Radiometer effect in the μSCOPE space mission

A.M. Nobili, D. Bramanti, G. Comandi, R. Toncelli
Space Mechanics Group, Department of Mathematics, University of Pisa, I-56127 Pisa, Italy

E. Polacco
Department of Physics, University of Pisa, and INFN, I-56127 Pisa, Italy

In space missions to test the Equivalence Principle (EP) the disturbing radiometer effect has exactly the same frequency and phase as the target signal. In [1] we have evaluated the radiometer effect for the three proposed experiments under investigation by space agencies: μSCOPE, STEP and GG. Motivated by a recent presentation of μSCOPE [2] (due to fly in 2004), we have re-examined the radiometer effect for this experiment carrying out a comparative analysis, on this issue, with the proposed heliocentric LISA mission for the detection of gravity waves. Should the spacecraft and payload be built to meet all the challenging requirements of LISA, but flying in the geocentric orbit of μSCOPE instead, temperature gradients along its test masses would still be so high as to make the radiometer effect larger than the target signal of an EP violation. We find no way to separate the radiometer systematic disturbance from the signal. Even the additional μSCOPE accelerometer, with test masses made of the same material for checking purposes, will not allow the radiometer effect to be distinguished from the signal because both of them vanish for test cylinders of equal composition and density. The option to increase the rotation speed of the spacecraft (now set at about 10^{-3} Hz) so as to average out temperature gradients and cancel the radiometer effect, is not viable in μSCOPE (neither in STEP). The axis of rotation not being the axis of symmetry of the test cylinders, they are constrained to move in one dimension and this makes them highly unstable if rotating faster than their (very low) natural frequencies. This analysis causes us to question the possibility for the μSCOPE space experiment, as currently designed, to achieve its goal of testing the equivalence principle to 1 part in 10^{15}. The issue is relevant, although to a less extent, also for STEP.

I INTRODUCTION

Three space missions (μSCOPE [2], STEP [3–5] and GG [6–8]) have been proposed for testing the Equivalence Principle (EP) in the gravitational field of the Earth by testing its most direct consequence: the universality of free fall, whereby all bodies fall with the same acceleration regardless of their mass and composition. The targets are: 10^{-15} for μSCOPE, 10^{-17} for GG, 10^{-17} [3, 4] or 10^{-18} [5] for STEP. While GG and μSCOPE are designed for room temperature, STEP requires a superfluid He dewar to maintain the experiment core at 1.8 K. In all cases the test bodies are concentric, hollow test cylinders (of different composition) forming an accelerometer sensitive to accelerations acting between them. In STEP and μSCOPE the sensitive axis of the accelerometer is the symmetry axis of the test cylinders and the spacecraft is actively controlled for it to lie in the orbital plane. Slow spacecraft rotation around the orbit normal will make the sensitive axis rotate relative to the Earth. An EP violation signal having the Earth as the source mass would therefore vary with the rotation period of the spacecraft with respect to the center of the Earth.

For µSCOPE −as reported in [2]− the orbital period is 5900 s (at an orbiting altitude of 700 km) and the rotation period of the spacecraft is 1570 s, resulting in a signal period of 1240 s (signal frequency $f_{EP} \simeq 8.06 \cdot 10^{-4}$ Hz).

A residual gas around the test cylinders and exposure of the spacecraft to the infrared radiation from the Earth (which causes non zero temperature gradients along the sensitive axes of the cylinders), give rise to a disturbing acceleration usually referred to as radiometer effect. Since the test cylinders have different composition, the radiometer effect −like an EP violation signal− is differential. For a spacecraft with space fixed attitude the radiometer effect varies with its orbital frequency, always pointing to the center of the Earth (just like the expected signal). The slow rotation of the spacecraft will up-convert the frequency of an EP violation to a frequency slightly higher than the orbital one (to help distinguish it from disturbances related to the orbital motion); but so will do with the frequency of the radiometer effect. This is therefore indistinguishable from the signal and its amplitude will limit the sensitivity of the mission in EP testing.

