1. THE SCIENTIFIC SIGNIFICANCE OF GG

1.1 RELEVANCE OF THE EQUIVALENCE PRINCIPLE

"GALILEO GALILEI" (GG) is a small satellite project devoted to testing the EQUIVALENCE PRINCIPLE (EP) to *1* part in 10^{17}, an improvement by *5* orders of magnitude over the best results obtained so far on Earth. It is the same target of the STEP mission proposal as evaluated twice by ESA at Phase A level within the competitions for the medium size missions M2 (Blaser *et al.,* 1993) and M3 (Blaser *et al.,* 1996).

Do bodies of different composition fall with the same acceleration in a gravitational field? If not, the so called Equivalence Principle is violated. The Equivalence Principle, expressed by Galileo and later reformulated by Newton, was assumed by Einstein as the founding Principle of general relativity, the best theory of gravitation available so far. In fact, it is not a Principle but a starting hypothesis unique to Gravity: no Equivalence Principle holds for the other fundamental forces of nature (the electromagnetic, weak and strong interaction) and almost all theories trying to unify gravity with these forces require the Equivalence Principle to be violated, thus indicating that general relativity may not be the *final truth* on gravitation, just as Newton's theory of gravitation was proved by Einstein not to be the *final truth* at beginning of this century. All tests of general relativity, except those on the Equivalence Principle, are concerned with specific predictions of the theory; instead, EP tests probe the basic assumption of general relativity, and this is why they are a much more powerful instrument of investigation. A high accuracy, unquestionable, experimental result on the Equivalence Principle –no matter whether it is violated or confirmed– will be a crucial asset for many decades to come. And this is how it has to be, because physics is an experimental science in which any theory, in spite of its internal consistency and beauty, has to confront experiments, and ultimately will stand or fall depending solely on experimental results.

Galileo questioned Aristotle's statement that heavier bodies should fall faster than lighter ones, arguing instead that all bodies fall at equal speeds regardless of their mass and composition (this is the *Universality of Free Fall*). His formulation of the universality of free fall, which lately became known as the Equivalence Principle, was first published in 1638*: "…veduto, dico questo, cascai in opinione che se si levasse totalmente la resistenza del mezzo, tutte le materie descenderebbero con eguali velocità " ("... having observed this I came to the conclusion that, if one could totally remove the resistance of the medium, all substances would fall at equal speeds ")*. It appeared in his *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti alla meccanica e ai movimenti locali*, which was published outside Italy (in Leiden) few years after completion due to Galileo's prosecution by the Church of Rome (Galileo, *Le Opere,* Vol. VIII, 1968 Edition). Aged 74, Galileo was blind and under house arrest; but the *Discorsi* are based on much earlier work, mostly on experiments with the inclined plane and the pendulum going back almost 40 years to the time when he was a young lecturer at the University of Pisa or had just moved to Padova. Galileo was well aware that experiments he made with inclined plane and pendula were much more accurate than just dropping masses from a tower; but mass dropping experiments allowed him to describe the universality of free fall in a very straightforward manner, not requiring a deep understanding of mechanics. This is how Galileo has become known worldwide for his mass dropping experiments. Indeed, no image of science has captured the imagination of ordinary people more than that of Galileo dropping masses from the leaning tower of Pisa, a symbol of the birth of the modern scientific method.

About *80* years after Galileo's first experiments Newton went further, actually recognizing the proportionality of *mass* and *weight*. Newton regarded this proportionality as so important that he devoted to it the opening paragraph of the *Principia* (Cajori Edition, 1934) where he stated*: " This quantity that I mean hereafter under the name of ... mass ... is known by the weight ...*

for it is proportional to the weight as I have found by experiments on pendulums, very accurately made... " . At the beginning of the 20th century, almost 300 years since Galileo's work, Einstein realized that because of the proportionality between the gravitational (passive) mass m_g and the inertial mass m_i, the effect of gravitation is locally equivalent to the effect of an accelerated frame and can be locally cancelled: this is known as the *Weak Equivalence Principle* which Einstein introduced in 1907 (Einstein, 1907) as the *hypothesis of complete physical equivalence* between a gravitational field and an accelerated reference frame: in a freely falling system all masses fall equally fast, hence gravitational acceleration has no local dynamical effects. Einstein then generalized this principle to the *Strong Equivalence Principle*, on which he based his theory of general relativity. The Strong Equivalence Principle states that in an electromagnetically shielded laboratory, freely falling and non rotating, the laws of physics –including their numerical content– are independent of the location of the laboratory. In such a laboratory all particles free of non gravitational forces move with the same acceleration. Therefore, according to Einstein, the effects of gravity are equivalent to the effects of living in a curved space-time. In this sense the Equivalence Principle expresses the very essence of General Relativity and as such it deserves to be tested as accurately as possible. In the last 30 years since the advent of the space age general relativity has been subject to extensive experimental testing as never before in its first 50 years of existence, and so far it has come out having no real competitors; the crucial area where experimental gravitation is likely to play an important role is in the verification of the universality of free fall as a test of the weak equivalence principle itself, since it is tantamount to testing whether gravitation can be ascribed to a metric structure of space-time.

