7.1 The STEP and μSCOPE space experiments

In all proposed space experiments a spacecraft orbiting the Earth at low altitude carries a system of concentric, coaxial test cylinders of different composition especially sensitive to differential accelerations acting between them. It also carries a read–out system of comparable sensitivity in order to detect, in the motion of the test cylinders, any deviation from the Universality of Free Fall in the gravitational field of the Earth.

In both STEP and μSCOPE the test cylinders are sensitive along their symmetry axis and the experimental apparatus moves around the Earth always maintaining a fixed orientation with respect to inertial space, as shown in Fig. 4. This is obtained by rigidly connecting to the spacecraft the supports on which the test cylinders are suspended and by actively controlling the attitude of the spacecraft by means of star sensors and thrusters. With this configuration, an Equivalence Principle violation (and deviation from the Universality of Free Fall) in the field of the Earth would generate a signal whose amplitude is modulated at the orbital frequency of the satellite ($\cong 1.7\cdot10^{-4}$ *Hz*, corresponding to an orbital period around the Earth of about 1 and a half hour; see Fig. 4).

During some selected time intervals the spacecraft can also be slowly rotated (with a rotation period of about 10^3 *s*) so as to rotate the sensitivity axis of the test cylinders with respect to the Earth, thus modulating an Equivalence Principle violation signal at the rotation frequency of the spacecraft. This was first proposed in [29], during a joint ESA–NASA study of STEP, to help distinguish the expected signal from some spurious effects characterized by the orbital period (note that in doing so, the modulation frequency is also further increased by almost a factor 6).

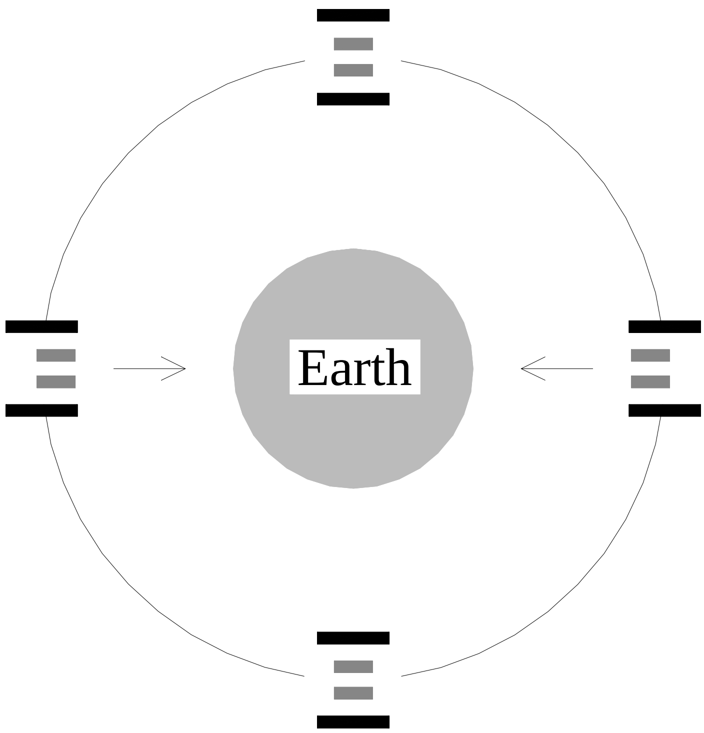

Fig. 4. – Schematic representation (not to scale) of the test cylinders –made of different materials– in the STEP and μSCOPE experiments as they orbit around the Earth. The figure is a section through the symmetry/sensitivity axis of the cylinders. The experimental apparatus is carried by a spacecraft (not shown in this figure) whose attitude is actively controlled in order to remain fixed with respect to the inertial space.

The test cylinders are free to move with respect to one another only along the symmetry axis. A deviation from the Universality of free fall in the gravitational field of the Earth would appear as indicated by the arrows, namely at the orbital frequency of the spacecraft and with a known phase angle (always pointing to center of the Earth). During the time intervals in which the spacecraft undergoes slow, controlled rotation around its center of mass in the plane of the figure (at about 10^{-3} *Hz* with respect to the Earth), this signal would also be modulated at the rotation rate.