The amplitude of the radiometer acceleration along the sensitive axis of the test masses in the µSCOPE accelerometer can be written as:

$$a_{TM} = \frac{pS}{2m} \frac{\Delta T_{TM}}{T} \tag{1}$$

where p is the pressure of the residual gas, T its average temperature and ΔT_{TM} the temperature difference between the two faces ($\simeq 0.1$ m apart) of the test cylinder of mass $m \simeq 0.5$ kg and cross section $S \simeq 10^{-3}$ m^2 The amplitude (1) must be proved to be smaller than the amplitude of the expected EP violation signal acceleration, namely $a_{EP} \simeq 7.96 \cdot 10^{-15}$ m/s^2 (at 700 km altitude and for the µSCOPE target in EP testing of 1 part in 10^{15}). With a value for the residual gas pressure of 10^{-5} Pa which has been experimentally demonstrated [9], the amplitude of the radiometer acceleration is $a_{TM} \simeq 3.33 \cdot 10^{-11} \Delta T_{TM}$ m/s^2. For this to be smaller than the signal, temperature differences between the opposite faces of the test cylinders (at the signal/radiometer frequency f_{EP}) must satisfy the following inequality:

$$\Delta T_{TM} < 2.39 \cdot 10^{-4} \quad \text{K} \tag{2}$$

The maximum temperature difference (2) agrees with the limit derived in [1] for µSCOPE taking into account that the residual gas pressure that we assumed in [1] was a factor 7.5 smaller than the value provided by the µSCOPE scientists in [2]. The mass and thermal capacity of the accelerometer itself will reduce thermal variations, caused by radiation impinging on the outer surface of the spacecraft, at frequencies higher than a threshold frequency depending on the timescale of its thermal inertia. First hand knowledge of the apparatus is required in order to establish how long is this timescale, and we therefore rely on the value $\tau \simeq 7200\,$s given in [2]. The resulting attenuation factor, at the rotation/signal/radiometer frequency f_{EP} is

$$\sqrt{1 + (2\pi\tau f_{EP})^2} \simeq 2\pi\tau f_{EP} \simeq 36.5 \tag{3}$$

thus setting the requirement on the amplitude of temperature differences in the apparatus close to the test masses (with frequency f_{EP}) at

$$\Delta T < 8.7 \cdot 10^{-3} \text{ K} \qquad (4)$$

As far as temperature gradients are concerned, over the 0.1 m size of interest they should be smaller than $2.39 \cdot 10^{-3}$ K/m along the symmetry axes of the test masses themselves. With a thermal inertia characterized by a timescale of 7200 s as reported in [2], they can be larger than that by the factor (3), i.e., not to exceed $8.7 \cdot 10^{-2}$ K/m. The thermal specifications given in [2, Table 1] report a value of $3 \cdot 10^{-3}$ K/m for thermal gradients. If this value refers to thermal gradients to be further attenuated by the factor (3), then the radiometer effect between the test masses of the μSCOPE accelerometer would be smaller than the target signal by a factor of 30. In any case, the issue as to what evidence is available that this specification will be met in the flight experiment, so that temperature differences across the test masses will fulfill the requirement (2), is a key issue.

Radiometer noise at frequencies other than f_{EP} can be expressed in terms of power spectral density (PSD) and its effect will decrease with the inverse square root of the integration time. Instead, the radiometer effect at the same frequency and phase as the signal is a systematic effect, not noise, and it does not decrease as the inverse square root of the integration time. Therefore formula [2, Sec 5], giving the PSD at the frequency f_{EP} of the signal, is inappropriate because it treats as noise the radiometer effect at the same frequency as the signal (the one which really matters).