The total mass-energy of a body can be expressed as the sum of many terms corresponding to the energy of all the conceivable interactions and components: $m = \Sigma_k m_k$. The adimensional Eötvös parameter $\eta = 2[(m_g/m_i)_A - (m_g/m_i)_B]/[(m_g/m_i)_A + (m_g/m_i)_B]$ which quantifies the violation of equivalence for two bodies of composition A and B, inertial mass m_i and gravitational mass m_g, can be generalized into

$$\eta_k = \frac{2[(m_g/m_i)_{A_k} - (m_g/m_i)_{B_k}]}{[(m_g/m_i)_{A_k} + (m_g/m_i)_{B_k}]} \tag{1.1}$$

such that a non-zero value of η_k would define the violation of equivalence between the inertial and gravitational mass-energy of the k-th type. For instance, the rest mass would contribute (as a fraction of the total) for $\cong 1$; the nuclear binding energy for $8 \cdot 10^{-3}$, the mass difference between neutron and proton for $8 \cdot 10^{-4} (A\text{-}Z)$ (A being the number of protons plus neutrons and Z the number of protons in the nucleus), the electrostatic energy of repulsion in the nuclei for $6 \cdot 10^{-4} Z^2 A^{-4/3}$, the mass of electrons for $5 \cdot 10^{-4} Z$, the antiparticles for $\cong 10^{-7}$, the weak interactions responsible of β decay for $\cong 10^{-11}$. From the point of view of conventional field theory, the verification of all these separate *Equivalence Principles* corresponds to a very peculiar coupling of each field to gravity; whether and why it should be so in all cases is a mystery. Let us consider the case of antiparticles. A peculiarity of gravity, strictly related to the Equivalence Principle, is that there is so far no evidence for antigravity, namely for the possibility that matter is gravitationally repelled by antimatter. A negative ratio of inertial to gravitational mass would obviously violate the Equivalence Principle and forbid any metric theory of gravity. Yet, there are theoretical formulations which would naturally lead to antigravity. Unfortunately, while experiments concerning the inertial mass of antiparticles have been highly successful, and these are very accurately known, gravitational experiments (i.e. involving the gravitational mass of antiparticles) are extremely difficult because of the far larger electric effects, such as those due to stray electric fields in the drift tube. In absence of such direct tests, an improvement by several orders of magnitude of current tests of the weak Equivalence Principle with ordinary matter would also be an important constraint as far as the relation between gravity and antimatter is concerned.

Nearly all attempts to extend the present framework of physics predict the existence of new interactions which are composition dependent and therefore violate the Equivalence Principle. Equivalence Principle tests are by far the most sensitive low energy probes of such new physics beyond the present framework. This is because any deviation from the universality of free fall –expressed as a fractional differential acceleration $\Delta a/a$ between falling bodies of different composition– is proportional to the post-Newtonian deviations from general relativity measured, for instance, by the adimensional parameter $\gamma^* \equiv \gamma - 1$ (γ the Eddington parameter) with a proportionality factor $<<1$ (from 10^{-5} to 10^{-3} depending on scalar or vector models). Therefore, while γ^* is constrained by post-Newtonian or pulsar tests below 10^{-3}, the current ground results on the Equivalence Principle giving $\Delta a/a \leq 10^{-12}$ already constrain γ^* below 10^{-7} or 10^{-9}, which clearly shows the superior probing power of Equivalence Principle tests.

No precise target accuracy at which a violation should occur has been predicted by these theories; an EP violation is expected, but only below the 10^{-12} level reached so far, probably well below it; whether this is really so, only high accuracy experiments can tell.

1.2 THE ADVANTAGES OF SPACE

There is a tendency in ground physics experiments, as well as in facilities for astronomical observations, to become big enterprises involving many scientists/engineers and large funding, often lasting many years. By contrast, scientific space missions tend to become smaller, faster and cheaper; so the gap between the two is decreasing. Yet, doing science in space is still a challenge in itself. Therefore, no space experiment should be proposed unless there is a very good reason for it to be done in space. As far as testing the Equivalence Principle is concerned, the crucial advantage of a space experiment in low Earth orbit (in essence two test bodies of different composition in the gravitational field of the Earth and a read-out system to monitor their relative displacements) is that the driving signal (over distances of the order of the radius of the Earth) is given by the entire value of the Earth gravitational acceleration, yielding a (differential) signal acceleration:

$$a_{EP} = \frac{GM_{\oplus}}{(R_{\oplus} + h)^2} \cdot \eta \tag{1.2}$$

for an EP violation of level η (the adimensional Eötvös parameter) and an orbiting altitude h around the Earth (G is the universal constant of gravity, $M_{\oplus}$, $R_{\oplus}$ are the mass and radius of the Earth). The *driving* acceleration from the Earth is therefore $GM_{\oplus}/(R_{\oplus}+h)^2$, whose value, at low altitudes, is close to $1\text{-}g$ ($8.4\ m/sec^2$ for $h \cong 520\ km$). If the test bodies, rather than orbiting around the Earth are suspended on its surface, the maximum strength of the driving signal (at $45°$ latitude) is only about $1.69{\cdot}10^{-2}\ m/sec^2$; if the Sun is taken as a source rather than the Earth, the corresponding driving signal is even weaker: $\cong 6{\cdot}10^{-3}\ m/sec^2$ at most (see Sec. 3.2). With a driving signal almost 3 orders of magnitude stronger, the advantage of testing the Equivalence Principle in space is unquestionable. By contrast, a short range EP experiment has nothing to gain from going into space since much bigger source masses can be used on the ground.