The goals for STEP and µSCOPE are 10^{-18} and 10^{-15} respectively in the measurement of the Eötvös parameter η. In terms of the differential acceleration to be detected this amounts to $8.4\cdot10^{-16}$ $cm\cdot s^{-2}$ and $8.2\cdot10^{-13}$ $cm\cdot s^{-2}$ respectively (STEP is designed to fly at an altitude of about 500 *km*, µSCOPE at 600–700 *km*). Just to have an idea of how small these accelerations are, the target for STEP corresponds to about the mutual attraction between two masses of 1 *g* each placed at a distance of 100 *m* from one another! Since the acceleration to be detected is differential, the test bodies should be insensitive to accelerations in common mode (i.e. to accelerations which are the same on both test bodies) while they should be as much as possible free to respond to differential effects.

In STEP the test bodies are suspended by superconducting magnetic bearings so as to move freely only along their symmetry axes. A feed back system, based on the measurement of the relative displacements as performed by one differential SQUID sensor (Superconducting Quantum Interference Device) counteracts any relative displacement recorded, so as to keep the test cylinders centered on one another as precisely as possible. The feed back signal itself is the output signal of the experiment. Motions of the test cylinders in common mode are measured (by another SQUID sensor), and rejected. This type of instrument is usually referred to as a differential accelerometer.

The experiment concept of µSCOPE is derived from STEP, but there are two important differences. While in STEP the experimental apparatus is maintained at 1.8 *K* inside a dewar filled with superfluid He (enclosed by another He dewar at 4.2 *K*), the µSCOPE mission is operated at room temperature, and therefore requires suspensions and read–out suitable for operation at these temperatures. The test bodies are suspended (independently) by electrostatic levitation (see Fig. 5); for each test cylinder there are radial and spin control electrodes to prevent radial motion and rotations, so that the symmetry axis is the only direction along which each cylinder is allowed to move. Each test cylinder has axis capacitors, forming a capacitance bridge: any movement of the test cylinder along its axis will unbalance the bridge and result in a detectable voltage signal which is used as the driving signal of a feedback system (also based on electrostatic forces). The difference with respect to STEP is that in µSCOPE the test bodies are neither coupled by the suspension nor by the read-out: both the common mode and differential mode displacements have to be derived from *independent* measurements of two *independent* test bodies, each one with its own *independent* read-out system. In STEP the test bodies are coupled by the read-out (there is one SQUID sensor for differential motions and one SQUID sensor for motions in common mode). As in classical torsion balance experiments (where the test bodies are physically coupled), this ensures that if there is no violation effect the response of the system must be zero (no effect, no signal).

Because of (19), the satellite altitude should be as low as possible. However, at very low altitude the satellite is subject to a strong atmospheric drag, caused by the resistance from residual air along the orbit. Air drag makes any satellite loose energy and spiral in, hence limiting the duration of the mission.