II   Sources of thermal variations in geocentric and heliocentric orbit: disturbances in LISA and in μSCOPE

The reason why the radiometer effect at the signal frequency is a serious matter of concern for μSCOPE, and needs to be singled out from the noise, becomes apparent by analyzing the same effect in the case of another proposed gravitational experiment in space, although far ahead in the future: the LISA (Laser Interferometry Space Antenna) mission for the detection of gravitational waves [10, 11]. Each LISA spacecraft (kept fixed with respect to inertial space by 3-axis active stabilization) carries an accelerometer whose test mass must be subject to accelerations not exceeding the level of $3 \cdot 10^{-15}$ m s^{-2}/√Hz in a frequency range between 0.1 mHz and 10 mHz, which requires spacecraft drag compensation to this level. For a measuring time $T_{EP} = 1/f_{EP}$ equal to the inverse of the signal frequency of μSCOPE, the target sensitivity of LISA is therefore $a_{lisa} \simeq 8.52 \cdot 10^{-17}$ m/s^2 This is a factor 100 smaller than the a_{EP} target of μSCOPE, which means that disturbances on the test mass of the LISA acceleration sensors must be 2 orders of magnitude smaller than in μSCOPE. The need for such a stringent requirement is not surprising, because the test masses in LISA are in fact "the mirrors" that will reflect back the laser beams of the interferometer on which the mission relies in order to detect geometrical changes caused by the passage of a gravitational wave. In the current LISA baseline [10] the accelerometer −named CAESAR− is based on capacitance sensing similarly to the μSCOPE accelerometer, and has been proposed by the same team at ONERA [12]. An interesting issue is therefore how relevant is the radiometer effect for LISA.

As far as the radiometer effect is concerned, the main difference between LISA and μSCOPE is that μSCOPE must fly in low Earth orbit while LISA's orbit will be heliocentric at the same distance from the Sun as the Earth (1 AU, with a 1 yr orbital period). In this orbit each LISA spacecraft will be exposed to the solar flux at the distance of the Earth ($\Phi_\odot \simeq 1.35 \cdot 10^3$ Watt/m^2 on average) and the main source of thermal variations will be due to changes in the solar irradiance around this mean value. Observed variations from 0.1 mHz to 10 mHz are described by a spectral density with a shallow frequency dependence [13]:

$$\frac{1.3 \cdot 10^{-3}}{(f/1mHz)^{1/3}} \frac{\Phi_{\odot}}{\sqrt{Hz}} \tag{5}$$

Thus, at the frequency f_{EP} of interest for µSCOPE, LISA would be subject to thermal variations of $\simeq 3.97 \cdot 10^{-5} \, \Phi_{\odot}$ and the resulting radiometer effect, having a random phase, is noise which therefore decreases with the inverse square root of the integration time. Instead, at the rotation/signal frequency f_{EP} of µSCOPE with respect to the Earth (the spacecraft rotates around the axis perpendicular to the orbit plane) most of its surface will be exposed to a radiation flux varying from zero to the full flux of the Earth infrared radiation. This is obtained from the solar radiation absorbed by the Earth (with a cross section $\pi R_{\oplus}^2$, $R_{\oplus}$ being the radius of the Earth) after subtracting the radiation re-emitted in the visible (due to an albedo of about 30%) and then considering that the radiation absorbed is re-emitted in the infrared from the entire surface of the planet, yielding an infrared flux $\Phi_{IR}^{\oplus} \simeq 0.17 \Phi_{\odot}$. The forcing thermal variation is therefore a factor 4300 larger on µSCOPE than it is on LISA, and the resulting radiometer effect has a constant amplitude which is determined better and better as the integration time increases −and mimics the signal. Unlike infrared radiation from the Earth, direct radiation from the Sun does not produce a systematic effect at the signal/spin frequency of the spacecraft with respect of the Earth, although it is obviously relevant for the thermal budget (the orbit is sun-synchronous).

