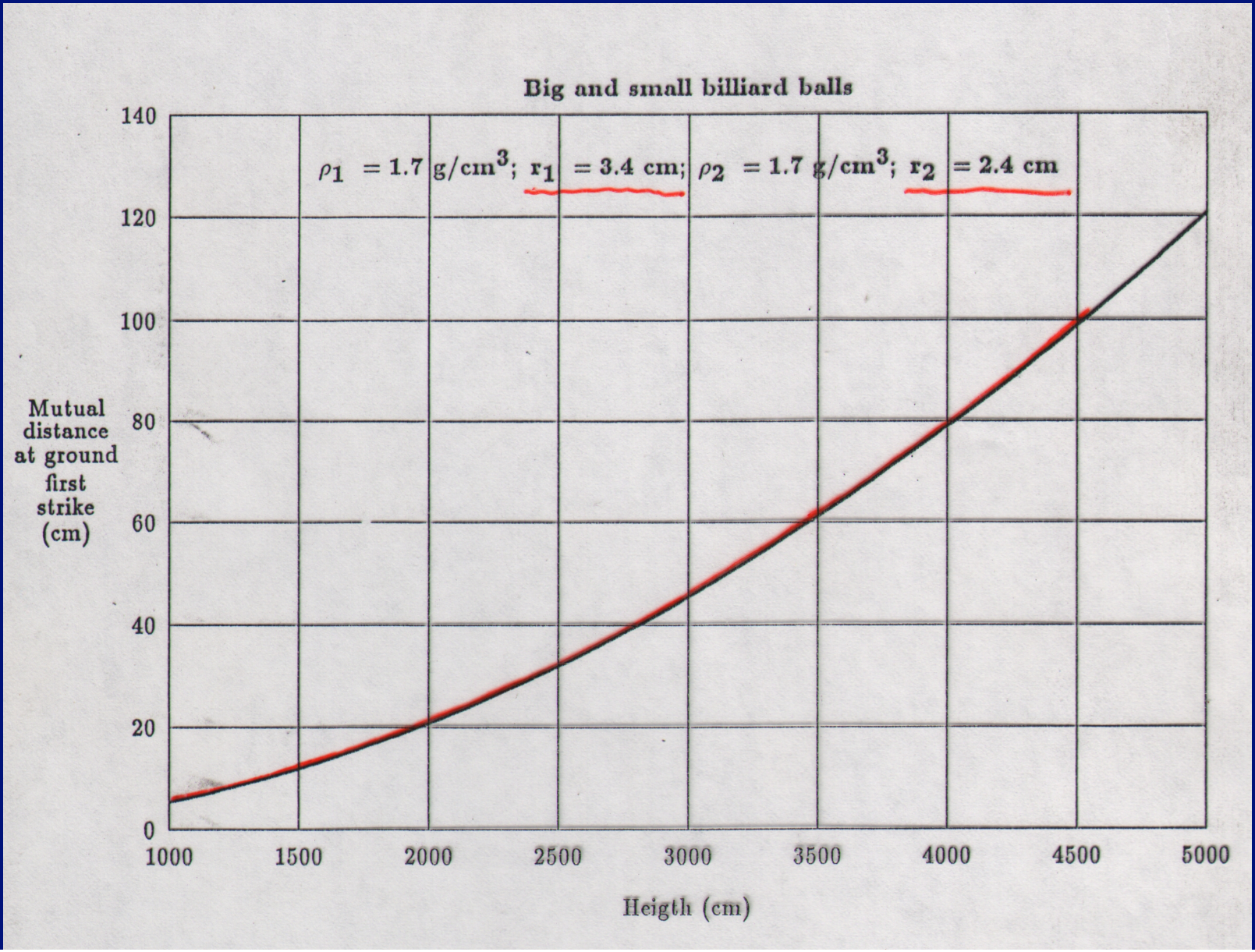
MASS DROPPING WITH AIR COMPENSATION

large billiard ball and small Al ball (about 4 cm difference)