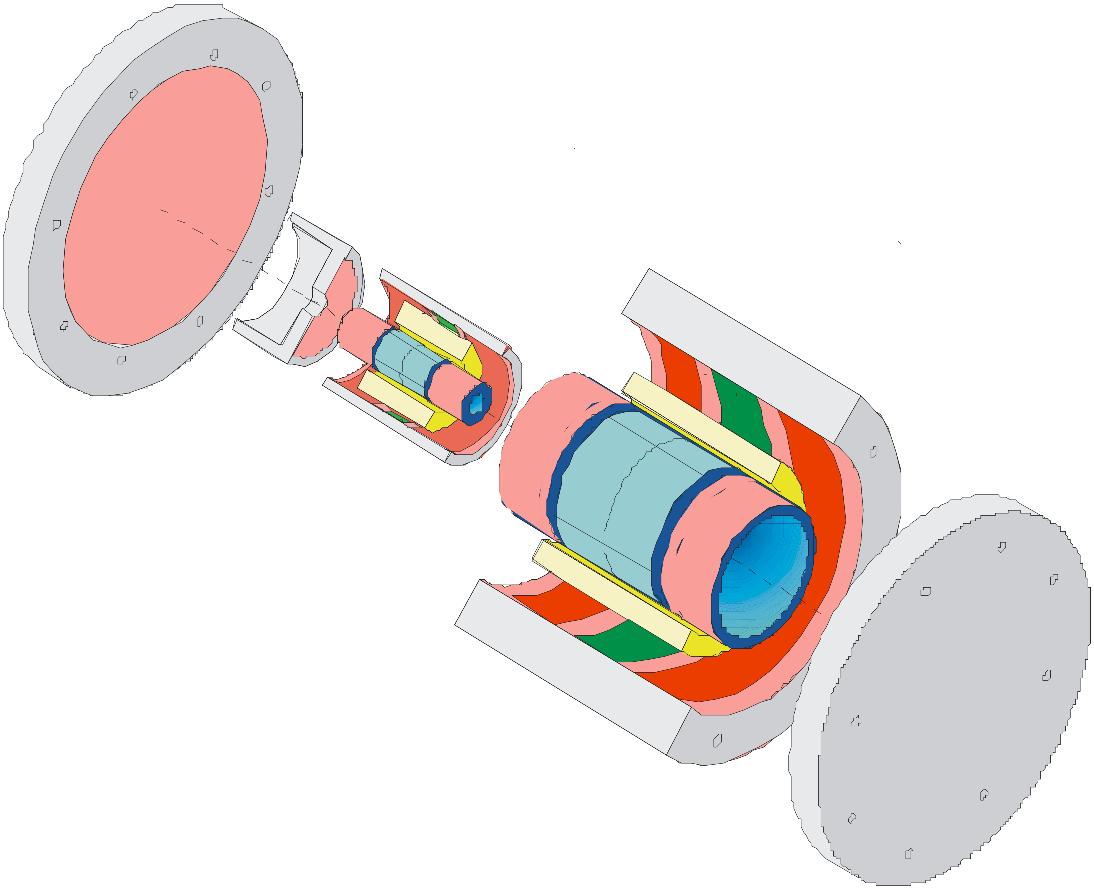

Fig. 5. – Cutaway view of the system of test cylinders designed for the μSCOPE experiment. (The figure is to scale; the length of the outer test cylinder is about 8 *cm*, the whole system assembled together is slightly more than 10 *cm* long). Each test body is a hollow cylinder surrounding a fixed cylindrical support (there are two separate supports, one for each of the two test cylinders) on which it is levitated by electrostatic forces, free to move only in the axial direction (radial and spin electrodes prevent any other movements). The axial movements of each test cylinder are sensed and controlled (independently) by axis electrodes. All cylinders (two test bodies –each one free to move along the axial direction– and two fixed supports) are all nested one inside the other. (Figure taken from the Website of μSCOPE [32])

In experiments aiming to detect small gravitational effects, air drag is also responsible for the largest disturbance on the experimental apparatus. Very weakly suspended test cylinders, such as those shown in Fig. 5, are screened from air drag by the spacecraft, but are subject to a corresponding –equal and opposite– inertial force, much bigger than the signal to be detected, given by:

$$a_{drag} \cong \frac{A}{M} \rho_{atm} \upsilon_{sc}^2 \tag{20}$$

where A/M is the so called area–to–mass ratio of the satellite, ρ_{atm} is the atmospheric density, and υ_{sc} is the velocity at which the atmospheric particles hit the spacecraft (there is also a numerical factor depending on the shape of the satellite, its aerodynamic coefficient, which is typically of order unity). For small and rather compact satellites orbiting at low altitudes (400 to 700 *km*) the resulting drag acceleration is much smaller than the local acceleration of gravity (by many orders of magnitude); however, it is still many orders of magnitude larger than the expected signal. The crucial difference with respect to the signal is that inertial accelerations resulting on the test cylinders from accelerations acting on the spacecraft outer surface and not on the test bodies themselves, are ideally the same on both of them (*common mode* accelerations), hence there should be no residual differential effect competing with the signal. However, this is true only if the suspension systems of the two bodies are perfectly identical; for instance, should the symmetry/sensitivity axes of the two cylinders not be perfectly aligned, the same inertial acceleration would result in two different axial components, and the system would detect a differential acceleration. Since the largest component of air drag acts *along–track*, i.e. tangent to the satellite orbit, the effect is also at the orbital frequency like the signal, although roughly 90° out of phase. In Fig. 4, the two arrows representing a differential signal due to air drag would be orthogonal to those depicting an Equivalence Principle violation. The difficulty, however, is that the drag effect itself is many orders of magnitude bigger than the signal and even a large phase difference is not sufficient to separate the two.