The acceleration due to the radiometer effect can also be written in terms of the temperature gradient dT/ds along the symmetry axis of the test body (of density ρ), i.e. $a_{re} = (p/2\rho T) \cdot (dT/ds)$ (as in [1, Eq. (1)]). For LISA it is $(a_{re})_{lisa} \simeq 8.33 \cdot 10^{-14} (dT/ds)_{lisa}$ m/s^2. This value is obtained (at 300 K environment temperature) taking into account that in LISA the proof mass of the sensors is a very dense mass made of Au-Pt ($\rho_{Au-Pt} \simeq 20 \cdot 10^3$ kg/m^3) and the residual gas pressure is expected not to exceed 10^{-6} Pa. If we now consider this disturbing acceleration over the time $T_{EP} = 1/f_{EP}$, and require that it is smaller than the corresponding target sensitivity of LISA, we obtain a requirement on temperature gradients in the proof mass of LISA because of the radiometer effect, namely:

$$\left(\frac{dT}{ds}\right)_{re,lisa} < 1.02 \cdot 10^{-3} \text{ K/m} \tag{6}$$

This means that, because of the radiometer effect, over the 0.05 m size of the Au-Pt proof mass in the direction of the laser beam, temperature differences must be $(\Delta T)_{re,lisa} < 5.15 \cdot 10^{-5}$ K. The analog of inequality (6) for µSCOPE can be written taking into account that the residual pressure is expected to be 10 times larger than in LISA, and that the relevant density is the one of the low density Ti test cylinder ($\rho_{Ti} \simeq 4.5 \cdot 10^3$ kg/m^3). A low density proof mass is obviously a disadvantage. However, when looking for a composition dependent differential effect as in µSCOPE, it is unavoidable to have a low density test cylinder and a high density one, and it is the low density one which dominates the differential radiometer effect. We obtain the condition:

$$\left(\frac{dT}{ds}\right)_{re,\mu scope} < 2.15 \cdot 10^{-3} \text{ K/m} \tag{7}$$

and recall that the radiometer effect in μSCOPE is systematic at the same frequency and phase as the signal. From (2), with a 0.1 m test mass, we have a value slightly larger than this, of $2.39 \cdot 10^{-3}$ K/m. However, the latter value was obtained from Eq. (1) using approximate values for mass, length and cross section of the proof mass, while (7) requires only to know its density.

We learn from [10, Sec. 9.5.1.1] that the goal for LISA is to maintain the fluctuations of the temperature difference across the proof mass cavity below $2 \cdot 10^{-5}$ K/√Hz. This means that over the time $T_{EP} = 1/f_{EP}$ of interest in μSCOPE temperature differences across the proof mass of the LISA sensors are required not to exceed $5.7 \cdot 10^{-7}$ K, and therefore:

$$\left(\frac{dT}{ds}\right)_{lisa, goal} < 1.14 \cdot 10^{-5} \text{ K/m} \tag{8}$$

By comparison with (6) we see that, with this goal of LISA on temperature gradients, the radiometer effect (over the time $T_{EP} = 1/f_{EP}$) is below the required sensitivity by a factor 100. Which amounts to saying that the LISA goal on thermal gradients in not set by the radiometer effect. Rather, it is likely to be set by the need to have a very stable laser cavity, because fluctuations in the heat load could lead to thermal gradients across the optical bench which would upset the stability of the laser cavity. This is also why in LISA most structural elements are made from carbon-epoxy, which has a very low coefficient of thermal expansion (about $4 \cdot 10^{-7}$ /K) and the optical bench is made from ULE. The purpose is to avoid active thermal control close to the test mass, because it is known to be a source of unwanted disturbances in small force gravitational experiments.

The goal (8) obviously refers to the LISA planned heliocentric orbit at 1 AU. Should it −the same spacecraft, with the same acceleration sensor− fly in a geocentric orbit like μSCOPE, just because of the larger heat load, this value would be 4300 times larger, namely:

$$\left(\frac{dT}{ds}\right)_{lisa, geo} < 4.9 \cdot 10^{-2} \text{ K/m} \tag{9}$$

