

THE NON-GRAVITATIONAL ACCELERATIONS OF THE CASSINI SPACECRAFT

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ABSTRACT

Accurate orbit determination of interplanetary spacecraft requires good knowledge of non-gravitational accelerations. Unfortunately, design and ground tests do not provide enough information for accurate modeling. Indeed, effects such as solar torque induced thruster firings and variations of thermo-optical coefficients, controlling solar radiation pressure, cannot be predicted very accurately and require an in-flight estimation. The accuracy of such determination relies on good quality radio-metric data. Cassini, a large planetary mission devoted to the study of Saturn and its satellites, is endowed with a unique radio system, enabling highly stable, coherent radio links to ground in X- and Ka-band. The leading Cassini non-gravitational accelerations are due to solar pressure and anisotropic thermal emission from the three on-board radioisotope thermoelectric generators (RTGs). With an initial area-to-mass ratio of about $0.0023 \text{ m}^2 \text{ kg}^{-1}$, the effect of the solar radiation pressure decreased below that of the RTGs at about 2.7 AU. At the distance of the first radio science cruise experiment (6.65 AU) it was about $5 \times 10^{-13} \text{ km s}^{-2}$, approximately 1/6 of the acceleration due to the RTGs. The separation of the two sources of non-gravitational accelerations is complicated by the use of thrusters to control the spacecraft attitude. In cruise, momentum wheels were employed only during radio science experiments, around three solar oppositions (2001, 2002, and 2003) and a solar conjunction (2002), and for approximately nine months prior to Saturn orbit insertion. These are the only dynamically clean periods where a good estimate of non-gravitational accelerations is possible. An additional difficulty is caused by the time variations of the thermo-optical coefficients of the 4m high gain antenna (HGA) coating, which exceeded the pre-launch expectations. These variations have been determined (and accounted for) thanks to telemetry data from two temperature sensors mounted on the HGA.

The orbital solutions singled out a large residual acceleration along the spacecraft longitudinal or ‘Z’ axis ($3 \times 10^{-12} \text{ km s}^{-2}$) that is consistent with anisotropic thermal emission from the RTGs. This acceleration requires a thermal emission of about 4.3 kW ($\sim 30\%$ of the 13 kW total thermal power) along this axis. Such a large anisotropy is fully compatible with the presence of heat shields protecting the spacecraft bus from the RTGs. After Saturn orbit insertion (July 2004) and the release of the Huygens probe (December 2004), the spacecraft mass decreased by 1.8 tons (40% of the total mass), leading to a significant increase of the non-gravitational accelerations. Using two years of Doppler data acquired during the Saturn tour, we determined updated values of Cassini’s non-gravitational accelerations and found that the changes are consistent with the mass variation.

We discuss these findings also in relation to possible inconsistencies caused by violations of Newton-Einstein gravity laws.

INTRODUCTION

On October 15, 1997 Cassini/Huygens, the largest and most complex planetary mission ever undertaken, began its seven year journey to the Saturn system, its final destination. In the early phase of the mission, Cassini carried out two Venus gravity assists (on Apr. 26, 1998 and June 24, 1999) and suffered the thermally adverse environment of the inner solar system. Later gravity assists with the Earth and Jupiter occurred on Aug. 18, 1999, and Dec. 30, 2000. The fixed, 4m diameter, high gain antenna (HGA) was constantly pointed to the Sun and worked as a heat shield, ensuring the safety of the bus instrumentation. Communications with NASA's Deep Space Network (DSN) were provided by two low gain antennas (LGAs), operating at X-band (7.2-8.4 GHz). During most of the cruise phase the spacecraft attitude was controlled by a set of balanced thrusters, therefore perturbing the dynamical coherence of the orbit. Thruster firings, combined with the unavailability of the HGA, made an accurate determination of non-gravitational accelerations impossible. Additional complications came from variations of thermo-optical coefficients of the HGA paint, controlling solar radiation pressure.