The effect of drag can be cured in two ways. One way is that the spacecraft compensates for the drag, in such a way that only a residual fraction of it will affect the test bodies. This requires the spacecraft to be equipped with sensors of the drag acceleration (a test body inside the spacecraft, as much as possible uncoupled from it, with a read out system to monitor its relative displacements with respect to the spacecraft) and thrusters on the spacecraft to actively force it in order to follow the motion of the test body (*drag–free* satellite). In STEP, the measurements of the common mode SQUID can be used to drive the thrusters; similarly, in μSCOPE it is possible to use the common mode signal obtained from the system of test bodies shown in Fig. 5. The signal being at the orbital frequency, drag compensation is required around this frequency.

Since drag is ideally a common mode effect, while the expected signal is differential, another way of reducing it is that the experimental apparatus be as good as possible in rejecting common mode effects. In STEP and μSCOPE common mode rejection is limited by the ability to make the symmetry/sensitivity axes of the test cylinders precisely aligned. In STEP it is expected that these axes can be aligned, and common mode effects can be rejected, to about 10^{-4}, but drag is so much bigger than the expected signal that the target sensitivity would be beyond reach unless drag were also partially compensated. The final choice, also in μSCOPE, is that drag is partially compensated and partially rejected .

Air drag is, at the altitudes of interest here, the largest non gravitational perturbation as well as the most relevant, because its main component is at the orbital frequency. The effect of solar radiation pressure is slightly smaller, but essentially DC. The perturbation caused by the fraction of solar radiation re-emitted by the Earth (depending on the albedo coefficient, hence also on cloud coverage) is modulated once per orbit in the presence of eclipses; the resulting inertial acceleration at the orbital frequency is partially compensated and partially rejected, together with air drag and any other inertial effect. Both STEP and μSCOPE are designed to fly along a sun–synchronous orbit, hence at high inclination angle over the Earth's equator; if the launch window is chosen properly, and for a short mission duration (about 6 months for STEP, limited by the amount of propellant to be carried on board for drag compensation), the spacecraft will be free from eclipses.

Perturbing accelerations with exactly the same frequency and phase as the signal cannot be separated from it and must therefore be reduced below the signal. If the STEP (or μSCOPE) spacecraft rotates around its center of mass, in the plane of Fig. 4, so that the sensitivity axis changes its orientation with respect to the Earth at a known frequency, these perturbing accelerations are modulated just like the signal, and cannot be distinguished from it (unless rotation can average out the perturbation; see Sec. 7.2 in the case of the radiometer effect in GG). However, spacecraft rotation will help separating those effects which do not have exactly the same signature as the signal but still act once per orbit (e.g. electric charging effects related to the satellite passing over the South Atlantic Anomaly).

The infrared radiation emitted by the Earth also hits the spacecraft, generating temperature differences in the residual gas surrounding the test cylinders that vary at the orbital frequency (because the attitude of the spacecraft is fixed with respect to inertial space while the source of the radiation is the Earth itself). This gives rise to the so-called *radiometer effect,* well known in space experiments to test the Equivalence Principle.