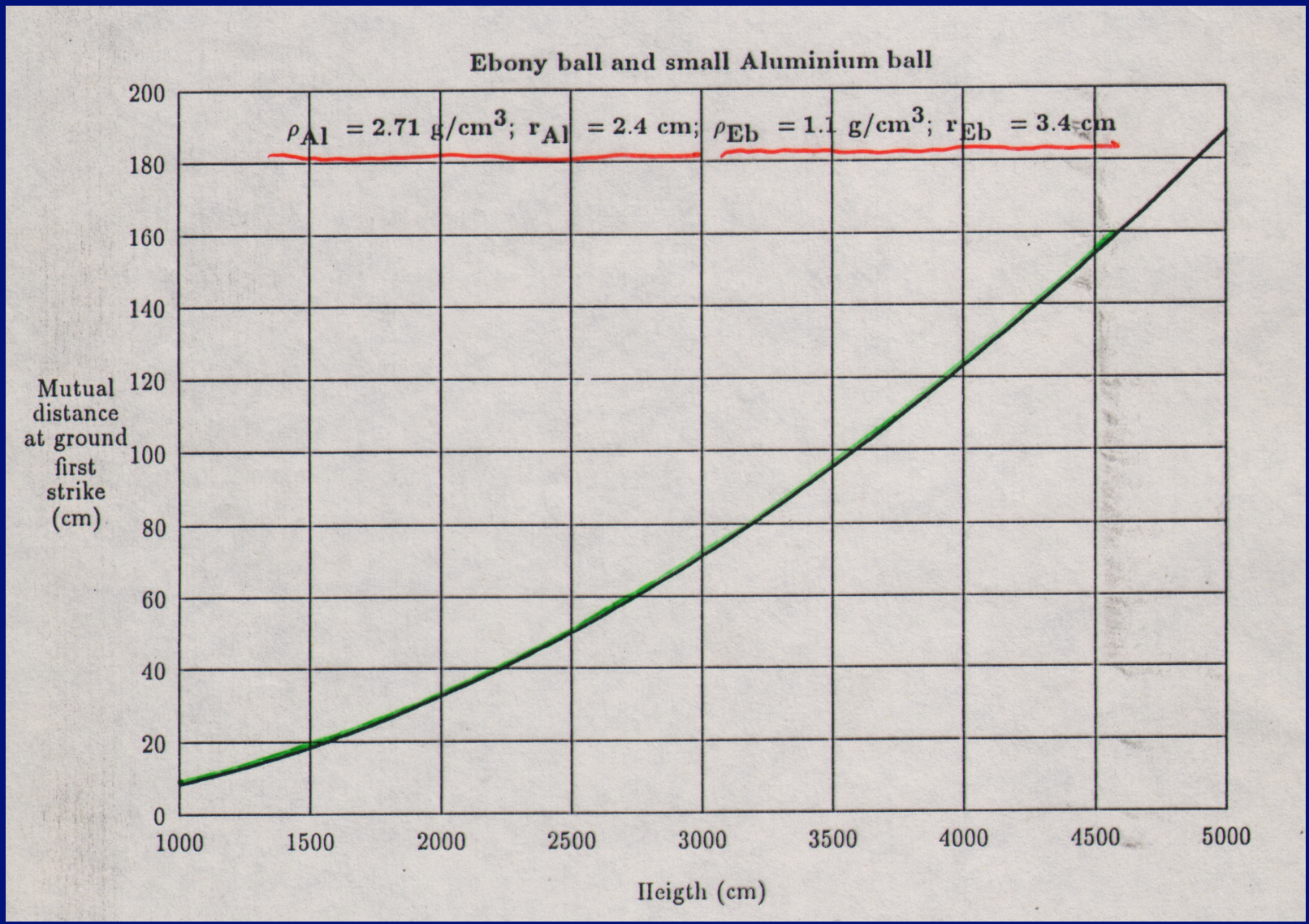
BIG AND SMALL BILLIARD BALL

The big one makes it first (about 1 m difference)