The first accurate measurement of non-gravitational accelerations was possible only after the Jupiter encounter, when four radio science experiments, carried out during a solar conjunction (June 6, 2002, to July 5, 2002) and three solar oppositions (Nov. 26, 2001 to Jan. 4, 2002, Dec. 6, 2002 to Jan. 14, 2003 and Nov. 10 to Nov. 30, 2003), required about 130 days of HGA Earth-pointed attitude. A full two-way Ka-band radio link, used for the first time in a planetary mission, required fine attitude control and the use of momentum wheels. The two leading non-gravitational accelerations were due to the solar pressure and non-isotropic thermal emission from the three on-board radioisotope thermoelectric generators (RTGs). As a first approximation, the latter can be assumed as constant in the body frame (the half-life of Pu^{238} is 87.7 years, much longer than the cruise duration), while solar pressure effects decrease as the inverse square of the heliocentric distance. During cruise, when the HGA was pointed to the sun and shielded the spacecraft bus, the initial area-to-mass ratio was $0.0023 \text{ m}^2 \text{ kg}^{-1}$, resulting in a solar pressure acceleration roughly equal to $2 \times 10^{-11} \text{ km s}^{-2}$ at 1 AU. The acceleration due to solar radiation decreased below that of the RTG's at about 2.7 AU, while at the distance of the first radio science experiment (6.65 AU) it was about $5 \times 10^{-13} \text{ km s}^{-2}$. As discussed later, the orbital solution singled out a residual acceleration along the spacecraft Z axis ($\approx 3 \times 10^{-12} \text{ km s}^{-2}$), which required a net thermal emission of about 4.3 kW ($\sim 30\%$ of the 13 kW total thermal power). Such a large anisotropy is fully compatible with the presence of heat shields protecting the spacecraft bus from RTGs.

On July 1, 2004 Cassini/Huygens successfully completed the Saturn orbit insertion (SOI) maneuver and was gravitationally captured by Saturn. About six months later (Dec. 25, 2004), the Huygens probe separated from the mother spacecraft and landed on Titan's surface just 20 days later (January 14, 2005). Cassini's large mass decrease led to a significant increase of all non-gravitational accelerations, providing the opportunity to test the force models and separate the contributions from different sources of non-gravitational accelerations. The Cassini navigation team (NAV) updated the estimates of the non-gravitational accelerations, finding them fully compatible with the variation of the mass. However, in the Saturn system the dynamical model is more

complex than in cruise and a variety of factors, such as planetary radiation pressure and incomplete knowledge of the gravity fields of Saturn system bodies, made the estimation less straightforward. In order to account for the inaccuracies of the dynamical model, the navigation filter made use of stochastic force models to limit adverse effects in the fit of radio-metric data. In this setup, data from each single Cassini orbit about Saturn (with periods ranging from 16 to 48 days) could be consistently fitted.

The estimate of the non-gravitational accelerations can avoid stochastic models if a multi-arc approach is used. Short data arcs (typically one six hour pass) are individually fitted and later combined in a solution (one for each orbit) providing both local and global parameters. Local parameters are all spacecraft state vector components at the beginning of the data arc, while global parameters include, as a minimum, the components of the RTG's acceleration in the spacecraft frame. All unmodeled accelerations are effectively absorbed by the over-parametrization of the solve-for parameters, i.e. the Cassini state vectors at the reference epochs of each arc. This approach allowed a good estimate of the spacecraft Z axis component of RTG acceleration. This appears to be by far the largest component, as expected from the geometry of the heat shields of the spacecraft. The results obtained with both methods are in good agreement, thus increasing the confidence in the estimation and the associated formal errors.

This paper shows the results accomplished to date in the analysis of two-years of Doppler data. Section 1 describes Cassini's orbit determination in the Saturn system and the multi-arc method; section 2 describes the general model of Cassini's non-gravitational accelerations, the data set used in the orbital fit and the results obtained so far.

