3 Brief history of past ground experiments

Aristotle’s view, that heavier bodies should fall faster than lighter ones, has been questioned –apparently for the first time– in the 6th century by Philoponus, who stated that

> *“if two bodies are released by the same altitude one can observe that the ratio of the times of fall of the bodies does not depend on the ratio of their weights, and the difference of the times is very small”.*

It was only much later that the issue was reconsidered, in 1553 by Benedetti, who stated that the velocity of fall does not depend on the weights of the falling bodies. Galileo started by showing the internal contradiction of Aristotle’s reasoning with a simple argument: [7]

> *“If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will partly be retarded by the slower, and the slower will be somewhat hastened by the swifter…. Hence the heavier body (made by the two tied together) moves with less speed than the lighter* (the former swifter one); *an effect which is contrary to your* (by Aristotle) *supposition”*

But the most important contribution of Galileo came from his being deeply aware of the need to provide *experimental evidence* that Aristotle's was wrong.

> *“The facts set forth by me up to this point and, in particular, the one which shows that difference of weight, even when very great, is without effect in changing the speed of falling bodies, so that as far as weight is concerned they all fall with equal speed: this idea is, I say, so new, and at first glance so remote from fact, that if we do not have the means of making it as clear as sunlight, it had better not be mentioned; but having once allowed it to pass my lips I must neglect no experiment or argument to establish it.”* [7]

As a typical application of *the scientific method,* Galileo started by a careful analysis of the phenomenon of falling bodies, trying to simplify it by retaining only its most important features. He knew that a simplified phenomenon is much easier to describe in mathematical terms than a complex one, and that only a precise mathematical description would allow him to make a prediction that could be tested by experiments, so as to subsequently improve –depending on the results of the experiments– the model upon which the prediction was based. A careful analysis led him to conclude that the differences –in some cases quite relevant– actually observed in the time of fall of different bodies (differing in weight as well as in composition), was in fact due to the different resistance they oppose to the medium (typically air) through which they move during fall. In order to *amplify* the resistance of the medium, so as to make sure that the hypothesis he made was right, Galileo performed a series of experiments in which he dropped bodies of different composition in media much denser than air, finding that indeed, the denser the medium, the bigger was the difference in time of fall [8, Vol. VIII p. 116]:

> “…nel *mezzo dell' argento vivo l'oro non solamente va in fondo più velocemente del piombo, ma esso solo vi descende, e gli altri metalli e pietre tutti si muovono in su e vi galleggiano, dove che tra palle d'oro, di piombo, di rame, di porfido, o di altre materie gravi, quasi del tutto insensibile sarà la disegualità del moto per aria …. ”*

The conclusion was that all bodies fall equally fast, any observed difference being due to the different resistance of the medium that different bodies are subject to:

"…veduto, dico questo, cascai in opinione che se si levasse totalmente la resistenza del mezzo, *tutte le materie* *descenderebbero con eguali velocità "*

"... having observed this I came to the conclusion that, if one could totally remove the resistance of the medium, *all substances* *would fall at equal speeds "*

This is indeed the first clear-cut statement of the Universality of Free Fall, and in this form it appeared for the first time in Galileo's *"Discorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti alla meccanica e ai movimenti locali"* [8, Vol. VIII], published outside Italy, in Leiden, in 1638. At the time Galileo was 74 years old, was blind and under house arrest in Arcetri (Florence) by order of the church of Rome, which also ordered that his works should not to be published. However, the *"Discorsi"* are in fact based on much earlier work, mostly on experiments with the inclined plane and the pendulum, going back almost 40 years, to the time when he was a young lecturer at the University of Pisa. Although this fact may seem very unusual nowadays, it was not uncommon at the time, and the evidence is in a letter addressed by Galileo to Guidobaldo dal Monte, a nobleman with a personal interest in Mechanics. In the letter [8, Vol. X p. 97], dated 1602, Galileo describes his pendulum experiments and notes –in passing– that the results would be the same for *different* suspended bodies.

Galileo was aware of the difficulty to provide evidence by dropping masses from a height (from a big height the accumulated effect of air resistance is too large to allow a reliable conclusion; from a small one any difference is too small to appreciate [8, Vol. VIII p. 128]). Most probably Galileo was not able to calculate precisely the effect of air resistance, but he knew that it is much smaller if the velocity of the body is small (in fact, it is $a_{drag} \propto \upsilon^2$). He therefore conducted experiments with bodies falling on inclined planes, where only a fraction of the gravitational acceleration is relevant, which reduces the velocity of fall –hence also the effect of air resistance– and eventually with pendula, where in addition to the slow velocities he could exploit the advantage of the periodic motion to obtain the first accurate test ever of the UFF:

"e finalmente ho preso due palle, una di piombo ed una di sughero, quella ben più di cento volte più grave di questa, e ciascheduna di loro ho attaccata a due sottili spaghetti eguali, lunghi quattro o cinque braccia, legati ad alto; allontanata poi l'una e l'altra palla dallo stato perpendicolare, gli ho dato l'andare nell'istesso momento, ed esse, scendendo per le circonferenze de' cerchi descritti da gli spaghi eguali, lor semidiametri, passate oltre al perpendicolo, son poi per le medesime strade ritornate indietro; e reiterando ben cento volte per lor medesime le andate e le tornate, hanno sensatamente mostrato come la grave va talmente sotto il passo della leggiera, che né in ben cento vibrazioni, né in mille, anticipa il tempo d'un minimo momento, ma camminano con passo egualissimo. Scorgesi anche l'operazione del mezzo, il quale, arrecando qualche impedimento al moto, assai più diminuisce le vibrazioni del sughero che quelle del piombo, ma non però che le renda più o meno frequenti; anzi quando gli archi passati dal sughero non fusser più che di cinque o sei gradi, e quei del piombo di cinquanta o sessanta, son eglin passati sotto i medesimi tempi." [8, Vol. VIII p. 128]

"Accordingly, I took two balls, one of lead and one of cork, the former more than a hundred times heavier than the latter, and suspended them by means of two equal fine

threads, each four or five cubits long. Pulling each ball aside from the perpendicular, I let them go at the same instant, and they, falling along the circumferences of circles having these equal strings for semi-diameters, passed beyond the perpendicular and returned along the same path. This free vibration repeated a hundred times showed clearly that the heavy body maintains so nearly the period of the light body that neither in a hundred nor even in a thousand will the former anticipate the latter by as much as a single moment, so perfectly do they keep step. We can also observe the effect of the medium which, by the resistance it offers to motion, diminishes the vibration of the cork more than that of the lead, but without altering the frequency of either; even when the arc traversed by the cork did not exceed five or six degrees while that of the lead was fifty or sixty, the swings were performed in equal times. " [7]

Newton made the same experiment, as he mentions in the opening paragraph of the *Principia,* and explicitly reported the accuracy achieved: 1 part in 10^3. In order to establish the accuracy that Galileo could have achieved with the pendulum experiment described here, the experiment has been repeated a few years ago [9] showing that it is difficult, by this method, to be less accurate than this. An accuracy of 10^{-3} is consistent with an error in length of 0.1, and it is in agreement with Galileo's claim that the bodies keep in step for hundred or even thousand swings, as these recent experiments have confirmed.

It may be worth noticing that the experiment does not require a clock. The argument, sometimes reported, that the first accurate pendulum experiments on the *Equivalence Principle* are due to Newton, and could not have been done by Galileo because a precise pendulum clock was only available after his death, is therefore not relevant (the pendulum clock was introduced by Huygens in 1657).

Yet, Galileo is famous worldwide for his tower experiments. Mass dropping experiments are often recalled in the *Discorsi* when arguing with Aristotle's point of view, because they allow him to describe the Universality of Free Fall in a very straightforward manner, with no need to discuss the relation between the motion of a falling body and the oscillations of a pendulum. At the University of Pisa in 1993 [10] we have compared modern calculations of the time of fall –taking into account all non gravitational effects– with the results of mass dropping experiments performed from the leaning tower of Pisa using a rather accurate, although quite simple, mass release device. The idea was to position the test masses at the far end of a horizontal platform, hinged at the other end, whose mass and dimension were such that, once released by a cut of wire, it would open up with a vertical acceleration –at the location of the test masses– larger than the local acceleration of gravity, that the test masses experience at the time of release (by a factor 5/4 for this specific device). In this way the disturbances of the dropping mechanism itself on the test masses, which are very important because of the short falling time, were reduced. Unless the effect of air resistance was compensated by an appropriate choice of density and dimension of the test bodies, so as to make their area–to–mass ratios equal (or very close), we could observe that the bodies do actually reach the ground at very different times, as predicted by the theoretical model. The mass dropping experiments mentioned by Galileo in the *Discorsi* would in fact have given different results from what he mentions. It was only in 1641 that the young scholar Vincenzo Renieri reported to Galileo the results of his experiments from the leaning tower of Pisa [8, Vol. XVIII p. 305] , and we have checked that the differences observed by Renieri are consistent with the effects of air resistance. While Galileo formulated very clearly the Universality of Free Fall and proved it with pendula many decade before Newton at a comparable accuracy, he most probably never dropped masses from the leaning tower of Pisa.