EBONY BALL AND SMALL Al BALL

The small Al ball makes it first (about 1.6 m difference)



FROM our EXPERIMENTS from the LEANING TOWER of PISA…

Zero-check test: two equal billiard balls. Ok, result close to that of test in which air drag was compensated

Many runs from water tower in the University courtyard before runs from the tower of Pisa (…people dressed in old costumes + national TV coverage of event…): effect of air resistance nicely demonstrated; results in agreement with theoretical calculations (only thanks to the release system used…impossible otherwise)

… but obviously less accurate than pendulum experiments described by Galileo (and re-made by Fuligni&Iafolla in Rome)

… the advantage of a null experiment could not be beaten!

WHAT ABOUT SALVIATI's QUOTATIONS of SPECIFIC RESULTS from TOWER EXPERIMENTS in GALILEO's "DISCORSI"? (I)

How is it that Galileo is best known worldwide for "his" mass dropping experiment from the leaning tower of Pisa?

Though the pendulum null experiment is perfectly described, the "plain" mass dropping experiments are much easier to be understood and "accepted" to question Aristoteles' authority and eventually replace his view on this matter…

In any case, Galileo was so sure of the result (from pendulum and also inclined plane experiments) -and since more than 30 years- that he would have no worry that figures quoted for mass dropping tests from a height might be incorrect…

..and in fact we have checked these quotations with our model calculations and found that they are incorrect!

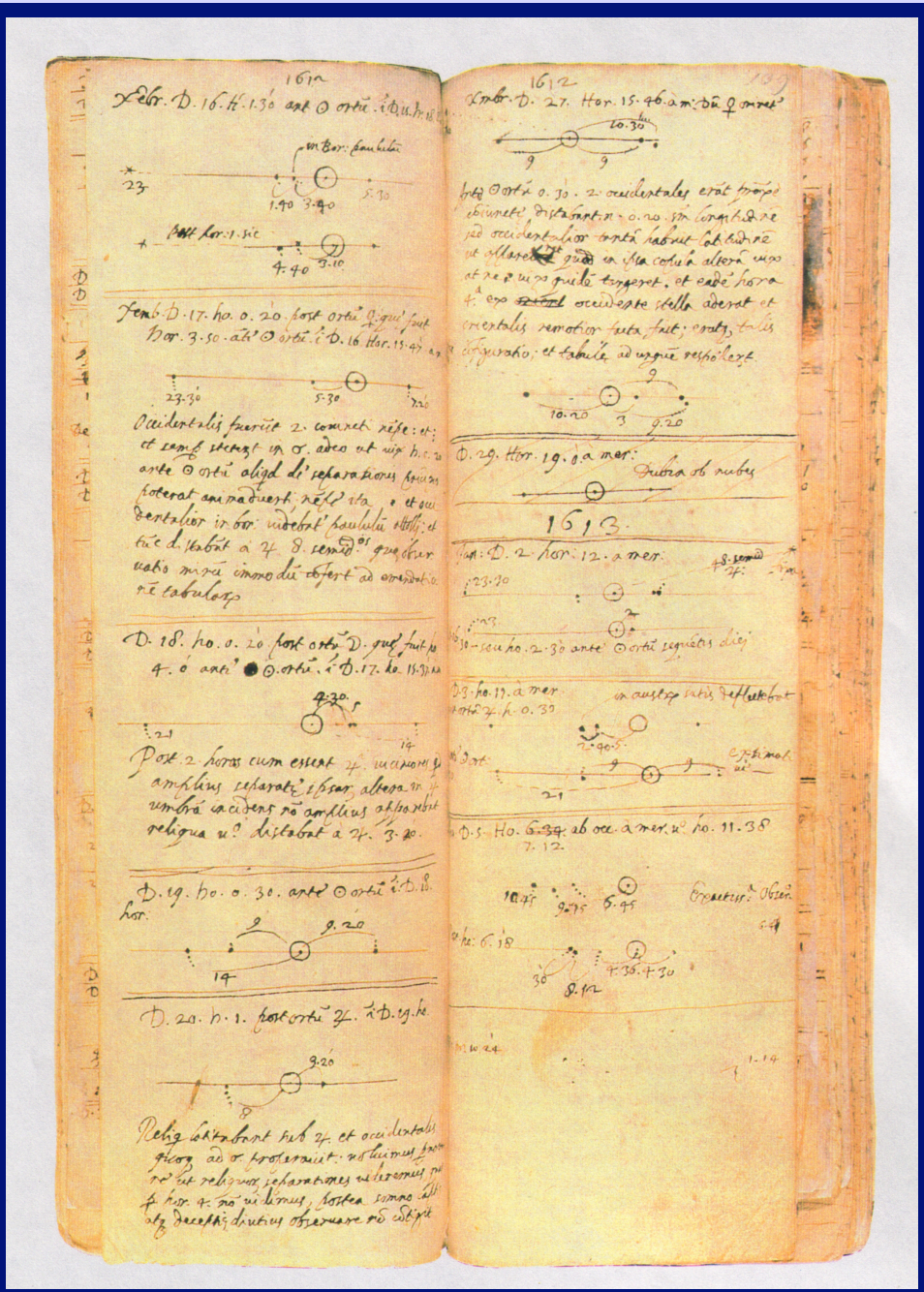
WHAT ABOUT SALVIATI's QUOTATIONS of SPECIFIC RESULTS from TOWER EXPERIMENTS in GALILEO's "DISCORSI"? (II)

