Data analysis of equivalence principle test in space. Advantage of measurements in 2D & sensitivity of the laboratory prototype

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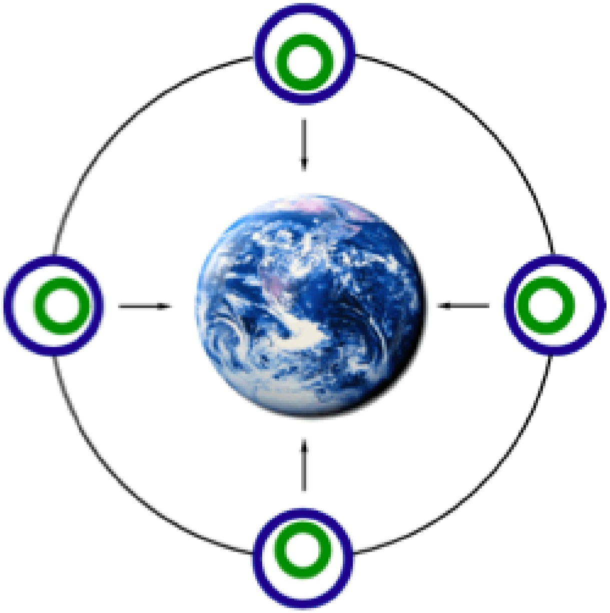
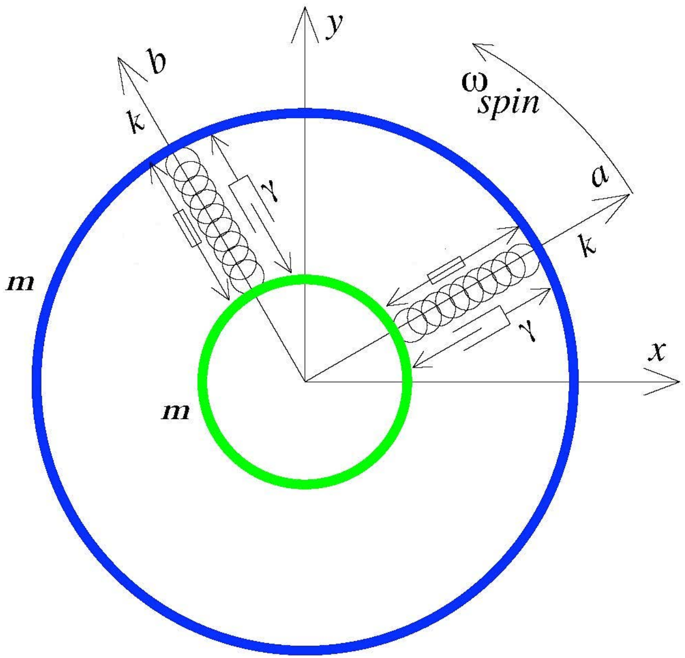
GG: violation signal in 2D



Satellite passively stabilized by one-axis rotation at $\nu_{spin} = 1\,\mathrm{Hz}$ around symmetry axis perpendicular to sensitive plane of test cylinders (blue & green indicate different composition).
Spin axis remains fixed in space (spin angular momentum conservation, very high spin energy).

Violation signal is a vector pointing to CM of Earth as the satellite orbits around it at $\nu_{orb} \simeq 1.7 \cdot 10^{-4}\,\mathrm{Hz}$

GG sensor: 2D rotating differential accelerometer

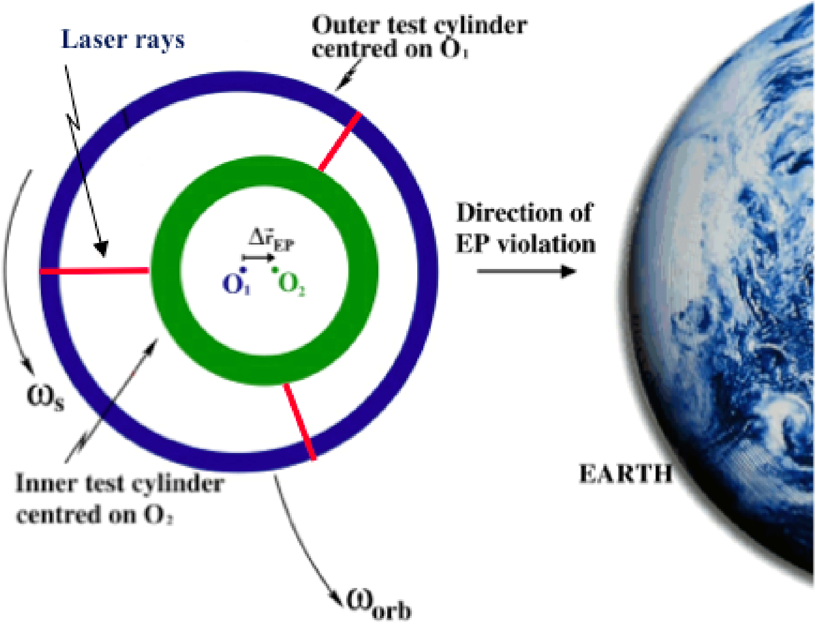


Two test cylinders of different composition rotate with the satellite at $\nu_{spin} = 1\,\mathrm{Hz}$. They are weakly coupled in the plane $\perp$ to the spin/symmetry axis to sense tiny differential accelerations.