1. ORBIT DETERMINATION IN THE SATURN SYSTEM

On July 1, 2004 a 627 m/s maneuver, requiring nearly one ton of bi-propellant (about 19% of the total mass), inserted Cassini/Huygens into a highly eccentric, 116 day orbit around Saturn. A periapsis raise maneuver (PRM) of 393 m/s was executed 53 days after SOI to eliminate another ring crossing and to set up the first targeted encounter with Titan. These events mark the beginning of an in-depth exploration of the Saturn system, which produced a wealth of scientific discoveries. The prime mission, completed on June 30, 2008, four years after SOI, included 74 orbits around Saturn, 45 Titan flybys and several encounters with other icy satellites. In the early phases of the tour Cassini released the Huygens probe, which made contact with Titan's atmosphere on Jan. 14, 2005 and successfully completed its two hour parachute descent. In June 2008 Cassini began a two-year extended mission, lasting to July 2010, with 60 additional orbits, 26 Titan flybys, 7 Enceladus flybys and one flyby each of Dione and Rhea. A further extension to Saturn's solstice, occurring in 2017 is currently under evaluation by NASA. Such a long and complex mission was made possible by careful design and accurate implementation of Titan's gravity assists, which minimized propellant consumption while meeting the scientific requirements of the mission. Titan was not only one of the main objectives of the mission, but was also the engine that made it possible. Indeed, a typical close flyby can change Cassini's velocity by as much as 800 m/s relative to Saturn, enough to allow a wide variety of orbital changes. SOI maneuver, PRM, and Huygens release decreased the spacecraft mass by about 1.8 tons (~ 40% of cruise mass), leading to a significant increase of all non-gravitational accelerations, whose knowledge is crucial for an accurate orbit determination. Updated estimates of RTG accelerations were produced by the orbit determination process and found consistent with the change in the spacecraft mass. However, an accurate estimation in the

complex Saturn system environment is complicated by inaccurate gravity field models, thruster firings, time variation of thermo-optic coefficients of spacecraft surfaces and unmodeled accelerations due to planetary radiation and non-isotropic thermal emission from spacecraft instrumentation. In order to compensate for model errors, the Cassini navigation team (NAV) includes stochastic dynamical processes. Accelerations up to $\sim 5 \times 10^{-13} \text{ km s}^{-2}$ (the magnitude of the solar pressure acceleration at Saturn) are modeled in 8 hour batches, except during close flybys or maneuvers, when this level can be increased by several orders of magnitude. These compensating noise processes could in principle affect RTG acceleration estimates and the associated uncertainties, requiring a more detailed analysis. We will however show that the estimates obtained by adopting state noise compensation are statistically equivalent to the ones provided by a deterministic, multi-arc model.

Cassini is endowed with a unique radio system, enabling highly stable, coherent radio links to ground in X- and Ka-band frequencies. During cruise radio science experiments Doppler data were acquired in a novel multi-link configuration, where a 34 m DSN antenna in Goldstone (DSS 25) transmitted and received simultaneously at X and Ka band frequencies (uplink at 7.2 and 34 GHz, downlink at 8.4 and 32.5 GHz, respectively). Three coherent links were established: X uplink/X downlink, X uplink/Ka downlink and Ka uplink/Ka downlink. With three Doppler observables available, plasma noise from the solar corona could be fully removed (in the geometric optic limit) even at impact parameters of 8 solar radii [1]. In addition, tropospheric wet path delay calibrations were available from an advanced water vapor radiometer at the DSS 25 site. The media calibration system (plasma noise cancellation and removal of tropospheric effects) proved crucial in a test of general relativity carried out during Cassini's 2002 solar conjunction. The test verified the validity of general relativity to one part in 50000, improving by a factor of 50 over previous tests [2]. Although the multi-link configuration was available only at one ground antenna (DSS 25), two-way coherent tracking at X-band was provided around the clock by the DSN complexes in Spain and Australia in all cruise radio science experiments.

The use of Ka band radio link provided Doppler measurements of unprecedented accuracy [3]. The Allan deviation of Doppler residuals at 1000 s integration time was 1.5×10^{-14} at solar conjunction and 1×10^{-14} or less at solar opposition. These frequency stabilities correspond to one-way range rate errors of $1.5\text{--}2 \times 10^{-6} \text{ m s}^{-1}$. During the four cruise radio science experiments the spacecraft was kept in a dynamically quiet state, with the antenna boresight constantly pointed to earth by using a momentum wheel control system. No onboard activities were allowed, and all instruments and subsystems remained in the same state throughout the whole duration of the experiments. This favorable configuration allowed an excellent estimation of the spacecraft state and the non-gravitational accelerations.

