Precise gravitation measurements on Earth and in space: Tests of the Equivalence Principle

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Summary

Experiments to test the *Universality of Free Fall* (UFF) –whereby, in a gravitational field, all bodies fall with the same acceleration regardless of their mass and composition– have played a major rôle in the history of experimental gravitation, alongside with experiments to measure the universal constant of gravity *G.* Indeed, experiments to test the UFF have started even earlier than big *G* experiments, with Galileo in Pisa already at the end of the 16th century. Newton's formulation of the law of gravitational attraction has brought the fundamental constant *G* and its measurement to the limelight of experimental physics. Ever since, the two lines of experiments have proceeded almost in parallel for about 300 years, sharing the difficulties of detecting extremely small forces and very often using similar experimental apparata. The birth of General Relativity, at the beginning of the 20th century, has put experiments on the UFF in a new perspective. The *Universality of Free Fall* is not only a relevant experimental fact, worth testing on its own right, but the direct consequence of the *Equivalence Principle* on which the theory of General Relativity is based. Hence, putting to test the *Universality of Free Fall* amounts to putting to test the foundations of General Relativity itself.

These lectures are devoted to the *Equivalence Principle*, its relevance and its testing, from the early work of Galileo through the major contributions of Eötvös, Dicke and Braginsky, till the recent successes of the "Eöt-Wash" group at the University of Washington. Yet, the emphasis of the lectures is on the leap forward in sensitivity which might come, were an Equivalence Principle experiment carried out in low orbit around the Earth. Three such missions are currently under investigation by space agencies around the world. The experimental apparata and mission designs are presented and compared.

The structure of the lectures is as follows: Scientific relevance of the Equivalence Principle (Section 1); Experiment principle and the expected signal (Section 2); Brief history of past ground experiments (Section 3); Equivalence principle tests by Lunar Laser Ranging (Section 4); Recent and ongoing laboratory experiments (Section 5); Advantages of an Equivalence Principle test in low Earth orbit (Section 6); Proposed space experiments to test the Equivalence Principle (Section 7), divided in two subsections, The STEP and μSCOPE space experiments (Section 7.1) and, The GG space experiment (Section 7.2); The GGG ("GG on the Ground") experiment: laboratory test of a proposed space apparatus (Section 8).

1 Scientific relevance of the Equivalence Principle

It is possible to ascribe two conceptually different kinds of masses to a body: an *inertial mass* and a *gravitational mass*. The inertial mass is the proportionality factor between a force (any kind of force) applied to the body and the acceleration it acquires in response to it in an inertial laboratory. The gravitational mass is a measurement of the property of the body to attract gravitationally any other body (gravitational *active* mass), or to be gravitationally attracted by any other body (gravitational *passive* mass). Assuming the validity of the action–reaction principle (which leads to conclude that the center of mass of an isolated system must move with constant velocity in an inertial frame of reference) also implies that the gravitational passive and active mass of a body must be the same. Since both concepts refer to the same physical interaction, this result appears to be quite natural. The gravitational mass is the analog in a gravitational field, of the electric charge in an electric field –it can be viewed as a *gravitational charge*– while it has no apparent relation (in spite of the name) with the concept of inertial mass.

Using Newton's law of gravitation to write the equation of motion of a body of inertial mass m_i and gravitational mass (both active and passive) m_g in the field of a source body of gravitational mass M_g (for instance, the Earth), we have:

$$m_i \ddot{\vec{r}} = -\frac{G M_g m_g}{r^3} \cdot \vec{r} \tag{1}$$

where $\ddot{\vec{r}}$ is the relative position vector between the centers of mass of the bodies and G the universal constant of gravity. If $m_i \propto m_g$ *(Equivalence of inertial and gravitational mass),* then the acceleration $\ddot{\vec{r}}$ is the same for all bodies. With the measured value of G and a proportionality factor of 1 $(m_i = m_g)$, the local acceleration of gravity on the surface of the Earth –the same for all bodies regardless of their mass and composition– amounts to about 980 $cm\ s^{-2}$. This is the so called *Universality of Free Fall* (UFF*).* No such thing holds for all other fundamental forces of Nature. For instance, a proton and an electron do not have –in the same electric field– the same (in modulus) acceleration, because the inertial mass of the proton is much bigger than the inertial mass of the electron and no proportionality holds between the inertial mass of a body and its *electric charge,* as it does between the inertial mass and the *gravitational charge*

Galileo was most probably the first one to provide experimental evidence for the UFF (see Sec. 3). However, he was not aware of the law which rules the gravitational interaction and which leads to writing Eq. (1). Therefore, he had no awareness of the equivalence between inertial and gravitational mass, and of the link between this concept and his own experimental results on the UFF.

The fact that the two concepts of inertial and gravitational mass refer in fact to the same physical quantity was first stated by Newton in the opening paragraph of the *Principia* [1]:

> "*This quantity that I mean hereafter under the name of ... mass ... is known by the weight ... for it is proportional to the weight as I have found by experiments on pendulums, very accurately made... "*

At the beginning of the 20th century, almost 300 years since Galileo's work, Einstein realized that because of the proportionality between the gravitational mass m_g and the inertial mass m_i, the effect of gravitation is locally equivalent to the effect of an accelerated frame and can be locally canceled. This is known as the *Weak Equivalence Principle* which Einstein introduced in 1907 [2] as the *hypothesis of complete physical equivalence* between a gravitational field and an accelerated reference frame: in a freely falling system all masses fall equally fast, hence gravitational acceleration has no *local* dynamical effects. Any test mass located inside the famous Einstein elevator –falling with the local acceleration of gravity g– and zero initial velocity with respect to it, remains motionless for the time of fall. An observer inside Einstein elevator will not be able to tell, before hitting the ground, whether he is moving with an acceleration g in empty space, far away from all masses, or else he is falling in the vicinity of a body (the Earth) whose local gravitational acceleration is also g (and in the same direction).

However, Einstein's formulation of the *Weak Equivalence Principle* whereby the effect of gravity disappears in a freely falling reference frame, holds only *locally.* The elevator is free falling *in the vicinity* of the Earth, which amounts to saying that $h<<R_\oplus$, namely, that the height of fall is much smaller than the radius of the Earth. The cancellation of gravity in a freely falling frame holds *locally* for each frame, but the direction of free fall is not the same in all of them. Which is the direct consequence of the fact that the gravitational field of a body (like Earth) is not uniform, giving rise to the so called *tidal forces* between test particles whose centers of mass are not coincident (and also inside a body of non zero dimensions). With the *Weak Equivalence Principle* Einstein has moved from Newton's concept of one global reference frame with gravitational forces and the UFF, to many free falling *loca*l frames *without* gravitational forces.

A step further in the development of General Relativity leads Einstein to a generalization of the *Weak Equivalence Principle,* known as the *Strong Equivalence Principle,* stating that in an electromagnetically shielded laboratory, freely falling and non rotating, the laws of physics –including their numerical content– are independent of the location of the laboratory. In such a laboratory all particles free of non gravitational forces have no relative accelerations. According to General Relativity and the *Strong Equivalence Principle* (which assumes the *Weak* one), all gravitational effects are replaced by the metric of a curved, 4-dimension space-time. In this sense the Equivalence Principle expresses the very essence of General Relativity and as such it deserves to be tested as accurately as possible.

The *Weak Equivalence Principle*, although obviously not known to Galileo, also leads –as in the case of Newton's *Equivalence of inertial and gravitational mass*– to the UFF*:* should bodies of different composition fall with different accelerations, the elevator and the test mass inside it would generally fall with different accelerations and the observer would be able to tell that he is close to the surface of the Earth and not in an accelerated frame in empty space.

In the last 30 years since the advent of the space age General Relativity has been subject to extensive experimental testing as never before in its first 50 years of existence, and so far it has come out having no real competitors; the crucial area where experimental gravitation is likely to play an important rôle is in the verification of the universality of free fall as a test of the weak equivalence principle itself, since it is tantamount to testing whether gravitation can be ascribed to a metric structure of space-time.

The adimensional parameter which quantifies a deviation from the UFF (hence, also a violation of Equivalence) for test bodies of different composition, A and B, inertial mass m_i and gravitational mass m_g , is the so-called Eötvös parameter η:

$$\eta = \frac{2[(m_g/m_i)_A - (m_g/m_i)_B]}{[(m_g/m_i)_A + (m_g/m_i)_B]} \tag{2}$$

The finding of a value $\eta \neq 0$ would disprove the UFF and indicate a violation of the Weak Equivalence Principle on which General Relativity ultimately relies. Instead, $\eta = 0$ –as reported by all experiments so far– confirms the basic assumption of General Relativity and has additional profound significance.

The total mass–energy of a body can be expressed as the sum of many terms, corresponding to the energy of all the conceivable interactions and components: $m = \Sigma_k m_k$. For instance, at the atomic level, the rest mass contributes (as a fraction of the total) for $\cong 1$; the nuclear binding energy for $8\cdot 10^{-3}$ (for light elements), the mass difference between neutron and proton for $1.4\cdot 10^{-3}$ $(A-Z)/A$ (A being the number of protons plus neutrons and Z the number of protons in the nucleus), the electrostatic energy of repulsion in the nuclei for $6\cdot 10^{-4} Z^2 A^{-4/3}$, the mass of electrons for $5\cdot 10^{-4} Z/A$, the antiparticles for $\cong 10^{-7}$, the weak interactions responsible of β decay for 10^{-9} or less. For an extended spherical body of radius R and (homogeneous) density ρ, the gravitational self–energy contributes by $-4/5\pi\rho G R^2/c^2$. The conventional Eötvös parameter (2) can therefore be generalized into:

$$\eta_k = \frac{2[(m_g/m_i)_{A_k} - (m_g/m_i)_{B_k}]}{[(m_g/m_i)_{A_k} + (m_g/m_i)_{B_k}]} \tag{3}$$

such that a non-zero value of η_k would define the violation of equivalence between the inertial and gravitational mass-energy of the *k-th* type.

From the point of view of conventional field theory, the verification of all these separate *Equivalence Principles* corresponds to a very peculiar coupling of each field to gravity; whether and why it should be so in all cases is a mystery.

Nearly all attempts to extend the present framework of physics predict the existence of new interactions which are composition dependent and therefore violate the Equivalence Principle. Equivalence Principle tests are by far the most sensitive low energy probes of such new physics beyond the present framework. Any deviation from the UFF –expressed as a fractional differential acceleration $\Delta a/a$ between falling bodies of different composition– is proportional to the post-Newtonian deviations from General Relativity measured, for instance, by the adimensional parameter $\gamma^* \equiv \gamma - 1$ (γ the Eddington parameter). The estimated value of the proportionality factor linking $\Delta a/a$ to γ^* changes depending on scalar (10^{-5}) or vector models, and in the latter case, on the kind of coupling expected for a new interaction $(10^{-2} \div 10^{-3})$. Since experimental tests of the UFF have shown that $\Delta a/a \leq 10^{-13}$ [3, 4, 5], they also constrain γ^* to much smaller limits than it has been obtained from post-Newtonian or pulsar tests, which provide only $|\gamma^*| \leq 10^{-3}$, clearly showing the superior probing power of *Equivalence Principle* tests. [see e.g. 6].

No precise target accuracy at which a violation should occur has been predicted by theories predicting new, composition dependent interactions. A violation is expected, but only below the level reached so far, probably well below it; whether this is really so, only high accuracy experiments can tell.

2 Experiment principle and the expected signal

An experiment to test the Universality of Free Fall requires two test bodies of different composition falling in the field of a source mass, and a read–out system to detect their motions relative to one another searching for a *differential* effect –pointing in the direction of the source mass and with a frequency determined by the relative motion of the test bodies with respect to it– which cannot be explained on the basis of known, classical phenomena. This requires that differential gravitational effects of classical origin (e.g. tidal effects or differential coupling due to different multipole moments of the test bodies as bodies of finite dimension), as well as non gravitational effects (e.g. due to residual air, radiation pressure, electric forces, magnetic forces), must be smaller than the signal expected in case of a deviation from the UFF (hence, of a violation of Equivalence). Which amounts to saying that, in order to be interpreted as a violation of Equivalence, the effect detected should go to zero for test bodies made of the same material.

In ground experiments the test bodies can be either free–falling (the so called *mass dropping* experiments) or suspended against the local acceleration of gravity; the source mass can be either the Earth or the Sun.

In *mass dropping* experiments the test bodies are released from a height and the *driving* acceleration acting upon them is the local acceleration of gravity $g=GM_\oplus/R_\oplus^2 \cong 980\ cm\ s^{-2}$ ($M_\oplus$, $R_\oplus$ being the mass and radius of the Earth). The differential acceleration expected because of a deviation from the UFF quantified by a given value η of the Eötvös parameter is the fraction η of g: $\Delta g=\eta\cdot g$. The smaller is η ($\eta << 1$), the better is the accuracy of the test, the smaller is the differential acceleration Δg that the apparatus must be able to detect. Δg is in the direction of free fall and its frequency depends on the rotation state of the free falling apparatus (it is a DC signal for not rotating falling bodies, while it is modulated at the frequency of rotation if the free falling apparatus rotates in the reference frame of the laboratory). Mass dropping experiments have the advantage of a large driving acceleration (the largest possible for an observer confined to the surface of the Earth or in orbit around it), but the disadvantage of a short duration of fall (half a second only for a dropping height of 10 m).

If a test body is suspended on the surface of the Earth against the local acceleration of gravity it is subject to the centrifugal force due to the diurnal rotation of the Earth at angular velocity $\omega_\oplus$, which acts in the meridian plane of the suspended body and is proportional to its inertial mass. The motion of the body is limited to the plane of the horizon; the component of the centrifugal force in this plane is directed in the North-South direction towards South and depends on the latitude ϑ:

$$f_c = m_i \omega_\oplus^2 \cdot R_\oplus \cdot cos\vartheta \cdot sin\vartheta \tag{4}$$

Equilibrium is reached at a position where this force is balanced by a component, in the same direction, of the local acceleration of gravity, which is proportional to the gravitational mass m_g of the body. This is the well known fact that a plumb line does not point to the center of the Earth but instead is displaced towards South by an angle which is maximum at 45° of latitude (where the deviation is of about 2 *milliradians*), is zero at the poles and along the equator. No deviation from the UFF (and no violation of Equivalence) means that all plumb lines at any given latitude are displaced by the same angle regardless of the material of the suspended body. Which is to say that two test masses of different composition suspended from two wires of the same length, when released from the local vertical should return to their maximum elongation angle at exactly the same time, always keeping in step. The experiment should monitor how accurately the pendula keep in step. In such pendulum experiments the driving acceleration is only 1.69 $cm \cdot s^{-2}$ –the maximum value (at $\vartheta=45^{\circ}$) of the centrifugal force per unit mass given by (4)– and it is a DC effect. However, the advantage is that the experiment is not limited by short duration.