In the early days of the space age ordinary people and scientists alike dreamed of going into space. Physicists in particular were fascinated by the *emptiness* and *quietness* of space, which appeared to be the ideal environment for many experiments, especially gravity experiments, limited by too many disturbances on Earth. As dreams faded away and reality began, it became apparent that space is not *empty* and spacecraft are not *quiet;* at least, not as much to automatically compensate for the disadvantage of the experiment becoming inaccessible to one's hands.

Nowadays the space environment is far better understood, and a few statements can be made which are not likely to be disproved. The absence of seismic noise can no longer be quoted as a reason for moving to space, since on-board of any space structure there is vibration noise instead, while down on Earth experimentalists have learned how to cope with seismic noise very effectively. Also, space in far from being *empty*: residual atmosphere (in low Earth orbits), photons from solar radiation, charged particles ... make physics experiments in space –particularly gravitational ones– far from straightforward. However, the main advantage of space is still there: the gravitational attraction of the Earth is largely compensated by the centrifugal force due to the orbital motion of the spacecraft and there is no such thing like the $g \cong 10\ m/sec^2$ local acceleration of gravity that shapes our everyday life on Earth by giving a *weight* to every object. In addition and more importantly, the largest acceleration on-board is many orders of magnitude smaller than $1\text{-}g$: how much depends on the orbit, the spacecraft, the body on-board that we are considering (e.g. whether it is suspended or free floating, or else rigidly connected to the spacecraft, whether it is close to the center of mass of the spacecraft or far away from it). Inside GG the largest acceleration –even not considering active drag compensation– is about a factor 10^8 smaller than 1-g. Evidently a ratio by one hundred million times between the force that has to be overcome in order to suspend the same body in a ground laboratory or inside the GG spacecraft makes a big difference in the problems to be faced; it is indeed possible to suspend *100 kg* in the spacecraft with the same suspensions that would be used on Earth for suspending *1 milligram*. *Weightlessness,* not the absence of seismic noise, is therefore the true advantage of space that must be exploited. GG has been specifically designed for weightlessness to provide numerous advantages: weak mechanical coupling, self-centering and balancing of the test bodies, electrical grounding, vibration isolation; all of them crucial for a sensitive EP experiment, and all of them deriving from the absence of weight in space.

An EP space experiment in low Earth orbit offers the crucial advantage of a signal about a factor of a thousand bigger than on Earth (see Sec. 3.2); the challenge of a fully automated remote controlled experiment can be traded off against the advantages of weightlessness. An accuracy of *1* part in 10^{17} in EP testing means five orders of magnitude improvement with respect to the best ground results. Even with further progress in ground experiments (e.g. to an accuracy of *1* part in 10^{13}, possibly 10^{14}) a space mission would undoubtedly mean a great leap forward allowing space scientists to probe a totally unknown, highly promising field of physics like no other ground experiment can even dream of.

1.3 HISTORY OF EQUIVALENCE PRINCIPLE TESTING

Aristotle's statement that heavier bodies should fall faster than lighter ones was already questioned in the *6th* century by Philoponus, who noted that: if two bodies are released by the same altitude one can observe that the ratio of the times of fall of the bodies does not depend on the ratio of their weights, and the difference of the times is very small. Amazingly enough, it was only in 1553 that Benedetti reconsidered the issue, stating that the velocity of fall does not depend on the weights of the falling bodies.

Galileo questioned Aristotle's view and even showed the internal contradiction of Aristotle's reasoning with a simple argument which goes as follows: *If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will partly be retarded by the slower, and the slower will be somewhat hastened by the swifter….Hence the heavier body (made by the two tied together) moves with less speed than the lighter* (the former swifter one)*; an effect which is contrary to your* (by Aristotle) *supposition.* More importantly, Galileo was well aware of the need to provide experimental evidence. By dropping bodies of different composition in media much denser than air Galileo came to the conclusion that all bodies fall equally fast and that any observed difference is due to the different resistance of the medium that different bodies are subject to. Galileo was also aware of the difficulty to prove this fact by dropping masses from a height. As he clearly argues in the *Discorsi*, from a big height the accumulated effect of air resistance is too large to allow a reliable conclusion, while from a small one any difference is too small to appreciate. Most probably Galileo was not able to calculate precisely the effect of air resistance, but he certainly knew that it was much smaller if the velocity of the body was small (we know it is proportional to the velocity squared). He therefore performed experiments with bodies falling on inclined planes where only a fraction of the gravitational acceleration is relevant, which reduces the falling velocity –hence also the effect of air resistance.

