RESIDUAL GAS EFFECTS IN SPACE-BORNE POSITION SENSORS

A. RÜDIGER
Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut
postal address: Hans-Kopfermann-Straße 1, 85748 Garching, Germany
E-mail: atr@mpq.mpg.de

Recent publications on space probes to test the Equivalence Principle pointed out the influence of the so-called radiometer effect on the sensitivity of space-borne position (or acceleration) sensors. Estimates for the space project LISA and its technology package LTP were also considering this effect. As will be shown, using the LISA sensors as an example, the usual derivation of this radiometer effect is hardly applicable to the sensors in question. Instead, a further noise source is described that can be expected to contribute much larger noise accelerations: the temperature dependence of outgassing from the sensor walls.

Temperature differences ΔT between the two electrodes facing the two sensitive surfaces of the test mass will introduce pressure differences Δp at these faces. In the case of the radiometer effect the (relative) pressure difference will be of the order $\Delta p/p \sim \Delta T/T$, whereas for the temperature dependent outgassing it will be of the order $\Delta p/p \sim \Theta \Delta T/T^2$, where the activation temperature Θ is by one to two orders of magnitude larger than the ambient temperature T. This makes the effect of the temperature dependent outgassing by one or two orders of magnitude larger than the radiometer effect hitherto considered.

1 Introduction

1.1 Space projects requiring position sensors

Many space projects are being planned that require a very accurate monitoring of the position of the test masses of the scientific payload. Deviations of their positions from a true geodesic trajectory are taken as the signal to be measured. Examples would be such effects as gravitational waves or violations of the Equivalence Principle (EP).

To keep these test masses protected from any disturbances is a major concern in the design of these space projects. It is evident that residual gas inside the sensors can give rise to sensitivity-limiting noise accelerations of the test masses.

1.2 Projects considered

The discussions below will mainly deal with the sensors to be used in the gravitational wave experiment LISA and its technology demonstrator LTP on

SMART-2. (Thus is will also be directly relevant for the planned Chinese project ASTROD.)

But the findings will furthermore find application in the missions (under construction, approved, or planned) that aim at testing the equivalence principle in space (STEP, MICROSCOPE, Galileo Galilei GG). As a matter of fact, it was a controversial discussion on the significance of the so-called radiometer effect in such EP missions that initiated the analysis given below.

1.3 The LISA sensor

LISA is a space mission to detect and measure gravitational waves. Such waves would change distances $\mathcal{L}$ between free-falling bodies by minute amounts, e.g. by as little as $\delta\mathcal{L}/\mathcal{L} \sim 10^{-22}$. In the LISA mission, there will be three spacecraft at huge distances $\mathcal{L}$ of 5 million km, and still the changes $\delta\mathcal{L}$ would be no larger than of the order 10^{-12} m. Needless to say that this requires very painstaking design of the measuring apparatus.

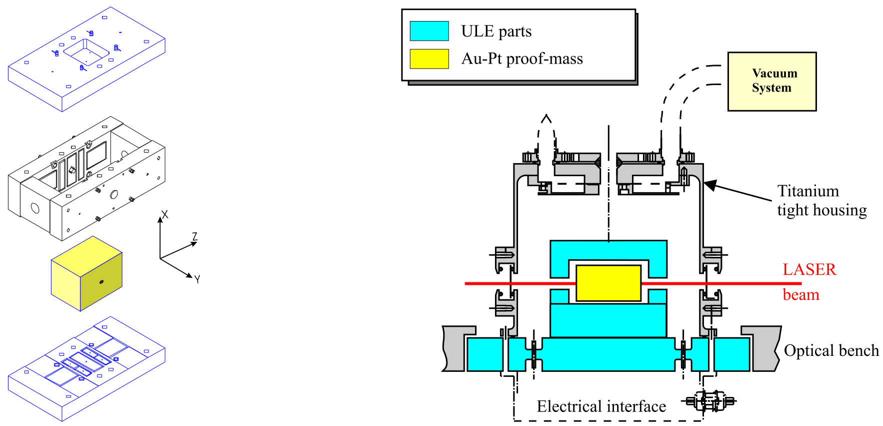
The fluctuations $\delta\mathcal{L}$ in the distances $\mathcal{L}$ are measured from test masses housed free-floating in the three spacecraft. These LISA spacecraft each contain two test masses, one for each arm forming the link to another LISA spacecraft. One such sensor is shown in Figure 1. The test mass, typically a 4 cm cube made of an Au/Pt alloy of low magnetic susceptibility, reflects the light coming from a YAG laser and defines the reference mirror of the interferometer arm.