Unfortunately a malfunction of the onboard Ka band frequency translator caused the loss of the Ka/Ka radio link after July 2003, so that high accuracy data were not available in the Saturnian phase and during the third cruise solar opposition (Nov. 2003). Orbit determination was carried out by using X band uplink/X band downlink Doppler and range data. Ka band Doppler data from the X/Ka radio link were sporadically available. Although the overall quality of Doppler data was at least a factor of 3 worse than in cruise, the availability of a large set of tracking data with the attitude controlled through momentum wheels allowed for good orbit determination and an estimate of the non-gravitational accelerations with accuracies not far from the ones obtained in cruise. The available radio-metric data also provided an accurate determination of the gravity fields of Saturn and its satellites, as well as a large improvement in the ephemerides of the Saturn system bodies [4].

In normal operations, Cassini is tracked from ground for about nine hours per day. The nearly 3 hour round trip light travel time reduces the amount of coherent tracking data to about 6 hours per day.

1.1. NAV Team Orbit Determination

The Cassini Navigation Team's estimates of cruise phase non-gravitational accelerations were based on a single 3.5 year data arc spanning the time period from just after the Jupiter flyby through SOI. Because of staffing constraints, software limitations, and more relaxed targeting requirements in interplanetary cruise, stochastic accelerations with large a priori uncertainties and eight hour batches were implemented over most of the arc in order to simplify modeling. Using stochastic accelerations, models of short-term attitude excursions became unnecessary and orbital perturbation models of thruster based attitude control were greatly simplified. While reducing workforce requirements, this lack of rigorous modeling also had the negative consequence of not fully exploiting the information content of the tracking data. Nevertheless, gravity wave experiments contained within the data arc enabled an accurate estimate of the acceleration due to non-isotropic thermal emission along the axis pointed toward the Earth. During three gravity wave experiments, Cassini's attitude was maintained on Earth point without thruster firings for 40, 41, and 20 continuous days, allowing accurate orbit modeling without the use of stochastic accelerations. The NAV estimate for this component of the acceleration is $(-3.01 \pm 0.02) \times 10^{-12} \text{ km s}^{-2}$ (referred to the cruise mass).

Targeting requirements during the Saturn orbital phase of the mission are much more stringent. Non-gravitational accelerations are estimated from data arcs spanning intervals of at least 1.5 revs about Saturn. Stochastic accelerations are once again estimated, but their a priori uncertainties are more than an order of magnitude smaller than during cruise, allowing small modeling errors, mostly from the solar pressure reflectivity model, to be absorbed. Spacecraft attitude is maintained with reaction wheels over the vast majority of the data arc and all attitude excursions are accurately modeled. Uncertainties in estimates of the non-isotropic non-gravitational acceleration radial component are typically reduced by a factor of 3 to 4 over the cruise based a priori uncertainties. Estimated values normalized to the spacecraft mass during the Titan 7 encounter are shown in Fig. 1.

1.2. Multi-arc analysis

A multi-arc approach was used to get independent estimates of the RTG's acceleration. In this method, orbital fits were obtained from short data arcs (~ 6 hours per tracking pass), without the need of dynamic noise compensation that may interfere with the estimates of the RTG's acceleration. Potential deficiencies of the dynamical model are absorbed by the over-parametrization of the problem. In the multi-arc technique the dynamical set of parameters is separated into two groups: global parameters common to all arcs, for which an explicit model is available, and local ones introduced to absorb effects of local dynamics, which cannot be adequately modeled on a global scale. In principle, the initial conditions are not constrained by a state propagation from the previous arc [8]. As our goal was the determination of the RTG's accelerations with a deterministic approach, state vectors at the beginning of each tracking pass

have been obtained by mapping Cassini's reconstructed trajectory (obtained using the method described in the previous section) at different epochs.