A co-rotating read-out (laser gauge) reads the relative (differential) displacements of the test cylinders, which provide the differential acceleration through the measured natural differential frequency ω_{diff}.

Sensitivity $\propto 1/\omega^2_{diff}$; the weaker the coupling, the lower the differential frequency, the higher the differential period, the higher the sensitivity to differential accelerations.

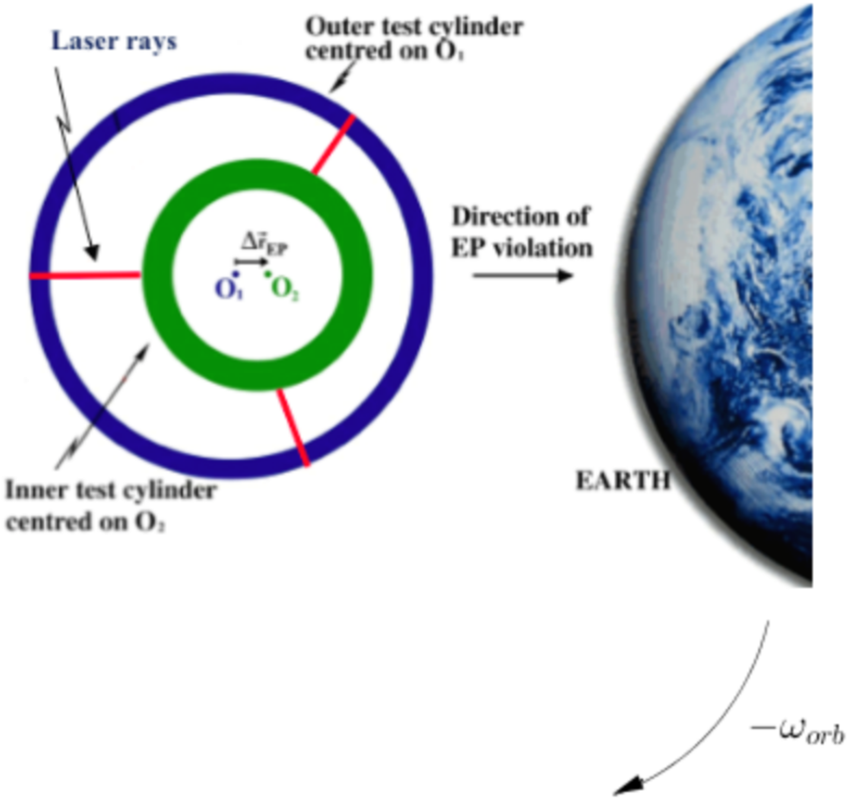
GG: up-conversion of signal to high frequency



Spin of test cylinders & laser gauge read-out up-converts the violation signal vector from orbital frequency to spin frequency.

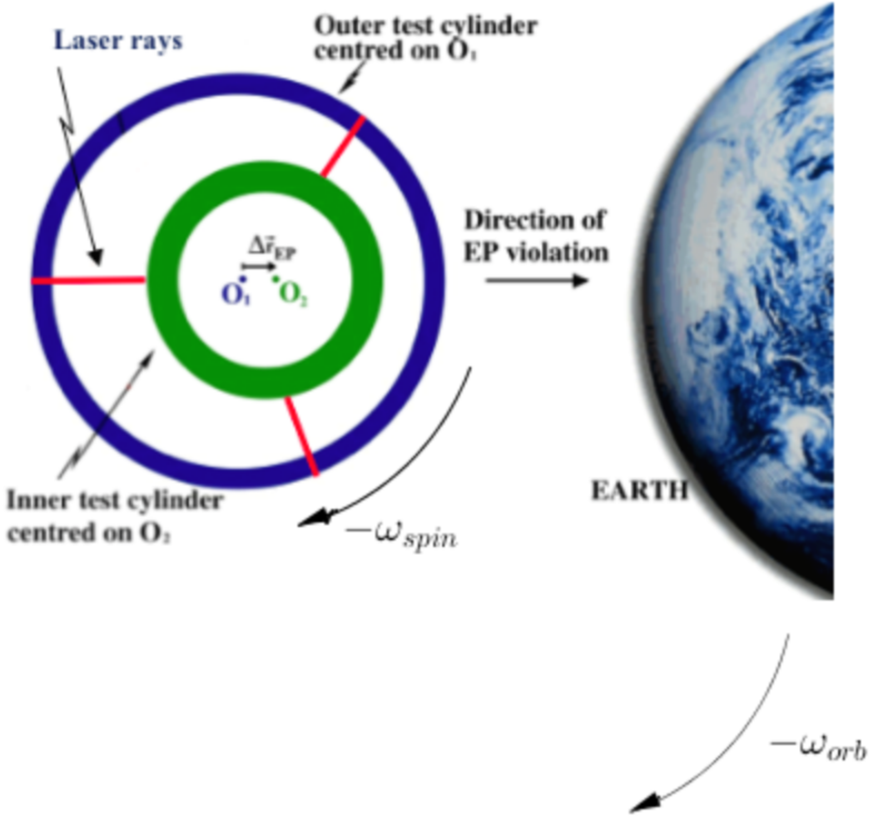
Signal frequency increased by factor $T_{orb}/T_{spin} \simeq 5800$

Violation signal in the non rotating satellite frame



In the **satellite frame centered on TM1 &** ***not spinning*** the Earth orbits at $-\omega_{orb}$ (if spin and orbital angular velocity vectors have same sign) and the violation signal points to its CM (or away from it; sign of violation unknown).