These test masses are to be freely floating in space. For this purpose these test masses are also used as inertial references for the drag-free control of the spacecraft that constitutes a shield to external forces. Development of these sensors is done at various institutions. Figure 1 shows a sensor modelled after already space-proven developments at ONERA[?], other configurations are being discussed[?].

These sensors feature a three-axis electrostatic suspension of the test mass with capacitive position and attitude sensing, as well as a (mild) electrostatic control, mainly in the insensitive x- and y-directions.

1.4 Sensor geometry

The relevant geometrical details can be seen from Figure **??**. The simplest test mass would be cubical, typically with sides of 4 cm. We will, for generality, include the possibility of a test mass that is elongated in z-direction (length L), but has identical dimensions b in the x- and y-directions. For distinction, we will call the cube surfaces that are normal to the z-direction the "faces",



(a) The sensor cage (b) Sensor configuration

Figure 1. Layout of one gravitational sensor: (a) exploded view, (b) with vacuum housing.

with area b^2, and the surfaces normal to the x- and y-direction the "sides", with area $b\,L$.

The test mass is housed in a cage only slightly larger, with gaps between test mass and cage of only few millimeters (1 to 4 mm), again possibly with gap dimensions d_z in the optically active z-direction that differ from the four gaps at the sides, d_s.

Some of the cage surfaces facing the 'sides' have holes (orifices) to evacuate the interior (the 6 gaps) of the sensor. These orifices would typically have diameters of 5 to 10 mm. The light used to measure the (longitudinal) z position of the test mass will enter through a similar opening, which allows these (sensitive) gaps to be efficiently (but not completely) evacuated.

The whole sensor (test mass inside cage) is by itself encased in a vacuum housing (Ti), and we assume that efficient pumping maintains a very good vacuum that is by an order of magnitude lower than the pressure building up inside the sensor.

1.5 Spurious accelerations

Spurious accelerations can result from asymmetries in the residual gas pressure inside the sensor, and we will here consider only the asymmetries in z-direction, i.e. differences between the two faces.

A few typical values will make clear what orders of magnitude are tolerable. The overall pressure p deemed tolerable in the LISA sensor was assumed to be $10^{-8}\,\mathrm{mbar} = 10^{-6}\,\mathrm{Pa}$, and for the technology demonstrator LTP this might be relaxed to $10^{-7}\,\mathrm{mbar} = 10^{-5}\,\mathrm{Pa}$.

If this pressure were acting on only one side of the test mass, the force F on the test mass would be $p\,b^2$, and the acceleration on the mass $m = \rho\,b^2\,L$ would accordingly be

$$a_1 \;=\; \frac{p\,b^2}{m} \;=\; \frac{p}{\rho\,L}\,, \tag{1}$$

which for LISA ($p = 10^{-6}\,\mathrm{Pa}$, $b = 4\,\mathrm{cm}$, density $\rho \approx 20\times 10^3\,\mathrm{kg/m^3}$) would amount to $8.3\times 10^{-10}\,\mathrm{m\,s^{-2}}$. On the other hand, the acceleration noise density allowed in LISA is only $1\times 10^{-15}\,\mathrm{m\,s^{-2}}/\sqrt{\mathrm{Hz}}$. This imposes a very stringent requirement on the symmetry between the pressures at the two faces, and the following sections will demonstrate what precautions have to be taken.

1.6 Temperature variations

This paper's main concern will be such asymmetries that result from differences in the temperatures at the cage surfaces opposite to the 'faces'. The very elaborate thermal shielding in the LISA (and LTP) spacecraft will make these temperature fluctuations very small, but even these minute variations can become disturbing.

The test mass itself, being very short and of an alloy (AuPt) with good thermal conductivity, will be considered as of equal temperature T_0 throughout.

In LISA, the random fluctuation of the temperature differences ΔT are assumed to be less than $10^{-6}\,\mathrm{K}/\sqrt{\mathrm{Hz}}$, with slightly relaxed requirement (10^{-5}) for LTP.

In the EP experiments, also a periodic variation will come into play, a thermal asymmetry due to the thermal radiation from the Earth, and this variation coincides in frequency and phase with the signal to be measured. This periodic temperature variation is estimated at $3\cdot 10^{-4}\,\mathrm{K}$ in the project MICROSCOPE.

The next sections will explain the mechanisms that can turn these temperature variations into asymmetries in pressure.