Better still than bodies falling on inclined planes are bodies suspended from a wire and brought to oscillation like a pendulum: if all bodies fall with the same acceleration in the gravitational field of the Earth, bodies suspended from wires of the same length which are released at the same time by the same angle must keep in step regardless of their mass or composition. Besides reducing the velocity –and air resistance– the periodic repetition of the motion allows a better measurement and improves the accuracy of the experiment. *…e finalmente ho preso due palle, una di piombo e una di sughero, quella ben più di cento volte più grave di questa, e ciascheduna di loro ho attaccata a due sottili spaghetti eguali, lunghi quattro o cinque braccia, legati ad alto; allontanata poi l'una e l'altra palla dallo stato perpendicolare, gli ho dato l'andare nell'istesso momento, ed esse, scendendo per le circonferenze de' cerchi descritti da gli spaghi eguali, lor semidiametri, passate oltre al perpendicolo, son poi per le medesime strade ritornate indietro e reiterando ben cento volte per lor medesime le andate e le tornate, hanno sensatamente mostrato, come la grave va talmente sotto il tempo della leggiera, che né in ben cento vibrazioni, né in mille, anticipa il tempo d'un minuto secondo, ma camminano con passo egualissimo.*

Pendulum experiments have provided the most accurate tests of the Equivalence Principle till the torsion balance was used for EP testing at the beginning of 1900. Newton is often reported as the first ever to perform pendulum tests of the Equivalence Experiment; in fact, Galileo's pendulum experiments as in the above quotation from the *Discorsi* had even been reported much earlier than the publication of the *Discorsi,* in a letter addressed by Galileo to Guidobaldo dal Monte in 1602 (*Le Opere,* Vol. X, 1968 Edition). It is sometimes argued that Galileo could not perform accurate pendulum tests of the Equivalence Principle because –although he had discovered the physical properties of the pendulum– he did not have an accurate pendulum clock, which was invented by Christian Huygens after Galileo's death, in 1657. Huygens clock was obviously available to Newton. The point is, however, that pendulum tests of the Equivalence Principle do not require a clock: the oscillating bodies must keep in step, and only differences between the two matter, not the precise value of the oscillation period. Newton quotes his pendulum tests of the Equivalence Principle to have an accuracy of 1 part in 10^3. The pendulum tests described by Galileo have been repeated by Fuligni and Iafolla (1993). They concluded that even without special care it is easy to reach an accuracy of 1 part in 10^3; an error by 0.1% in the length of the suspensions is a realistic assumption, yielding a similar accuracy for the outcome of the experiment, and this is in agreement with Galileo's observation that the bodies keep in step for hundred or even thousands swings.

Experiments to test the Equivalence Principle require to measure a *differential* effect between the test bodies; a pendulum experiment is better than a mass dropping experiment because it makes it easier to record differences, but it is not a *null* experiment. An apparatus for EP testing should be differential by its own design, i.e. such that it gives a non-zero signal only in case of violation; otherwise it should give a null result. The torsion balance is one such differential device: two objects of different composition are connected by a rod and suspended in a horizontal orientation by a thin wire. Any suspended body is subject to the gravitational attraction of the Earth and to the centrifugal force due to the Earth daily rotation about its axis. In the horizontal plane the two reach equilibrium along the North-South direction. The gravitational attraction is proportional to the gravitational mass m_g, while the centrifugal force is proportional to the inertial mass m_i. If the ratio m_g/m_i is different for the two bodies of the torsion balance there is differential force in the North-South direction; with the rod of the balance aligned along the East-West direction the entire differential force gives a torque tending to twist the torsion balance. The differential force and its torque are constant; by exchanging the bodies on the balance (or rotating the apparatus by $180°$), the sense of twist should reverse. If the ratio m_g/m_i is the same for the two bodies (no violation) the torsion balance should give no twist (null result).

The first careful tests of the Equivalence Principle with a torsion balance were performed over many years at the beginning of this century by Roland von Eötvös and his colleagues in Budapest. Their results were published three years after Eötvös' death (Eötvös *et al.,* 1922) and more details on the apparatus and methods of measurement appeared in Eötvös' collected works (Eötvös, 1953); the experiments were later repeated by Renner (1935) using Eötvös' apparatus. Most precise experiments are null experiments, and the Eötvös experiment is no exception: they reported no EP violation to the level of several parts in 10^9, an accuracy much better than in all previous experiments.

