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Radiometer & outgassing: Notes on the manuscript "Residual gas effects in space-borne position sensors", by A. Rüdiger, April 2002

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In manuscript [1] Albrecht Rüdiger points out the presence of a noise source stronger than the radiometer effect due to the temperature dependence of the outgassing rate of the sensor cage. He also questions the validity of the way the radiometer effect has been computed so far. I notice that if there are no openings in the sensor cavity (hence, there is no gas pumping) the radiometer effect dominates and it is correctly expressed by the classical formula (as given in [2]). Instead, if gas emitted by the materials inside the cavity is pumped out by means of holes to open space (in order to reduce the pressure), then the effect pointed out in [1] is correct. At room temperature, the issue is a serious one for μSCOPE, and should be carefully investigated for LISA; it is not relevant for GG because the temperature gradients which generate the disturbance are totally negligible due to fast rotation. In STEP extremely low pressure is made possible by very low temperature and it reduces the disturbing effect of temperature gradients.

I. THE CASE OF A SENSOR CAVITY WITH NO OPENINGS

Let us first consider the case (at room temperature) that there are no openings on the surfaces of the sensor cavity to avoid gas pressure build up due to outgassing from material inside it. Equilibrium will be reached (between gas emitted from any outgassing surface and the gas pressure in the surroundings) at a pressure higher than the value one would expect to be able to reach in space. A pressure of 10^{-4} Pa can reasonably be reached. Touboul reports having reached a pressure of 10^{-5} Pa (I guess this value refers to the case that there are no openings in the cavity -no gas pumping- but I wait for confirmation). In absence of openings, and in the presence of a radiation source, the dominant effect is the classical radiometer effect.

It can be derived from the equation

$$\frac{p}{\sqrt{T}} = const \qquad (1)$$

which holds assuming only stationary equilibrium and energy equipartition. From (1), by differentiation, one gets the relationship between pressure gradient and temperature gradient (any temperature gradient caused by a radiation source will generate a pressure gradient, hence a net acceleration on the proof mass)[1]:

[1] In [1, Sec. 2] Rüdiger claims that (1) holds only provided that the mean free path of the gas molecules is much smaller than the dimensions of the vessel, which is not the case for the sensor cavities we are considering where the residual pressure is low enough for the mean free path to be more than hundreds of meters. In order to sort out this issue I have looked in the literature. The most relevant reference is [5] where,

$$\frac{\Delta p}{p} = \frac{1}{2}\frac{\Delta T}{T} \tag{2}$$

For a proof mass of density ρ, a temperature gradient dT/ds along an axis s, will produce a net acceleration along that axis:

$$a_s = \frac{p}{2\rho}\frac{1}{T}\frac{dT}{ds} \tag{3}$$

(this is Eq. 1 in [2]). En passant, this equation shows that, assuming a constant temperature gradient along the mass, and the same surrounding gas pressure (both hypotheses are reasonable), the radiometer acceleration a_s depends only on the density of the proof mass, and therefore gives no differential effect in a 2-body accelerometer where the proof masses have the same composition and density (i.e. such an accelerometer is unable to check for a systematic radiometer effect). In [1, p. 4] Rüdiger agrees that, in experiments to test the equivalence principle, temperature gradients caused by the infrared radiation from the Earth, give a radiometer effect with the same signature as the target signal.

Note that in [3, Ch. 6] (1) is reported in the case that the mean free path is much larger than the dimensions of the vessel. It is definitely not true that (1), hence (3) for the radiometer acceleration, are valid only in the assumption that the mean free path is much smaller than the dimensions of the vessel. Sec. 2 and Sec. 4.1 of [1] should be corrected accordingly. Instead, I agree that, for all space experiments under consideration, the mean free path is very large, as computed by Rüdiger in [1, Sec. 4.1]. Indeed, this fact is very important in GG because it makes heat radiative transfer dominate over heat conduction by the gas, thus contributing significantly to the reduction of temperature gradients [2]

In absence of openings the monotonic pressure profile resulting from the presence of a temperature gradient will dominate and I see no objections to (1), hence neither to (3), and therefore –in absence of any pumping out of the gas– I conclude that the radiometer effect dominates and is correctly expressed by (3).

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starting from the Section on "Thermal molecular pressure", Knudsen discusses both cases (small and large mean free path), and demonstrates that (1) holds in the case of a large mean free path (rarefied gases) <u>and not</u> in the other. His analysis refers to room temperature. The theoretical and experimental evidence provided is so wide and thorough that it deserves to be looked at directly. At the very end of his book, Knudsen notes: "The results of my measurements at high pressure [i.e., small mean free path] are in fairly good agreement with the theory given by A. Einstein [Zeitschrift für Physik. Bd. 52, 1928] and others". It is also worth noting that the work by Knudsen was checked and confirmed extensively by Loeb [6] (Ch. VII. "The laws of rarefied gases and surface phenomena", in particular Sec. 83 on "Thermal transpiration"). White [3], whose work on this issue was the only one I had previously considered, briefly reports the results by Knudsen and Loeb because, being interested in cryostats, he has also to deal with low pressure (rarefied gases). The analysis by Knudsen and followers applies in full to the case of sensor cavities having no direct openings to outer space, hence with no gas flow in that direction (as it is the case in GG and, I guess also of μSCOPE and STEP) and for the pressures involved.