2 The Radiometer Effect, a first glimpse

The radiometer effect as a source of noise in position sensors has been recognized quite early[?,?,?], and estimates of its implication on the sensitivity of these sensors have been made[?,?,?]. A temperature gradient in the residual gas surrounding the test mass would be the origin of sensitivity-limiting acceleration noise.

The origin of the radiometer effect can readily be seen from an idealized model of a volume of gas with a linear temperature distribution, i.e. a constant temperature gradient $\mathrm{d}T/\mathrm{d}z$ in the relevant z-direction, and no spatial dependence in the other directions. The volume is assumed large, i.e. the dimensions are assummed to be greater than the mean free path λ of the molecules. Under these conditions the volume of gas is stationary, i.e. from both sides of a plane normal to the z axis an equal number of molecules crosses that plane per time unit τ.

The faster molecules from the hotter side can draw from a more extended depth τv_+ than the slower molecules from the cooler side: τv_-. Thus, for equal number of impinging molecules, the density $\rho(T)$ must be inversely proportional to the velocity $v(T)$.

The pressure $p(T)$ is the impulse (momentum) per time unit τ imparted on the plane by $\tau\, v\, \rho$ molecules, each with a momentum proportional to $v(T)$. Thus, the pressure $p(T)$ is proportional to $v(T)$, and therefore proportional to the square root of T:

$$p(T) = p_0 \sqrt{\frac{T}{T_0}}\,. \tag{2}$$

This leads to a pressure gradient of

$$\frac{\mathrm{d}p(T)}{\mathrm{d}x} \;=\; \frac{\mathrm{d}p(T)}{\mathrm{d}T}\cdot\frac{\mathrm{d}T}{\mathrm{d}x} \;=\; \frac{1}{2}\frac{1}{\sqrt{T}}\,p_0\sqrt{\frac{1}{T_0}}\,\frac{\mathrm{d}T}{\mathrm{d}x} \;=\; \frac{1}{2}\,p(T)\,\frac{1}{T}\,\frac{\mathrm{d}T}{\mathrm{d}x}\,. \tag{3}$$

An extended body, with length L in z-direction, would experience a temperature difference at its two faces of $\Delta T = L\,\mathrm{d}T/\mathrm{d}z$, leading to a (relative) net pressure $\Delta p/p$ in z-direction of

$$\frac{\Delta p}{p} \;=\; \frac{1}{2}\,\frac{\Delta T}{T}\,. \tag{4}$$

This result, in essence, is what has been used to estimate the noise arising in position sensors due to temperature gradients $\mathrm{d}T/\mathrm{d}z$ in the presence of a residual gas pressure p. The validity of this relation will undergo further scrutiny in a later section (Section 4).

3 The Outgassing Effect

The pressure inside the sensor is upheld, against continuous pumping, by the outgassing from the sensor surfaces. Let us assume that openings for pumping out are situated at two of the four side surfaces of the electrode cage, and that there is also a wide opening each opposite the two sensitive faces normal to the z-direction.

3.1 The stationary pressure

First let us estimate the static pressure building up inside the sensor. The pressure at the sensitive faces will be of the order

$$p \approx q \cdot R \cdot A \,, \tag{5}$$

where R is the (averaged) flow impedance from the outgassing area A to the pump, usually measured in $[rms/\ell]$, (ℓ for 'liter'), and q is the outgassing rate, usually measured in

$$\frac{\mathrm{mbar}\,\ell}{\mathrm{cm}^2\,\mathrm{s}} \,, \tag{6}$$

or in SI units:

$$\frac{\mathrm{Pa\,m}}{\mathrm{s}} \,. \tag{7}$$

Excellent vacuum material, after baking out, may have outgassing rates as low as

$$10^{-12}\,\frac{\mathrm{mbar}\,\ell}{\mathrm{cm}^2\mathrm{s}} \;=\; 10^{-9}\,\frac{\mathrm{Pa\,m}}{\mathrm{s}} \,. \tag{8}$$

This means that inside the time of 1 s a pressure of 10^{-9} Pa would build up in a column of height 1 m, or of 10^{-6} Pa in a gap of width 1 mm.

But the sensor will consist of an assortment of different materials, so efficient bake-out (at temperatures above 100 °C) will hardly be possible, and thus much higher outgassing rates q must be expected.

The two cage surfaces opposite to the 'faces' will have a hole in the middle, perhaps of 10 mm in diameter, in a wall of thickness of perhaps also 10 mm. the flow impedance R from these surfaces to the pumps can be expected to be in the order of $1\,\mathrm{s}/\ell$ for air at room temperature; for hydrogen H_2 we would have one quarter of that value. Some more quantative analysis of the flow impedances will be found in Section 5.2.