Eötvös results have remained unchallenged until the 60's, when an Eötvös-type experiment was performed in Princeton (Roll *et al.* 1964) with an essential novelty. On the footsteps of Eötvös outstanding results the Princeton scientists led by Robert Dicke also used a torsion balance. However, Eötvös original experiment, despite its high quality, has an essential weakness: the signal is a constant twist produced by a constant torque (DC effect), hence the experiment lacks a suitable control, for there is no way of turning off the centrifugal force of the Earth's diurnal rotation. Only a $180°$ rotation of the apparatus by the experimentalists (with the inevitable consequence of disturbing the delicate balance) gives the signal a signature. Dicke suggested to look at the acceleration of the apparatus towards the Sun, rather than the Earth, and compare it with the centrifugal force due to the annual rotation of the Earth around the Sun. Were the ratio m_g/m_i different for the weights of the balance, an anomalous torque would appear; at 6 a.m. and 6 p.m., and for a torsion balance beam in the North-South direction, an anomalous gravitational pull upon one of the weights at an end of the beam would produce a turning force. The resulting twist would be periodic with a 24-hr period (the solar day) due to the diurnal rotation of the Earth. The horizontal component of the acceleration towards the Sun (as well as of the annual centrifugal acceleration) is –at most– *3/8* of the diurnal centrifugal acceleration, but it is modulated by the rotation of the Earth with a *24-hr* period and therefore the measurement contains its own zero check. Thanks to this frequency modulation and in spite of the weaker signal Roll *et al.* (1964) improved significantly Eötvös result, finding no EP violation to about 1 part in 10^{11} for aluminum and gold. Braginsky and Panov (1973) also used the Sun as a source and the modulation of the Earth's rotation. They improved the experiment further using 8 masses (of aluminum and platinum) at the vertices of an octagon, instead of two. The torsion-balance experiment being a small force experiment, any spurious gravitational field due to nearby masses can cause significant disturbances. The geometrical structure realized by Braginsky and Panov is less sensitive to the effects of such fields, since only a gravitational potential with a non-zero fifth derivative does affect their balance. The result of their experiment is that the inertial and gravitational mass (for aluminum and platinum) are the same to about *1* part in 10^{12}.

The advantage of a space experiment (with a much stronger signal) was recognized soon after the beginning of the space age. While designing a space experiment scientists also tried to provide a frequency modulation. Chapman and Hanson (1970) proposed to test the Equivalence Principle in space using a fast rotating platform so as to modulate the signal at its rotating frequency; however, in their apparatus the test bodies were constrained to move along one diameter of the rotating platform, and it is well known that any such rotating system is always strongly unstable above the critical speed (Den Hartog, 1985). Worden and Everitt (1973) proposed instead that the orbital motion of the spacecraft enclosing the test bodies would provide the modulation. This requires the spacecraft (which carries the coaxial test cylinders) to be kept fixed with respect to inertial space by accurate active control. This is the STEP proposal (Worden and Everitt, 1973; Worden, 1976; Worden, 1987; Blaser *et al.*, 1993; Blaser *et al.* 1996). The symmetry axis of the test cylinders is the sensitive axis, and lies in the orbital plane (the system is very stiff in the plane perpendicularly to the symmetry axis). If one cylinder is attracted by the Earth more than the other there is a relative movement of the two one inside the other; the effect is maximum when the symmetry axis is directed towards the center of the Earth (changing sign as the satellite moves by *180°* around the Earth) and it is zero when the symmetry axis is perpendicular to the satellite-to-Earth direction. Hence, the signal has an intensity varying at the orbital frequency of the satellite (corresponding to a period, in low Earth orbit, of about *6,000 sec*). Any higher frequency signature, higher than the orbital frequency, that one would wish to impress on the signal requires the spacecraft to be spun around its actively controlled space-fixed attitude. Due to the STEP design these can only be slow rotations and require a careful active control.

In the meantime scientists have achieved a higher frequency modulation of the putative EP violation signal in classical torsion balance experiments where the apparatus is mounted on a turntable that rotates with a period of *1÷2 hr*. These are the very careful EP experiments by the "Eöt-Wash" group at the University of Seattle (Su *et al.,* 1994) whose results are at present the most accurate (to about *1* part in 10^{12}). Their modulation frequency is the highest achieved so far, and their ongoing attempts to improve the sensitivity by one order of magnitude include a faster and smoother rotation of the turntable. The only alternative to attempting faster rotation is pursued by the R. Cowsik group in India, with a much heavier torsion balance in a very low noise environment (*25 m* under the ground) and a very good thermal stability (by means of two, very large, concentric vacuum chambers) (Unnikrishnan, 1994; Cowsik *et al.,* 1997)

It is apparent how all EP differential experiments based on the direct measurement of extremely small displacements between two test bodies have been driven, in Earth-based experiments as well as in space proposals, by the need to provide a modulation frequency of the expected tiny signal at the highest possible frequency. GG tries to go much beyond in this trend by modulating the signal at *2 Hz*, an increase by more than a factor 10^4 in comparison to all previous experiments. A modulation frequency of $\cong 1\ Hz$ has been proposed also for an Equivalence Principle experiment inside a capsule in sub-orbital flight with a free falling time of $\cong 30\ sec$ (Lorenzini *et al.,* 1994; Iafolla *et al.*,1998).