The radiometer acceleration along the symmetry axis s of a cylinder of density ρ is given by:

$$a_s = \frac{p}{2\rho}\frac{1}{T}\frac{dT}{ds} \tag{21}$$

(see, e.g. [38]) where p is the pressure of the residual gas and T its temperature. In space experiments on the Equivalence Principle the test cylinders have different composition (and typically also different dimensions); they are therefore subject to a different radiometer acceleration. If in addition the radiation source that the satellite is exposed to is the Earth, the resulting radiometer acceleration on the test cylinders will have the same frequency and phase of a possible violation of Equivalence having the Earth as the source mass, thus making it indistinguishable from the expected signal. The additional rotation of the STEP and μSCOPE spacecraft, whose purpose is to modulate an EP violation signal at a frequency different from (and larger than) the orbital frequency around the Earth, will modulate the radiometer differential acceleration as well.

In the STEP experiment the residual pressure is extremely low: $p \cong 10^{-13}$ *Torr* ($1.33\cdot 10^{-10}$ dyn/cm^2) because the apparatus is maintained at 1.8 K inside a dewar filled with superfluid He (enclosed by another He dewar at 4.2 K). An Equivalence Principle violation to the level of 1 part in 10^{18} (the target of this mission) would give a differential acceleration of $8.4\cdot 10^{-16}$ $cm\cdot s^{-2}$; the radiometer effect resulting from (21) must be below this value. The test body of lower density (Be, with a density of 1.85 g/cm^3) is subject to a larger radiometer effect and will dominate in the differential one because the other body is much denser. For this to be smaller than the target signal, the condition on temperature gradients across the Be test mass of the STEP experiment (10÷15 cm size) is:

$$\left(\frac{dT}{ds}\right)_{STEP} < 4.2\cdot 10^{-5} \quad K/cm \tag{22}$$

which must be ensured over the period of the signal (i.e. the orbital period around the Earth) or –in case of signal modulation– over the rotation period of the spacecraft. The superfluid He dewar, besides ensuring extremely low pressure, is pivotal in reducing temperature gradients, so as to meet the requirement given by (22).

In the case of μSCOPE the experiment is run at room temperature, the satellite is designed to fly at higher altitude than STEP (600-700 km) and the target is to test the Equivalence Principle to 10^{-15}, corresponding to a differential acceleration of $8.2\cdot 10^{-13}$ $cm\cdot s^{-2}$. The less dense test cylinder is made of Ti, about 8 *cm* long (with a density 4.5 g/cm^3). The residual gas pressure as reported for a predecessor of the current instrument (STAR [39]) is of $3.7\cdot 10^{-6}$ *Torr.* A pressure of $7.5\cdot 10^{-9}$ *Torr* is foreseen for the LISA mission [40, 41], far ahead in the future. Assuming that a value of 10^{-8} *Torr* is achieved with μSCOPE, the condition on temperature gradients analog to (22), (but referring to an Equivalence Principle test 3 orders of magnitude less sensitive) is:

$$\left(\frac{dT}{ds}\right)_{\mu SCOPE} < 1.7\cdot 10^{-4} \quad K/cm \tag{23}$$

from which it is apparent that a room temperature mission of this kind will find it very hard to achieve an Equivalence Principle test of high accuracy (temperature differences of 0.5 *K* are reported for STAR [39]; see also [38] and references therein).

Having chosen, for STEP, to operate the experiment at very low temperatures, other advantages besides very low pressure can be exploited. The main advantage, discussed above, is the possibility to use SQUID read-out sensors, allowing differential displacements to be measured independently of the ones in common mode; which is crucial in experiments to test the Equivalence Principle. As we have seen, if the STEP experiment design is modified only for operation at room temperature, as it is the case with μSCOPE, the capacitance read–out used in substitution of the SQUID read-out is adequate in terms of sensitivity but no longer allows differential and common mode effects to be uncoupled.