..the relevant experiments were anyway reported and the results were anyway correct…

Galileo was always extremely careful in reporting his measurements and findings correctly (..no cheating!).. And we have **a beautiful example from his handwritten systematic observations of the "galilean" satellites of Jupiter** Io, Europa, Ganimede and Callisto (after he had discovered them). Observation reproduced in his book on the discovery (**"Sidereus Nuntius" - Star Messenger)** but also available in his original manuscript (at Biblioteca Nazionale di Firenze) as he made the observations night after night **in order to recognize which satellite was which, to measure their orbital motion and to make predictions for other people to observe** (his observations were questioned by the Church as "artifacts" of his telescope..)

…checked by several authors, including E.M. Standish&A.M.N…

GALILEO's OBSERVATIONS OF JUPITER's SATELLITES (+ Neptune…)



GAL: A MODERN DIFFERENTIAL MASS DROPPING EXPERIMENT (I)

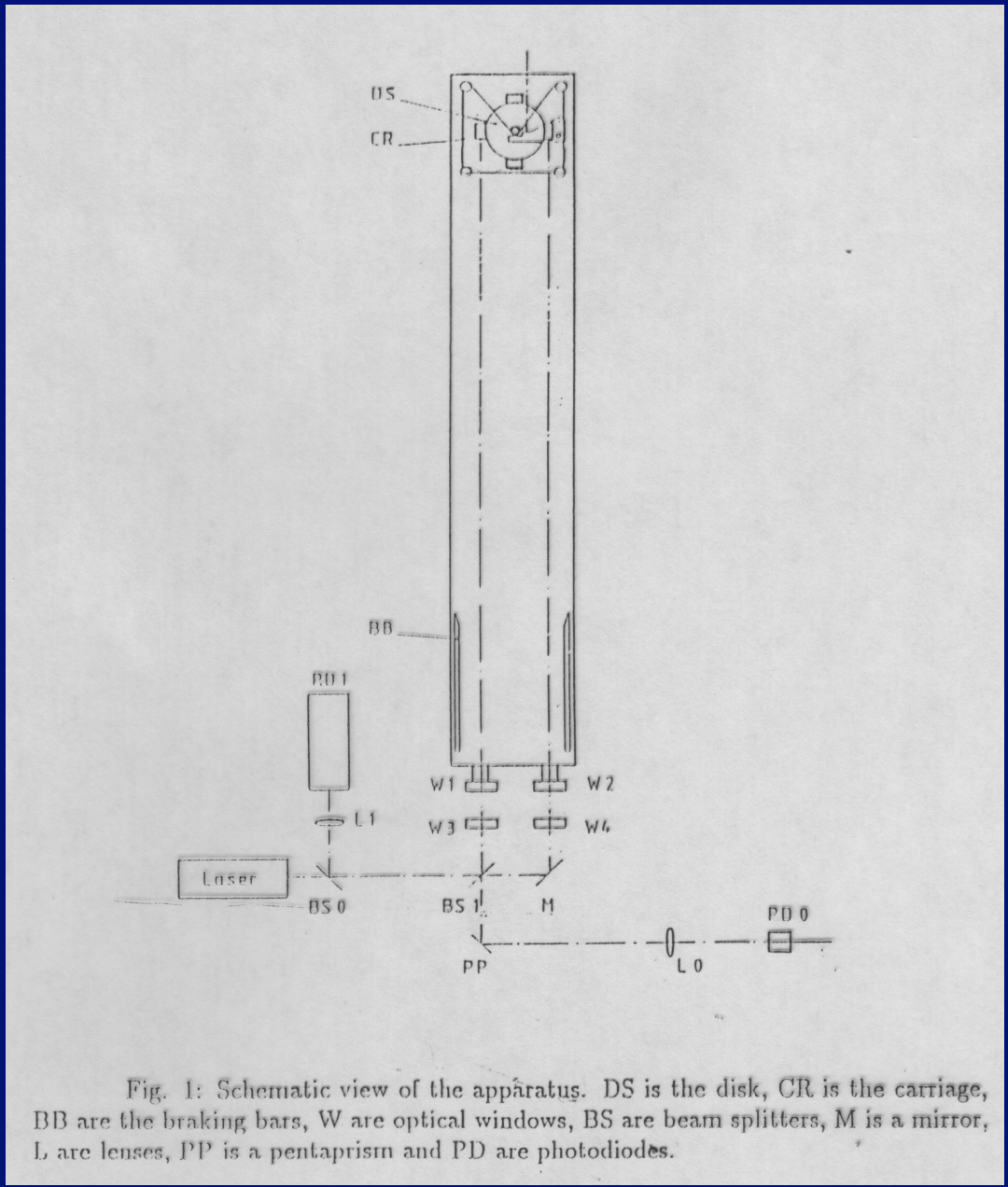


Fig. 1: Schematic view of the apparatus. DS is the disk, CR is the carriage, BB are the braking bars, W are optical windows, BS are beam splitters, M is a mirror, L are lenses, PP is a pentaprism and PD are photodiodes.

Clever idea + **modern interferometer read-out available …** while torsion balance angular measurement could already be done by Cavendish and Eötvös

