$$\Delta T_{cage} = \frac{\sigma \cdot S_{cage} \cdot T^3 \varepsilon_{mylar} \cdot P}{c_{cage} M_{cage}} \Delta T_{mylar} \tag{26}$$

where σ is the Stefan-Boltzmann constant, $S_{cage}=1\ m^2$, $M_{cage}=40\ kg$, $c_{cage}=3.9\cdot10^6\ erg\ g^{-1}\ K^{-1}$ are the mass, surface and specific heat of the cage, $\varepsilon_{mylar}=0.05$ is the emissivity of Mylar.

It is apparent that temperature differences as small as these make the radiometer effect totally negligible in the GG mission. The result holds at room temperature and shows no need for a cryogenic experiment. The radiometer effect is not a limiting factor in Equivalence Principle testing for this mission. Thermal noise is higher at higher temperature, but since it is proportional to $\sqrt{(T/M)}$ (with M the mass of a test body), more massive test cylinders (10 *kg* in GG, about 100 *g* in STEP) compensate for the higher temperature.

Fast spin of the whole GG system has other advantages. Any local disturbing source rotates with the sensors, and therefore gives a DC effect. This helps considerably in dealing with perturbations such as those caused by local mass anomalies, parasitic capacitances, or the so called “patch effects”. They can be separated from the expected signal (modulated at the spin frequency) without posing severe requirements on the experiment design. The differential acceleration caused by the Earth tidal perturbation, due to the centers of mass of the test cylinders not being exactly coincident in the plane perpendicular to the spin/symmetry axis, appears at twice the frequency of spin.

In the present baseline the GG satellite is designed to fly on an equatorial orbit with the spin axis almost perpendicular to the orbit plane. This configuration maximizes the signal for the entire duration of the mission without any active control of the satellite. However, the GG satellite will be in the shadow of the Earth for about 2000 *s* each orbit, each orbit going from sunlight to darkness and viceversa. Fast spin helps reduce thermal effects and it is found that requirements on thermal stability can be met by means of passive isolation only. However, it appears that launchers capable to inject small satellites into low equatorial orbits are few and expensive, while the situation is more favorable for sun-synchronous, high inclination orbits such as those of STEP and µSCOPE. For this reason a new GG baseline, based on a sun-synchronous orbit, is being investigated within an advanced study funded by the Italian space agency (ASI). An additional accelerometer is also incorporated in the new design, whose test cylinders are made of the same material, for zero check. A unique property of the new accelerometer is that of being concentric to the original one, so that they are both close to the center of mass of the spacecraft and do not undergo different tidal effects from Earth, which would make the zero check more problematic.

Finally, a peculiarity of the GG design –based on mechanical suspensions and sensitive in the plane perpendicular to the symmetry axis of the test cylinders rather than along it, as in the case of STEP and µSCOPE– is that the system proposed for flight can be tested on the ground looking for a signal of violation in the horizontal plane of the laboratory while using the vertical direction to suspend the rotating cylinders against local gravity.

8 The GGG (“GG on the Ground”) experiment: laboratory test of a proposed space apparatus

The GG test of the Equivalence Principle is designed for space for two reasons: a driving signal stronger than on Earth (by about 3 orders of magnitude) and the absence of weight (the largest perturbation is $\cong 10^8$ times smaller than 1-g). Yet, a 1-g version of GG can be conceived. This is possible because in the GG design the expected signal lies in the plane perpendicular to the spin/symmetry axis of the test cylinders, as shown in Fig. 7: if on the ground the apparatus is suspended against local gravity along this axis, an Equivalence Principle violation signal has a component in the horizontal plane (as shown in Sec. 2) to which the system is sensitive. If the mechanical suspensions are stiff along the vertical (enough to withstand weight) and soft in the horizontal plane (in order to provide a very weak mechanical coupling of the test cylinders), the rotor is safe and at the same time it is very sensitive to differential forces in the horizontal plane, such as the force due to a possible violation of Equivalence.