Pendulum experiments to test the *Equivalence Principle* have been repeated and improved throughout the 19^{th} century (by Bessel in 1826), and in the 1920s by Potter, only to reach an accuracy of a few 10^{-5}. However, a big jump in sensitivity, by almost 4 orders of magnitude, took place at the end of the 19^{th} century when a torsion balance was used for the first time to test the Equivalence between inertial and gravitational mass. Around 1888 in Budapest Loránd Eötvös built a sensitive torsion balance for gravimetric measurements. The instrument, generally known as the *Eötvös balance,* was used for extensive field measurements and later on in prospecting for oil and natural gas.

It was indeed a very subtle idea that a deviation from the proportionality between inertial and gravitational mass –a fundamental concept in Newtonian Mechanics– should show up as a rotation of the balance. It is also not clear what originally motivated Eötvös to start a long series of experiments on this issue (note that it was before Einstein's work). It was probably when he realized the huge capabilities of his instrument as compared to the limited experimental evidence previously available in support of the Equivalence between inertial and gravitational mass, that he decided to go ahead (see Fig. 1).

Fig. 1. – The torsion balance used by Loránd Eötvös to test the proportionality between inertial and gravitational mass. The experiments were carried out starting in1888, and then in the period between 1905 and 1908. (Picture downloaded from the Eötvös virtual museum available on the World Wide Web).

The experiments rapidly demonstrated the superior capabilities of the torsion balance with respect to pendula in testing the UFF. In the work published in 1922 [11], 3 years after Eötvös death, the result reported was a test to about $5{\cdot}10^{-9}$. A result which remained unchallenged till the 1960s despite the advent of General Relativity at the beginning of the century and consequent relevance of the Equivalence Principle.

In spite of their novelty and success, Eötvös' experiments are limited by the fact that the differential acceleration (5) expected for a violation of Equivalence η in the field of the Earth is (at a given latitude) constant both in direction (North–South) and in modulus. It therefore results in a constant twist angle of the balance (twist angle is maximum if the balance is aligned in the East–West direction). There is no zero internal check, unless the test masses themselves are physically swapped on the balance; which inevitably perturbs the measurements. A way to overcome this limitation came in the 1960s from Princeton [12] when another subtle idea was put forward: if the UFF is tested for the same test bodies in the gravitational field of the Sun rather than the Earth, the effect on the balance of possible deviation from the UFF (although slightly weaker for the same value of η) would have a modulation with the period of the day, as we have shown in Sec. 2. This modulation naturally provides an internal zero check of the experiment with no need to modify the apparatus: a balance aligned in the East–West direction should give zero twist at sunrise and sunset and maximum twist (in modulus) at noon and midnight. Such a modulation proved very successful, yielding an improvement by almost 3 orders of magnitude in sensitivity, to about 10^{-11} [12]. Another torsion balance was designed and manufactured in Moscow a few years later, also referring to the Sun as the source mass of a possible violation of Equivalence. The experiment was carried out in the basement of the Physics Department (on a deeply rooted rock, for reduced seismic noise); special care was devoted to improving the mechanical quality of the suspension wire and to reducing the effects of the local mass anomalies. The sensitivity reported was about one order of magnitude better, to about 10^{-12} [13], again demonstrating the vital importance of a frequency modulation of the expected signal.

4 Equivalence Principle tests by Lunar Laser Ranging

Test bodies of laboratory size have a negligible fraction *f* of their mass coming from gravitational binding energy:

$$f \equiv -\frac{3}{5} \cdot \frac{GM^2}{R} \Big/ Mc^2 \tag{17}$$

while for the Earth and the Moon this fraction is:

$$f_{earth} \cong -4.64 \cdot 10^{-10} \qquad f_{moon} \cong -1.9 \cdot 10^{-11} \tag{18}$$

The Equivalence Principle which Einstein puts at the basis of General Relativity requires that all bodies fall with the same acceleration in an external gravitational field, with the gravitational binding energy contributing equally to the gravitational and the inertial masses. With the Moon orbiting the Earth and both moving in the gravitational field of the Sun, a violation of the Equivalence Principle would cause the orbit of the Moon around the Earth–Moon center of mass to be “polarized” in the direction of the Sun (as sketched in Fig. 2). This effect would have the synodic period of 29.53 *days* and is usually referred to as the Nordtvedt effect [14, 15]. It adds up to the classical variations of the lunar orbit which result from the Sun’s tidal acceleration of the Moon relative to the Earth, first estimated by Newton himself. Laplace later found that Newton’s distorted orbit was also slightly polarized towards the Sun, with the center of the Earth shifted in the direction of full Moon. It is the so called *parallactic inequality*, named in this way because it is proportional to the ratio of lunar to solar distance, and therefore its measurement could be viewed as a way of determining this ratio. Should the Earth and the Moon accelerate at different rates toward the Sun, this would result in a small *additional* polarization of the orbit. For it to be observed, the motion of the Moon should be very accurately monitored.