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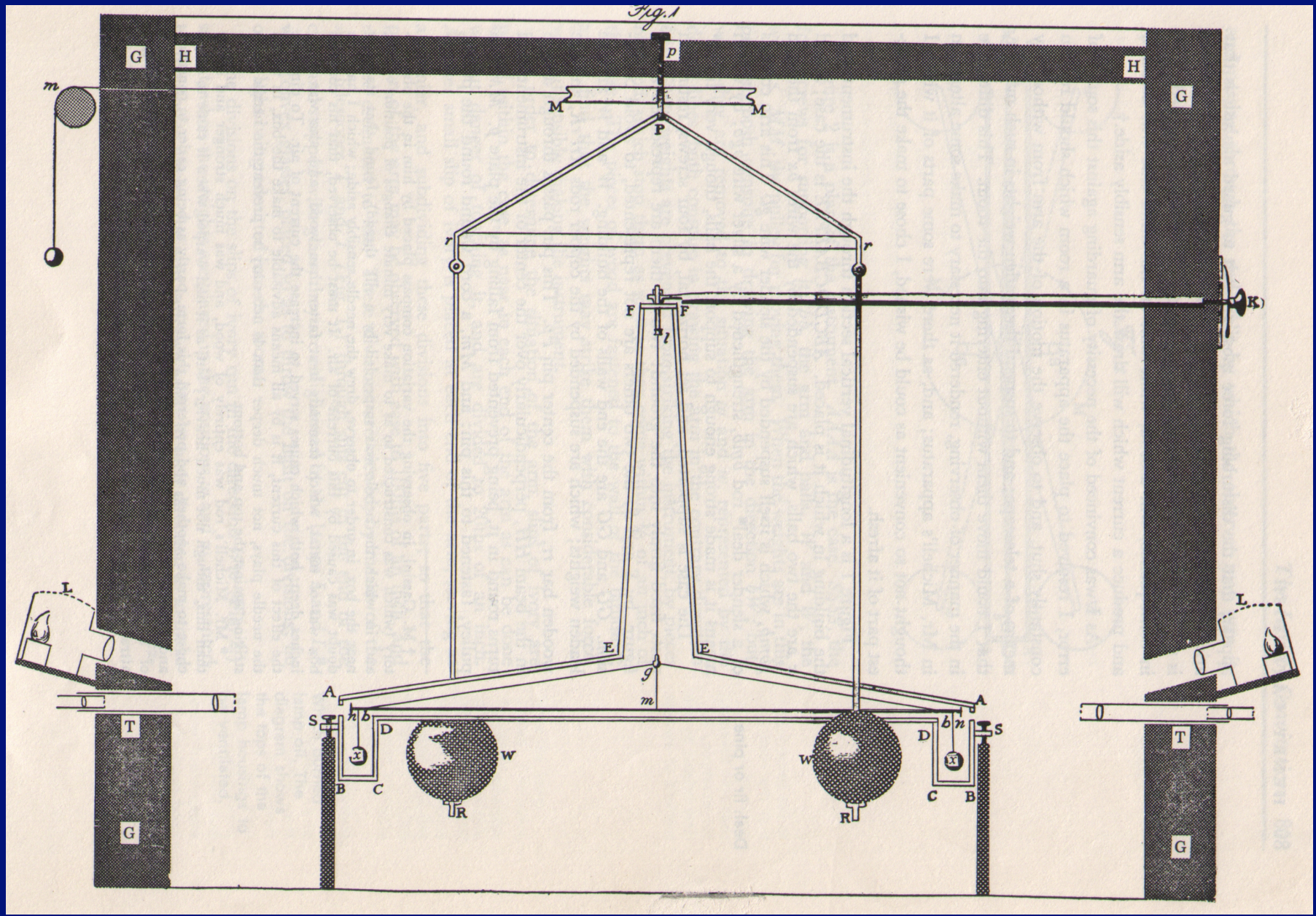
MITCHELL-CAVENDISH TORSION BALANCE to MEASURE G



EÖTVÖS TORSION BALANCE to TEST the EQUIVALENCE PRINCIPLE (I)

Are plumb lines of different composition deflected towards South by the same amount?

If not, the equivalence principle is violated. The torsion balance is a *differential* instrument for detecting this *differential* effect