Rather than being suspended independently, the test bodies can be attached at the opposite ends of a horizontal beam which is then suspended with a wire from its midpoint. Any force acting differently on the test bodies and perpendicular to the plane formed by the beam and the wire, gives rise to a non zero net torque which will twist the wire by an angle proportional to the differential force itself. This is the *torsion balance* originally designed by John Mitchell and used by Henry Cavendish at the end of the 18th century to determine the universal constant of gravity G from the measurement, and the theoretical prediction, of the gravitational effect produced on the balance by two large source masses suspended outside the balance on opposite sides of the test masses. In experiments to test the UFF the test masses at the opposite ends of the beam have different composition and equal mass (although having equal mass is not strictly needed) and there are no artificial source masses outside the balance. If the beam is oriented in the East–West direction the force (4) gives a non zero torque on each test mass. The UFF implies a zero twist angle for any composition of the test bodies and any direction of the beam; a deviation from the UFF results in a non zero net torque and consequent twist of the balance by a constant angle (maximum for the beam in the East–West direction) if the balance is stationary in the laboratory (DC effect). The driving acceleration is the same as in pendulum experiments; the differential acceleration whose effect on the torsion balance (twist angle) should be detected in order to test the Equivalence Principle to the level η is

$$a_{EP}^{\oplus} = \eta \cdot \omega_{\oplus}^2 \cdot R_{\oplus} \cdot cos\vartheta \cdot sin\vartheta \qquad (a_{EP}^{\oplus} \cong \eta \cdot 1.69\ cm \cdot s^{-2} \quad at \quad \vartheta = 45^{o}) \tag{5}$$

A constant angle of twist means that there is no zero–check on the result of the experiment, unless the test bodies themselves are swapped on the balance (to prove a violation of Equivalence the balance should twist by the same angle –but in the opposite direction– when the masses are swapped). If the balance is located on a rotating tray the twist angle is modulated at the rotation frequency of the tray. The torsion constant of the wire should be as weak as possible (given the need to withstand the weight of the bodies) in order to result in a detectable twist angle even for a very small differential acceleration acting on the bodies of the balance, i.e. for $\eta << 1$ in (5).

Taking the Sun as the source mass instead of the Earth, the signal expected from a violation of Equivalence is derived by writing the equations of motion of the test bodies (of different composition A and B) in the (non inertial) reference frame centered at the center of the Earth and orbiting around the Sun at the annual angular velocity $\Omega_{\oplus}$ (the gravitational attraction from the Sun is proportional to the *gravitational mass* of the test bodies, while the centrifugal force due to the rotation of the Earth around the Sun –or the rotation of the Sun around the Earth in a geocentric frame of reference– is proportional to their inertial mass):

$$
\begin{aligned}
m_i^A \vec{a}^A &= -m_g^A \cdot \frac{GM_{sun}}{r_A^3} \vec{r}_A + m_i^A \cdot (\vec{\Omega}_\oplus \times (\vec{\Omega}_\oplus \times \vec{R})) \\
m_i^B \vec{a}^B &= -m_g^B \cdot \frac{GM_{sun}}{r_B^3} \vec{r}_B + m_i^B \cdot (\vec{\Omega}_\oplus \times (\vec{\Omega}_\oplus \times \vec{R}))
\end{aligned}
\tag{6}
$$

where $\vec{r}_A$, $\vec{r}_B$ are the relative position vectors of the test bodies with respect to the Sun, $\vec{R}$ is the Earth–to–Sun vector, and M_{sun} is the mass of the Sun. If the test bodies are concentric: $\vec{r}_A = \vec{r}_B \equiv \vec{r}$. Otherwise, $\vec{r}_A \cong \vec{r}_B \equiv \vec{r}$ where the relative distance of the test bodies has been neglected with respect to their distance from the Sun. Any classical tidal effect due to the centers of mass of the bodies not being exactly coincident, and depending on the specific experimental set–up, must be smaller than the signal expected for the level of violation the experiment is aiming to detect. From this, and the centrifugal acceleration of the Earth at its distance form the Sun, we have:

$$
\begin{aligned}
m_i^A \vec{a}^A &= -m_g^A \cdot \frac{GM_{sun}}{r^3} \vec{r} + m_i^A \cdot \frac{GM_{sun}}{R^3} \vec{R} \\
m_i^B \vec{a}^B &= -m_g^B \cdot \frac{GM_{sun}}{r^3} \vec{r} + m_i^B \cdot \frac{GM_{sun}}{R^3} \vec{R}
\end{aligned}
\tag{7}
$$

By defining:

$$
\begin{aligned}
m_g^A &= m_i^A \cdot \left(1 + \frac{\eta}{2}\right) \\
m_g^B &= m_i^B \cdot \left(1 - \frac{\eta}{2}\right)
\end{aligned}
\tag{8}
$$

we also get:

$$
\begin{aligned}
m_i^A \vec{a}^A &= -GM_{sun} m_i^A \cdot \left(\frac{\vec{r}}{r^3} + \frac{\eta}{2} r \frac{\vec{r}}{r^3} - \frac{\vec{R}}{R^3}\right) \\
m_i^B \vec{a}^B &= -GM_{sun} m_i^B \cdot \left(\frac{\vec{r}}{r^3} - \frac{\eta}{2} \frac{\vec{r}}{r^3} - \frac{\vec{R}}{R^3}\right)
\end{aligned}
\tag{9}
$$

and therefore, for the differential acceleration between the test bodies resulting from a violation of Equivalence η, we get:

$$
\vec{a}_{EP}^{sun} = \vec{a}^B - \vec{a}^A = \eta \cdot \frac{GM_{sun} \vec{r}}{r^3} \qquad a_{EP}^{sun} \cong \eta \cdot 0.6 cm \cdot s^{-2}
\tag{10}
$$

In a geocentric reference frame the position vector of the Sun, can be written as:

$$\vec{R}_{sun} = R\ (cos\delta\, sinH,\, cos\delta\, cosH,\, sin\delta\,) \tag{11}$$

where δ and H are its declination and hour angle giving its angular position on the celestial sphere at any time of the year and the day (e.g. $\delta = 0$ at the equinoxes, H=0 when the Sun is at the local meridian). In the same frame, the position vector of the laboratory, at latitude ϑ, where the test bodies are located is:

$$\vec{R}_{lab} = R_{\oplus}\,(0,\, cos\vartheta,\, sin\vartheta) \tag{12}$$

and the position vector $\vec{r}$ of Eq. (10), giving the relative position of the Sun with respect to the laboratory, is:

$$\begin{aligned}\vec{r} = \vec{R}_{sun} - \vec{R}_{lab} &= \left(R\ cos\delta\, sinH,\, R cos\delta\, cosH - R_{\oplus}\, cos\vartheta,\, R sin\delta - R_{\oplus}\, sin\vartheta\right) \\ &\equiv (\,r_x, r_y, r_z\,)\end{aligned} \tag{13}$$

We now rewrite it in the reference frame of the laboratory itself in which the first coordinate is the East–West direction, the second coordinate is the North–South direction (the resulting plane is the plane of the horizon) and the third coordinate in the direction of the Zenith (the vertical plane is the meridian plane):

$$\vec{r} = (\,R_{sun} - R_{lab}\,) = (\,r_x\,, r_y sin\vartheta - r_z\, cos\vartheta, r_z\, sin\vartheta + r_y\, cos\vartheta\,) \tag{14}$$

which immediately gives the North–South and East–West components (in the horizontal plane of the laboratory) of the differential acceleration (10) which would result from a violation of Equivalence η having the Sun as the source mass:

$$a_{EP,NS}^{sun} = \eta \cdot G\frac{M_{sun}}{R^3} \cdot (\,r_y sin\vartheta - r_z\, cos\vartheta\,) \tag{15}$$

$$a_{EP,EW}^{sun} = \eta \cdot G\frac{M_{sun}}{R^3} \cdot r_x \tag{16}$$

showing a dependence from the *daily* and annual motion of the Sun (contained in r_x, r_y, r_z as defined by (13)), as well as from the latitude ϑ of the laboratory. It is therefore apparent that using the Sun as the source mass ensures a frequency modulation of the signal (with a 24-*hr* period) even for a stationary apparatus. This fact was successfully exploited in the 1960s and 1970s (see Sec. 5*)* yielding a considerable improvement over past torsion balance experiments having the Earth as the source mass, despite a weaker driving acceleration (0.6 $cm\ s^{-2}$ instead of 1.69 $cm\ s^{-2}$).

It is worth noticing that a different source mass (the Sun instead of the Earth) changes the distance range of the test (from the radius of the Earth to the Astronomical unit); a fact that should be taken into account when using the result of the tests to place limits on the existence of new composition-dependent interactions.

3 Brief history of past ground experiments

Aristotle’s view, that heavier bodies should fall faster than lighter ones, has been questioned –apparently for the first time– in the 6th century by Philoponus, who stated that

> *“if two bodies are released by the same altitude one can observe that the ratio of the times of fall of the bodies does not depend on the ratio of their weights, and the difference of the times is very small”.*

It was only much later that the issue was reconsidered, in 1553 by Benedetti, who stated that the velocity of fall does not depend on the weights of the falling bodies. Galileo started by showing the internal contradiction of Aristotle’s reasoning with a simple argument: [7]

> *“If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will partly be retarded by the slower, and the slower will be somewhat hastened by the swifter.... Hence the heavier body (made by the two tied together) moves with less speed than the lighter* (the former swifter one); *an effect which is contrary to your* (by Aristotle) *supposition”*

But the most important contribution of Galileo came from his being deeply aware of the need to provide *experimental evidence* that Aristotle's was wrong.

> *“The facts set forth by me up to this point and, in particular, the one which shows that difference of weight, even when very great, is without effect in changing the speed of falling bodies, so that as far as weight is concerned they all fall with equal speed: this idea is, I say, so new, and at first glance so remote from fact, that if we do not have the means of making it as clear as sunlight, it had better not be mentioned; but having once allowed it to pass my lips I must neglect no experiment or argument to establish it.”* [7]

As a typical application of *the scientific method,* Galileo started by a careful analysis of the phenomenon of falling bodies, trying to simplify it by retaining only its most important features. He knew that a simplified phenomenon is much easier to describe in mathematical terms than a complex one, and that only a precise mathematical description would allow him to make a prediction that could be tested by experiments, so as to subsequently improve –depending on the results of the experiments– the model upon which the prediction was based. A careful analysis led him to conclude that the differences –in some cases quite relevant– actually observed in the time of fall of different bodies (differing in weight as well as in composition), was in fact due to the different resistance they oppose to the medium (typically air) through which they move during fall. In order to *amplify* the resistance of the medium, so as to make sure that the hypothesis he made was right, Galileo performed a series of experiments in which he dropped bodies of different composition in media much denser than air, finding that indeed, the denser the medium, the bigger was the difference in time of fall [8, Vol. VIII p. 116]:

> “…nel *mezzo dell' argento vivo l'oro non solamente va in fondo più velocemente del piombo, ma esso solo vi descende, e gli altri metalli e pietre tutti si muovono in su e vi galleggiano, dove che tra palle d'oro, di piombo, di rame, di porfido, o di altre materie gravi, quasi del tutto insensibile sarà la disegualità del moto per aria …. ”*

The conclusion was that all bodies fall equally fast, any observed difference being due to the different resistance of the medium that different bodies are subject to:

> *"…veduto, dico questo, cascai in opinione che se si levasse totalmente la resistenza del mezzo, tutte le materie descenderebbero con eguali velocità "*

> *"... having observed this I came to the conclusion that, if one could totally remove the resistance of the medium, all substances would fall at equal speeds "*

This is indeed the first clear-cut statement of the Universality of Free Fall, and in this form it appeared for the first time in Galileo's *"Discorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti alla meccanica e ai movimenti locali"* [8, Vol. VIII], published outside Italy, in Leiden, in 1638. At the time Galileo was 74 years old, was blind and under house arrest in Arcetri (Florence) by order of the church of Rome, which also ordered that his works should not to be published. However, the *"Discorsi"* are in fact based on much earlier work, mostly on experiments with the inclined plane and the pendulum, going back almost 40 years, to the time when he was a young lecturer at the University of Pisa. Although this fact may seem very unusual nowadays, it was not uncommon at the time, and the evidence is in a letter addressed by Galileo to Guidobaldo dal Monte, a nobleman with a personal interest in Mechanics. In the letter [8, Vol. X p. 97], dated 1602, Galileo describes his pendulum experiments and notes –in passing– that the results would be the same for *different* suspended bodies.

Galileo was aware of the difficulty to provide evidence by dropping masses from a height (from a big height the accumulated effect of air resistance is too large to allow a reliable conclusion; from a small one any difference is too small to appreciate [8, Vol. VIII p. 128]). Most probably Galileo was not able to calculate precisely the effect of air resistance, but he knew that it is much smaller if the velocity of the body is small (in fact, it is $a_{drag} \propto \upsilon^2$). He therefore conducted experiments with bodies falling on inclined planes, where only a fraction of the gravitational acceleration is relevant, which reduces the velocity of fall –hence also the effect of air resistance– and eventually with pendula, where in addition to the slow velocities he could exploit the advantage of the periodic motion to obtain the first accurate test ever of the UFF:

> *"e finalmente ho preso due palle, una di piombo ed una di sughero, quella ben più di cento volte più grave di questa, e ciascheduna di loro ho attaccata a due sottili spaghetti eguali, lunghi quattro o cinque braccia, legati ad alto; allontanata poi l'una e l'altra palla dallo stato perpendicolare, gli ho dato l'andare nell'istesso momento, ed esse, scendendo per le circonferenze de' cerchi descritti da gli spaghi eguali, lor semidiametri, passate oltre al perpendicolo, son poi per le medesime strade ritornate indietro; e reiterando ben cento volte per lor medesime le andate e le tornate, hanno sensatamente mostrato come la grave va talmente sotto il passo della leggiera, che né in ben cento vibrazioni, né in mille, anticipa il tempo d'un minimo momento, ma camminano con passo egualissimo. Scorgesi anche l'operazione del mezzo, il quale, arrecando qualche impedimento al moto, assai più diminuisce le vibrazioni del sughero che quelle del piombo, ma non però che le renda più o meno frequenti; anzi quando gli archi passati dal sughero non fusser più che di cinque o sei gradi, e quei del piombo di cinquanta o sessanta, son eglin passati sotto i medesimi tempi."* [8, Vol. VIII p. 128]

> *"Accordingly, I took two balls, one of lead and one of cork, the former more than a hundred times heavier than the latter, and suspended them by means of two equal fine*

> *threads, each four or five cubits long. Pulling each ball aside from the perpendicular, I let them go at the same instant, and they, falling along the circumferences of circles having these equal strings for semi-diameters, passed beyond the perpendicular and returned along the same path. This free vibration repeated a hundred times showed clearly that the heavy body maintains so nearly the period of the light body that neither in a hundred nor even in a thousand will the former anticipate the latter by as much as a single moment, so perfectly do they keep step. We can also observe the effect of the medium which, by the resistance it offers to motion, diminishes the vibration of the cork more than that of the lead, but without altering the frequency of either; even when the arc traversed by the cork did not exceed five or six degrees while that of the lead was fifty or sixty, the swings were performed in equal times. "* [7]

Newton made the same experiment, as he mentions in the opening paragraph of the *Principia,* and explicitly reported the accuracy achieved: 1 part in 10^3. In order to establish the accuracy that Galileo could have achieved with the pendulum experiment described here, the experiment has been repeated a few years ago [9] showing that it is difficult, by this method, to be less accurate than this. An accuracy of 10^{-3} is consistent with an error in length of 0.1, and it is in agreement with Galileo's claim that the bodies keep in step for hundred or even thousand swings, as these recent experiments have confirmed.