For the vacuum considered sufficient in LTP, i.e. for 10^{-5} Pa, we can thus afford to have an outgassing rate of the order

$$10^{-6}\,\frac{\mathrm{Pa\,m}}{\mathrm{s}}, \tag{9}$$

a seemingly very relaxed requirement. But in reality, even that will require some effort to achieve it.

The very rough assumptions on the value of the flow impedance are of significance only for the relation between outgassing rate q and resulting residual pressure p, they do not enter in the mathematical form of the resulting $\Delta p/p$.)

3.2 Temperature dependence

What we are interested in is the pressure changes δp due to fluctuations δT in temperature.

The pumping down is so efficient that no pressure equilibrium between the gas phase and the solution phase in the wall material will be reached. Then the outgassing can be expected to be governed by a simple 'activation energy' law:

$$q(T) \;=\; Q\,\exp(-E_{\mathrm{act}}/kT) \;=\; Q\,\exp(-\Theta/T)\,, \tag{10}$$

where Q is a constant factor of dimension [Pa m/s], and Θ is an activation temperature describing the temperature dependence.

This Θ is very strongly dependent on the gas, the host material, and also on the temperature range (see Section 5), but it can safely be assumed to be within the range of 3000 K to 30 000 K. Thus it is by one to two orders of magnitude higher than the ambient temperature T. This high value causes a very strong temperature dependence of the outgassing, and this will have its implications on the noise budget.

A temperature fluctuation of δT will change the outgassing rate according to

$$\delta q \;=\; Q\,\frac{\Theta\delta T}{T^2}\,\exp(-\Theta/T)\,, \tag{11}$$

or by a relative amount of

$$\frac{\delta q}{q} \;=\; \frac{\Theta\delta T}{T^2}\,. \tag{12}$$

So we see that a temperature difference ΔT between the two active faces would cause a relative outgassing difference

$$\frac{\Delta q}{q} \;=\; \frac{1}{2}\,\frac{\Theta\Delta T}{T^2}\,, \tag{13}$$

in which a factor 1/2 was introduced because only the sensor cage surfaces would undergo the temperature variations, whereas the surfaces of the test mass would remain at a constant (mean) temperature T_0.

To calculate the pressure difference, on the other hand, we should return to Equation 10, which states the rate at which gas molecules are emitted from the surface. The pressure will be proportional to this number, but also to the momentum they carry: and that is proportional to their velocity, i.e. to the square root of T. Thus, from

$$p(T) \;=\; \sqrt{\frac{T}{T_0}}\, p_0 \, \exp(-\Theta/T),, \tag{14}$$

we derive a relative pressure difference using the derivative with respect to temperature T,

$$\frac{\mathrm{d}p(T)}{\mathrm{d}T} \;=\; \sqrt{\frac{T}{T_0}}\, p_0 \, \exp(-\Theta/T) \left(\frac{1}{2} + \frac{\Theta}{T} \right) . \tag{15}$$

Again attaching the factor 1/2 to account for only the cage surface undergoing the temperature changes, we arrive at the (relative) pressure difference

$$\frac{\Delta p}{p} \;=\; \frac{1}{2} \left(\frac{1}{2} + \frac{\Theta}{T} \right) \frac{\Delta T}{T} . \tag{16}$$

So we get a factor $(1/2 + \Theta)/T$, by which the new effect of temperature dependent outgassing is greater than the alleged radiometer effect. The first term is, in its form, reminiscent of the radiometer effect, but the physics is different. The second term is by a significant factor Θ/T larger than what had previously been assumed for the temperature behaviour of residual gas effects.

We find, therefore, that we have to heed the effect of the temperature dependence of outgassing much more than the radiometer effect, and it will be this new effect that will govern the choice of material, of bakeout treatment, and of the pumping requirements in the position sensors.

4 Radiometer Effect Revisited

But the arguments against the radiometer effect as the principal noise contribution will go yet further.

The simple derivation of the radiometer effect, as sketched in Section 2, is not an adequate description of the residual gas physics in the sensor. Some of the "flaws" of that treatment will be enumerated below.

4.1 Mean free path

The assumption of a sufficiently large gas volume, with dimensions larger than the mean free path λ is missed by many orders of magnitude: at our assumed pressure of 10^{-5} Pa, water vapor would have a mean free path of 400 m, and hydrogen of even 1200 m.

As it turned out, this assumption of the dimensions was not really very important in deriving the estimate for $(\Delta p/p)_{\mathrm{radiom}}$, as the underlying temperature difference ΔT, and not the temperature gradient, was used in Eq. 4.