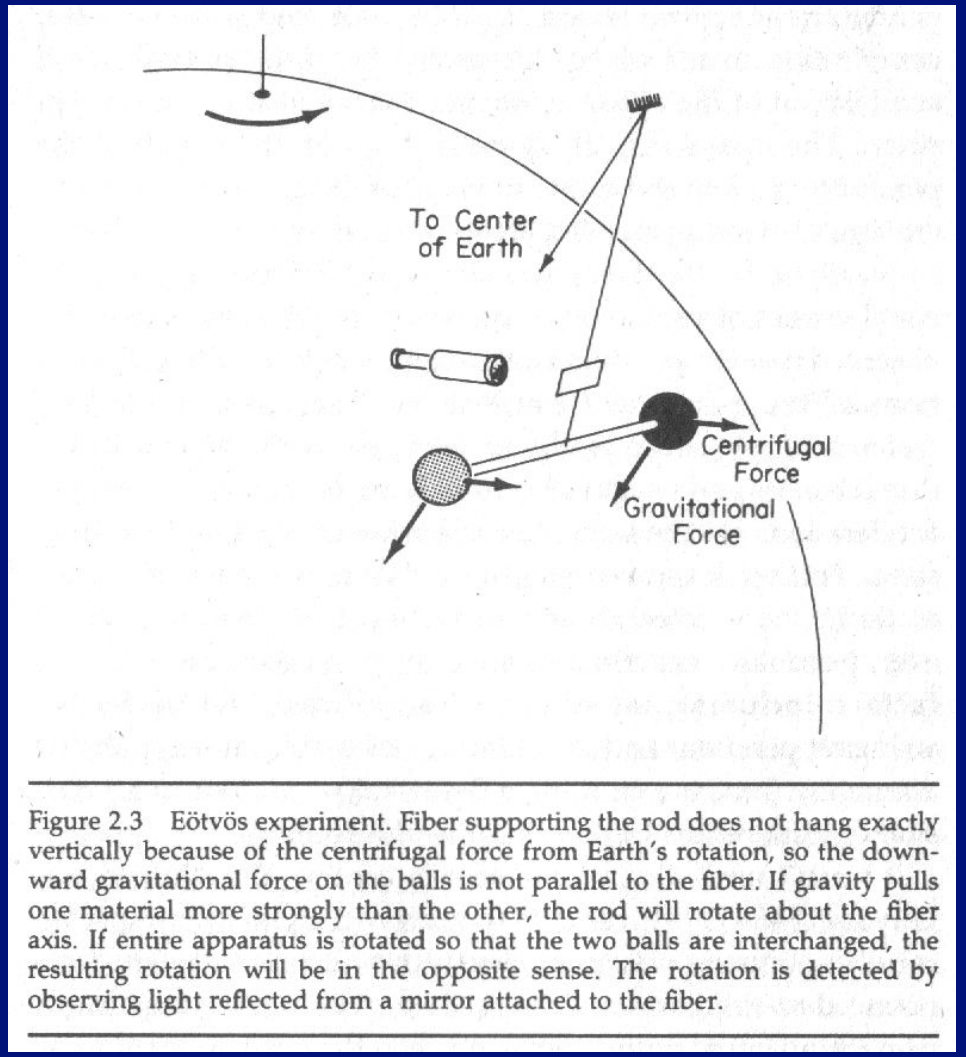


Figure 2.3 Eötvös experiment. Fiber supporting the rod does not hang exactly vertically because of the centrifugal force from Earth's rotation, so the downward gravitational force on the balls is not parallel to the fiber. If gravity pulls one material more strongly than the other, the rod will rotate about the fiber axis. If entire apparatus is rotated so that the two balls are interchanged, the resulting rotation will be in the opposite sense. The rotation is detected by observing light reflected from a mirror attached to the fiber.

$$a_{NS}^{\oplus} = \omega_{\oplus}^2 R_{\oplus} \cos\theta \sin\theta \; ; \; 1.7 \cdot 10^{-2} \; ms^{-2} \quad (\theta = 45^{o} \; latitude)$$

EÖTVÖS TORSION BALANCE to TEST the EQUIVALENCE PRINCIPLE (II)

- **Eötvös (1888-1905/08s): ≅ confirms UFF to 1 part in 10^8 by torsion balance tests** (improves by 3 orders of magnitude over pendulum tests)

  It was indeed a very subtle idea that a deviation from the proportionality between inertial and gravitational mass should show up as a rotation of the balance (inherently differential instrument !!)