It may be worth noticing that the experiment does not require a clock. The argument, sometimes reported, that the first accurate pendulum experiments on the *Equivalence Principle* are due to Newton, and could not have been done by Galileo because a precise pendulum clock was only available after his death, is therefore not relevant (the pendulum clock was introduced by Huygens in 1657).

Yet, Galileo is famous worldwide for his tower experiments. Mass dropping experiments are often recalled in the *Discorsi* when arguing with Aristotle's point of view, because they allow him to describe the Universality of Free Fall in a very straightforward manner, with no need to discuss the relation between the motion of a falling body and the oscillations of a pendulum. At the University of Pisa in 1993 [10] we have compared modern calculations of the time of fall –taking into account all non gravitational effects– with the results of mass dropping experiments performed from the leaning tower of Pisa using a rather accurate, although quite simple, mass release device. The idea was to position the test masses at the far end of a horizontal platform, hinged at the other end, whose mass and dimension were such that, once released by a cut of wire, it would open up with a vertical acceleration –at the location of the test masses– larger than the local acceleration of gravity, that the test masses experience at the time of release (by a factor 5/4 for this specific device). In this way the disturbances of the dropping mechanism itself on the test masses, which are very important because of the short falling time, were reduced. Unless the effect of air resistance was compensated by an appropriate choice of density and dimension of the test bodies, so as to make their area–to–mass ratios equal (or very close), we could observe that the bodies do actually reach the ground at very different times, as predicted by the theoretical model. The mass dropping experiments mentioned by Galileo in the *Discorsi* would in fact have given different results from what he mentions. It was only in 1641 that the young scholar Vincenzo Renieri reported to Galileo the results of his experiments from the leaning tower of Pisa [8, Vol. XVIII p. 305] , and we have checked that the differences observed by Renieri are consistent with the effects of air resistance. While Galileo formulated very clearly the Universality of Free Fall and proved it with pendula many decade before Newton at a comparable accuracy, he most probably never dropped masses from the leaning tower of Pisa.

Pendulum experiments to test the *Equivalence Principle* have been repeated and improved throughout the 19th century (by Bessel in 1826), and in the 1920s by Potter, only to reach an accuracy of a few 10^{-5}. However, a big jump in sensitivity, by almost 4 orders of magnitude, took place at the end of the 19th century when a torsion balance was used for the first time to test the Equivalence between inertial and gravitational mass. Around 1888 in Budapest Loránd Eötvös built a sensitive torsion balance for gravimetric measurements. The instrument, generally known as the *Eötvös balance,* was used for extensive field measurements and later on in prospecting for oil and natural gas.

It was indeed a very subtle idea that a deviation from the proportionality between inertial and gravitational mass –a fundamental concept in Newtonian Mechanics– should show up as a rotation of the balance. It is also not clear what originally motivated Eötvös to start a long series of experiments on this issue (note that it was before Einstein's work). It was probably when he realized the huge capabilities of his instrument as compared to the limited experimental evidence previously available in support of the Equivalence between inertial and gravitational mass, that he decided to go ahead (see Fig. 1).

Fig. 1. – The torsion balance used by Loránd Eötvös to test the proportionality between inertial and gravitational mass. The experiments were carried out starting in1888, and then in the period between 1905 and 1908. (Picture downloaded from the Eötvös virtual museum available on the World Wide Web).

The experiments rapidly demonstrated the superior capabilities of the torsion balance with respect to pendula in testing the UFF. In the work published in 1922 [11], 3 years after Eötvös death, the result reported was a test to about $5 \cdot 10^{-9}$. A result which remained unchallenged till the 1960s despite the advent of General Relativity at the beginning of the century and consequent relevance of the Equivalence Principle.

In spite of their novelty and success, Eötvös' experiments are limited by the fact that the differential acceleration (5) expected for a violation of Equivalence η in the field of the Earth is (at a given latitude) constant both in direction (North–South) and in modulus. It therefore results in a constant twist angle of the balance (twist angle is maximum if the balance is aligned in the East–West direction). There is no zero internal check, unless the test masses themselves are physically swapped on the balance; which inevitably perturbs the measurements. A way to overcome this limitation came in the 1960s from Princeton [12] when another subtle idea was put forward: if the UFF is tested for the same test bodies in the gravitational field of the Sun rather than the Earth, the effect on the balance of possible deviation from the UFF (although slightly weaker for the same value of η) would have a modulation with the period of the day, as we have shown in Sec. 2. This modulation naturally provides an internal zero check of the experiment with no need to modify the apparatus: a balance aligned in the East–West direction should give zero twist at sunrise and sunset and maximum twist (in modulus) at noon and midnight. Such a modulation proved very successful, yielding an improvement by almost 3 orders of magnitude in sensitivity, to about 10^{-11} [12]. Another torsion balance was designed and manufactured in Moscow a few years later, also referring to the Sun as the source mass of a possible violation of Equivalence. The experiment was carried out in the basement of the Physics Department (on a deeply rooted rock, for reduced seismic noise); special care was devoted to improving the mechanical quality of the suspension wire and to reducing the effects of the local mass anomalies. The sensitivity reported was about one order of magnitude better, to about 10^{-12} [13], again demonstrating the vital importance of a frequency modulation of the expected signal.

4 Equivalence Principle tests by Lunar Laser Ranging

Test bodies of laboratory size have a negligible fraction f of their mass coming from gravitational binding energy:

$$f \equiv -\frac{3}{5} \cdot \frac{GM^2}{R} \Big/ Mc^2 \tag{17}$$

while for the Earth and the Moon this fraction is:

$$f_{earth} \cong -4.64 \cdot 10^{-10} \qquad f_{moon} \cong -1.9 \cdot 10^{-11} \tag{18}$$

The Equivalence Principle which Einstein puts at the basis of General Relativity requires that all bodies fall with the same acceleration in an external gravitational field, with the gravitational binding energy contributing equally to the gravitational and the inertial masses. With the Moon orbiting the Earth and both moving in the gravitational field of the Sun, a violation of the Equivalence Principle would cause the orbit of the Moon around the Earth–Moon center of mass to be “polarized” in the direction of the Sun (as sketched in Fig. 2). This effect would have the synodic period of 29.53 *days* and is usually referred to as the Nordtvedt effect [14, 15]. It adds up to the classical variations of the lunar orbit which result from the Sun’s tidal acceleration of the Moon relative to the Earth, first estimated by Newton himself. Laplace later found that Newton’s distorted orbit was also slightly polarized towards the Sun, with the center of the Earth shifted in the direction of full Moon. It is the so called *parallactic inequality*, named in this way because it is proportional to the ratio of lunar to solar distance, and therefore its measurement could be viewed as a way of determining this ratio. Should the Earth and the Moon accelerate at different rates toward the Sun, this would result in a small *additional* polarization of the orbit. For it to be observed, the motion of the Moon should be very accurately monitored.

On 21 July 1969, during the Apollo 11 first manned mission to the Moon, the first retroreflector array was placed on the surface of the Moon enabling highly accurate measurements of the Earth-Moon separation by means of laser ranging.

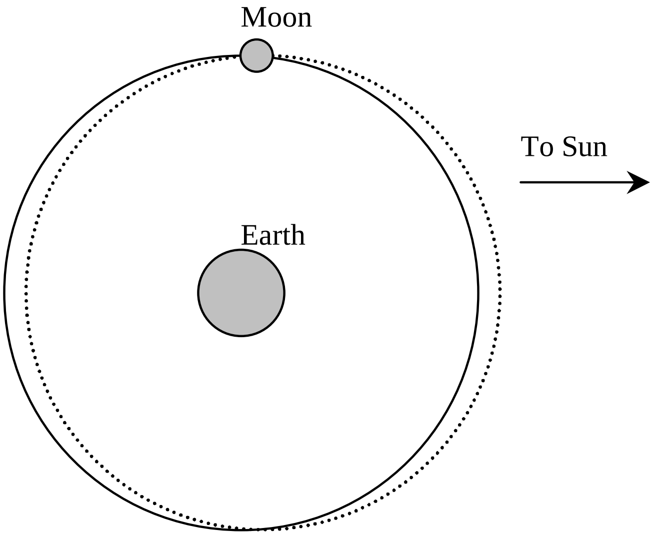

Fig. 2. – Schematic representation (figure not to scale) of the polarization of the lunar orbit toward the Sun. The polarization is mostly due to a classical effect known as the *parallactic inequality* (the Earth–Moon system is not an isolated 2–body system, because of perturbations by the Sun), first pointed out by Laplace. However, the amplitude of the polarization would be sligltly larger than calculated by Laplace should the Earth and the Moon accelerate differently toward the Sun because of a violation of the Equivalence Principle. A violation might occur either because of the different gravitational binding energy of the Earth and the Moon (as given by (18)) or because of their different composition, or both (see text).

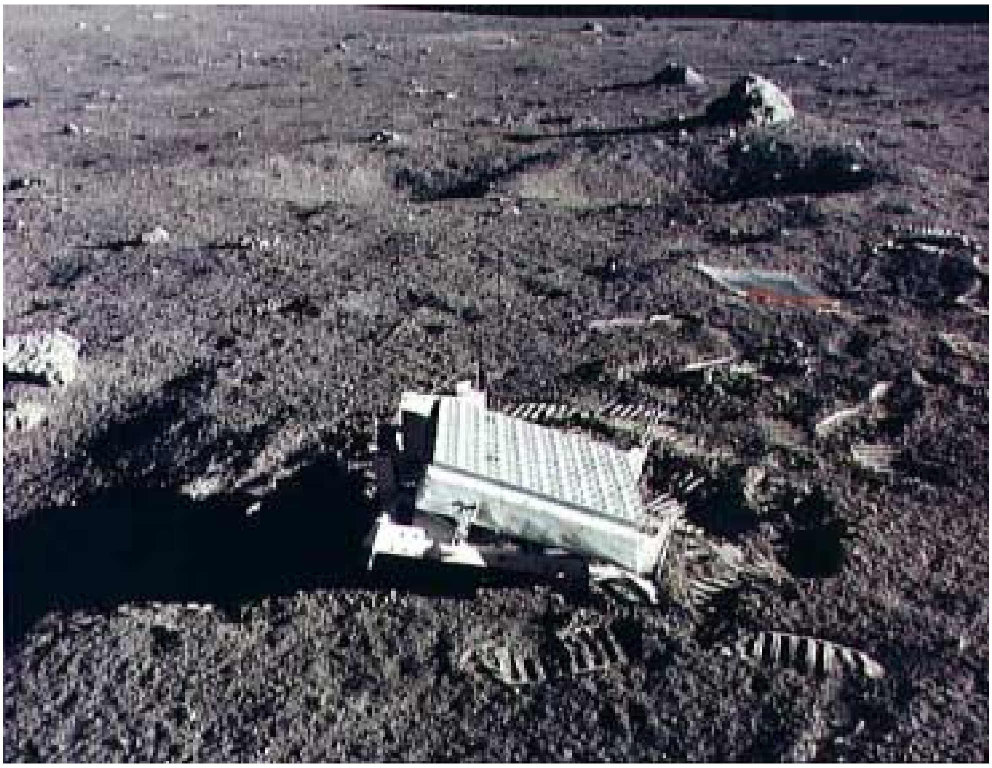

Fig. 3. – Photograph of a retroreflector array that astronauts from the Apollo 14 left on the surface of the Moon (100 corner cubes of 3.8 cm diameter mounted in an aluminum panel).

Within the next two years, two more retroreflectors were brought to the Moon by astronauts from Apollo 14 and Apollo 15 (see Fig. 3). A fourth one came in 1973 with the Soviet Lunakhod II spacecraft. Since then, laser stations on the ground can fire to these reflectors and get enough photons back (in spite of an overall signal loss of about 10^{-21} !) to be able to measure the separation distance from measurement of the round–trip travel times (photons are received about 2.6 s after they are sent). The accuracy currently achieved for a range measurement observing session (corresponding to tens of minutes of photon returns) is of 2–3 cm. After almost 30 years of ever–improving quality of data and data analysis, the amplitude of the parallactic inequality relevant for testing the Equivalence Principle is measured to a precision of 1.3 cm, allowing scientists to conclude that any deviation of the Earth and the Moon from the Universality of Free Fall (i.e. any fractional difference of their accelerations toward the Sun) must be less than $5{\cdot}10^{-13}$ [16, 17].

If one could rule out any composition–dependent violation of Equivalence for the Earth and the Moon, then this result could only be interpreted as proving that the gravitational binding energies of the Earth and the Moon contribute equally to their gravitational and inertial masses to about 1 part in 10^3. However, the Earth and the Moon do have different composition. The average composition of the Earth is dominated by its iron–nickel core, while the composition of the Moon is closer to that of the (less dense) Earth mantle, primarily made of silicates. Unless iron–nickel on one side and silicates on the other do accelerate the same in the gravitational field of the Sun (and to an accuracy comparable to that of the laser ranging test), Lunar Laser Ranging data cannot be interpreted without ambiguity as a test of how equally gravitational binding energies of the Earth and the Moon contribute to their gravitational and inertial masses

This ambiguity has recently been removed thanks to experiments carried out at the University of Washington, in Seattle, with "miniature" earths and moons placed on a continuously rotating torsion balance whose twist data have been analyzed for any deviations from the Universality of Free Fall with respect to the Sun [5]. The composition of the test bodies was chosen for them to resemble, to the best of current knowledge, the core of the Earth and its mantle. Improvements in the apparatus have allowed a sensitivity of a few 10^{-13} in the measurement of fractional differences in the acceleration of the test bodies toward the Sun, which is good enough for the ambiguity of Lunar Laser Ranging data to be resolved

5 Recent and ongoing laboratory experiments

A re–analysis of the Eötvös experiments carried out by [18] in 1986 has had the merit to draw the attention of a large number of scientists from all over the world to the Equivalence Principle and to the measurement of the universal constant of gravity. In relation to the Equivalence Principle the authors made the point that the most accurate tests available at the time –those carried out in Princeton [12] and in Moscow [13]– had checked for violation of Equivalence in the gravitational field of the Sun, thus over a range of 1 AU, while the old Eötvös tests at the turn of the century were still the most accurate ones as far as tests in the gravitational field of the Earth are concerned.