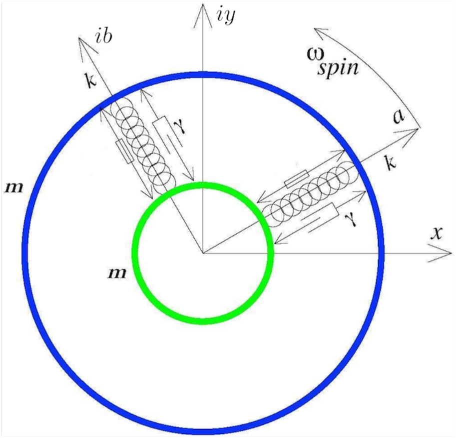
Violation signal in the rotating satellite frame



In the **satellite frame centered on TM1** & ***spinning*** the violation vector rotates at $-\omega_{spin} - \omega_{orb}$.

In 2D a rotating vector can be distinguished from an oscillating one with the same frequency

Complex Fourier analysis and separation of effects



2D rotating read-out gives relative displacement in complex rotating plane:

$$\zeta = a + ib$$

Sign of spin is known, and can be distinguished by complex Fourier analysis (includes frequency sign): FFT^-, FFT^+

Signal vs non rotating effects at same frequency

Violation signal appears in FFT^- only on one side of spin frequency line (left in case shown) at frequency distance ω_{orb}:

$$\zeta_{WEP} = \varrho_{WEP} e^{i(-\omega_{spin} - \omega_{orb})t}$$

An oscillating spurious effect at same frequency ω_{orb} as the signal appears in FFT^- but on both sides of spin frequency line:

$$\zeta_{osc} = \frac{\rho_{osc}}{2}(e^{i(-\omega_{spin} - \omega_{orb})t} + e^{i(-\omega_{spin} + \omega_{orb})t})$$

...read-out noise appears both in FFT^- and FFT^+ (half each)...

Signal versus whirl motion

Whirl forward

Weak instability due to losses in the suspensions at spin frequency causing growing orbital motion of the CM of test cylinders around common center of mass at the natural coupling frequency $\omega_w \simeq 10\omega_{orb}$ in the same direction as spin

$$\zeta_{wf} = \varrho_{wf} e^{i(-\omega_{spin} - \omega_w)t}$$

Line appears in FFT$^-$ left of spin frequency, like signal but about 10 times farther away

Whirl backward

Orbital motion of the CM of test cylinders around common center of mass at the natural coupling frequency ω_w in the opposite direction w.r.t spin; damps naturally by physics laws

Line appears in FFT$^-$ on the opposite side of the spin frequency line (to the right)

$$\zeta_{wb} = \varrho_{wb} e^{i(-\omega_{spin} + \omega_w)t}$$

This separation of lines is exploited in the control of whirl forward, to avoid amplifying whirl backward (same frequency, damps naturally)

Careful....

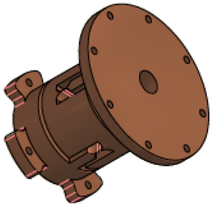
Not all systematics can be separated from signal this way...

Recipe on most dangerous systematics: have short integration time, have plenty of signal-to-noise ratio to burn (i.e. very low read-out noise), make many measurements to target sensitivity during mission lifetime, and then
$\Rightarrow$ *let physics laws discriminate errors from signal...*