Other advantages of low temperature, relevant for testing the Equivalence Principle, are a reduced level of all disturbances related to thermal expansion, and a reduced level of thermal noise. Reducing thermal expansion close to the test bodies is important. However, a cryogenic experimental apparatus does not prevent the spacecraft outer shell from expanding and contracting in response to the infrared radiation from the Earth while orbiting around it, and the spacecraft mass so displaced will act on the test bodies giving rise to a differential acceleration (because the test cylinders cannot be exact monopoles) with the same frequency and phase as an Equivalence Principle violation. In STEP this thermal expansion effect turns out to be much bigger than the expected signal, posing a very stringent requirement on the knowledge (by direct measurement) of the residual quadrupole mass moments of the test cylinders. Residual, non zero quadrupole moments are unavoidable because of machining errors. As for thermal noise, it depends on the temperature–to–mass ratio, and could be reduced not only by operating at lower temperatures but also by using more massive test bodies.

A cryogenic experiment like STEP has disadvantages too. An apparatus aiming to detect the effects of extremely small forces is naturally limited by nearby mass anomalies, more so if these masses move at the same frequency as the expected signal. The best would be to have no moving masses at all on board the spacecraft, which is practically impossible if it needs propellant to feed the thrusters in order to compensate for drag. As far as drag compensation is concerned, STEP has another constraint: being cryogenic, it needs to carry sufficient He on board to keep the experimental apparatus cool for the entire duration of the mission, and needs to get rid of the resulting boiled off He in a controlled way, so as not to disturb the experiment itself. This requires proportional He thrusters (impulsive thrusters would be too noisy), and the resulting natural choice is to use these thrusters with boiled off He as the propellant to compensate for drag. However, He itself in a not perfectly full or perfectly empty dewar will respond with zero friction to the varying tidal potential of the Earth (see Fig. 6). As a result, it will move around the test bodies at exactly the orbital frequency [29, 30], and if the test bodies have non zero (and different) residual mass moments the resulting differential effect competes directly with the signal. It is not an easy task to stop completely this motion inside a large dewar which contains hundred liters of He.

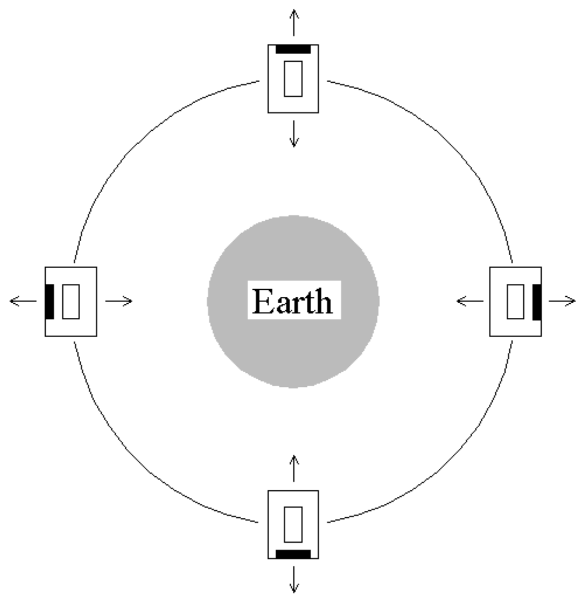

Fig. 6. – Skematic representation of the STEP He dewar while orbiting around the Earth carried by a spacecraft (not shown) in space–fixed attitude. The arrows represent the outward tidal acceleration of the Earth acting in the dewar along the satellite–to–Earth axis. The inward components along the axis perpendicular to it are not shown; the tangential component is zero at these locations and is not shown at other angles, where its effect is to form a tidal bulge either on the side of the dewar facing Earth or on the one away from it –only the latter is shown here. Unless the dewar is completely full (or completely empty, of course) any He mass anomaly will follow (with zero friction) the effect of Earth tides, and in doing so it will move around the test cylinders of the Equivalence Principle experiment (located close to the center of mass of the spacecraft) at the orbital frequency. If the spacecraft is rotated around its center of mass, in the plane of the figure, this effect appears at the rotation frequency with respect to the center of the Earth, which is the same as the modulation frequency of an Equivalence Principle violation signal (see Fig. 4). In all cases, if the He tide interacts differently with the test cylinders (i.e. because of their different mass momnets) its disturbance competes with the signal to be detected (figure not to scale).