Since then, the most systematic and successful experiments on the Equivalence Principle are the so called "Eöt–Wash" experiments carried out by the group of E. Adelberger at the University of Washington in Seattle. The *Eöt–Wash* apparatus is a torsion balance operated at room temperature with small test cylinders (10 *g* each) placed on a turntable whose rotation provides a modulation of the expected signal with the period of about 1*hr*. Frequency modulation is crucial in order to check for violation in the field of the Earth (or of a local mass nearby), otherwise the signal would be DC and therefore very hard to detect, like in the Eötvös experiments. Moreover, the turntable can rotate quite fast, thus increasing the modulation frequency of the signal by about 24 times with respect to [12, 13]. However, should any local mass anomaly couple differently with the test bodies (i.e. because of their different multipole mass moments), the corresponding differential acceleration will also be modulated at the rotation frequency of the turntable. This would obviously not happen in the experiments [12, 13], where the modulation of the signal (in the field of the Sun) is provided by the rotation of Earth itself and the effect of any local mass anomaly is DC. The procedure set up by the *Eöt–Wash* group to deal with this issue is to first use *ad-hoc* test cylinders in which the various multipole moments (starting from those of lower degree) have been amplified in order to amplify the corresponding effects caused by mass anomalies close by (but also in the region around the laboratory). Then, a variable distribution of masses is set up around the torsion balance to be adjusted (on the basis of numerical calculations as well as of direct measurements) until those effects are canceled. By a careful procedure of successive iterations it is possible to select an appropriate mass distribution that minimizes the perturbations from local mass anomalies on the torsion balance, and the instrument is ready to mount the cylinders devoted to testing the Equivalence Principle. Helmholtz coils and μ–metal shielding are needed in order to reduce the torque caused by the magnetic field of the Earth interacting with the residual magnetic moment of the tray. This is a typical need of torsion balance experiments because they are based on the measurement of a torque.

No violation has been detected, and the sensitivity of the experiments has steadily improved: from 1 part in 10^{11} in 1990 [3] to about 1 part in 10^{12} in 1994 [4], to about 1 part in 10^{13} in 1999 [5]. In [3, 4] data were checked for violation of Equivalence in the field of the Earth with test bodies made of Be/Al and Be/Cu. In [5] the source mass was the Sun and the test bodies were manufactured to simulate the difference in composition between the Earth and the Moon, in order to remove the ambiguity of Lunar Laser Ranging tests of the Equivalence Principle, as discussed in the previous Section.

Other research groups are carrying out torsion balance experiments to test the Equivalence Principle. In India [19] the apparatus is located underground at a remote site characterized by a very low level of seismic noise; the torsion balance has the shape of a ring, with two halves of different materials, much more massive than in the *Eöt–Wash* balance (1.5 *kg* each). The balance does not rotate and data (measurements of the deflection angle) are checked for violation of Equivalence in the gravitational field of the Sun, as in [12, 13]. The instrument is very sensitive and the laboratory itself has been very carefully constructed for such sensitive measurements. However, systematic effects apparently related to daily variation of atmospheric pressure still need to be taken care of.

R. Newman [20], from the University of California at Irvine, leads the only group that is attempting to set up a cryogenic torsion pendulum for gravitational experiments, the expected advantages being: reduced thermal noise, reduced temperature sensitivity, improved temperature control and possibly also improved characteristics of the suspension fibre. In addition, the experiment is operated at a very remote site, in a former missile bunker where the microseismic noise power spectrum is about two orders of magnitude less than that at Irvine.

At the University of Washington, Seattle, P. Boynton [21] is working on a torsion pendulum for testing Newtonian gravity at room temperature. The accent in this case is on the use of a new observable (the second harmonic amplitude of the pendulum motion) that provides measurement of extremely small torques with significant freedom from effects that may limit the traditional, non-cryogenic applications of the torsion pendulum.

A new version [22] of the classical mass dropping Galileo–type experiment has been initiated by Italian scientists from the University of Pisa after the publication of [18]. Rather than dropping separate masses of different composition, the authors drop a (vertical) disk whose two halves are made of Al and Cu respectively (350 *g* each, with a drop height of about 4 *m*): any deviation from the Universality of Free Fall would cause an angular acceleration of the free falling disk around its axis and such a motion can be accurately measured by means of a modified Michelson interferometer in which the two arms terminate at two corner–cube reflectors mounted on the rim of the disk. The original goal of the experiment was to test for any composition dependent effect in the range between 10 *km* and the radius of the Earth. The experiment reached the sensitivity $\Delta g/g=7.2\cdot 10^{-10}$ finding no deviation from the Universality of Free Fall. It is interesting to note that the same experiment has been performed also with a homogeneous disk, which should obviously result in a zero signal.

A short distance test of the Equivalence Principle based on the torsion balance has been performed more recently [23] by the *Eöt–Wash* group using a large rotating source mass (3 *ton* of ^{238}U) and checking for its effect on a torsion balance with test cylinders in Cu/Pb. The balance was sensitive to a relative differential acceleration $\Delta a/a \cong 10^{-9}$ from the rotating source mass and the result helped to set better limits on new composition-dependent interactions in the distance range from 10 to 10000 *km*.

6 Advantages of an Equivalence Principle experiment in low Earth orbit

The advantage of performing an Equivalence Principle test in low Earth orbit is a *driving signal* about *3* orders of magnitude bigger than in ground laboratories. If test bodies of different composition orbit around the Earth at an altitude *h,* a violation of Equivalence η would make them fall differently toward the Earth, with a differential acceleration (one with respect to the other):

$$a_{EP} = \eta \cdot \frac{GM_{\oplus}}{(R_{\oplus}+h)^2} \qquad (a_{EP} \cong \eta \cdot 840\ cm \cdot s^{-2} \quad for \quad h \cong 500\,km) \tag{19}$$

By comparison with Equivalence Principle experiments in which the test bodies are suspended against local gravity on the surface of the Earth this effect is –for the same value of the Eötvös parameter η – 500 or 1400 times bigger, depending on whether ground experiments consider the Earth or the Sun as the source mass (see (5) and (10)).

Another advantage of a space experiment is the absence of weight in orbit. The largest acceleration inside the orbiting laboratory (the spacecraft) is about 100 *million* times smaller than the local acceleration of gravity on the surface of the Earth. The goal of the experiment being the detection of an extremely small acceleration ($\eta \ll 1$ in (19)), this is obviously an advantage. In addition, in a space experiment the only nearby mass that can disturb the experiment is the mass of the spacecraft itself, which can be better controlled.

As a consequence, space missions can potentially test the Equivalence Principle to a considerable higher accuracy than ground tests, deeply probing a so far totally unknown field of physics where a violation is likely to occur. On these grounds space missions on the Equivalence Principle have been proposed since the early 1970s, and they have attracted more and more attention from space agencies around the world in the last 10 years.

Just as an additional example on the relevance of an Equivalence Principle test to a very high accuracy, let us consider the case of antiparticles. A peculiarity of gravity, strictly related to the Equivalence Principle, is that there is so far no evidence for antigravity, namely for the possibility that matter is gravitationally repelled by antimatter. A negative ratio of inertial to gravitational mass would obviously violate the Equivalence Principle and forbid any metric theory of gravity. Yet, there are theoretical formulations which would naturally lead to antigravity. Unfortunately, while experiments concerning the inertial mass of antiparticles have been highly successful, and these are very accurately known, gravitational experiments (i.e. involving the gravitational mass of antiparticles) are extremely difficult because of the far larger electric effects, such as those due to stray electric fields in the drift tube. In absence of such direct tests, an improvement by several orders of magnitude of current tests of the Equivalence Principle with ordinary matter would also be an important constraint as far as the relation between gravity and antimatter is concerned.

Yet, one should not undermine the difficulties of a space experiment. It must be operated from remote, with no direct access to the apparatus after launch. Experiment parts must be tested at $1-g$, while they have been designed and optimized for weightlessness. Space is not as empty and quiet as one might think at first glance. Although there is no seismic noise of the kind we are used to on Earth, the spacecraft is subject to air resistance along its orbit, as well as to pressure from solar photons, both of them giving rise to disturbances on the test bodies. There are electric charges inside the spacecraft, whose interactions could completely mask gravitational effects. The spacecraft is exposed to heat sources (from the Sun and the Earth itself), which might produce relevant disturbances on the test bodies. All these are matters of concerns for any space experiment aiming to test the Equivalence Principle.

Data from a space experiment can be analyzed for checking any deviation from the Universality of Free Fall not only toward the Earth but also toward the Sun, or the center of our galaxy. However, except in the case of the Earth, the corresponding driving signal would be no bigger than it is in ground based experiments. Short range tests, which require an artificial source mass nearby (possibly a big one), are also not suitable candidates for a space experiment, and are much better carried out on the ground [23]

7 Proposed space experiments to test the Equivalence Principle

Ground tests of the Equivalence Principle based on the torsion balance have so far achieved the best accuracy. However, is the torsion balance the best instrument to fly? Test bodies on a torsion balance are sensitive to differential forces in the plane perpendicular to the suspension wire of the balance, which on the surface of the Earth aligns itself along the direction of the local vertical. Indeed, any deviation of the suspension wire from the verticality (e.g. due to terrain tilts related to microseismicity and Earth tides) is a disturbance and a serious matter of concern in these experiments. In an almost 0-g environment inside a spacecraft orbiting around the Earth there is no natural “vertical”, and the wire of the balance must be aligned by an active system of sensors and actuators. A good active alignment is possible; however, the *passive nature* of the instrument as it has been used so far in ground experiments, which proved vital in the detection of extremely small gravitational forces, would no longer be there.

Test bodies orbiting around the Earth at a non zero separation distance would be subject to varying tidal forces that are differential by nature. In addition, if the bodies have non zero (and different) multipole moments, they are also subject to a differential gravitational attraction from nearby mass anomalies and from the Earth itself. It is mandatory to make these classical effects as small as possible, and this suggests that the test bodies should be concentric –in order to reduce tidal effects–, spherical and homogenous –in order to reduce the effects of differential coupling to multipole moments. The closest practical solution, also taking into account that there must be a read–out system in between the test bodies to monitor their differential motions, is to have concentric, coaxial, hollow cylinders with appropriate dimensions to reduce their multipole moments while maintaining the cylindrical symmetry. Indeed, in all experiments proposed so far to test the Equivalence Principle in space the test bodies are concentric cylinders, differing only in the way they are arranged and suspended.

The first experiment to test the Equivalence principle in low Earth orbit was proposed in the USA in 1970 [24]. The authors suggested to place the test cylinders on a rotating aluminum wheel with their symmetry/sensitive axes in the radial direction and the rotation axis perpendicular to the plane of the wheel. The rotation speed was to be quite high: 100 *rpm* (1.7 *Hz*), the purpose being to modulate a putative signal of Equivalence Principle violation at high frequency. After the success of the Princeton and Moscow experiments [12, 13], which improved by a few orders of magnitude over the Eötvös result mostly thanks to a 24–*hr* modulation, the importance to modulate the signal at higher frequency had became apparent and motivated the experiment design proposed by [24]. The idea was abandoned, probably because of the disturbances that fast rotation would cause on the test bodies in this design. The authors did not seem to be aware that, with only one dimension available for the motion of the test cylinders, such a system is indeed known to be unstable [25]. If this limitation is removed, fast rotation can in fact be exploited to reduce perturbations related to the rotation itself, but this became known only many years later (see Sec. 7.2).

Shortly after [24] another American proposal was put forward for a space experiment to test the Equivalence Principle, named STEP (Satellite Test of the Equivalence Principle) [26]. Although the idea of a rapid rotation of the test masses, and consequent high frequency modulation of the expected signal was abandoned, the STEP experiment could still offer a modulation of the signal at a frequency more than a factor of 10 higher than the modulation frequency of the Princeton and Moscow experiments (see Sec. 7.1). STEP has dominated the field of space experiments on the Equivalence Principle for over 25 years by now [26–31]. A simplified version of it, based on the same concepts (named μSCOPE), has been proposed in France [32]. A different experiment and mission design, based on fast rotation and high frequency modulation (at 2 *Hz*, close to the original proposal of [24]) has come from Italy in the 1990s [33–37], named “GALILEO GALILEI” (GG). STEP, μSCOPE and GG are all under investigation by national space agencies. The goals are: 10^{-15} for the French μSCOPE, 10^{-17} for the Italian “GALILEO GALILEI” (GG), 10^{-18} for the American STEP. Their main features and the expected sensitivity are discussed in the next two sub Sections. Ground testing of a prototype of the GG apparatus proposed for flight is described in Sec. 8.

7.1 The STEP and μSCOPE space experiments

In all proposed space experiments a spacecraft orbiting the Earth at low altitude carries a system of concentric, coaxial test cylinders of different composition especially sensitive to differential accelerations acting between them. It also carries a read–out system of comparable sensitivity in order to detect, in the motion of the test cylinders, any deviation from the Universality of Free Fall in the gravitational field of the Earth.

In both STEP and μSCOPE the test cylinders are sensitive along their symmetry axis and the experimental apparatus moves around the Earth always maintaining a fixed orientation with respect to inertial space, as shown in Fig. 4. This is obtained by rigidly connecting to the spacecraft the supports on which the test cylinders are suspended and by actively controlling the attitude of the spacecraft by means of star sensors and thrusters. With this configuration, an Equivalence Principle violation (and deviation from the Universality of Free Fall) in the field of the Earth would generate a signal whose amplitude is modulated at the orbital frequency of the satellite ($\cong 1.7 \cdot 10^{-4}$ *Hz*, corresponding to an orbital period around the Earth of about 1 and a half hour; see Fig. 4).

During some selected time intervals the spacecraft can also be slowly rotated (with a rotation period of about 10^{3} *s*) so as to rotate the sensitivity axis of the test cylinders with respect to the Earth, thus modulating an Equivalence Principle violation signal at the rotation frequency of the spacecraft. This was first proposed in [29], during a joint ESA–NASA study of STEP, to help distinguish the expected signal from some spurious effects characterized by the orbital period (note that in doing so, the modulation frequency is also further increased by almost a factor 6).

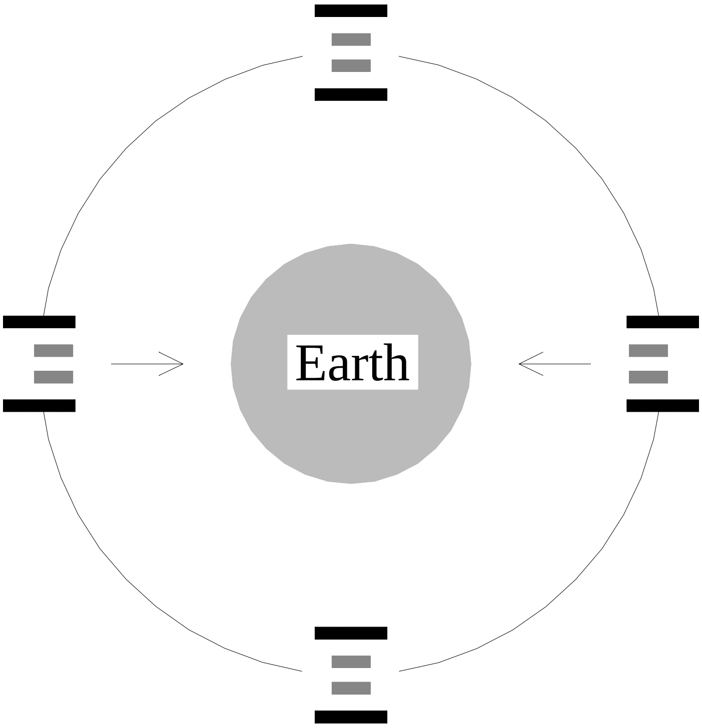

Fig. 4. – Schematic representation (not to scale) of the test cylinders –made of different materials– in the STEP and μSCOPE experiments as they orbit around the Earth. The figure is a section through the symmetry/sensitivity axis of the cylinders. The experimental apparatus is carried by a spacecraft (not shown in this figure) whose attitude is actively controlled in order to remain fixed with respect to the inertial space.