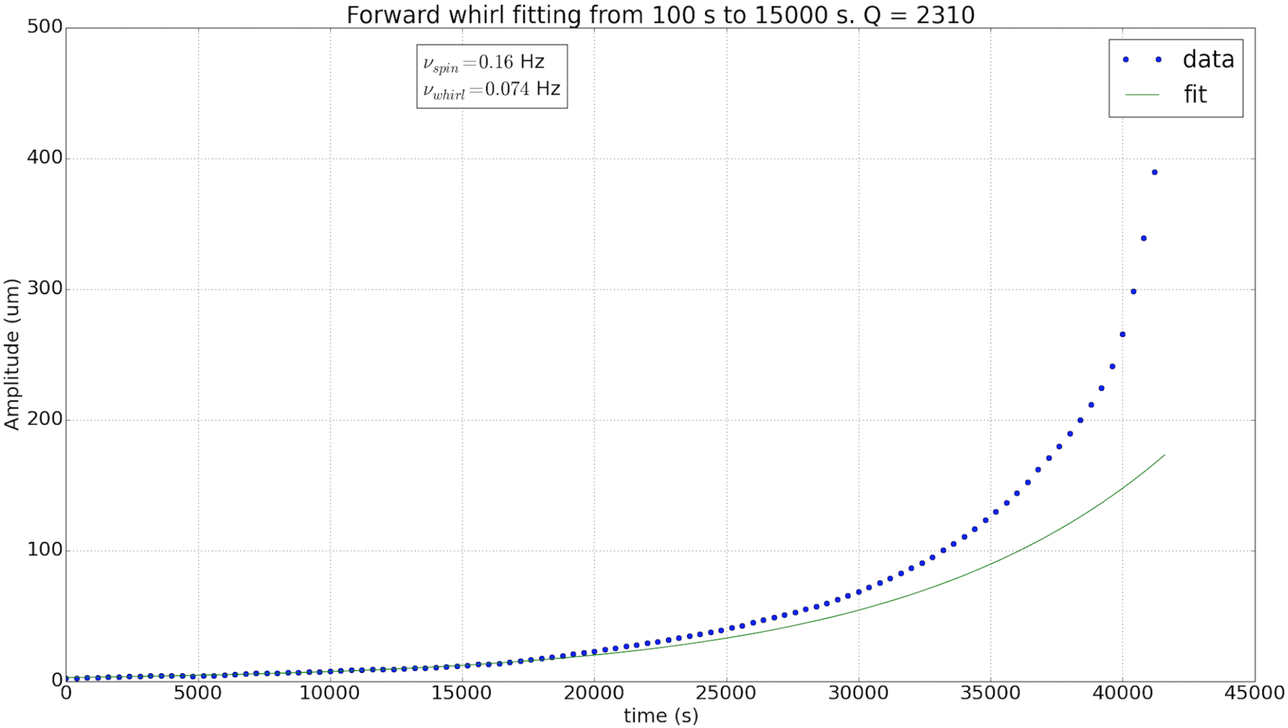
... but this is another story...

Pegna et al, PRL 2011, Nobili et al. PRD 2014

GGG prototype: Q measured from whirl growth

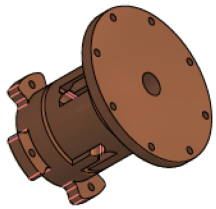


Whirl growth due to losses in the 2D CuBe joints which couple the test cylinders to form a vertical beam balance sensitive in the horizontal plane

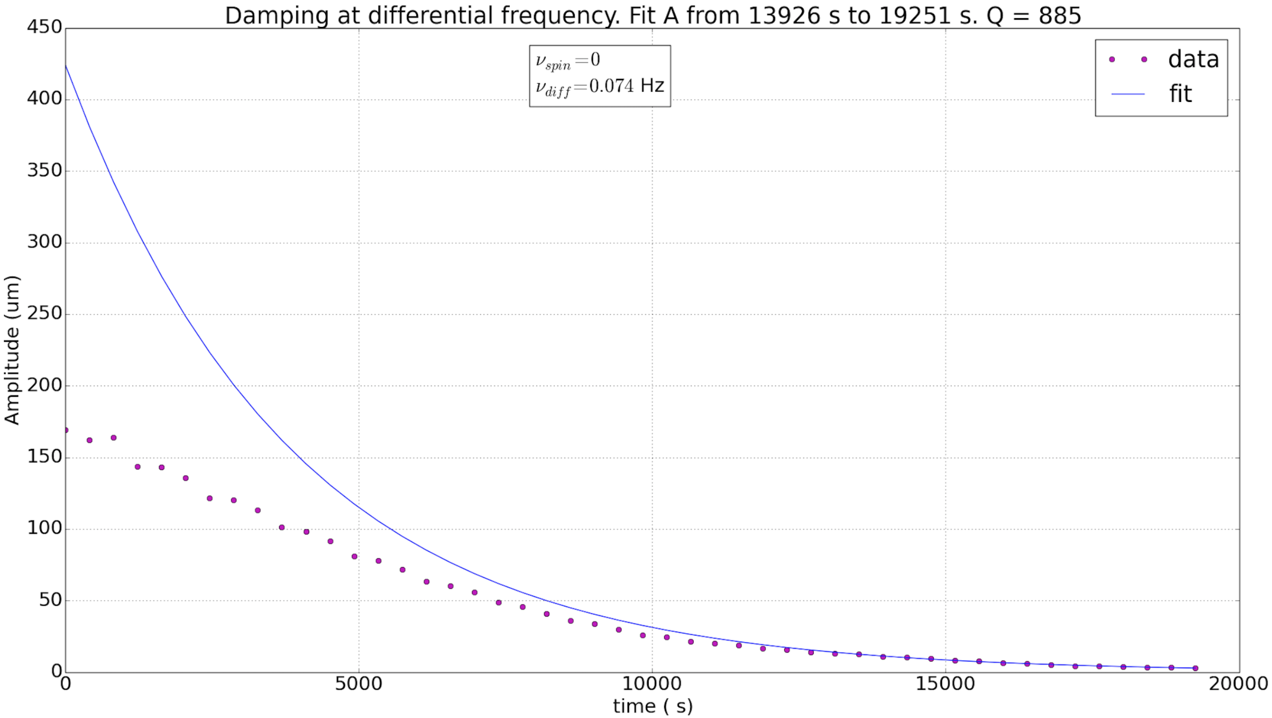


$Q_w = 2310$ from whirl growth at $\nu_w = 0.074\,\mathrm{Hz}$ while spinning at $\nu_{spin} = 0.16\,\mathrm{Hz}$