But the gross violation of that assumption on the dimensions makes the concept of a temperature gradient rather meaningless.

(see also the publication by Einstein)

4.2 Non-monotonic pressure profile

The derivation of the radiometer effect relied on the monotonic pressure profile that would result from the exclusive cause of the temperature gradient. The pumping down at the (inactive) sides of the sensor effectively decouples the two gas volumes at the (active) faces of the test mass. This will preclude the pressure law of Eq. 2 from being reached.

4.3 Gas flow

In a similar vein, this pumping at the sides causes a continuous gas flow from the two active faces to the pump openings, again a condition in contrast to the assumptions of Section 2. There is, at no point in the system, the assumed equilibrium between gas flows in the two opposite directions. As a matter of fact, all the dimensions are extremely small compared with the mean free paths, and thus there is only molecular flow in the sensor gaps, and no collisions between the residual gas molecules.

4.4 Radiometer effect is not the issue

The conclusion of this Section is that the consideration of a radiometer effect as a source of acceleration noise in the sensor is beside the point. Rather it will be more important and meaningful to concentrate on the noise due to the temperature dependence of the outgassing rate, as sketched in Section 3.

5 Outgassing Rate, Revisited

In consideration of its much greater importance, the effect of the outgassing rate and its temperature dependence is to be treated now in more detail than in Section 3. This more detailed analysis will, however, in no way change the result of Section 3, and the comparison of the Equations 4 and 13.

5.1 Details of geometry

As can be seen from the equations in Sections 2 and 3, the precise geometry enters in identical ways in both the presumed radiometer effect and the effect of outgassing. The ratio between the two is not dependent on geometrical assumptions. Nevertheless it will be of great interest for the *absolute* pressure values that can be reached.

The currently proposed geometry of the sensors for the LTP is shown in Figure **??**, and the salient values are given in Table **??**. For LISA proper, the geometry has not yet been decided, but it will most probably not differ greatly from LTP's.

5.2 Flow impedance

(will give some details on the flow resistances inside the LISA and LTP sensors)

5.3 Activation temperatures

The above discussions have made it clear that a precise knowledge of the outgassing mechanisms would be desirable. But the same literature damps all hopes for a simple answer. Too many are the processes involved, and too idealized are most of the experimental evidences.

In all of the processes, an activation energy in the form of Equation 10 plays a decisice role, and these activation energies can be found under a wide spectrum of names: heat of desorption, adsorption energy, activation energy, enthalpy of desorption, etc.

Furthermore, the processes involved may differ widely: whether it is a desorption of a sheer surface layer (the surface coverage θ also playing a role), whether the gas atoms (or molecules) are only physically bound (physisorption) or more strongly (chemisorption), or whether diffusion (or even penetration) through the substrate is involved.

The residual gas behavior of the LTP or LISA sensors will certainly be a mixture of all of these processes. The literature, understandably, deals mainly with cases of good reproducibility: only one single gas, surfaces that are very clean, or even preferred monocrystalline surfaces.

In our sensor, on the other hand, we have an assortment of surfaces: possibly a substrate of ULE, with metal electrodes applied by some process that does not guarantee a simple clean surface. Also the handling during assembly, and the long period from launch till the beginning of measurements will influence the values of outgassing.

So it becomes apparent that a reliable knowledge of the outgassing characterstics can only be gained through realistic experimental investigations in the laboratory. The temperature dependence of all of the processes involved in the outgassing is so steep that with reasonably small temperature swings one can have great effects: temperature cycles with a swing amplitude of 50 K will probably produce more than a factor of ten in outgassing rate.

Such experimens should in all cases be done before a final decision on the vacuum of the sensors is made.

6 Conclusion

The requirements on the quality of the vacuum inside the sensors are not dictated by what is termed the 'radiometer' effect. Rather, the temperature dependence of the outgassing of the cage walls imposes a requirement that is by a large factor Θ/T above earlier estimates based on the radiometer effect, where Θ is an activation temperature of the order 3000 to 30 000 K, and T the ambient temperature, $T \approx 300\,\mathrm{K}$.

This significantly higher noise contribution will have its implications on the design and choice of materials in the space projects LISA and LTP, and it should also be carefully considered for the space projects aiming at testing the validity of the Equivalence Principle.

Theoretical predictions on the precise temperature law of outgassing are not expected to yield realistic values. Rather, laboratory experiments should be made to measure the outgassing, as a function of temperature, for materials involved in the design of the sensors.