GAL: A MODERN DIFFERENTIAL MASS DROPPING EXPERIMENT (II)

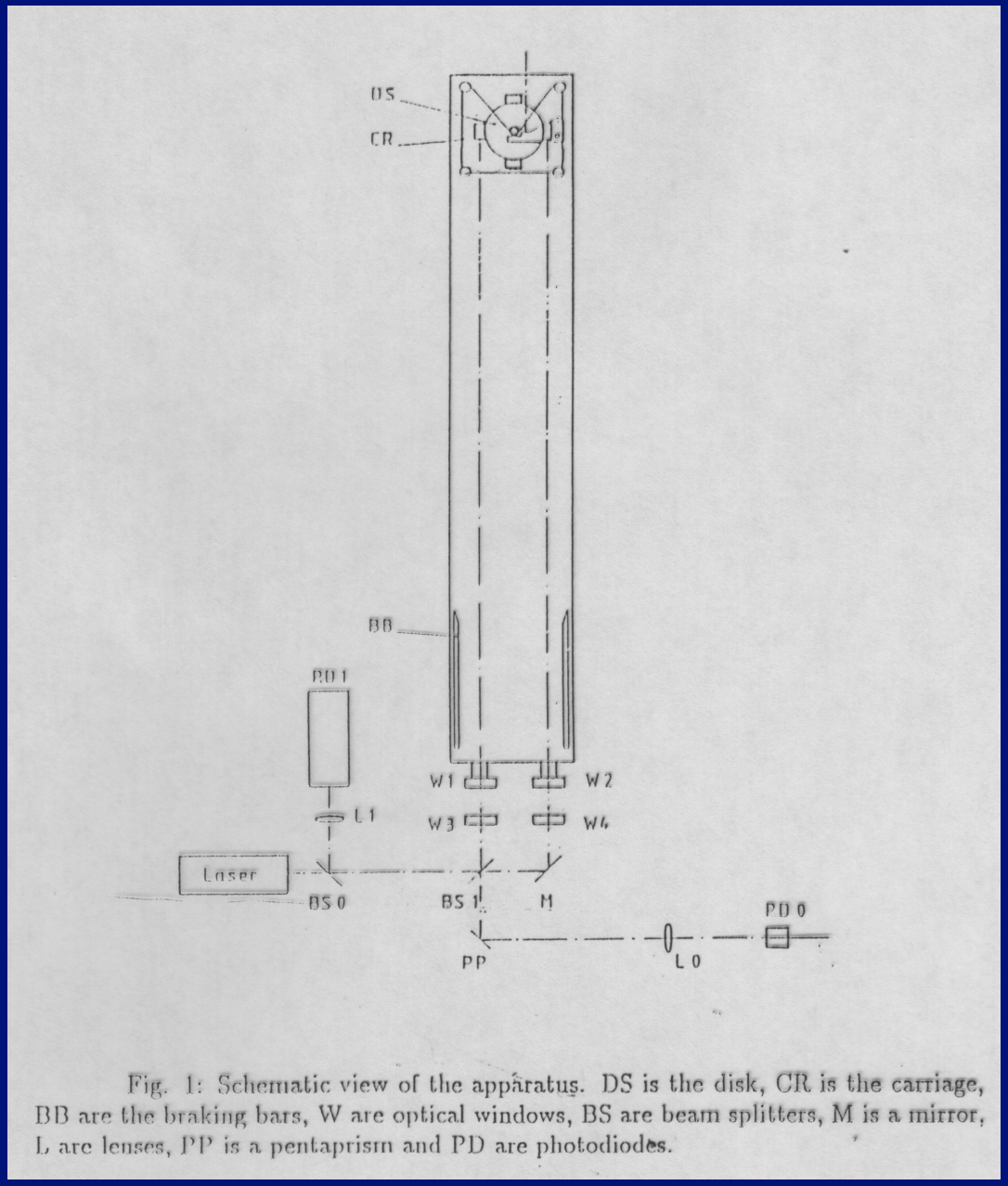


Fig. 1: Schematic view of the apparatus. DS is the disk, CR is the carriage, BB are the braking bars, W are optical windows, BS are beam splitters, M is a mirror, L are lenses, PP is a pentaprism and PD are photodiodes.

…but the disturbing rotational acceleration of the disk caused by its unlocking at 1-g limited the sensitivity of the experiment to:

$$7.2 \cdot 10^{-10}$$

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