By comparison with the μSCOPE requirement (7), in that case set by the radiometer effect, this is a factor 23 too large. Assuming that μSCOPE will have a residual gas pressure 10 times lower than the current value of 10^{-5} Pa used to derive (7), reaching 10^{-6} Pa as expected for LISA, the requirement (7) is relaxed by a factor 10. As a consequence, the discrepancy with (9) would be by a factor 2.3. However, a disturbance with the same frequency and phase as the signal, but 2.3 times larger, appears to be unacceptable. Moreover, one should not underestimate that the inequality (9) has been obtained by scaling only for the higher heat load in geocentric orbit, and then comparing it with (7) under the assumption that the μSCOPE spacecraft and payload are built to meet the standards currently set for LISA. With the LISA project more than 10 years into the future, and a technology demonstration preparatory mission (SMART2) planned for testing its acceleration sensors and drag free technology, we are led to question that μSCOPE, as it is currently designed, will be able to get close to its planned target of testing the equivalence principle to 1 part in 10^{15}.

The analysis [12] of the CAESAR sensor for LISA, carried out prior to [10], reported a temperature difference of 0.01 K with variations of 10^{-3} K/√Hz at 10^{-4} Hz for the selected sensor configuration along the direction of the laser beam. This does not appear to comply to the more stringent

requirement set in [10, Sec. 9.5.1.1] for fluctuations of the temperature difference across the proof mass of LISA,

In low Earth orbit, it is interesting to consider the STAR accelerometer (manufactured by ONERA), currently flying onboard the low altitude ($\simeq 450$ km) geodetic spacecraft CHAMP, and designed to reach a sensitivity less good than the one µSCOPE is aiming to. At the time of its integration for flight on CHAMP, the test mass of STAR was expected to be surrounded by a residual gas at $5 \cdot 10^{-4}$ Pa pressure, and to reach temperature differences of 0.5 K [14]. A malfunctioning of the accelerometer has been reported [15], involving the linear acceleration along the radial direction of the satellite and two angular accelerations about its roll and pitch axes (CHAMP's attitude is 3-axis stabilized, Earth pointing). It is reported that because of the malfunctioning these accelerations are disturbed. Luckily, the linear direction disturbed is less relevant for the geodetic mission (which needs best sensitivity along track), but it is the direction of the radiometer effect, and it is therefore unfortunate that no direct measurement is available to provide evidence for pre-launch expectations [14].

III   CAN THE RADIOMETER EFFECT BE DISTINGUISHED FROM AN EP VIOLATION SIGNAL?

By getting close to its target µSCOPE will detect a differential acceleration between the test cylinders corresponding to an EP violation at the level of 10^{-15}, a level which might still be unexplored at the time of flight (currently planned for 2004). Whether the detected acceleration is an EP violation, or else a perturbing effect fully accountable within known physics, will be of crucial importance. The best torsion balance experiments on the ground have tested the equivalence principle to about 10^{-13} [16, 17]. An EP test in vertical free fall (GREAT) inside a vacuum capsule to be released from a balloon at an altitude of about 40 km (30 s free fall time) has been proposed with the same target as µSCOPE [18, 19] and is under study at Harvard-Smithsonian Center for Astrophysics. It has the advantage of easy repeatability. For a check at higher accuracy, one would need no less than another dedicated space mission like GG or STEP.

How to ensure that a detected residual acceleration with the frequency and phase expected for the signal is not in reality due to the radiometer effect? A non zero eccentricity of the satellite in its orbit around the Earth will not help, because both the EP driving signal (the gravitational attracting acceleration of the Earth monopole on the satellite) and the radiometer effect decrease as the inverse square of the distance (so does the flux of infrared radiation from the Earth heating the spacecraft). It is also not possible to count on daily or seasonal variations of the radiometer effect, because the spacecraft is always affected by an average of day/night and summer/winter infrared radiation, also helped by the very high thermal inertia of the oceans. Measuring temperature differences (and residual gas pressure) and having the instrument calibrated for response to temperature variations, would provide knowledge of the radiometer effect, thus allowing it to be subtracted away. Another option might be to actively control thermal gradients. This requires an appropriate distribution of thermometers and heaters in order to measure and reduce temperature differences (at the rotation/signal frequency) until the radiometer effect becomes smaller than the sensitivity. Once it is too small to be detected, it would also be too small to mask the target signal. However, it is a risky choice to have thermometers and heaters (even thermometers alone) close to the test masses, and even more so to control them at the frequency of the signal.