The GGG apparatus, located at the LABEN laboratories in Florence, is shown in Fig. 11. It is mounted on a metal frame fixed inside a vacuum chamber of 1*m* diameter. The test cylinders, of which only the outer one is visible in the picture, weigh 10 *kg* each (as in GG). The capacitance plates of the read out, which measure the relative displacements of the test cylinders in the horizontal plane, are located in between the test cylinders. The suspension tube, held by a shaft turning inside ball bearings is shown. Inside this tube is the most delicate mechanical part of the apparatus: the coupling arm, and 3 laminar cardanic suspensions (shown Fig. 12), one for carrying the total weight (at the center of the arm), one for suspending the outer test cylinder (at the top of the arm), and one for suspending the inner test cylinder (at the bottom of the arm). Such a mechanical system is very similar to an ordinary beam balance (with a vertical beam), so that masses and lengths can be adjusted to make it most sensitive to differential effects. A simple scheme of the system is shown in Fig. 13, while a section of it is given in Fig. 14.

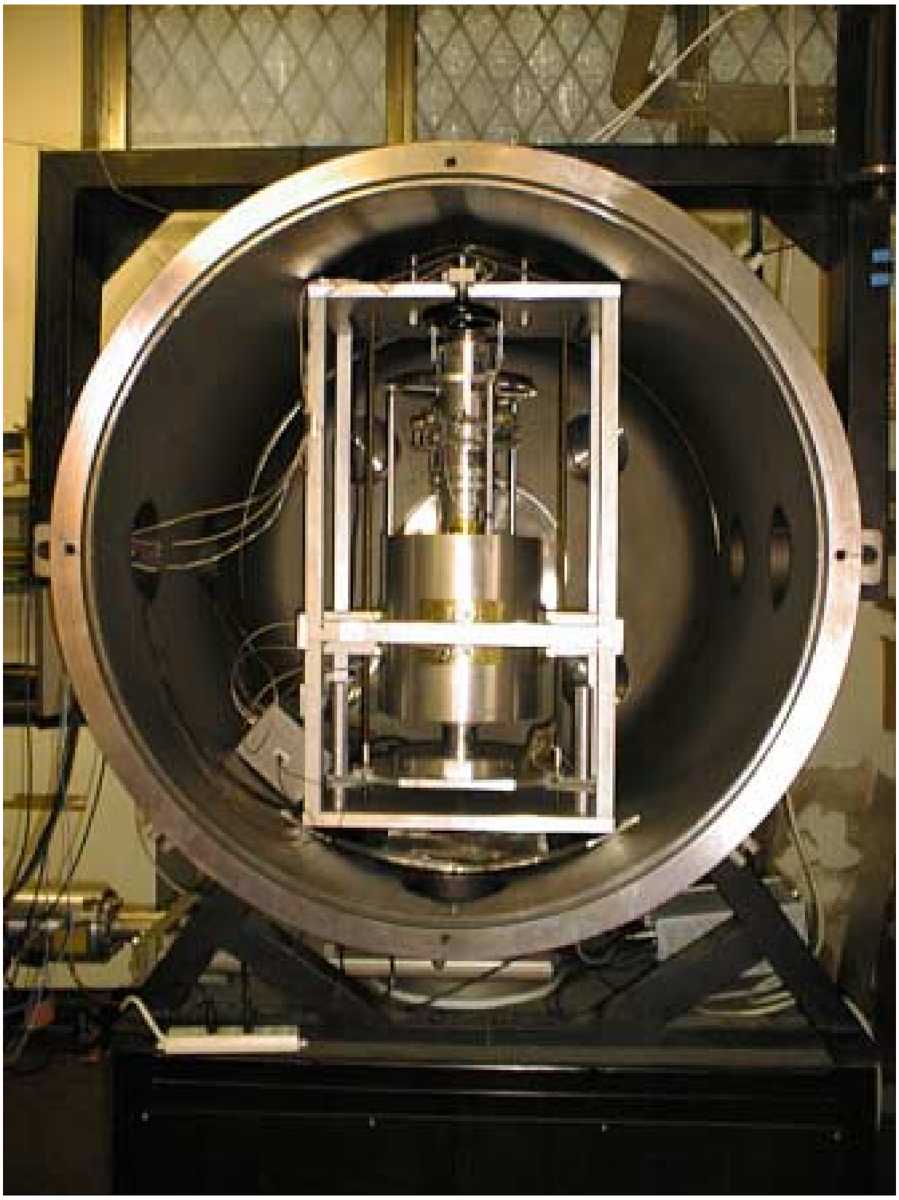

Fig. 11. – The GGG apparatus inside the vacuum chambe (LABEN laboratories, Florence)

