8. EXPERIMENT AND MISSION REQUIREMENT RELEVANT FOR ATTITUDE AND DRAG FREE CONTROL DESIGN

8.1 From scientific requirement to DFC requirement

8.1.1 XY plane

To derive the major DFM requirements starting from scientific requirements, it is necessary to introduce several assumptions related to the approach that will be followed to detect and estimate the Equivalence Principle violation, taking into account the payload characteristics.

Let be:

T : interval of observation;

Δf : frequency resolution,

$$\Delta f = \frac{1}{T}$$

A : amplitude of the proof masses differential acceleration due to the EP violation that it is necessary to detect in reliable way (8.37 10-17 m/s^2);

D : amplitude of the proof masses differential displacement due to the EP violation that it is necessary to detect in reliable way;

δD : residual of all other environmental harmonic disturbances (space environment and spacecraft) at spinning rate;

$\hat{D}$: estimated amplitude of the EP violation that it is necessary to detect in reliable way;

N : spectral density of the overall environmental disturbances noise (space environment and spacecraft, capacitors measurement noise) affecting the EP violation measurement,

χ_{CMRR} : balance displacement common mode rejection ratio (CMRR) rejection in XY spacecraft axes

χ_{DFC} : drag-free displacement rejection on XY spacecraft axes

N_T : thrusters force noise spectral density (N/√Hz) around spin rate

N_{MDF} : spectral density of the measurement error of the relative position between PGB and spacecraft (m/√Hz) around spin rate

N_{MS} : spectral density of the measurement of the relative position variation between the test masses

α_1 , α_2 : apportionment coefficients

β_1 , β_2 , β_3 : apportionment coefficients

Without loss in the generality, it has been considered that the detection of the EP violation will be based on the computation of the spectrum of the signal acquired by PGB capacitors in real time. The use of other approaches like Maximum likelihood provides the same conclusions.

The estimated amplitude of the EP violation is affected by residual of environmental disturbances and measurement:

$$\hat{D} = \sqrt{D^2 + \delta D^2 + 2N^2 \Delta f}$$

$$\hat{D} = D\sqrt{1 + \left(\frac{\delta D}{D}\right)^2 + \frac{2N^2 \Delta f}{D^2}}$$

The amplitude of the proof masses displacement is related to the acceleration induced by EP violation according to the relation:

$$D = \frac{A}{(\omega_{dm})^2}$$

$$\omega_{dm} = \frac{2\pi}{T_{dm}}$$

D is as much lower as ω_{dm} is higher and as T_{dm} is lower.

Since it shall be $\frac{2N^2 \Delta f}{D^2} << 1$ and $\left(\frac{\delta D}{D}\right)^2 << 1$, a first order approximation may be considered:

$$\hat{D} \cong D\left(1 + \frac{1}{2}\left(\frac{\delta D}{D}\right)^2 + \left(\frac{N^2 \Delta f}{D^2}\right)\right)$$

$$\frac{\hat{D}}{D} \cong 1 + \frac{1}{2}\left(\frac{\delta D}{D}\right)^2 + \frac{N^2}{D^2 T}$$

$$\frac{\hat{D} - D}{D} = \frac{\Delta D}{D} \cong \frac{1}{2}\left(\frac{\delta D}{D}\right)^2 + \frac{N^2}{D^2 T}$$

From above formulas, each error contributors shall be a fraction of the allowed relative error. Let α_1 and α_2 be these fractions. The following formulas read:

$$\frac{1}{2}\left(\frac{\delta D}{D}\right)^2 \le \alpha_1^2 \frac{\Delta D}{D}$$

$$\frac{N^2}{D^2 T} \le \alpha_2^2 \frac{\Delta D}{D}$$

$$\alpha_1^2 + \alpha_2^2 \le 1$$

The upper bounds for harmonic and random disturbances at and around spinning frequency shall be such that:

$$\delta D \leq D\sqrt{2\alpha_1^2 \frac{\Delta D}{D}}$$

$$N \cong D\sqrt{T\alpha_2^2 \frac{\Delta D}{D}}$$

δD results from:

- Residual drag and solar pressure compensation left by drag-free controller and balance CMRR rejection;
- Residual harmonic disturbance at spinning rate introduced by the spacecraft (e.g. thrusters mechanical noise due to open/close of valves if any, etc.) and left by drag-free controller and balance displacement common mode rejection ratio (CMRR) rejections.

$$\chi_{CMRR}\,\chi_{DFM}\frac{(a_D + a_T)}{(\omega_{dm})^2} \leq \delta D$$

N depends on:

- displacement measurement noise (electronic chain, ADC quantization, etc.);
- residual effects of drag-free control :
  - residual effects after CMRR rejection and DFC rejection of thrusters noise;
  - residual effects after CMRR rejection and DFC rejection of thrusters resolution.