GGG: Q from decay at $\nu_{diff} = \nu_w$, zero spin

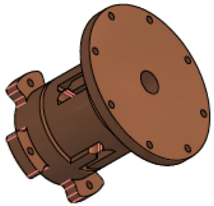


At zero spin amplitude of oscillations at the natural differential frequency decays due to losses in the same 2D CuBe joints



$Q_{diff} = 885$ at zero spin from decay of oscillation amplitude at $\nu_{diff} = 0.074\,\mathrm{Hz}$

Theory confirmed



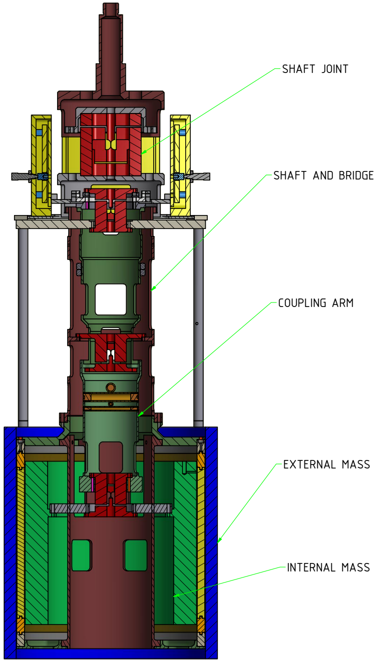
- In low dissipation: $\nu_w = \nu_{diff}$
- Backward whirl damps naturally
- Forward whirl growth depends on losses at spin frequency, not at natural differential (whirl) frequency $\Rightarrow$ Q from whirl growth in rotation must be higher because spin frequency is higher than differential frequency and at higher frequencies losses are smaller

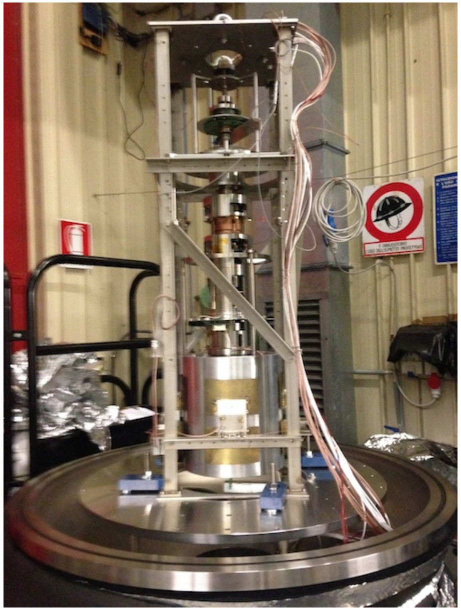
$$A(t) = A(t_o) e^{i(\omega_w (t-t_o)/2Q)} \qquad t - t_o = \frac{Q}{\pi} T_w \ln \frac{A(t)}{A(t_o})$$

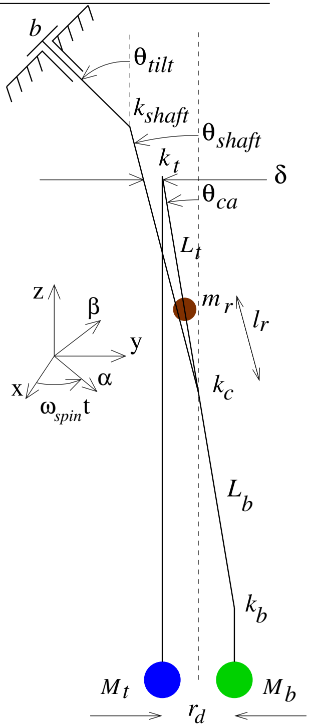
For GG sensor in space: Q=2310, whirl period $T_w = T_{diff} = 540\,\mathrm{s}$ $\Rightarrow$ once whirl has been damped a factor 10 growth needs 10.6 days!