The dynamical model used in the multi-arc orbital fit includes the gravitational accelerations from Saturn and its satellites, as well as those from other planets and satellites; the gravitational parameters and Saturn system ephemerides, as well the first four zonal coefficients of Saturn, were adopted from the satellite ephemerides consistent with NAV's reconstruction. Satellite ephemerides are continuously updated by the NAV team, and new releases are provided after each revolution of the spacecraft about Saturn. After August 2005, the Saturn GM estimate stabilized close to the final value of [4], with a remarkable variation of about $10^2 \text{ km}^3 \text{ s}^{-2}$ from the initial estimates. While the NAV filter, making use of state noise compensation, absorb this effect without biasing the estimates of the non-gravitational accelerations, the multi-arc approach (with the given initial conditions and state covariance matrices) leads to corrupted estimates from Jan. 2005 to Aug. 2005 until the improved Saturn system ephemerides were used.

The dynamical model also includes non-gravitational accelerations from the RTG's anisotropic thermal emission and from solar pressure. Arcs including orbital maneuvers or thruster operations were excluded from the combined, multi-arc, solution. Relativistic perturbations of Saturn, Jupiter and the Sun are also taken into account, while the trajectories have been integrated in the Saturn-system barycentric reference frame by means of the JPL Orbit Determination Program (ODP). The computed observables and the propagated partial derivatives were also obtained from the same code. The remaining part of the process has been carried out using separate numerical codes (developed under contract from the Italian Space Agency). The information matrices from each arc are linearly accumulated in a global information matrix, whose inversion is part of the estimation process which provides central values of estimated parameters along with their uncertainties in the form of a full covariance matrix. Since this inversion is one of the main problem in the orbit determination procedure (the information matrix is often ill-conditioned) and requires special numerical techniques, we used a square root formulation of the weighted least square method for the multi-arc batch data filter [12].

In order to exploit the improved knowledge of Saturn system ephemerides and provide a coherent basis for the multi-arc solution, a new set of single arc orbital fit was obtained by using the same ephemerides (sat303 [7]) for all data arcs. In addition, updated values of the high gain antenna thermo-optical coefficients were used in the solar pressure model, as discussed below. As the thrust from the RTG's anisotropic thermal emission is nearly constant over the time span considered, a single estimate of global parameters over the entire set data span would be possible, were it not for the dependence of the non-gravitational accelerations on the time-varying mass of the spacecraft. As the ODP allows the estimate of accelerations, and not forces, we obtained a separate multi-arc solution for each revolution of Cassini about Saturn, where the spacecraft mass was considered to be constant. This data processing produces an estimate of the RTG accelerations per revolution, by combining up to 38 separate tracking passes.

The vector of estimated parameters is made up by Cassini state vectors (local parameters) and RTG accelerations (global parameters). Although the multi-arc method may provide a single value for the RTG accelerations, estimates were obtained for each spacecraft orbit about Saturn. This choice allows an immediate comparison with the results obtained in the NAV setup. Other parameters (see Tab. 2) are not solved-for but treated as consider parameters and their uncertainty is accounted for in the final covariance matrix. We used very large a priori uncertainties for RTG accelerations (more than 100% of their expected value) in the form of a diagonal matrix. The a

priori covariance for the state vector components at the beginning of each arc was generated by mapping the NAV orbital reconstruction to the epoch of interest. In order to avoid excessive constraints on the multi-arc solution, the a priori uncertainties derived from the NAV state covariance matrices have been scaled by a factor of 5^2 . The uncertainty of magnetometer boom optical coefficients is approximately 100% of their nominal value, while the a priori uncertainty of the HGA's diffuse reflectivity coefficient is large enough to be compliant with the value used by NAV (in the orbit reconstruction), different from that one used in the multi-arc solution and inferred by temperature sensors (see next section and Appendix). Also the gravitational parameters (GM) of Saturn and its major satellites have been included in the final solution as consider parameters and their uncertainty accounted for (see Tab. 2 and [5]).

Doppler observables have been weighted with the mean standard deviation of the residuals from the single tracking pass, increased by 10%, such that the SOS (Sum of Squares) is never greater than the number of Doppler data, which are never over-weighted.