The test cylinders are free to move with respect to one another only along the symmetry axis. A deviation from the Universality of free fall in the gravitational field of the Earth would appear as indicated by the arrows, namely at the orbital frequency of the spacecraft and with a known phase angle (always pointing to center of the Earth). During the time intervals in which the spacecraft undergoes slow, controlled rotation around its center of mass in the plane of the figure (at about 10^{-3} *Hz* with respect to the Earth), this signal would also be modulated at the rotation rate.

The goals for STEP and μSCOPE are 10^{-18} and 10^{-15} respectively in the measurement of the Eötvös parameter η. In terms of the differential acceleration to be detected this amounts to $8.4 \cdot 10^{-16}$ $cm \cdot s^{-2}$ and $8.2 \cdot 10^{-13}$ $cm \cdot s^{-2}$ respectively (STEP is designed to fly at an altitude of about 500 *km*, μSCOPE at 600–700 *km*). Just to have an idea of how small these accelerations are, the target for STEP corresponds to about the mutual attraction between two masses of 1 *g* each placed at a distance of 100 *m* from one another! Since the acceleration to be detected is differential, the test bodies should be insensitive to accelerations in common mode (i.e. to accelerations which are the same on both test bodies) while they should be as much as possible free to respond to differential effects.

In STEP the test bodies are suspended by superconducting magnetic bearings so as to move freely only along their symmetry axes. A feed back system, based on the measurement of the relative displacements as performed by one differential SQUID sensor (Superconducting Quantum Interference Device) counteracts any relative displacement recorded, so as to keep the test cylinders centered on one another as precisely as possible. The feed back signal itself is the output signal of the experiment. Motions of the test cylinders in common mode are measured (by another SQUID sensor), and rejected. This type of instrument is usually referred to as a differential accelerometer.

The experiment concept of μSCOPE is derived from STEP, but there are two important differences. While in STEP the experimental apparatus is maintained at 1.8 *K* inside a dewar filled with superfluid He (enclosed by another He dewar at 4.2 *K*), the μSCOPE mission is operated at room temperature, and therefore requires suspensions and read–out suitable for operation at these temperatures. The test bodies are suspended (independently) by electrostatic levitation (see Fig. 5); for each test cylinder there are radial and spin control electrodes to prevent radial motion and rotations, so that the symmetry axis is the only direction along which each cylinder is allowed to move. Each test cylinder has axis capacitors, forming a capacitance bridge: any movement of the test cylinder along its axis will unbalance the bridge and result in a detectable voltage signal which is used as the driving signal of a feedback system (also based on electrostatic forces). The difference with respect to STEP is that in μSCOPE the test bodies are neither coupled by the suspension nor by the read-out: both the common mode and differential mode displacements have to be derived from *independent* measurements of two *independent* test bodies, each one with its own *independent* read-out system. In STEP the test bodies are coupled by the read-out (there is one SQUID sensor for differential motions and one SQUID sensor for motions in common mode). As in classical torsion balance experiments (where the test bodies are physically coupled), this ensures that if there is no violation effect the response of the system must be zero (no effect, no signal).

Because of (19), the satellite altitude should be as low as possible. However, at very low altitude the satellite is subject to a strong atmospheric drag, caused by the resistance from residual air along the orbit. Air drag makes any satellite loose energy and spiral in, hence limiting the duration of the mission.

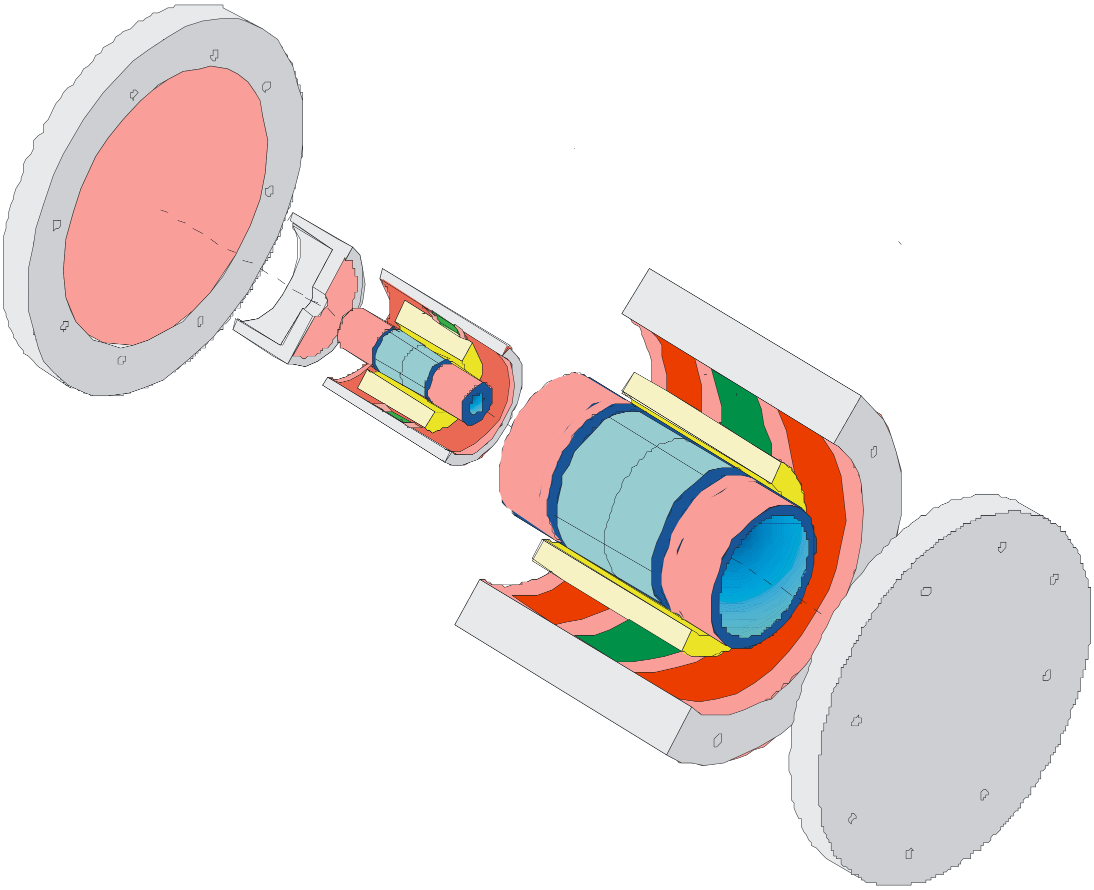

Fig. 5. – Cutaway view of the system of test cylinders designed for the μSCOPE experiment. (The figure is to scale; the length of the outer test cylinder is about 8 *cm*, the whole system assembled together is slightly more than 10 *cm* long). Each test body is a hollow cylinder surrounding a fixed cylindrical support (there are two separate supports, one for each of the two test cylinders) on which it is levitated by electrostatic forces, free to move only in the axial direction (radial and spin electrodes prevent any other movements). The axial movements of each test cylinder are sensed and controlled (independently) by axis electrodes. All cylinders (two test bodies –each one free to move along the axial direction– and two fixed supports) are all nested one inside the other. (Figure taken from the Website of μSCOPE [32])

In experiments aiming to detect small gravitational effects, air drag is also responsible for the largest disturbance on the experimental apparatus. Very weakly suspended test cylinders, such as those shown in Fig. 5, are screened from air drag by the spacecraft, but are subject to a corresponding –equal and opposite– inertial force, much bigger than the signal to be detected, given by:

$$a_{drag} \cong \frac{A}{M} \rho_{atm} \upsilon_{sc}^{2} \tag{20}$$

where A/M is the so called area–to–mass ratio of the satellite, ρ_{atm} is the atmospheric density, and υ_{sc} is the velocity at which the atmospheric particles hit the spacecraft (there is also a numerical factor depending on the shape of the satellite, its aerodynamic coefficient, which is typically of order unity). For small and rather compact satellites orbiting at low altitudes (400 to 700 *km*) the resulting drag acceleration is much smaller than the local acceleration of gravity (by many orders of magnitude); however, it is still many orders of magnitude larger than the expected signal. The crucial difference with respect to the signal is that inertial accelerations resulting on the test cylinders from accelerations acting on the spacecraft outer surface and not on the test bodies themselves, are ideally the same on both of them (*common mode* accelerations), hence there should be no residual differential effect competing with the signal. However, this is true only if the suspension systems of the two bodies are perfectly identical; for instance, should the symmetry/sensitivity axes of the two cylinders not be perfectly aligned, the same inertial acceleration would result in two different axial components, and the system would detect a differential acceleration. Since the largest component of air drag acts *along–track*, i.e. tangent to the satellite orbit, the effect is also at the orbital frequency like the signal, although roughly 90° out of phase. In Fig. 4, the two arrows representing a differential signal due to air drag would be orthogonal to those depicting an Equivalence Principle violation. The difficulty, however, is that the drag effect itself is many orders of magnitude bigger than the signal and even a large phase difference is not sufficient to separate the two.

The effect of drag can be cured in two ways. One way is that the spacecraft compensates for the drag, in such a way that only a residual fraction of it will affect the test bodies. This requires the spacecraft to be equipped with sensors of the drag acceleration (a test body inside the spacecraft, as much as possible uncoupled from it, with a read out system to monitor its relative displacements with respect to the spacecraft) and thrusters on the spacecraft to actively force it in order to follow the motion of the test body (*drag–free* satellite). In STEP, the measurements of the common mode SQUID can be used to drive the thrusters; similarly, in μSCOPE it is possible to use the common mode signal obtained from the system of test bodies shown in Fig. 5. The signal being at the orbital frequency, drag compensation is required around this frequency.

Since drag is ideally a common mode effect, while the expected signal is differential, another way of reducing it is that the experimental apparatus be as good as possible in rejecting common mode effects. In STEP and μSCOPE common mode rejection is limited by the ability to make the symmetry/sensitivity axes of the test cylinders precisely aligned. In STEP it is expected that these axes can be aligned, and common mode effects can be rejected, to about 10^{-4}, but drag is so much bigger than the expected signal that the target sensitivity would be beyond reach unless drag were also partially compensated. The final choice, also in μSCOPE, is that drag is partially compensated and partially rejected .

Air drag is, at the altitudes of interest here, the largest non gravitational perturbation as well as the most relevant, because its main component is at the orbital frequency. The effect of solar radiation pressure is slightly smaller, but essentially DC. The perturbation caused by the fraction of solar radiation re-emitted by the Earth (depending on the albedo coefficient, hence also on cloud coverage) is modulated once per orbit in the presence of eclipses; the resulting inertial acceleration at the orbital frequency is partially compensated and partially rejected, together with air drag and any other inertial effect. Both STEP and μSCOPE are designed to fly along a sun–synchronous orbit, hence at high inclination angle over the Earth's equator; if the launch window is chosen properly, and for a short mission duration (about 6 months for STEP, limited by the amount of propellant to be carried on board for drag compensation), the spacecraft will be free from eclipses.

Perturbing accelerations with exactly the same frequency and phase as the signal cannot be separated from it and must therefore be reduced below the signal. If the STEP (or μSCOPE) spacecraft rotates around its center of mass, in the plane of Fig. 4, so that the sensitivity axis changes its orientation with respect to the Earth at a known frequency, these perturbing accelerations are modulated just like the signal, and cannot be distinguished from it (unless rotation can average out the perturbation; see Sec. 7.2 in the case of the radiometer effect in GG). However, spacecraft rotation will help separating those effects which do not have exactly the same signature as the signal but still act once per orbit (e.g. electric charging effects related to the satellite passing over the South Atlantic Anomaly).

The infrared radiation emitted by the Earth also hits the spacecraft, generating temperature differences in the residual gas surrounding the test cylinders that vary at the orbital frequency (because the attitude of the spacecraft is fixed with respect to inertial space while the source of the radiation is the Earth itself). This gives rise to the so-called *radiometer effect,* well known in space experiments to test the Equivalence Principle.

The radiometer acceleration along the symmetry axis s of a cylinder of density ρ is given by:

$$a_s = \frac{p}{2\rho}\frac{1}{T}\frac{dT}{ds} \tag{21}$$

(see, e.g. [38]) where p is the pressure of the residual gas and T its temperature. In space experiments on the Equivalence Principle the test cylinders have different composition (and typically also different dimensions); they are therefore subject to a different radiometer acceleration. If in addition the radiation source that the satellite is exposed to is the Earth, the resulting radiometer acceleration on the test cylinders will have the same frequency and phase of a possible violation of Equivalence having the Earth as the source mass, thus making it indistinguishable from the expected signal. The additional rotation of the STEP and µSCOPE spacecraft, whose purpose is to modulate an EP violation signal at a frequency different from (and larger than) the orbital frequency around the Earth, will modulate the radiometer differential acceleration as well.

In the STEP experiment the residual pressure is extremely low: $p \cong 10^{-13}$ *Torr* ($1.33\cdot 10^{-10}$ dyn/cm^2) because the apparatus is maintained at 1.8 K inside a dewar filled with superfluid He (enclosed by another He dewar at 4.2 K). An Equivalence Principle violation to the level of 1 part in 10^{18} (the target of this mission) would give a differential acceleration of $8.4\cdot 10^{-16}$ $cm\cdot s^{-2}$; the radiometer effect resulting from (21) must be below this value. The test body of lower density (Be, with a density of 1.85 g/cm^3) is subject to a larger radiometer effect and will dominate in the differential one because the other body is much denser. For this to be smaller than the target signal, the condition on temperature gradients across the Be test mass of the STEP experiment (10÷15 cm size) is:

$$\left(\frac{dT}{ds}\right)_{STEP} < 4.2\cdot 10^{-5} \quad K/cm \tag{22}$$

which must be ensured over the period of the signal (i.e. the orbital period around the Earth) or –in case of signal modulation– over the rotation period of the spacecraft. The superfluid He dewar, besides ensuring extremely low pressure, is pivotal in reducing temperature gradients, so as to meet the requirement given by (22).

In the case of µSCOPE the experiment is run at room temperature, the satellite is designed to fly at higher altitude than STEP (600-700 km) and the target is to test the Equivalence Principle to 10^{-15}, corresponding to a differential acceleration of $8.2\cdot 10^{-13}$ $cm\cdot s^{-2}$. The less dense test cylinder is made of Ti, about 8 *cm* long (with a density 4.5 g/cm^3). The residual gas pressure as reported for a predecessor of the current instrument (STAR [39]) is of $3.7 \cdot 10^{-6}$ *Torr.* A pressure of $7.5 \cdot 10^{-9}$ *Torr* is foreseen for the LISA mission [40, 41], far ahead in the future. Assuming that a value of 10^{-8} *Torr* is achieved with µSCOPE, the condition on temperature gradients analog to (22), (but referring to an Equivalence Principle test 3 orders of magnitude less sensitive) is:

$$\left(\frac{dT}{ds}\right)_{\mu SCOPE} < 1.7 \cdot 10^{-4} \quad K/cm \tag{23}$$

from which it is apparent that a room temperature mission of this kind will find it very hard to achieve an Equivalence Principle test of high accuracy (temperature differences of 0.5 *K* are reported for STAR [39]; see also [38] and references therein).