µSCOPE is designed to carry one accelerometer for EP testing (inner cylinder in Pt, 0.5 kg; outer cylinder in Ti, 0.4 kg) and an additional one with test bodies made of the same material for checking purposes (inner cylinder in Pt, 0.5 kg; outer cylinder in Pt, 1.7 kg) [20]. A "candidate" EP violation signal shall be discarded if it is detected also by the zero-check accelerometer.

Unfortunately, this accelerometer provides no check if the "candidate" EP violation signal is due to the radiometer effect. If we express the radiometer acceleration in terms of the (average) density of the test cylinder −as in [1, Eq. (1)]− it is apparent that, for a constant temperature gradient along the sensitive/symmetry axis of the cylinder and a given residual gas pressure, it depends only on the mass density of the test body, and therefore vanishes between bodies of equal composition (deviations of local gradients from a constant would produce second order effects). A way to make the zero-check accelerometer of μSCOPE as sensitive to the radiometer effect as the EP testing one is to reduce the average density of the 1.7 kg Pt outer cylinder to the same density as the 0.4 kg Ti outer cylinder of the EP accelerometer. Given the comparable volume, this requires to excavate the outer Pt cylinder enough to take away 1.3 kg of its mass (i.e. almost 77% of it). This should be done in such a way to maintain the cylindrical symmetry, but also to minimize the relative difference $\Delta I/I$ of the principal moments of inertia. A non zero value of $\Delta I/I$ makes each test cylinder sensitive to external mass anomalies in the spacecraft and −most importantly− to mass movements with the signal/rotation frequency of the spacecraft relative to the Earth. Such movements are due to the fact that (at slow rotation) the Earth-facing side of the spacecraft will always be hotter than the opposite side that is facing cold space, causing expansion-contraction of spacecraft masses relatively close to the test cylinders at the frequency and phase of the signal. The box-like, small structure (0.6x0.6x0.8 m) with flat panels of μSCOPE [2, Fig. 5] indicates that mass deformations will occur close to the test masses, leading to accelerations even a few orders of magnitude larger than the signal. Only if both test cylinders are sufficiently "spherical" (i.e. their $\Delta I/I$ are sufficiently small), the resulting differential effect (directly competing with the signal) can be neglected. Whether this can be achieved in manufacturing the outer Pt test cylinder, taking into account that it needs to be substantially excavated according to an appropriate theoretically computed design, remains to be investigated.

Even in STEP, whose test masses are surrounded by a residual gas with extremely low pressure and good thermal stability, both properties ensured by a superfluid He dewar at 1.8 K, the radiometer effect is estimated by the mission team to be among the 6 largest perturbations [21], less than 1 order of magnitude below the target signal. Also in this case, because of the higher target sensitivity, the accuracy of the thermal model used to rule out the radiometer effect and the uncertainty in the physical parameters involved are of crucial importance for an EP violation detected by a single experiment to be accepted beyond question. The more so in absence of a systematic check.

The radiometer effect becomes unimportant if rotation relative to the Earth is fast enough to average out temperature gradients [1]. μSCOPE (as well as STEP) relies on a slow rotation of the spacecraft in order to modulate the signal (which has the orbital frequency) at a higher frequency, and to separate it from the inevitable disturbances which occur over a complete revolution of the satellite around the Earth. However, this does not help with the radiometer effect, because slow spin modulates the signal as well as the radiometer. Instead, fast spin modulates an EP violation signal but averages out the temperature gradients which generate the radiometer effect, and therefore makes it to vanish, as we have demonstrated for the GG proposed experiment [1].