The main part of the electronics of the experiment is arranged on an annular support around the suspension tube. It contains the 2 capacitance bridges, their preamplifiers, the analog to digital converters and the driver of the optical emitter. The digitized signal produced by these circuits (giving the relative displacements of the test cylinders as an electric signal) is optically transmitted to the non rotating system, and eventually outside the vacuum chamber to the computer. Sliding contacts are used for power transmission inside the rotor.

Fig. 12. – The 3 laminar suspensions of the GGG apparatus. They have been manufactured in CuBe by electroerosion in 3-D to ensure good mechanical quality.

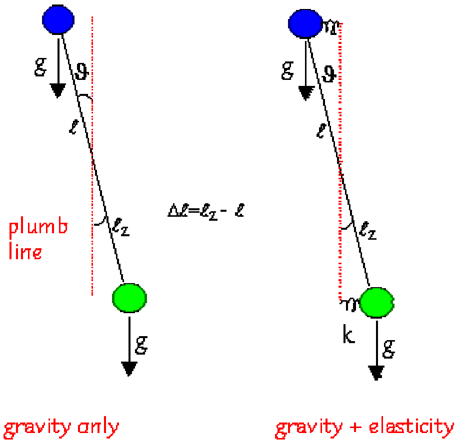

Fig. 13. – Simple dynamical scheme showing how the GGG test bodies, two hollow cylinders, are coupled. Although they are suspended so as to make them concentric (see Fig. 14), they are coupled in such a way as to behave as in an ordinary balance, except for the fact that the beam of the balance is vertical and that there is a weak elastic coupling in the horizontal plane. Dynamical systems of this kind can be treated similarly to very well known models and long periods of differential oscillation can be obtained by accurately balancing the masses and their arm lengths.

An Equivalence Principle violation with the Earth as the source mass would cause a constant displacement of the GGG test rotors in the North-South direction. However, due to the diurnal rotation of the Earth, a gyroscopic effect arises between the test rotors (proportional to the spin rate of the apparatus) which is also in the North-South direction and much larger: in the current design it amounts to about a few μm. A non zero tilt angle between the spin axis and the direction of local gravity would also give rise to a constant relative displacement (in the direction of the residual tilt).