Q in GG will be higher because: i) much less complex flexures at zero g; ii) higher spin frequency; iii) smaller displacements. Requirement is $Q = 20000$ ***and bench tests give values close to it.***

Application to GGG prototype

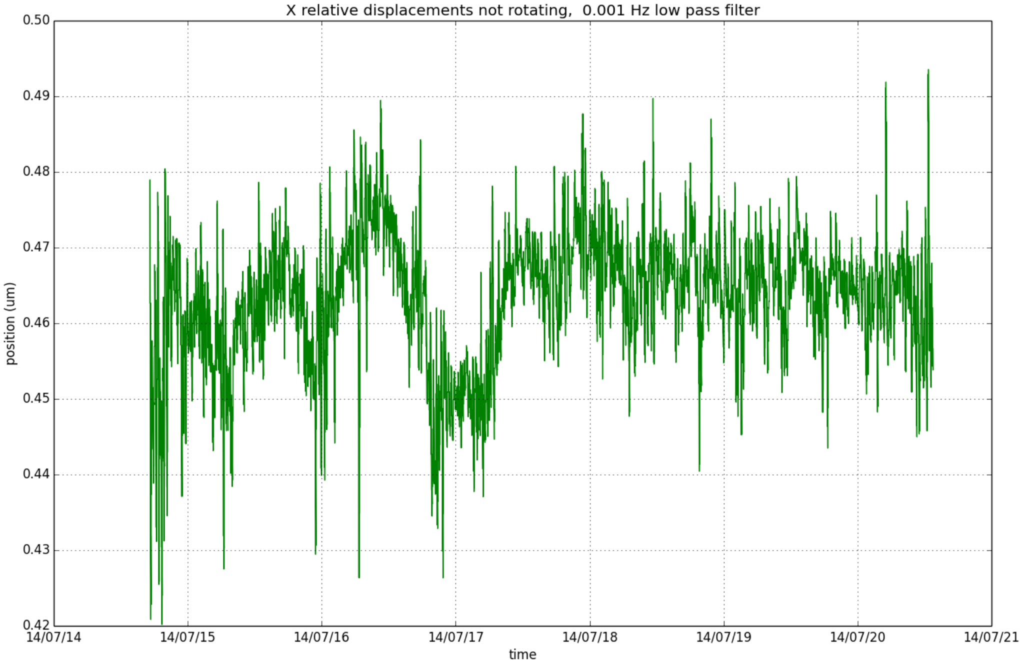






GG in space needs no motor no bearings, is isolated in space (no "terrain" tilts...), has weaker coupling and higher sensitivity by more than 3 orders of magnitude ... GG must deal with drag but know how is available...

Time series of relative displacements (I)

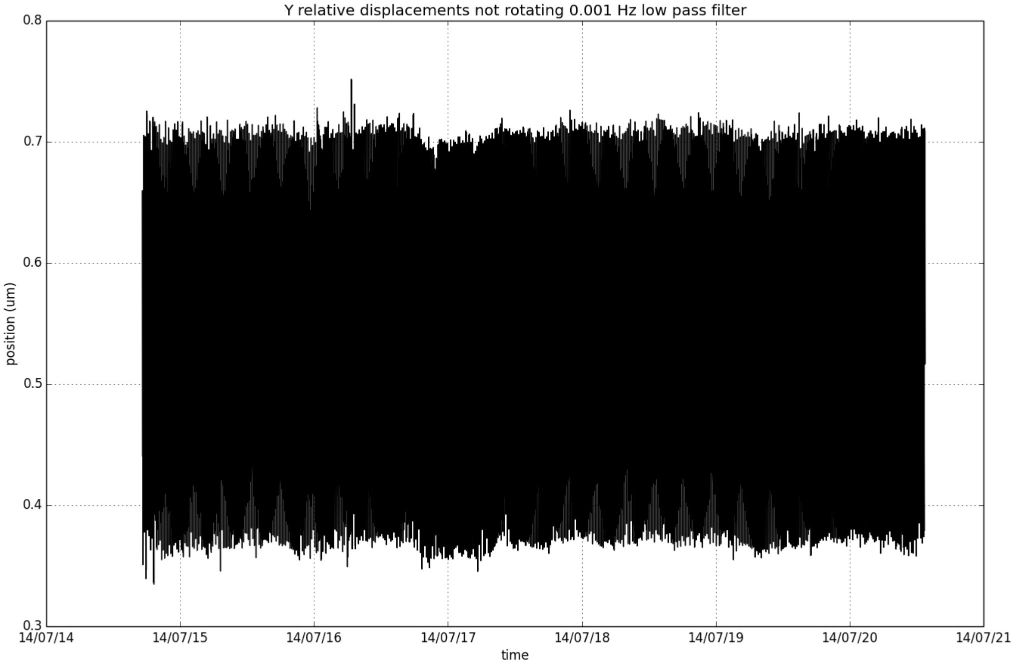


Time series of the relative displacements of the test cylinders; x axis of lab horizontal plane (non rotating frame).