2. NON-GRAVITATIONAL ACCELERATIONS

The good quality of Cassini's radio-metric observables allowed an accurate estimation of the main sources of non-gravitational accelerations. Although engineering models can be developed for the non-gravitational accelerations, their accuracy is generally limited by the incomplete knowledge of some crucial quantities, such as optical coefficients of the external surfaces. Almost invariably, the models contain adjustable parameters to be estimated in-flight, and Cassini makes no exception. In general one can distinguish between two different classes of non-gravitational accelerations: impulsive and continuous.

Impulsive accelerations, due essentially to thruster firings, are by far the largest ones, but with a typical duration of few minutes they have a peculiar signature in the observation residuals; nevertheless they affect the dynamical coherence of the orbit and if not properly estimated can affect the estimation of other parameters. In the NAV orbit determination all impulsive and finite burn maneuvers are part of the estimation process. In the multi-arc approach, where the goal is the determination of global parameters, all data arcs containing thruster operations have been discarded.

Continuous non-gravitational accelerations are due to solar radiation pressure, planetary albedo and anisotropic thermal emission from the RTGs and the spacecraft bus. As Cassini's Saturn pericenter passes are never closer than 3.5-5 Saturn radii and most of the time is spent at much larger distances from the planet, albedo is by far the smallest source and its magnitude is [13]

$$a_{PR} = \frac{A}{M} \frac{\Phi}{c} \left(\frac{1}{9.2} \right)^2 A_S \left(\frac{R_S}{r} \right)^2 \approx 10^{-16} \text{ km s}^{-2} \quad (1)$$

where A_S and R_S are the albedo and radius of Saturn, r is Saturn-Cassini distance and A is the frontal surface. The numerical value is obtained assuming $A_S = 0.5$, $r = 10^6$ km and $A=10 \text{ m}^2$. Near Saturn pericenter passes this acceleration can increase at most to $\sim 5 \times 10^{-15} \text{ km s}^{-2}$, resulting always three-four orders of magnitude less than that from the RTG. Both in the NAV and multi-arc approach the

pressure from planetary albedo is not modeled. In the NAV model its effects are accounted for as stochastic accelerations.

With a magnitude of $3\text{--}4 \times 10^{-13} \text{ km s}^{-2}$ solar pressure acceleration has appreciable effects on the spacecraft dynamics. Although both approaches (NAV and multi-arc) contain a model of the solar pressure, their accuracy is limited by two factors, namely the imperfect knowledge of the thermo-optical coefficients of the exposed surfaces, and the variable attitude of the spacecraft (Cassini's optical remote sensing instruments are rigidly mounted on the spacecraft and are pointed to their target by rotating the whole spacecraft). The latter effect, in particular, introduces a time variability in the solar pressure that is particularly difficult to model. However, during tracking passes the effect of the solar pressure is much simpler, as the spacecraft bus is shielded from solar radiation by the large, 4 m diameter high gain antenna. The small, annual variation of the sun aspect angle (about 6 degrees) due to the orbital motion of the earth produces a change in the solar pressure by less than 1%. (Such a change is easily accounted for in the model.) In the multi-arc method the accuracy in the solar pressure acceleration is therefore only limited by the knowledge of the HGA thermo-optical coefficients.

The variations of the HGA diffuse reflectivity as inferred from temperature sensors were taken into account in the multi-arc method. During tracking passes, due to the constant pointing of the HGA to earth and the small sun aspect angle, the solar pressure effect is almost entirely directed in the radial direction, which nearly coincides with the z-axis of the spacecraft frame. One has therefore to expect a high correlation with the correspondent RTG's acceleration component. Fortunately the thermo-optical coefficients can be obtained either from the specifications of the HGA paint (emissivity), or from telemetry data of two temperature sensors (diffuse reflectivity coefficient; see Appendix). In a simple order of magnitude calculation, the solar pressure acceleration is given by [13]

$$a_{sp} = (\alpha + 2\varepsilon) \frac{\Phi A \cos \delta}{cMD^2} \quad (2)$$

where α and ε are, respectively, the absorption and reflectivity coefficient of the element, A and M its area and mass, Φ the solar constant, D the distance from the sun and $\cos \delta \approx 1$. At Saturn (~ 9.2 AU) Cassini's solar pressure acceleration (with $M=2700 \text{ kg}$ and $A=12 \text{ m}^2$) is

$$a_{sp} \approx (\alpha + 2\varepsilon) \cdot 2.4 \times 10^{-13} \text{ km s}^{-2} \quad (3)$$