Then

$$\sqrt{(\chi_{CMRR} N_{DFM})^2 + N_{MS}^2} \leq N$$

where

$$(\chi_{CMRR} N_{DFM})^2 = \left(\chi_{CMRR}\chi_{DFM}\frac{1}{(\omega_{dm})^2}\frac{N_T}{m_S}\right)^2 + (\chi_{CMRR} N_{MDF})^2$$

The random error shall be

$$\sqrt{\left(\chi_{CMRR}\,\chi_{DFM}\frac{1}{(\omega_{dm})^2}\frac{N_T}{m_S}\right)^2 + (\chi_{CMRR} N_{MDF})^2 + N_{MS}^2} \leq N$$

$$\chi_{CMRR}\,\chi_{DFM}\frac{1}{(\omega_{dm})^2}\frac{N_T}{m_S} \leq \beta_1 N$$

$$\chi_{CMRR} N_{MDF} \leq \beta_2 N$$

$$N_{MS} \leq \beta_3 N$$

$$\beta_1^2 + \beta_2^2 + \beta_3^2 \leq 1$$

Considering the balance characteristics and the apportionment rules provided in Table 8-1, it is possible to derive:

$$\chi_{DFM} < 2.4 10^{-5}$$
$$N_{MS} \leq 0.5 10^{-12} \text{ m}/\sqrt{Hz}$$
$$N_{MDF} \leq 0.5 10^{-6} \text{ m}/\sqrt{Hz}$$
$$N_T \leq 0.06 \text{ N}/\sqrt{Hz}$$

The maximum thruster's noise shall also be compatible with the requirement on gap between PGB and spacecraft (0.15mm). The spectral density of the thrusters noise shall be:

$$N_T \leq \frac{\sqrt{2\pi}\, d_{PGB\_SP}\, \omega_O^{3/2} m_S}{\sqrt{p}} \quad \text{N}/\sqrt{Hz}$$

where:

d_{PGB_SP} : standard deviation of the allowed displacement

p : multiple of the suspension bandwidth (p>2)

$$N_T \leq \quad 120\,\mu\text{N}/\sqrt{Hz}\ (p = 4)$$

Table 8-1 – Parameter considered for DFM requirement derivation at 2Hz spin rate

No	Parameter	Symbol	Unit	Value	Comments
1	Minimum observable differential acceleration due to the EP violation	A	m/s^2	$8.37\ 10^{-17}$	
2	Balance differential period	T_{dm}	s	≥ 500	The lower admissible value shall be considered
3	Balance differential angular frequency		rad/s	0.0126	
4	Proof masses differential displacement due to the minimum observable EP violation	D	m	$0.53\ 10^{-12}$	

5	Maximum allowed relative uncertainty acceptable in the EP violation detection	$\Delta D / D$		0.3	TBC
6	Maximum perturbing environmental force	a_D	m/s^2	$\leq 0.2\ 10^{-6}$	
7	Maximum perturbing spacecraft force	a_T	m/s^2	$\leq 0.02\ 10^{-6}$	
8	Apportionment coefficient	α_1		0.7071	
9	Apportionment coefficient	α_2		0.7071	
10	Apportionment coefficient	β_1		0.07053	
11	Apportionment coefficient	β_2		0.07053	
12	Apportionment coefficient	β_3		0.00705	
13	PGB-spacecraft displacement variation due to overall thrusters noise	d_{PGB_SP}	m	$30\ 10^{-6}$	
14	Satellite mass	m_S	kg	400	20% margin

8.1.2 Z spin axis

The movement of the test-masses along Z direction, due to coupling with gravity gradient, introduces an additional differential acceleration in the plane XY that is not separable with respect to the EP violation.

The acceleration along Z axis is due to the fact that the satellite spin-axis is not orthogonal to the orbital plane. In preliminary way, for requirement derivation the assumed worst case value of the tilt angle is 10deg (about 5deg is the orbit plane inclination, 5deg as very conservative maximum spin-axis de-pointing). At 10deg, the maximum expected magnitude for the acceleration along Z axis is equal to 3.4 10^{-8} m/s^2.

According to [RD 10], the relation between the acceleration along Z axis and the differential one in XY plane is about:

$$A_Z = \frac{A_{XY}}{\left(2\ 10^{-6}\right)\chi_{DFM\_Z}\ \chi_{CMRR\_Z}}$$

The same relation is applicable to the displacement:

$$D_Z = \frac{D_{XY}}{\left(2\ 10^{-6}\right)\chi_{DFM\_Z}\ \chi_{CMRR\_Z}}$$

Considering:

- The maximum allowed acceleration A_{XY} due to the Z-axis movement and coupling with gravity gradient not greater than 5 10^{-10} m/s^2;
- $\chi_{CMRR\_Z} < 0.02$;

- The magnitude of the acceleration along Z axis A_Z < 4 10^{-8} m/s^2

then χ_{DFM_Z} shall be not greater than 1/200.

Starting from the apportionment for the random error considered for XY plane, but considering the effect of random error lower than 1/10 of the measurement error (negligible), it is possible to derive:

$$D_Z = \frac{0.05\,10^{-12}\ \mathrm{m}/\sqrt{Hz}}{\left(2\ 10^{-6}\right)\ 2 10^{-2}} < 1.25\ 10^{-6}\,\mathrm{m}/\sqrt{Hz}$$