Surprisingly enough, completely different tests of the Equivalence Principle (for the Earth and the Moon falling towards the Sun) have achieved an accuracy close to that of torsion balance experiments, even about one order of magnitude better (Dickey *et al.* 1994; Williams *et al.*, 1996). The Earth–Moon distance is measured by lunar laser ranging (LLR) to the corner cube laser reflectors left by the astronauts on the surface of the Moon, accurate to better than 1 cm. Were the Earth and the Moon to be attracted differently by the Sun because of their different composition ($1/3$ iron core and $2/3$ silicate mantle the Earth; entirely silicate mantle the Moon), a physical model based on conventional Newtonian gravity with general relativistic corrections would not be able to make predictions reconcilable with the observed LLR data. This is a test of the Equivalence Principle for different composition, but also for gravitational self–energy effects in the Earth (testing gravity's pull on gravitational energy), effects which are obviously absent in test bodies of laboratory size. According to Einstein, all forms of matter and energy, including the gravitational binding energy, accelerate at the same rate in a uniform gravitational field, and the gravitational binding energy of the earth amounts to $5{\cdot}10^{-10}$ of its mass and is therefore not negligible to the current achieved accuracy. However, it should be emphasized that –despite the high quality of the analysis– LLR tests of the Equivalence Principle are based on highly complex physical models of many perturbing effects on the orbit of the Moon (such as tides) whose signature can be the same as that of an EP violation, and which involve many unknown parameters to be adjusted. To the contrary, EP experiments with test bodies of laboratory size can always provide a zero check: no sensitivity can be claimed better than the one which is obtained using in the same apparatus test bodies of the same composition.

1.4 NOVELTIES AND ADVANTAGES OF THE GG DESIGN

A torsion balance is not well suited for testing the Equivalence Principle in space, where scientists agree to use coaxial test cylinders, with a read out system to detect relative displacements between them. The main novelty of GG is to modulate the expected EP violation signal by spinning the entire spacecraft (also of cylindrical symmetry, enclosing the test bodies and the read–out sensors) at *2 Hz*, which in addition provides *1-axis* passive stabilization of the satellite.

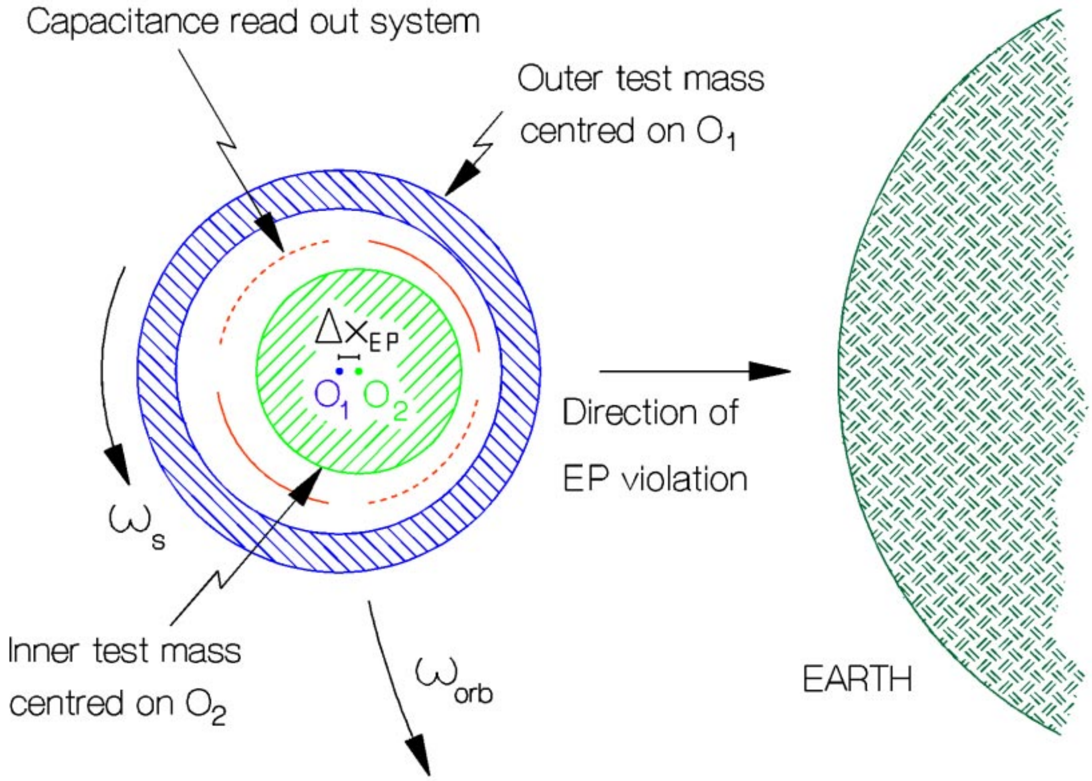

Figure 1.1 (not to scale) Section of the GG coaxial test cylinders and capacitance sensors in the plane perpendicular to the spin axis. They spin at angular velocity ω_s while orbiting around the Earth at angular velocity ω_{orb}. The capacitance plates of the read-out are shown in between the test bodies, in the case in which the centers of mass of the test bodies are displaced from one another by a vector $\Delta\vec{x}_{EP}$ due to an Equivalence Principle violation in the gravitational field of the Earth (e.g., the inner test body is attracted by the Earth more than the outer one because of its different composition). Under the (differential) effect of this new force the test masses, which are weakly coupled by mechanical suspensions, reach equilibrium at a displaced position where the new force is balanced by the weak restoring force of the suspension, while the bodies rotate independently around O_1 and O_2 respectively. The vector of this relative displacement has constant amplitude (for zero orbital eccentricity) and points to the center of the Earth (the source mass of the gravitational field). The signal is therefore modulated by the capacitors at their spinning frequency with respect to the center of the Earth.