As previously noted, the thermo-optical coefficients are not well known. The main uncertainty is in the diffuse reflectivity, which generally changes in space, under the effect of radiation and outgassing. The solar absorptance of the HGA, related to the optical reflectivity, has been estimated and monitored throughout the mission using temperature readings and a simple thermal model of the element. In any case, an error in the thermo-optic coefficients biases the estimate of the

remaining contributions to the non-gravitational acceleration in the radial direction (due almost entirely to the anisotropic thermal emission from the RTGs). In the multi-arc analysis the diffuse reflectivity has been treated as a consider parameter, with a conservative 70% a priori uncertainty (see Tab. 2).

The three on-board RTGs are the core components of the power and pyrotechnic subsystem (PPS). They provide power to all onboard subsystems through thermoelectric effect driven by the natural radioactive decay of Pu^{238} , whose half life time is 87.74 years. As mentioned in the introduction, before launch the three RTGs had a total thermal fuel inventory of about 13 kW, which requires about 30% of non-isotropic thermal emission to produce the accelerations measured during the cruise phase. The precise fraction of such anisotropy is difficult to predict even if a model of the acceleration due to exponential decay is available. The ODP software models this acceleration as [15]

$$\vec{r} = R_M (A_r \hat{Z}^* + A_x \hat{X}^* + A_y \hat{Y}^*) e^{-\beta \Delta t} \quad (4)$$

where R_M is the ratio of the spacecraft mass at injection to current mass and $\hat{X}^*, \hat{Y}^*, \hat{Z}^*$ are the spacecraft coordinate system vectors. The exponential scale factor β is 2.5^{-10} s^{-1} . An important aspect of the RTG force (force is constant, but acceleration varies inversely with mass) is its constancy in the spacecraft frame (put aside the small exponential decay), making its modeling and estimation easier. To a large extent the anisotropic thermal emission from the other spacecraft components shares this characteristic, so that, de facto, the two sources of non-gravitational acceleration are confused in our analysis. The dissipated thermal power from the spacecraft subsystems amounts to about 700 W, and is emitted in a largely isotropic pattern about the spacecraft long axis (coinciding with the optical axis of the HGA).

Fig. 1 shows the estimates of the radial component (A_r) of the RTG-induced acceleration using NAV and multi-arc analysis. In the multi-arc approach only A_r could be estimated, in the sense that the post-fit uncertainty for the other components (orthogonal to spacecraft long axis) is of the same order of the expected value. The NAV approach, which also indirectly accounts for the effects of non-gravitational accelerations also when the spacecraft is not being tracked, provides a rough estimate (to 30-50% of the central value) of the non-radial components. However, since the radial component dominates, the good agreement found provides a further validation of NAV estimates. The first few Cassini orbits have been excluded from the comparison as Saturn's gravitational parameter was not known with sufficient accuracy. (Reprocessing of all data with the most recent satellite ephemerides is under way).

The central values of the two estimates are always in agreement within the formal accuracies and consistent for all revolutions, with the only exception of H1 (Sept. 2005), where the central value of the multi-arc estimate deviates from the weighted average ($-5.45 \times 10^{-12} \text{ km s}^{-2}$, referred to T7 mass) by 3.3 formal standard deviations.

CONCLUSIONS

The analysis of two years of radio-metric data from the Cassini mission using state noise compensation and deterministic multi-arc model has provided consistent results in the estimate of the radial component of the spacecraft non-gravitational acceleration. This acceleration is attributed almost entirely to anisotropic thermal emission from the three on-board RTGs, with much smaller contributions from the solar radiation pressure and anisotropic thermal emission from the spacecraft bus. The variation of the radial acceleration between the cruise and Saturnian phase of the mission, when the spacecraft mass underwent a 40% change, is fully consistent with known physical laws.