II. THE CASE OF A SENSOR CAVITY WITH OPENINGS

Let us consider the case of the LISA sensor, as described in [1, Sec. 1.4], with appropriate openings to open space so as to pump out the gas emitted inside the cavity. If such pumping is not symmetric at the opposite sides of the sensitive axis there will be a differential pressure, hence a net disturbing acceleration on the proof mass in that direction. According to [1, Eq. (10)], the outgassing rate $q(T)$ at average temperature T (having the dimensions of a pressure divided by velocity) is expressed by the exponential law

$$q(T) = Q(T)e^{(-\Theta/T)} \tag{4}$$

where the value of the scale temperature Θ (depending on the gas, the host material and the temperature range, as reported in [1]) is derived from experiments. In [1, Sec. 3.2] Rüdiger quotes as a reasonable range $3000\ K \le \Theta \le 30000\ K$; if there is a reference at hands, that would help. The resulting pressure is proportional to $q(T)$, namely:

$$p(T) = const \cdot \sqrt{T} e^{(-\Theta/T)} \tag{5}$$

as in Eq. (14) of [1]. From this exponential dependence we get (including the ½ factor introduced by Rüdiger) the same as Eq. (16) of [1], namely:

$$\frac{\Delta p}{p} = \frac{1}{2}\left(\frac{1}{2}\frac{\Delta T}{T} + \frac{\Theta}{T}\frac{\Delta T}{T}\right) \tag{6}$$

from which it is apparent, by comparison with (2) and given the values expected for Θ, that in the presence of gas pumping the additional term at the right hand side is much more relevant than the first one.

III. RELEVANCE FOR μSCOPE, LISA, STEP AND GG

The disturbing effect expressed by (6) is more relevant in space experiments devoted to testing the equivalence principle (low Earth orbit) than it is for LISA (heliocentric orbit at 1 AU) because of the infrared radiation from the Earth, which is large and causes temperature gradients with the same frequency and phase as the signal. However, the three cases of STEP, GG and μSCOPE are all different.

At very low temperature (STEP) it is possible to achieve extremely low pressures by freezing out the gas, and I guess there are no openings in the sensor cavity. Temperature gradients caused by the infrared radiation from the Earth give a radiometer effect competing with the signal, and must therefore be sufficiently small (note the presence of the average temperature in the denominator of (3)). The requirement on temperature gradients along the sensitive axis of the STEP proof mass, as computed in [2] for the goal $\eta_{\mathrm{STEP}} = 10^{-18}$ in testing the equivalence principle, is:

$$\left(\frac{dT}{ds}\right)_{\mathrm{STEP}} < 4.2\times10^{-3}\ \mathrm{K/m} \qquad (7)$$

At room temperature, in the case of GG ($\eta_{\mathrm{GG}} = 10^{-17}$), it has been demonstrated that the fast rotation of the system makes temperature gradients in the sensitivity plane totally negligible (see the last three Eqs. in [2]). Thus, there is no need to have a very low pressure, and the value achievable in the presence of outgassing shall be sufficient. Disturbances casued by temperature gradients are not an issue in the GG experiment even at room temperature.

In the case of μSCOPE, the recent analysis [4] shows that, with a residual pressure of 10^{-5} Pa, and assuming that the LISA requirement on fluctuations of temperature differences is met also in μSCOPE at low Earth orbit, the radiometer effect (3) would be 23 times larger than the target signal (corresponding to $\eta_{\mu\mathrm{SCOPE}} = 10^{-15}$). Indeed, LISA plans to reach a residual pressure 10 times smaller than that, and μSCOPE could try to reach the same value. However, if this is done by continuous gas pumping as described in [1], than the much larger effect (6) comes into play, which would nullify the advantage of a lower pressure. It is also very important to check that the reported value of 10^{-5} Pa for μSCOPE has been measured in a situation of no gas pumping from the sensor cavity; otherwise the problem is even more serious. Since the effects due to the infrared Earth radiation–either ruled by (2) or by (6)– have the same signature as the signal and are not checked by the accelerometer with equal composition test masses, the issue is definitely of major importance.

As far as LISA is concerned, the choice of gas pumping appears to have been made already, but according to [1] the effect (6) has not yet been taken into account. From the analysis [4, Sec. 2] the current LISA requirement on fluctuations of the temperature difference across the proof mass cavity may be dictated by the need to have a very stable laser cavity (fluctuations in the heat load could lead to thermal gradients across the optical bench which would upset the stability of the laser cavity), rather than by the classical radiometer effect given by (3). Thus, taking into account the effect (6), may not change the picture significantly. However, this should be checked over the entire frequency range of LISA, with a more firm evaluation of the parameter Θ, and possibly by the LISA scientists who have calculated the requirements reported in [4]

[1] A. Rüdiger, "Residual gas effects in space-borne position sensors", Manuscript, April 2002 http:// eotvos.dm.unipi.it/nobili/opendiscussion/manuscript_ruediger_april2002.pdf

[2] A.M. Nobili *et al.*, Phys. Rev. D **63**, 101101(R) (2001) http://eotvos.dm.unipi.it/nobili/opendiscussion/radiometer_PRD_2001.pdf

[3] G.K. White, Experimental Techniques in Low Temperature Physics, Clarendon Press, Oxford, 1959

[4] A.M. Nobili *et al.*, "Radiometer effect in the μSCOPE space mission", submitted (2001) http://eotvos.dm.unipi.it/nobili/opendiscussion/radiometer_microscope_revised.pdf

[5] M. Knudsen, *The kinetic theory of gases*, London Methuen & Co. Ltd, New York John Wiley & Sons Inc., 1952 (first published in 1934 following Lectures given at University of London in 1933)

[6] L. Loeb, *The kinetic theory of gases*, McGraw Hill Book Company Inc., New York and London 1934