The figure above shows, in the plane perpendicular to the spin/symmetry axis, how the GG coaxial test cylinders (of different composition) would move one with respect to the other were they attracted differently by the Earth because of an EP violation. The Figure shows the test cylinders one inside the other and two pairs (for doubling the output data) of capacitance plates in between them to measure any relative displacement. If one of the bodies is attracted by the Earth more than the other, the two centers of mass move away from one another always towards the center of the Earth. In GG the test cylinders are coupled by very weak mechanical suspensions so that even a tiny differential force (in the plane perpendicular to the spin/symmetry axis) causes a mechanical displacement which is detectable once transformed into an electric potential signal by the capacitance read-out. The weaker the coupling, the longer the natural period of differential oscillations of the test bodies, the more sensitive the system is to differential forces such as the one caused by an EP violation. It is apparent from Fig. 1.1 that spinning capacitance plates modulate the amplitude of the electrical signal caused by an EP violation at their spinning frequency with respect to the Earth ($2\ Hz$ in the current baseline), with a well defined phase (the vector does always point towards the center of the Earth). In absence of spin the EP violation force has constant intensity (except for the effect of orbital eccentricity of the satellite, which is close to zero) and a direction changing at the orbital frequency of the satellite around the Earth ($\cong 1.75 \cdot 10^{-4}\ Hz$); so in GG the electric signal is modulated at a frequency about 10^4 times higher than the frequency of the EP violation force, the advantage being the reduction of low frequency noise both mechanical (the suspensions have higher mechanical quality factor Q at higher frequency) and electrical (lower $1/f$ electronic noise). The spinning state of the GG spacecraft is a stable *1-axis* rotation and needs no active control.

An important consequence of the fact that in GG the expected EP violation force lies in the plane normal to the spin/symmetry axis of the test cylinders is that a major perturbation due to the so called "radiometer effect" is zero also at room temperature. It is known that, in low pressure conditions where the mean free path of the gas particles is much larger than the dimensions of the vessel, a cylinder whose faces are not at the same temperature is subject to an acceleration along its symmetry axis whose value is exceedingly large unless the residual gas pressure is extremely low, down to values which can only be obtained at very low temperatures. In STEP this radiometer effect along the symmetry/sensitive axis of the test cylinders competes directly with the signal, and is reduced thanks to the extremely low level of residual pressure, which can be obtained by operation at superfluid He temperature (about $2\ K$). Instead, a hollow cylinder whose inner and outer surfaces were not exactly at the same temperature, would have zero radiometer effect in the plane perpendicular to its axis, for pure symmetry reasons. In reality, azimuth asymmetries as well as the radiometer effect along the symmetry axis of the cylinders must be taken into account in GG, since it is a non cryogenic experiment; however, the requirements they impose on the amount of acceptable temperature gradients are compatible with a pure passive thermal control of the GG experimental apparatus (see Sec. 2.2.3). If the radiometer effect is dealt with at room temperature, one of the main reasons why a high accuracy EP experiment in space should be operated in cryogenic conditions is no longer valid.

Low temperature is certainly helpful in reducing thermal noise. Since the dependence of thermal noise acceleration on the experiment temperature T and the mass m of the test bodies is $\propto (T/m)^{1/2}$, in GG we use more massive test bodies: test masses of $10\ kg$ each at $300\ K$, as we have in GG, result in the same thermal noise as with test masses of $0.1\ kg$ at a temperature of $3\ K$. However, a future, lower temperature version of the GG experiment can be envisaged for which the rapid spin gives an important advantage: the very high centrifugal force at the periphery of the spacecraft would dominate the motion of the refrigerating (movable) material and largely reduce, by symmetry, its perturbations on the bulk of the experiment; evaporation can take place along the spin axis for symmetry reasons too. This is unlike non-spinning or slowly spinning satellite experiments for EP testing in which perturbations from the nearby refrigerant mass (a few hundred liters of He in STEP) is known to be a serious source of perturbation.