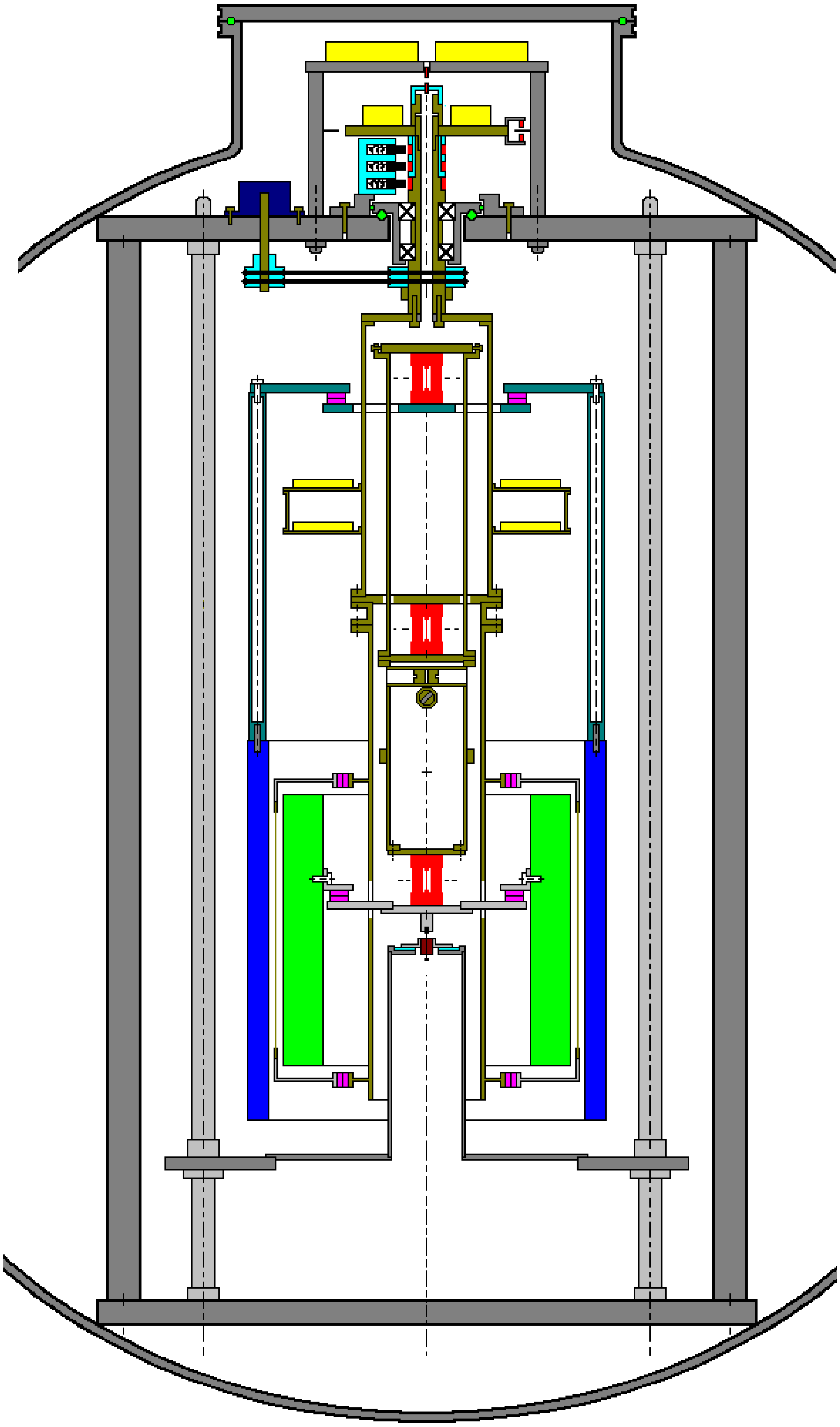

Fig. 14. – Section through the spin axis of the GGG experiment set up inside the vacuum chamber (see picture in Fig. 11). The drawing is to scale, the diameter of the vacuum chamber being 1 *m*.

The GGG apparatus must therefore look for a possible Equivalence Principle violation signal with the Sun as the source mass, as in the experiments by [12, 13]. This means a driving signal 1400 times weaker than it is in space for GG. Note that none of these difficulties (verticality of the rotation axis and gyroscopic effect) would affect the GG space experiment, in which there is no local gravity preferential direction and no gyroscopic effect (the rotation axis is almost perfectly fixed in space, and moreover –unlike in GGG– the test cylinders are suspended by their centers of mass).

The GGG apparatus described here has been tested for stable rotation (typically from *2* to 6 *Hz*) and for the measurement of small relative displacements between the centers of mass of the test cylinders. The sensitivity of the GGG capacitance read-out has been tested on bench, finding that it can detect 5 *picometer* displacements in 1*s* integration time. This is fully adequate for the GG space experiment, which must be sensitive to slightly less than 1 *picometer* displacements in order to fulfill its goal of an Equivalence Principle test to 10^{-17}; less than 100 *s* integration time is enough for the GGG read out to complete the task. The sensitivity of GGG to differential effects, as obtained from measurements carried out so far with the system in rapid rotation, is similar to that reported in the literature for the other accelerometers designed for space which have been subject to intensive testing [42,43]. It is expected that the sensitivity of GGG can be significantly improved because, unlike general-purpose accelerometers, it has been designed and optimized for the detection of an Equivalence Principle violation signal.

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