Having chosen, for STEP, to operate the experiment at very low temperatures, other advantages besides very low pressure can be exploited. The main advantage, discussed above, is the possibility to use SQUID read-out sensors, allowing differential displacements to be measured independently of the ones in common mode; which is crucial in experiments to test the Equivalence Principle. As we have seen, if the STEP experiment design is modified only for operation at room temperature, as it is the case with µSCOPE, the capacitance read–out used in substitution of the SQUID read-out is adequate in terms of sensitivity but no longer allows differential and common mode effects to be uncoupled.

Other advantages of low temperature, relevant for testing the Equivalence Principle, are a reduced level of all disturbances related to thermal expansion, and a reduced level of thermal noise. Reducing thermal expansion close to the test bodies is important. However, a cryogenic experimental apparatus does not prevent the spacecraft outer shell from expanding and contracting in response to the infrared radiation from the Earth while orbiting around it, and the spacecraft mass so displaced will act on the test bodies giving rise to a differential acceleration (because the test cylinders cannot be exact monopoles) with the same frequency and phase as an Equivalence Principle violation. In STEP this thermal expansion effect turns out to be much bigger than the expected signal, posing a very stringent requirement on the knowledge (by direct measurement) of the residual quadrupole mass moments of the test cylinders. Residual, non zero quadrupole moments are unavoidable because of machining errors. As for thermal noise, it depends on the temperature–to–mass ratio, and could be reduced not only by operating at lower temperatures but also by using more massive test bodies.

A cryogenic experiment like STEP has disadvantages too. An apparatus aiming to detect the effects of extremely small forces is naturally limited by nearby mass anomalies, more so if these masses move at the same frequency as the expected signal. The best would be to have no moving masses at all on board the spacecraft, which is practically impossible if it needs propellant to feed the thrusters in order to compensate for drag. As far as drag compensation is concerned, STEP has another constraint: being cryogenic, it needs to carry sufficient He on board to keep the experimental apparatus cool for the entire duration of the mission, and needs to get rid of the resulting boiled off He in a controlled way, so as not to disturb the experiment itself. This requires proportional He thrusters (impulsive thrusters would be too noisy), and the resulting natural choice is to use these thrusters with boiled off He as the propellant to compensate for drag. However, He itself in a not perfectly full or perfectly empty dewar will respond with zero friction to the varying tidal potential of the Earth (see Fig. 6). As a result, it will move around the test bodies at exactly the orbital frequency [29, 30], and if the test bodies have non zero (and different) residual mass moments the resulting differential effect competes directly with the signal. It is not an easy task to stop completely this motion inside a large dewar which contains hundred liters of He.

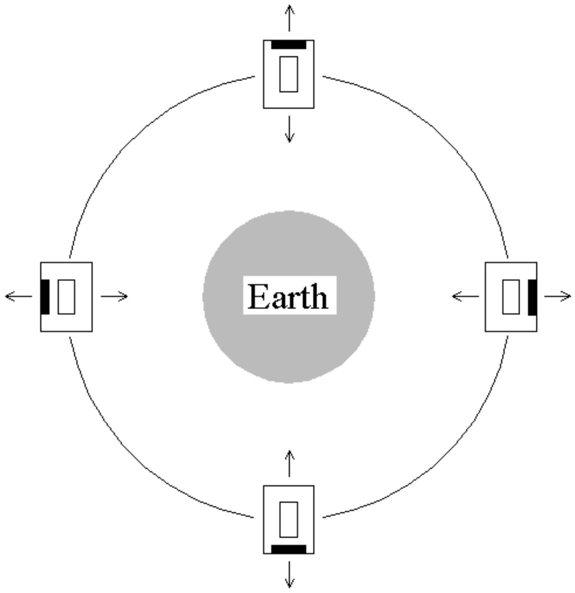

Fig. 6. – Skematic representation of the STEP He dewar while orbiting around the Earth carried by a spacecraft (not shown) in space–fixed attitude. The arrows represent the outward tidal acceleration of the Earth acting in the dewar along the satellite–to–Earth axis. The inward components along the axis perpendicular to it are not shown; the tangential component is zero at these locations and is not shown at other angles, where its effect is to form a tidal bulge either on the side of the dewar facing Earth or on the one away from it –only the latter is shown here. Unless the dewar is completely full (or completely empty, of course) any He mass anomaly will follow (with zero friction) the effect of Earth tides, and in doing so it will move around the test cylinders of the Equivalence Principle experiment (located close to the center of mass of the spacecraft) at the orbital frequency. If the spacecraft is rotated around its center of mass, in the plane of the figure, this effect appears at the rotation frequency with respect to the center of the Earth, which is the same as the modulation frequency of an Equivalence Principle violation signal (see Fig. 4). In all cases, if the He tide interacts differently with the test cylinders (i.e. because of their different mass momnets) its disturbance competes with the signal to be detected (figure not to scale).

Not being limited by the presence of He, μSCOPE is designed –unlike STEP– to use thrusters based on ion propulsion (e.g. FEEP – Field Emission Electric Propulsion). As compared to He thrusters, FEEP have two important advantages: i) a very high specific impulse, whereby only a minute amount of propellant (Cs in the case of FEEP) is sufficient to ensure drag compensation for the entire mission duration; ii) a fine electric tuning of the thrust, ensuring a high level of proportionality hence a more accurate compensation of drag and lower noise as compared to He mechanical thrusters. In addition, while the STEP mission duration is limited by the large amount of He required for the cryostat (in the order of hundred liters), the small amount of Cs propellant needed by FEEP (in the order of ten grams) makes a longer mission duration possible, thus allowing data taking for longer integration times.

Objects in space get charged because of cosmic rays and solar wind. Charging is especially intense if the spacecraft passes over the so called South Atlantic Anomaly (once per orbit), as it is the case with the STEP and μSCOPE sun-synchronous orbits. Electric interactions effects are huge compared to the small gravitational signal to be detected in these experiments. For STEP a radiation sensor has been proposed in order to discard contaminated data [29], or else a 130 *kg* tungsten shield to reduce the flux of dangerous particles [30]. Another solution is to discharge each test cylinder, which in STEP and μSCOPE are levitated (by electrostatic forces in μSCOPE, by superconducting magnetic bearings in STEP). The solution chosen for μSCOPE is passive discharging (by means of a thin conducting gold wire), while the current solution for STEP is active electric discharging. The effect of the stiffness and damping of the wire in one case [42], and that of acting directly on the test masses in order to first measure the charge it has acquired and then to neutralize it, are serious matters of concerns in both experiments whose original design, being based on levitated test bodies, is naturally prone to electric charging.

To summarize, with the STEP experiment concept it is mandatory to operate at very low temperatures in order to achieve a high accuracy test of the Equivalence Principle. In turn, this choice has profound consequences in terms of the mission complexity, hence inevitably on its risks and cost. The μSCOPE variant of STEP at room temperature is severely limited by the radiometer effect and by a read-out system which does not exploit the differential nature of an Equivalence Principle violation signal. It is also worth recalling that a free fall experiment has been proposed for testing the Equivalence Principle inside a vacuum capsule to be released from a balloon at an altitude of 40 *km*, allowing a free fall time of 30 *s* [43]. The gravity detector to be used is a differential version (with zero baseline) of ISA–Italian Spring Accelerometer–built at IFSI–CNR in Rome and tested in the Gran Sasso Laboratory by measuring Luni-Solar tides. ISA is based on torsional spring suspensions and capacitive pick ups. At release inside the capsule ISA would be given a rotation rate of about 1 *Hz,* so as to modulate the signal of a possible violation of Equivalence at this frequency. The experiment can be performed also at low temperature. The expected sensitivity is of 1 part in 10^{14} at room temperature and one order of magnitude better at low temperature. This is competitive with the target of μSCOPE, with the additional advantages that the experiment can be easily repeated (to modify and improve the apparatus) and that suborbital flights are obviously much less expensive than free flyers, even if they need low altitudes.

7.2 The GG space experiment

In the case of GG the concentric test cylinders spin around the symmetry axis at a rather high frequency (2 *Hz* with respect to the center of the Earth) and are sensitive to differential effects in the plane perpendicular to the spin/symmetry axis (see Fig. 7, to be compared with Fig. 4). A cylindrical spacecraft encloses, in a nested configuration, a cylindrical cage and the test bodies inside it; the whole system has a dominant moment of inertia with respect to the symmetry axis and is (passively) stabilized by 1-*axis* rotation around it. The suspensions are all mechanical, and very weak, thanks to weightlessness. The test cylinders are weakly coupled, forming a mechanical system similar to an ordinary beam balance with the beam along the symmetry axis of the cylinder, and therefore sensitive to differential effects in the plane perpendicular to the beam. The weaker the coupling, the longer the natural period of differential oscillations of the test bodies, the more sensitive the system is to differential forces such as the one caused by an Equivalence Principle violation. The mechanical suspensions provide passive electric discharging of the test cylinders.

As shown in Fig. 7, an Equivalence Principle violation in the field of the Earth would generate a signal of constant amplitude (for zero orbital eccentricity) whose direction is always pointing to the center of the Earth, hence changing orientation with the orbital period of the satellite.

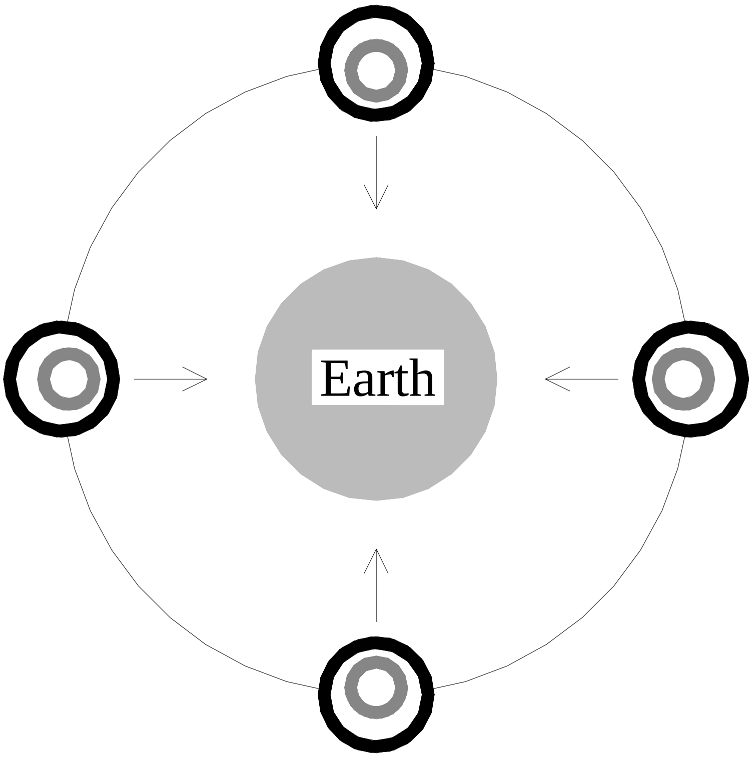

Fig. 7. – Section across the spin/symmetry axis of the GG outer and inner test cylinders (of different composition) as they orbit around the Earth inside a co-rotating, passively stabilized spacecraft (not shown). The centers of mass of the test cylinders are shown to be displaced towards the center of the Earth as in the case of a violation of Equivalence in the field of the Earth (indicated by the arrows). The read-out system (a capacitance bridge with plates located in between the test cylinders; not shown in figure) rotates at the same frequency as the test cylinders and the spacecraft, and therefore modulates the violation signal at this frequency (2 *Hz* with respect to the center of the Earth) (figure not to scale).

The read-out is made of 2 capacitance bridges (1 is for redundancy). In each bridge the two plates are placed half way in between the test cylinders, 180° from one another, and rotate with the system. Any differential force in the plane perpendicular to the spin/symmetry axis, as shown in Fig. 8, causes a mechanical displacement which unbalances the bridge and is transformed into an electric potential signal. A differential force in a fixed direction is modulated at the spin frequency of the bridge. The target of the GG experiment (10^{-17} in the Eötvös parameter η) requires to detect, at the orbiting altitude of the satellite ($\cong 500$ *km*), a differential acceleration of $\cong 8.4 \cdot 10^{-15}$ $cm\ s^{-2}$ yielding, in the GG system, a differential displacement of almost 10^{-10} *cm* amplitude. This mechanical displacement is transformed by the capacitance bridge (with the specific capacitances and gap of the GG system) into an electric potential signal of about 1 nV.

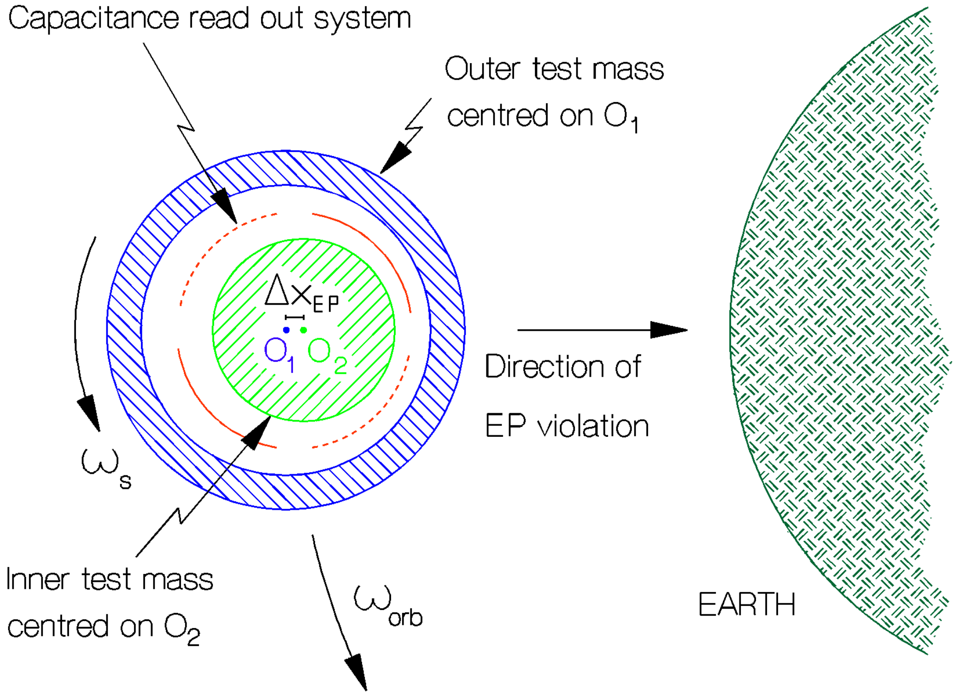

Fig. 8. – Section of the GG coaxial test cylinders and capacitance sensors in the plane perpendicular to the spin axis (not to scale). The capacitance plates of the read-out are shown in between the test bodies, in the case in which the centers of mass of the test bodies are displaced from one another by a vector $\Delta\vec{x}_{EP}$ due to an Equivalence Principle violation in the gravitational field of the Earth (e.g., the inner test body is attracted by the Earth more than the outer one because of its different composition). Under the (differential) effect of this new force the test masses, which are weakly coupled by mechanical suspensions, reach equilibrium at a displaced position where the new force is balanced by the weak restoring force of the suspension, while the bodies rotate independently around O_1 and O_2 respectively. The vector of this relative displacement has constant amplitude (for zero orbital eccentricity) and points to the center of the Earth (the source mass of the gravitational field); it is therefore modulated by the capacitors at their spinning frequency with respect to the center of the Earth.