$\nu_{spin} = 0.16\,\text{Hz}$ ($\nu_{diff} = 0.074\,\text{Hz}$ natural differential frequency)

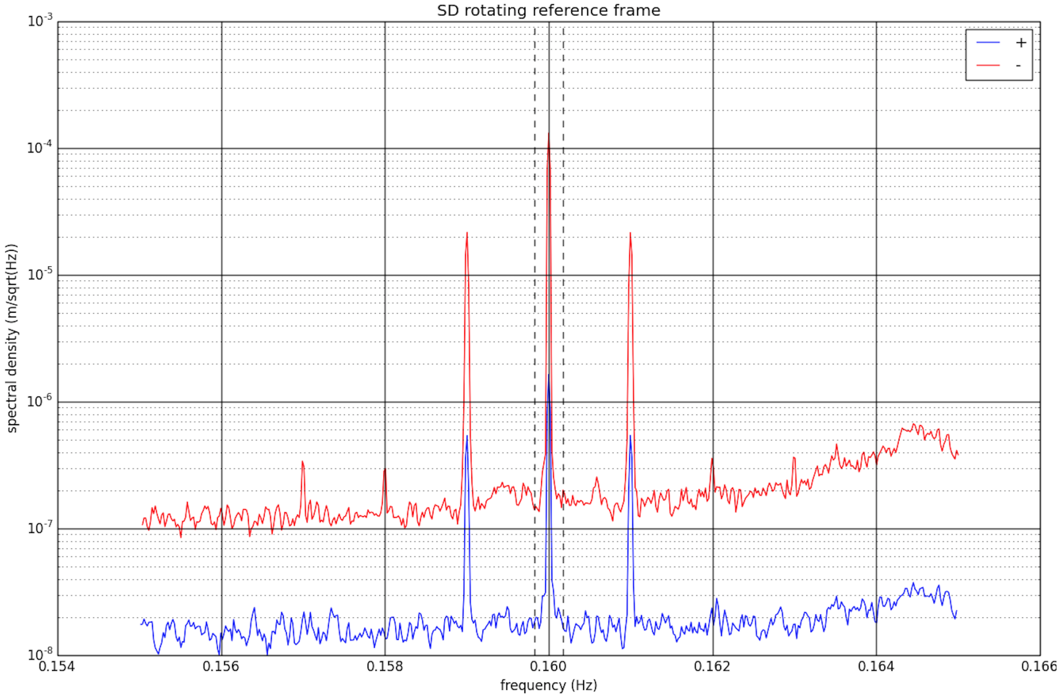
The centers of mass stay within $0.08\,\mu\text{m}$ from each other.

Time series of relative displacements (II)



Time series of relative displacements along y axis of lab (non rotating frame) a 1 mHz. A calibration signal at 1 mhz is applied in y which dominates motion .

Complex Fourier analysis: Spectral Density



Applied signal @ $1\,\mathrm{mHz}$ appears on both sides of $\nu_{spin} = 0.16\,\mathrm{Hz}$, but should be only in the red curve ($\mathbf{SD^-}$). Leakage to the blue curve (SD^+) is due to bearings/motor rotation noise which makes "real" SD^-, SD^+ differ from ideal ones $\Rightarrow$ bearings/motor rotation noise partially rejected in SD^+.

GG in space has no motor/no bearings $\Rightarrow$ blue (SD^+) curve gives sensitivity to GG target signal at one of two dashed lines close to $\nu_{spin} = 0.016\,\mathrm{Hz}$ by $\nu_{orb} = 1.7 \cdot 10^{-4}\,\mathrm{Hz}$.

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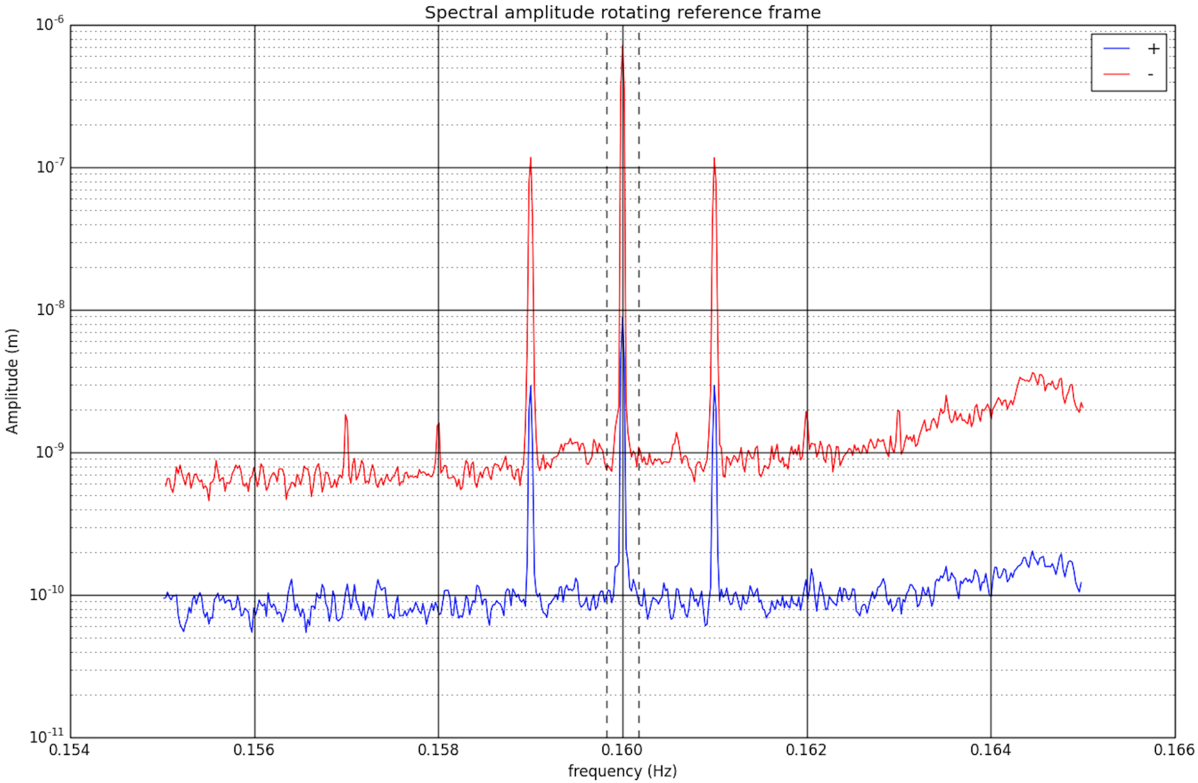
Complex Fourier analysis: spectral amplitude