The weak mechanical coupling of the GG test bodies (e.g. obtained with a simple and effective use of helical springs and flat gimbals pivoted on thin torsion wires) is the key feature which allows us to cope with a major dangerous effect: the effect of residual air drag along the satellite orbit. Air resistance acting on the spacecraft surface is experienced by the test bodies suspended inside it as a translational inertial acceleration equal and opposite to the one caused by air drag on the center of mass of the whole satellite (spin axes are stable due to the extremely high energy of spin). This acceleration is about 8 orders of magnitude weaker than 1-g on Earth but about as many orders of magnitude larger than the expected signal; it should be the same on both test cylinders, but only in the ideal case that their masses and suspensions were exactly the same. Drag-free control (with FEEP ion thrusters) of the GG spacecraft reduces the corresponding inertial acceleration on the payload. In order to further reduce its differential effect on the test cylinders due to small differences in their suspensions, the test cylinders are coupled similarly to the two weighs of an ordinary balance whose arms can be adjusted (by means of piezoceramic actuators) so as to eliminate differential effects. Balancing of weights (under local gravity) is obviously well known on Earth, where weights can be balanced to 1 part in 10^8 or better, much more accurately than it is required for the GG test bodies. It is also well known that small forces are much easier to balance than large ones; hence, since in the GG spacecraft the largest force (due to air drag) is many orders of magnitude weaker than local gravity on Earth, balancing the test bodies must be easier than on Earth by far, where it has been tested on the payload prototype. To be balanced is a property of the system, not of the particular force acting on it; hence, all other common mode perturbations beside drag (e.g. solar radiation pressure) are also balanced once the main drag effect is balanced. Balancing the drag does not eliminate an EP violation signal because it is a differential and constant signal while drag produces common mode forces, is variable in time and about $90°$ out of phase with respect to the EP violation signal. Vibration noise close to the spin frequency (e.g. from the FEEP thrusters) is attenuated by the suspensions of the laboratory enclosing the test bodies.

Drag–free control of GG with FEEP thrusters requires a negligible amount of propellant, amounting to only several grams for the entire duration of the mission. By contrast, the He thrusters planned for STEP require a few hundred liters of He on board, whose mass is in itself a disturbing source for the experiment. FEEP thrusters are electrically tuned; instead, He thrusters are tuned mechanically. Because of their numerous advantages, particularly the extremely low propellant mass, FEEP have been proposed also for STEP (Blaser *et al.,* 1994), but the presence on board of a considerable mass of superfluid He and the consequent need to eliminate the boiled off He from the dewar in a carefully controlled manner, clearly require *He* thrusters, which are therefore used also for drag–free control rather than adding the FEEP. The superiority of FEEP in small force gravitational missions is beyond question; they are the current baseline choice not only for GG but also for LISA and OMEGA, two mission proposals for the detection of gravitational waves in space.

The GG bodies all spin at a frequency much higher than their natural frequencies of oscillation (which are very low because of the very weak suspensions that can be used in absence of weight). This state of rotation is very close to that of ideal, unconstrained, rotors and allows the test cylinders to self-center very precisely (the center of mass of an ideal free rotor would be perfectly centered on the spin axis). This is how it is possible to reconcile a high frequency of spin (hence a high frequency modulation of the signal) with the need to measure extremely small relative displacements. However, suspensions are not perfect, which means that, as they undergo deformations at the frequency of spin, they also dissipate energy. The higher the mechanical quality of the suspensions, the smaller the energy losses. Small energy losses are known to produce slow whirl motions of the suspended bodies one around the other which must be damped actively, with small capacitance sensors/actuators and appropriate control laws (see Chap. 6). In fact, whirl motions of the GG test cylinders are so slow that they can be damped at time intervals long enough to allow data taking in between, when active damping can be switched off.

Unlike STEP, GG has mechanical suspensions and no floating bodies; they are all connected by mechanical conductive suspensions which once coated with the same conductive material provide a "Faraday cage" and consequent electrical discharging of the experimental apparatus. This is a major advantage because the electric forces caused by even a limited amount of charge (in the Van Allen belts and South Atlantic Anomaly) are enormous compared to the very small gravitational force to be detected; apart from the discharging mechanism to be devised, the electric charge acquired by the test bodies needs to be measured before discharging, which in itself is a source of perturbation. No such devices are needed in GG.

It is clear that fast rotation and weak mechanical suspensions are the main features of the GG experiment design, distinguishing it from the STEP design. Other advantages of fast rotation beside the modulation of the signal are that a large number of perturbing effects (e.g. due to in-homogeneity of the test bodies, spacecraft mass anomalies, non-uniform thermal expansion, parasitic capacitances, etc.) appear as DC because the entire system is spinning.

Last but not the least, the GG payload design can be tested on the ground (see Chap. *3*). An important novel feature of the GG space experiment is that it is sensitive to differential forces in the plane perpendicular to the spin/symmetry axis of the test cylinders (as in the case of an Equivalence Principle violation). If the test masses are suspended on the surface of the Earth a violation of the Equivalence Principle can be detected as a force in the horizontal plane along the North/South direction; the GG test cylinders can be suspended along the vertical/symmetry axis and yet be sensitive to differential forces in the horizontal plane. The suspensions must necessarily be strong along the vertical, to sustain the weight of the bodies, but it is possible to couple the cylinders weakly so that they can respond by a measurable displacement (measurable with a capacitance read–out at room temperature, like in GG) to tiny differential forces acting between them in the horizontal plane. The result is very important: despite the weaker EP violation signal (by about 3 orders of magnitude) and the unfavorable 1-g environment, the GG payload design can be tested on the ground to the same level of measurement sensitivity which is required by the GG mission in space.