As in an ordinary balance, the two arms can be adjusted (with PZT actuators) in order to reject forces acting in *common mode*, such as the inertial forces generated by air drag or by other non gravitational forces acting on the outer surface of the spacecraft. Mechanical beam balances can reject common mode forces very effectively, hence are very sensitive to differential effects. In this respect the GG design of the test bodies differs from both the STEP and μSCOPE designs in which each test cylinder is suspended independently. In addition, the GG capacitance bridge is a differential sensor, similarly to the STEP differential SQUID. However, no feed back is required because of the restoring force provided by the mechanical suspensions. For the same reason, common mode motions need not be controlled either. A scheme of the mechanical system formed by the GG test cylinders is given in Fig. 9.

The effects of air drag –and of any other non gravitational forces– are partially rejected by the coupled system of the test bodies and partially compensated by means of FEEP thrusters. They require only a few tens of grams of Cs propellant, allowing long mission duration and data taking. Vibration noise due to the thrusters firing at the spin frequency is attenuated by the mechanical suspensions of the cage enclosing the test cylinders.

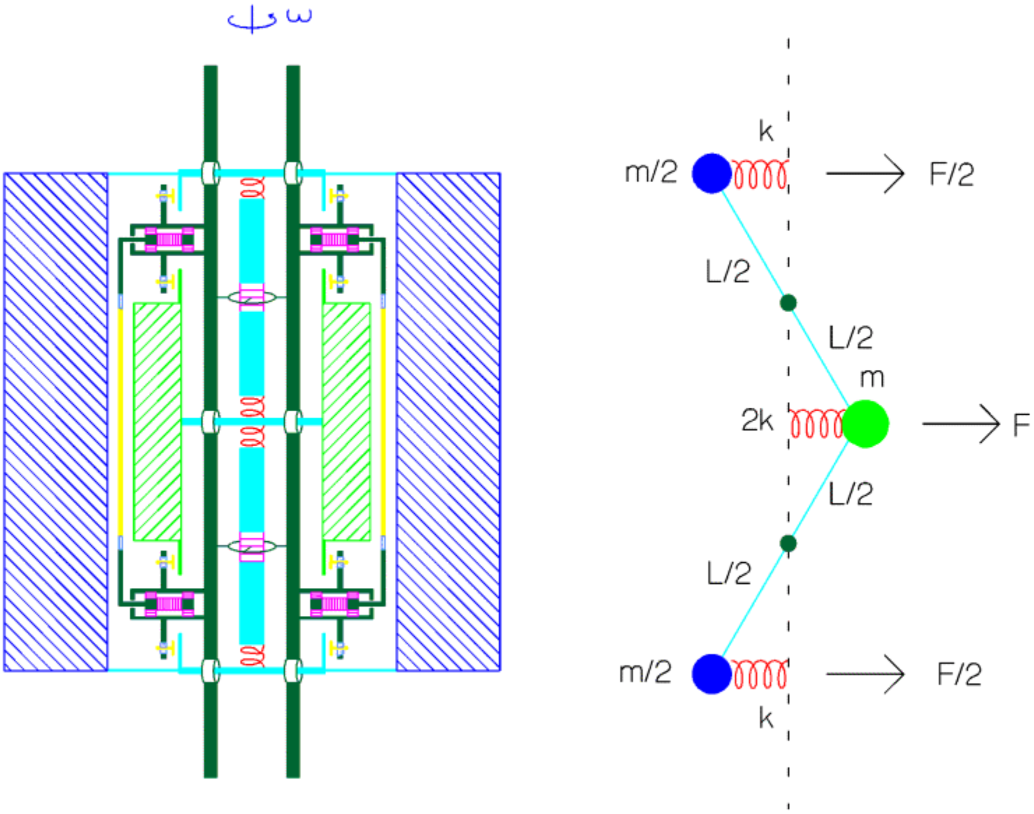

Fig. 9. – (Figure to scale) Section through the spin axis of the GG test cylinders (10 *kg* each) and the capacitance plates of the read-out in between. The lower density cylinder (21 *cm* in height) encloses the higher density one. Inside the inner cylinder is a narrow tube rigidly connected to a laboratory not shown here (also of cylindrical shape, and in its turn suspended inside the spacecraft) enclosing the test bodies and the read-out capacitance plates shown here. Two arms inside the tube, but not in contact with it, are used to couple the test bodies; the only connection between the coupling arms and the laboratory is via two flat gimbals at the midpoints of each arm. The inner test cylinder is suspended from the coupling arms at its center by means of two helical springs; the outer one is also suspended from the arms with helical springs, one at the top and one at the bottom of its symmetry axis. Being pivoted on torsion wires the gimbals allow conical movements of the coupling arms around their midpoints, e.g. in response to a differential force between the test bodies. The PZT actuators shown next to the gimbals are for adjusting the length of the two halves of each coupling arm. The capacitance plates of the read–out are shown in between the test cylinders; they are connected to the laboratory tube and have inch–worms for adjusting their distance from the surfaces of the test cylinders.

The GG bodies all spin at a frequency much higher than their natural frequencies of oscillation (which are very low because of the very weak suspensions that can be used in the absence of weight). This state of rotation is very close to that of ideal, unconstrained, rotors and allows the test cylinders to self-center very precisely (the center of mass of an ideal free rotor would be perfectly centered on the spin axis). Self–centering is possible if the suspended bodies are free to move in a plane; it is not possible if they are constrained to a 1-dimensional motion, as in the rotating experiment proposed by [24] (see [25]). However, suspensions are not perfect, which means that, as they undergo deformations at the frequency of spin, they also dissipate energy. The higher the mechanical quality of the suspensions, the smaller the energy losses. Energy dissipation causes the spin rate to decrease, hence also the spin angular momentum will decrease; and since the total angular momentum must be conserved, the suspended bodies will develop slow whirl motions one around the other. Although very slow, whirl motions must be damped. In GG they are damped actively with small capacitance sensors/actuators and appropriate control laws which have been developed, implemented and tested in a numerical simulation of the full GG satellite dynamical system using the software package DCAP of Alenia Spazio (also checked by simulating the GG dynamical system in Matlab). Simulations include drag free control as well. They demonstrate that the system can be fully controlled and that the control does not affect the expected sensitivity of the GG experiment (10^{-17} in the Eötvös parameter η). In fact, with the measured value of the quality factor of the suspensions ($\cong$ 20000), whirl motions of the test cylinders are so slow that they can be damped at time intervals long enough to allow data taking in between, when active damping is switched off and will therefore produce no disturbance at all on the Equivalence Principle test. Laboratory tests of the GG prototype (sect. 8) performed with the damping devices switched off, have confirmed that the growth rates of whirl motions are very slow.

As a novel subject, damping of whirl motions in the GG experiment has been the subject of extensive analysis. Doubts have been expressed and a paper has been published [44] arguing that the GG proposed test of the Equivalence Principle would be limited, because of whirl motions instabilities, to a sensitivity of 1 part in 10^{14}, which is 3 orders of magnitude worse than the sensitivity expected by the GG Team. The issue has now been settled [35, 36 Ch. 6].

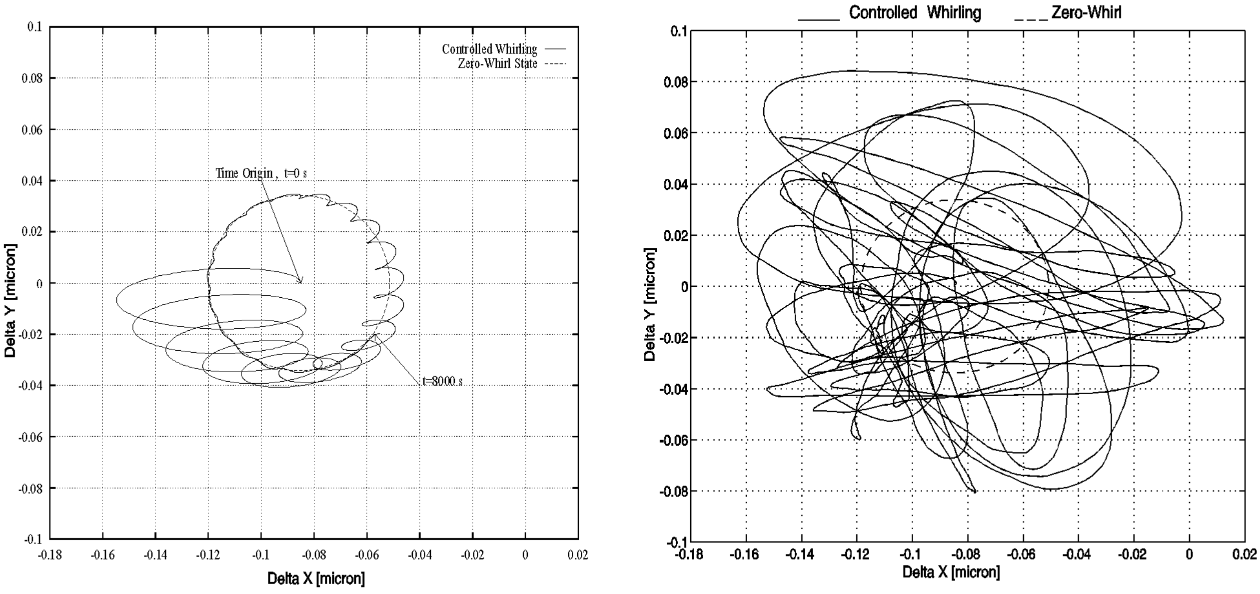

Fig. 10. – (Taken from [35]) Trajectory of the relative motion of the centers of mass of the GG outer spacecraft and the laboratory which encloses the test cylinders in the plane perpendicular to the spin axis in a 2-body model (coupling constant 0.02 *N/m*, Q=90). The Y axis is pointed to the center of the Earth, hence the largest effect of the residual atmospheric drag, assumed to be of $5\cdot10^{-9}$ *N*, is a constant displacement along the X axis (of $\cong$ 0,08 μm); its second harmonic (assumed to be 40% of it) appears in this system as a variation at the orbital period (5700 *s*). This is the dashed circle, showing –in both plots– the stationary state that the system would reach if the whirl motion were perfectly damped. The plot on the left is obtained with the control laws of the GG team assuming the following errors: initial bias of 10 μm linear and 1° angular; fractional error in spin rate measurements $\Delta\omega_s/\omega_s=10^{-4}$; offset (by construction and mounting) of 10 μm; errors in the capacitors of 0.1 μm r.m.s. Whirl oscillations with the natural period of 314 *s* (around the points of the dashed circle) and of decreasing amplitude are apparent as the system is brought to its stationary state in 8,000 *s* only. Note that at this point the relative distance of the two centers of mass is below 5 *Å*. The plot on the right shows, for the same system, but under much more ideal assumptions (perfect knowledge of spin

rate; perfect centering of the rotor; an initial linear bias of $1\mu m$ and no angular bias; an error in the sensors/actuators 10 times smaller, i.e. of $10^{-2}\ \mu m$) the results obtained by applying the control laws proposed by [44]. It is apparent that even in a much more favorable situation the same system has been unwittingly transformed into one dominated by very large active forces for which there is in fact no need, as the plot on the left demonstrates. Note that the dissipation has been assumed to be the same in both cases (Q=90), hence failure to stabilize the whirl motion (right-hand plot) has to be ascribed only to the control laws implemented in that case. Regarding the plot on the left, note that the assumptions for the various error sources are conservative. For instance, small capacitors like those designed for GG can be shown in the laboratory to be sensitive to relative displacements of $10^{-2}\ \mu m$.

The plots of Fig. 10 are worth showing; although they refer to a simplified 2–*body* model (in this case the GG outer spacecraft and the cylindrical laboratory suspended inside it), they show a very clear comparison between the two approaches. Results from simulations of the complete system are reported in [36 Ch. 6]. The basic physical principles which govern the behavior of weakly coupled fast spinning rotors in space may also be of interest to the reader due to the novelty of the subject [45].

The GG experiment allows signal modulation at high frequency (much higher than in other experiments of its kind); moreover –similarly to the torsion balance and unlike a general purpose accelerometer– it has been designed as a *differential* instrument specifically to detect a violation from the Universality of Free Fall. It is operated at room temperature. Does the radiometer effect limit the GG sensitivity as severely as in the case of μSCOPE? In GG the radiometer effect must be computed in the plane perpendicular to the symmetry axis of the test cylinders (Fig. 7). Any temperature gradient in this plane in the direction of the Earth will result in a radiometer effect indistinguishable from the signal. However, azimuthal temperature asymmetries on the surface of the spacecraft itself are reduced by its 1-*axis* rotation according to the formula:

$$\Delta T_{sc} = \frac{\alpha \cdot \Phi \cdot h \cdot r \cdot P}{c \cdot m / 2} \tag{24}$$

yielding a temperature difference ΔT_{sc} of a few mK for a spinning spacecraft exposed to the infrared radiation flux of the Earth ($\Phi \cong 2.4 \cdot 10^5\ erg\ cm^{-2}\ s^{-1}$) and resembling the outermost shell of GG (radius r=0.5 m, height h=1.3 m, mass m=33 kg, specific heat $c=0.9 \cdot 10^7\ erg\ g^{-1}\ K^{-1}$ ($0.2\ cal\ g^{-1}\ K^{-1}$), absorption coefficient α =0.73, spin period P=0.5 s). A thin (0.2 cm) insulating layer (Mylar or Kapton) inside the spacecraft shell, whose timescale of thermal inertia is τ = 40 s, will further reduce the temperature difference to ΔT_{mylar} of the order of 10 μK:

$$\Delta T_{mylar} \cong \frac{\Delta T_{sc} P}{2\tau} \tag{25}$$

Finally, vacuum between this layer and the cage enclosing the test cylinders (made in Cu and coated with Mylar) does ensure radiative transfer of heat and consequent very effective reduction of temperature differences in the direction of the Earth, to ΔT_{cage} of the order of $10^{-10}\ K$ (comparable to quantum fluctuations over half the spin period) according to:

$$\Delta T_{cage} = \frac{\sigma \cdot S_{cage} \cdot T^3 \varepsilon_{mylar} \cdot P}{c_{cage} M_{cage}} \Delta T_{mylar} \tag{26}$$

where σ is the Stefan-Boltzmann constant, S_{cage}=1 m^2, M_{cage}=40 kg, c_{cage}=3.9·10^6 $erg\ g^{-1}\ K^{-1}$ are the mass, surface and specific heat of the cage, ε_{mylar}=0.05 is the emissivity of Mylar.