Amplitude of spectral lines from previous spectral density: $\nu_{sampl} = 32\nu_{spin}$, $T_{res} = 86400\,\mathrm{s}$

GGG current sensitivity (I)

@ GG signal frequency $1.7 \cdot 10^{-4}\,\mathrm{Hz}$ **:**

- Lowest relative displacement/$\sqrt{\mathrm{Hz}}$ noise: $\simeq 2 \cdot 10^{-8}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$
- Lowest relative displacement noise (20 days): $\simeq 2.2 \cdot 10^{-11}\,\mathrm{m}$
- Lowest differential acceleration noise/$\sqrt{\mathrm{Hz}}$ (with $0.074\,\mathrm{Hz}$ natural differential frequency):
  $\simeq 2 \cdot 10^{-8} \cdot (2\pi \cdot 0.074)^2\,\mathrm{ms}^{-2}/\sqrt{\mathrm{Hz}} \simeq 4.3 \cdot 10^{-9}\,\mathrm{ms}^{-2}/\sqrt{\mathrm{Hz}}$
- Lowest differential acceleration noise (20 days): $\simeq 4.76 \cdot 10^{-12}\,\mathrm{m/s}^2$

GGG: where does it stand as prototype of GG?

$$\eta_{GGGprototype@1.7\cdot10^{-4}\mathrm{Hz}} \simeq \frac{4.76\cdot10^{-12}\,\mathrm{m/s^2}}{8.1\,\mathrm{m/s^2}} \simeq 5.9\cdot10^{-13}$$

$$\eta_{GGtarget} = 10^{-17}$$

$$\frac{\eta_{GGGprototype@1.7\cdot10^{-4}\mathrm{Hz}}}{\eta_{GGtarget}} = 5.9\cdot10^{4}$$

$$\frac{sensitivityGG@zero-g}{sensitivityGGG@one-g} = (0.074\,\mathrm{Hz}/1.85\cdot10^{-3}\,\mathrm{Hz})^2 = (540\ \ \mathrm{s}/13.5\,\mathrm{s})^2 \simeq 1600$$

no way to bridge this gap on ground!

$\Downarrow$

The only factor that GGG can still gain is: $\frac{5.9\cdot10^4}{1600} = 37$

(Note: target displacement signal in space is 0.6 pm and read-out noise must be $1\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ @ 1 Hz ... laser gauge, for other reasons too...)

GGG: where does it stand compared to others?

Best GGG sensitivity to WEP/UFF violation in the field of the Sun (*Nobili et al., CQG 2012*):

$$\eta_{GGG_{\odot}@1.16\cdot10^{-5}\mathrm{Hz}} \simeq \frac{3.4\cdot10^{-10}\,\mathrm{m/s^2}}{a_{\odot-Pisa}} \simeq \frac{3.4\cdot10^{-10}\,\mathrm{m/s^2}}{0.0057\,\mathrm{m/s^2}} \simeq 6\cdot10^{-8}$$

GGG sensitivity to differential accelerations @ very low frequencies:

i) $6\cdot10^4$ *times* **worse** *than torsion balances* (*...they cannot fly*)
Braginsky & Panov, JEPT 1972 (Univ. Moscow)
Baessler et al., PRL 1999 (UW Seattle, USA) $\eta\simeq10^{-12}$ (in the field of the Sun...)

ii) $2.9\cdot10^3$ *times* **better** *than* :
Fray et al., PRL 2004 (Max Planck, DE) ^{85}Rb, ^{87}Rb
Schlippert et al., PRL 2014 K, ^{87}Rb
Tarallo et al., 2014 ^{85}Sr, ^{87}Sr ...all at $\eta=10^{-7}$

iii) 202 *times* **better** *than* Cs, SiO_2 test
Peters et al., Nature 1999 (Stanford, USA)

iv) 124 *times* **better** *than* ^{87}Rb, SiO_2 test
Merlet et al., Metrologia 2010 (LNE-SYRTE, Paris, FR)

v) 20 *times* **better** *than* Al, Cu test
Carusotto, Polacco et al., PRL 1992 (CERN)
Still the best test of WEP/UFF by mass dropping : $\eta=7.2\cdot10^{-10}$!!!!

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