Not being limited by the presence of He, μSCOPE is designed –unlike STEP– to use thrusters based on ion propulsion (e.g. FEEP – Field Emission Electric Propulsion). As compared to He thrusters, FEEP have two important advantages: i) a very high specific impulse, whereby only a minute amount of propellant (Cs in the case of FEEP) is sufficient to ensure drag compensation for the entire mission duration; ii) a fine electric tuning of the thrust, ensuring a high level of proportionality hence a more accurate compensation of drag and lower noise as compared to He mechanical thrusters. In addition, while the STEP mission duration is limited by the large amount of He required for the cryostat (in the order of hundred liters), the small amount of Cs propellant needed by FEEP (in the order of ten grams) makes a longer mission duration possible, thus allowing data taking for longer integration times.

Objects in space get charged because of cosmic rays and solar wind. Charging is especially intense if the spacecraft passes over the so called South Atlantic Anomaly (once per orbit), as it is the case with the STEP and μSCOPE sun-synchronous orbits. Electric interactions effects are huge compared to the small gravitational signal to be detected in these experiments. For STEP a radiation sensor has been proposed in order to discard contaminated data [29], or else a 130 *kg* tungsten shield to reduce the flux of dangerous particles [30]. Another solution is to discharge each test cylinder, which in STEP and μSCOPE are levitated (by electrostatic forces in μSCOPE, by superconducting magnetic bearings in STEP). The solution chosen for μSCOPE is passive discharging (by means of a thin conducting gold wire), while the current solution for STEP is active electric discharging. The effect of the stiffness and damping of the wire in one case [42], and that of acting directly on the test masses in order to first measure the charge it has acquired and then to neutralize it, are serious matters of concerns in both experiments whose original design, being based on levitated test bodies, is naturally prone to electric charging.

To summarize, with the STEP experiment concept it is mandatory to operate at very low temperatures in order to achieve a high accuracy test of the Equivalence Principle. In turn, this choice has profound consequences in terms of the mission complexity, hence inevitably on its risks and cost. The μSCOPE variant of STEP at room temperature is severely limited by the radiometer effect and by a read-out system which does not exploit the differential nature of an Equivalence Principle violation signal. It is also worth recalling that a free fall experiment has been proposed for testing the Equivalence Principle inside a vacuum capsule to be released from a balloon at an altitude of 40 *km*, allowing a free fall time of 30 *s* [43]. The gravity detector to be used is a differential version (with zero baseline) of ISA–Italian Spring Accelerometer–built at IFSI–CNR in Rome and tested in the Gran Sasso Laboratory by measuring Luni-Solar tides. ISA is based on torsional spring suspensions and capacitive pick ups. At release inside the capsule ISA would be given a rotation rate of about 1 *Hz,* so as to modulate the signal of a possible violation of Equivalence at this frequency. The experiment can be performed also at low temperature. The expected sensitivity is of 1 part in 10^{14} at room temperature and one order of magnitude better at low temperature. This is competitive with the target of μSCOPE, with the additional advantages that the experiment can be easily repeated (to modify and improve the apparatus) and that suborbital flights are obviously much less expensive than free flyers, even if they need low altitudes.

7.2 The GG space experiment

In the case of GG the concentric test cylinders spin around the symmetry axis at a rather high frequency (2 *Hz* with respect to the center of the Earth) and are sensitive to differential effects in the plane perpendicular to the spin/symmetry axis (see Fig. 7, to be compared with Fig. 4). A cylindrical spacecraft encloses, in a nested configuration, a cylindrical cage and the test bodies inside it; the whole system has a dominant moment of inertia with respect to the symmetry axis and is (passively) stabilized by 1-*axis* rotation around it. The suspensions are all mechanical, and very weak, thanks to weightlessness. The test cylinders are weakly coupled, forming a mechanical system similar to an ordinary beam balance with the beam along the symmetry axis of the cylinder, and therefore sensitive to differential effects in the plane perpendicular to the beam. The weaker the coupling, the longer the natural period of differential oscillations of the test bodies, the more sensitive the system is to differential forces such as the one caused by an Equivalence Principle violation. The mechanical suspensions provide passive electric discharging of the test cylinders.