Indeed, the first proposal for testing the equivalence principle in space was based on fast rotation [22]. They suggested to place the test cylinders on a rotating aluminum wheel with the symmetry/sensitive axis in the radial direction and the rotation axis perpendicular to the plane of the wheel. The rotation speed of the wheel was to be quite high: 100 rpm (1.7 Hz), much higher than the mHz frequency planned for μSCOPE and STEP, and close to the 2 Hz nominal spin rate of GG. The proposed experiment [22] would have test cylinders with low natural frequencies, i.e. weakly coupled (this is mandatory in EP testing, because the test masses must be sensitive to small forces)

and −at the same time− a fast spin frequency. Although the authors may not have been aware of it, this mechanical system is known to be highly unstable [23, Ch. 6]. The test cylinder, because it is constrained to move on a straight line, will not be able to self center on the rotation axis, and is therefore destined to "fly away", as shown by a simple example in [23, Ch. 6 p. 227]. No matter how precisely it is manufactured and mounted, any tiny offset of its center of mass from the spin axis is destined to grow (till the experiment is terminated by the test masses hitting their cage) because no equilibrium position exists for this system.

For the same reason, based on physical laws (and not for technical difficulties which might eventually be overcome), neither µSCOPE nor STEP can use fast rotation, because the test cylinders would be highly unstable if spinning faster than their (low) natural frequencies. As a matter of fact, the rotation of µSCOPE in the current mission baseline is very slow (1570 s period relative to inertial space). In STEP, where the test mass suspension is based on magnetic levitation, the common mode and differential mode natural oscillations have periods of 1470 s and 1130 s respectively; as for spacecraft rotation, it is set to be a factor 2.72 faster than its revolution around the Earth, resulting in a rotation period of 2000 s (according to a recent numerical simulation of the system, and for an orbiting altitude of 500 km [21]). Thus, a rotation frequency of the STEP spacecraft slightly slower than its common and differential mode frequencies avoids instability; but there is almost no margin left for faster rotation because this is limited to be (strictly) slower than 1/1470 Hz. Otherwise, active control would be needed in order to maintain the masses at a given (arbitrary) relative position. At fast spin this would require compensation of centrifugal forces (proportional to the spin frequency squared) many orders of magnitude larger than the signal.

Weak coupling and fast rotation can be reconciled if the test bodies spin around their symmetry axis −which after all is the natural choice for bodies of cylindrical symmetry− with the plane perpendicular to it sensitive to small forces and the centers of mass free to move in this plane (not in one dimension only). In this case an equilibrium position is known to exist, and to be closer to the spin axis than the original offset by construction and mounting (by the ratio, squared, between the natural and the spin frequency) [23, Ch. 6]. Whirl unstable motions are also known to develop around this equilibrium position, but they have the natural (not the spin) frequency and their growth rate decreases as 1 over the quality factor of the system at the spin frequency [24]. For GG, evidence that whirl instabilities are very slow and can be controlled by very small forces proportional to the natural (not the spin) frequency squared, and inversely proportional to the quality factor of the system, comes from theoretical analysis [25-26, 8], numerical simulations [8, 6 Ch. 6] and laboratory tests of a ground prototype [27].

IV   CONCLUSIONS

We have analyzed the radiometer effect in the µSCOPE space experiment, due to fly in 2004 for testing the equivalence principle to 1 part in 10^{15} in the gravitational field for the Earth. By comparison with the heliocentric LISA mission we conclude that in µSCOPE the radiometer effect is likely to be larger than the signal, and directly competing with it. We point out that the second µSCOPE accelerometer whose test cylinders are made of the same material for checking of systematic effects, will not check −in its current design− for the radiometer one. It is therefore impossible to know with confidence at what level an "observed" signal should be discarded because of the radiometer effect instead of being announced as an EP violation, which would invalidate General Relativity. It seems appropriate for the µSCOPE mission to be reconsidered, with specific attention to this issue and possibly with a less ambitious goal in testing the equivalence principle, closer to the results obtained by ground based torsion balance experiments [17].

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