It is apparent that temperature differences as small as these make the radiometer effect totally negligible in the GG mission. The result holds at room temperature and shows no need for a cryogenic experiment. The radiometer effect is not a limiting factor in Equivalence Principle testing for this mission. Thermal noise is higher at higher temperature, but since it is proportional to $\sqrt{(T/M)}$ (with M the mass of a test body), more massive test cylinders (10 *kg* in GG, about 100 *g* in STEP) compensate for the higher temperature.

Fast spin of the whole GG system has other advantages. Any local disturbing source rotates with the sensors, and therefore gives a DC effect. This helps considerably in dealing with perturbations such as those caused by local mass anomalies, parasitic capacitances, or the so called "patch effects". They can be separated from the expected signal (modulated at the spin frequency) without posing severe requirements on the experiment design. The differential acceleration caused by the Earth tidal perturbation, due to the centers of mass of the test cylinders not being exactly coincident in the plane perpendicular to the spin/symmetry axis, appears at twice the frequency of spin.

In the present baseline the GG satellite is designed to fly on an equatorial orbit with the spin axis almost perpendicular to the orbit plane. This configuration maximizes the signal for the entire duration of the mission without any active control of the satellite. However, the GG satellite will be in the shadow of the Earth for about 2000 *s* each orbit, each orbit going from sunlight to darkness and viceversa. Fast spin helps reduce thermal effects and it is found that requirements on thermal stability can be met by means of passive isolation only. However, it appears that launchers capable to inject small satellites into low equatorial orbits are few and expensive, while the situation is more favorable for sun-synchronous, high inclination orbits such as those of STEP and µSCOPE. For this reason a new GG baseline, based on a sun-synchronous orbit, is being investigated within an advanced study funded by the Italian space agency (ASI). An additional accelerometer is also incorporated in the new design, whose test cylinders are made of the same material, for zero check. A unique property of the new accelerometer is that of being concentric to the original one, so that they are both close to the center of mass of the spacecraft and do not undergo different tidal effects from Earth, which would make the zero check more problematic.

Finally, a peculiarity of the GG design –based on mechanical suspensions and sensitive in the plane perpendicular to the symmetry axis of the test cylinders rather than along it, as in the case of STEP and µSCOPE– is that the system proposed for flight can be tested on the ground looking for a signal of violation in the horizontal plane of the laboratory while using the vertical direction to suspend the rotating cylinders against local gravity.

8 The GGG ("GG on the Ground") experiment: laboratory test of a proposed space apparatus

The GG test of the Equivalence Principle is designed for space for two reasons: a driving signal stronger than on Earth (by about 3 orders of magnitude) and the absence of weight (the largest perturbation is $\cong 10^8$ times smaller than 1-g). Yet, a 1-g version of GG can be conceived. This is possible because in the GG design the expected signal lies in the plane perpendicular to the spin/symmetry axis of the test cylinders, as shown in Fig. 7: if on the ground the apparatus is suspended against local gravity along this axis, an Equivalence Principle violation signal has a component in the horizontal plane (as shown in Sec. 2) to which the system is sensitive. If the mechanical suspensions are stiff along the vertical (enough to withstand weight) and soft in the horizontal plane (in order to provide a very weak mechanical coupling of the test cylinders), the rotor is safe and at the same time it is very sensitive to differential forces in the horizontal plane, such as the force due to a possible violation of Equivalence.

The GGG apparatus, located at the LABEN laboratories in Florence, is shown in Fig. 11. It is mounted on a metal frame fixed inside a vacuum chamber of 1*m* diameter. The test cylinders, of which only the outer one is visible in the picture, weigh 10 *kg* each (as in GG). The capacitance plates of the read out, which measure the relative displacements of the test cylinders in the horizontal plane, are located in between the test cylinders. The suspension tube, held by a shaft turning inside ball bearings is shown. Inside this tube is the most delicate mechanical part of the apparatus: the coupling arm, and 3 laminar cardanic suspensions (shown Fig. 12), one for carrying the total weight (at the center of the arm), one for suspending the outer test cylinder (at the top of the arm), and one for suspending the inner test cylinder (at the bottom of the arm). Such a mechanical system is very similar to an ordinary beam balance (with a vertical beam), so that masses and lengths can be adjusted to make it most sensitive to differential effects. A simple scheme of the system is shown in Fig. 13, while a section of it is given in Fig. 14.

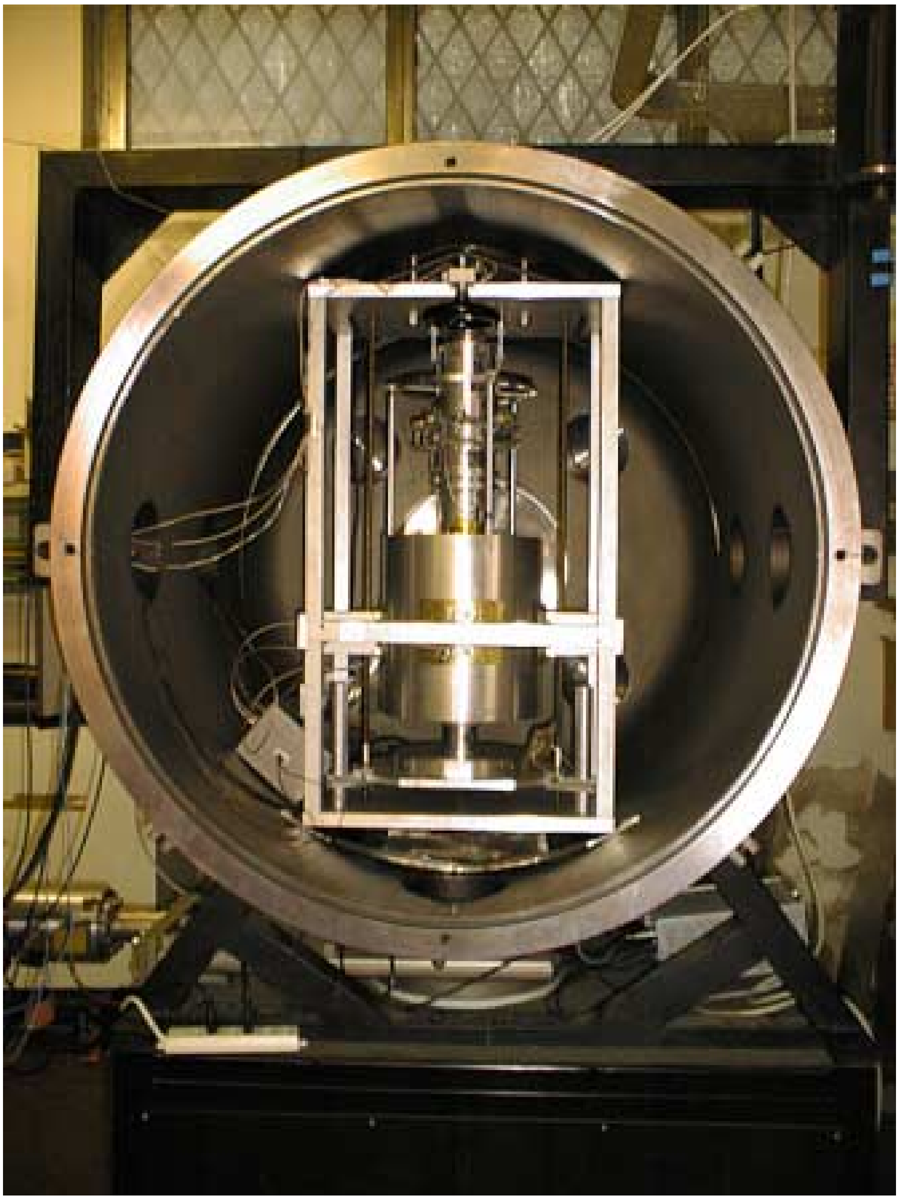

Fig. 11. – The GGG apparatus inside the vacuum chambe (LABEN laboratories, Florence)

The main part of the electronics of the experiment is arranged on an annular support around the suspension tube. It contains the 2 capacitance bridges, their preamplifiers, the analog to digital converters and the driver of the optical emitter. The digitized signal produced by these circuits (giving the relative displacements of the test cylinders as an electric signal) is optically transmitted to the non rotating system, and eventually outside the vacuum chamber to the computer. Sliding contacts are used for power transmission inside the rotor.

Fig. 12. – The 3 laminar suspensions of the GGG apparatus. They have been manufactured in CuBe by electroerosion in 3-D to ensure good mechanical quality.

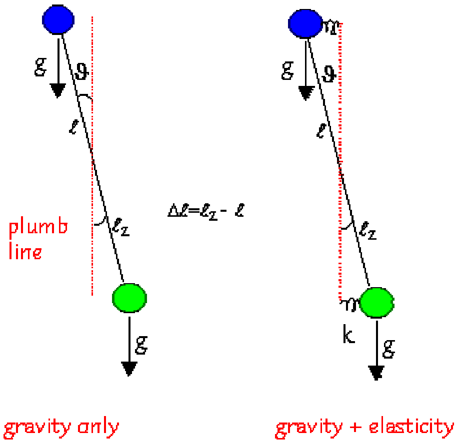

Fig. 13. – Simple dynamical scheme showing how the GGG test bodies, two hollow cylinders, are coupled. Although they are suspended so as to make them concentric (see Fig. 14), they are coupled in such a way as to behave as in an ordinary balance, except for the fact that the beam of the balance is vertical and that there is a weak elastic coupling in the horizontal plane. Dynamical systems of this kind can be treated similarly to very well known models and long periods of differential oscillation can be obtained by accurately balancing the masses and their arm lengths.

An Equivalence Principle violation with the Earth as the source mass would cause a constant displacement of the GGG test rotors in the North-South direction. However, due to the diurnal rotation of the Earth, a gyroscopic effect arises between the test rotors (proportional to the spin rate of the apparatus) which is also in the North-South direction and much larger: in the current design it amounts to about a few μm. A non zero tilt angle between the spin axis and the direction of local gravity would also give rise to a constant relative displacement (in the direction of the residual tilt).

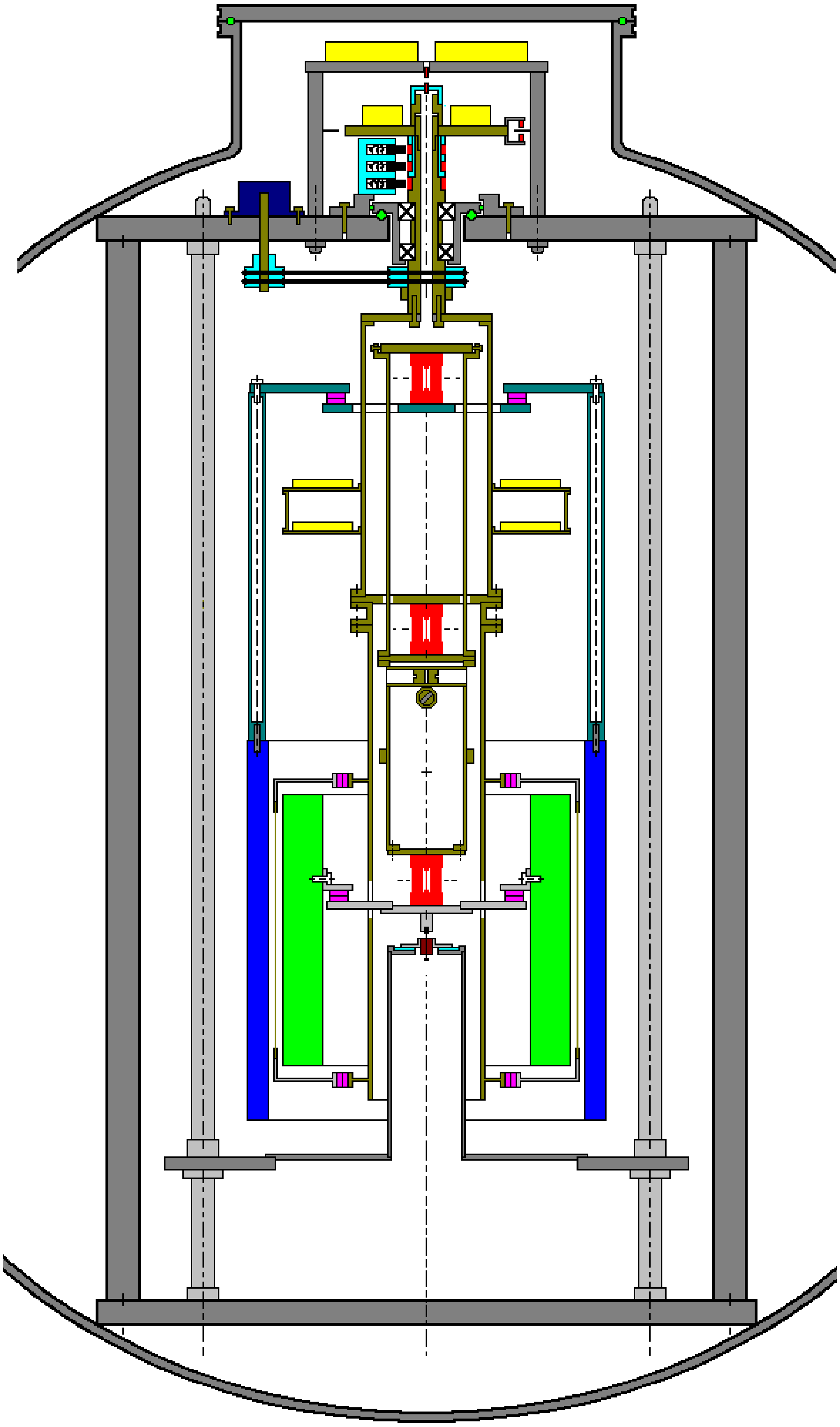

Fig. 14. – Section through the spin axis of the GGG experiment set up inside the vacuum chamber (see picture in Fig. 11). The drawing is to scale, the diameter of the vacuum chamber being 1 *m*.

The GGG apparatus must therefore look for a possible Equivalence Principle violation signal with the Sun as the source mass, as in the experiments by [12, 13]. This means a driving signal 1400 times weaker than it is in space for GG. Note that none of these difficulties (verticality of the rotation axis and gyroscopic effect) would affect the GG space experiment, in which there is no local gravity preferential direction and no gyroscopic effect (the rotation axis is almost perfectly fixed in space, and moreover –unlike in GGG– the test cylinders are suspended by their centers of mass).

The GGG apparatus described here has been tested for stable rotation (typically from *2* to 6 *Hz*) and for the measurement of small relative displacements between the centers of mass of the test cylinders. The sensitivity of the GGG capacitance read-out has been tested on bench, finding that it can detect 5 *picometer* displacements in 1*s* integration time. This is fully adequate for the GG space experiment, which must be sensitive to slightly less than 1 *picometer* displacements in order to fulfill its goal of an Equivalence Principle test to 10^{-17}; less than 100 *s* integration time is enough for the GGG read out to complete the task. The sensitivity of GGG to differential effects, as obtained from measurements carried out so far with the system in rapid rotation, is similar to that reported in the literature for the other accelerometers designed for space which have been subject to intensive testing [42,43]. It is expected that the sensitivity of GGG can be significantly improved because, unlike general-purpose accelerometers, it has been designed and optimized for the detection of an Equivalence Principle violation signal.

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