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APPENDIX

Cassini's HGA is fixed in the spacecraft body-fixed reference frame and directed along the $-Z^*$ axis. When the spacecraft is tracked from ground, the solar radiation pressure is therefore almost entirely determined by its geometric and optical characteristics. The HGA is a 4 m diameter paraboloid coated with the PCBZ paint. The pre-launch solar absorptance was approximately 0.15. An in-flight estimation of its post-launch value is possible by means of two temperature sensors, mounted on the HGA back side. These two sensors were never exposed to sunlight as the HGA acted as a thermal shield and was always pointed to either the sun or the earth before reaching Saturn. In a simple model, the estimate is carried out by assuming that the HGA is in thermal equilibrium and by neglecting the finite thermal conductivity of the structure. Under these assumptions, immediately after launch the thermal emissivity ε can be estimated by using the spec values of the paint absorptance α and the temperature readings:

$$\varepsilon = \frac{\Phi \alpha}{\sigma T^4} \quad (5)$$

Referring to an emission in the long-wavelength, thermal infrared, ε can be assumed as constant (the thermal emission properties of a body in space are largely unaffected by radiation and outgassing). Later in the mission, the change in the absorptance can be obtained from the known value of ε and the temperature readings [10]. The relation between the absorptance and the diffuse reflectivity coefficient ν can be inferred considering the flat plate model, as described in [15]. Neglecting the degradation factors and the specular reflectivity coefficient, the radial force due to solar radiation depends on ν and the area A in the combination

$$F_r \propto A(1 + 2 \cdot \nu) \quad (6)$$

Since the total momentum exchange between a solar photon impinging on a surface is proportional to the absorbed flux $\alpha\Phi$ and twice the reflected one, $2(1 - \alpha)\Phi$, the thrust F_r due to solar radiation pressure is proportional to

$$F_r \propto A(2 - \alpha) \quad (7)$$

Finally, comparing the two previous expression, the relation between ν and α is

$$\nu = \frac{(1-\alpha)}{2} \quad (8)$$

As a conclusive remark, the HGA absorptance exceeded the end-of-life specifications, leading to a diffuse reflectivity coefficient and a solar pressure acceleration smaller than expected.

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<i>Orbit</i>	<i>Epoch</i>		<i>N° pass</i>	<i>N° Doppler data</i>	<i>Mass</i>
T7	Aug. 13, 2005	Sep. 1, 2005	18	1106	2781
H1	Sep. 2, 2005	Sep. 13, 2005	10	1140	2767
D1	Sep. 14, 2005	Oct. 1, 2005	16	1234	2767
T8	Oct. 2, 2005	Oct. 19, 2005	18	1362	2739
R1	Oct. 20, 2005	Nov. 15, 2005	22	1842	2726
T9	Nov. 16, 2005	Dec. 13, 2005	25	1701	2710
T10	Dec. 14, 2005	Jan. 4, 2006	20	1539	2710
T11	Jan. 5, 2006	Feb. 3, 2006	29	3572	2709
T12	Feb. 5, 2006	Mar. 9, 2006	33	4514	2709
T13	Mar. 10, 2006	Apr. 8, 2006	28	2144	2708
T14	Apr. 9, 2006	May 9, 2006	26	1979	2707
T15	May 10, 2006	Jun. 10, 2006	28	2148	2706
T16	Jun. 11, 2006	Aug. 27, 2006	38	3799	2704
T18	Aug. 28, 2006	Sep. 16, 2006	14	2794	2698
T19	Sep. 17, 2006	Oct. 2, 2006	14	1390	2689
T20	Oct. 3, 2006	Nov. 28, 2006	23	1535	2683
T21	Nov. 29, 2006	Dec. 7, 2006	8	833	2678
T22	Dec. 8, 2006	Dec. 21, 2006	13	739	2678
T23	Dec. 22, 2006	Jan. 8, 2007	16	2964	2670
T24	Jan. 8, 2007	Jan. 31, 2007	18	1322	2667

Tab. 1 Epochs, n° of passes, Doppler data points and initial mass of the multi-arc solutions.

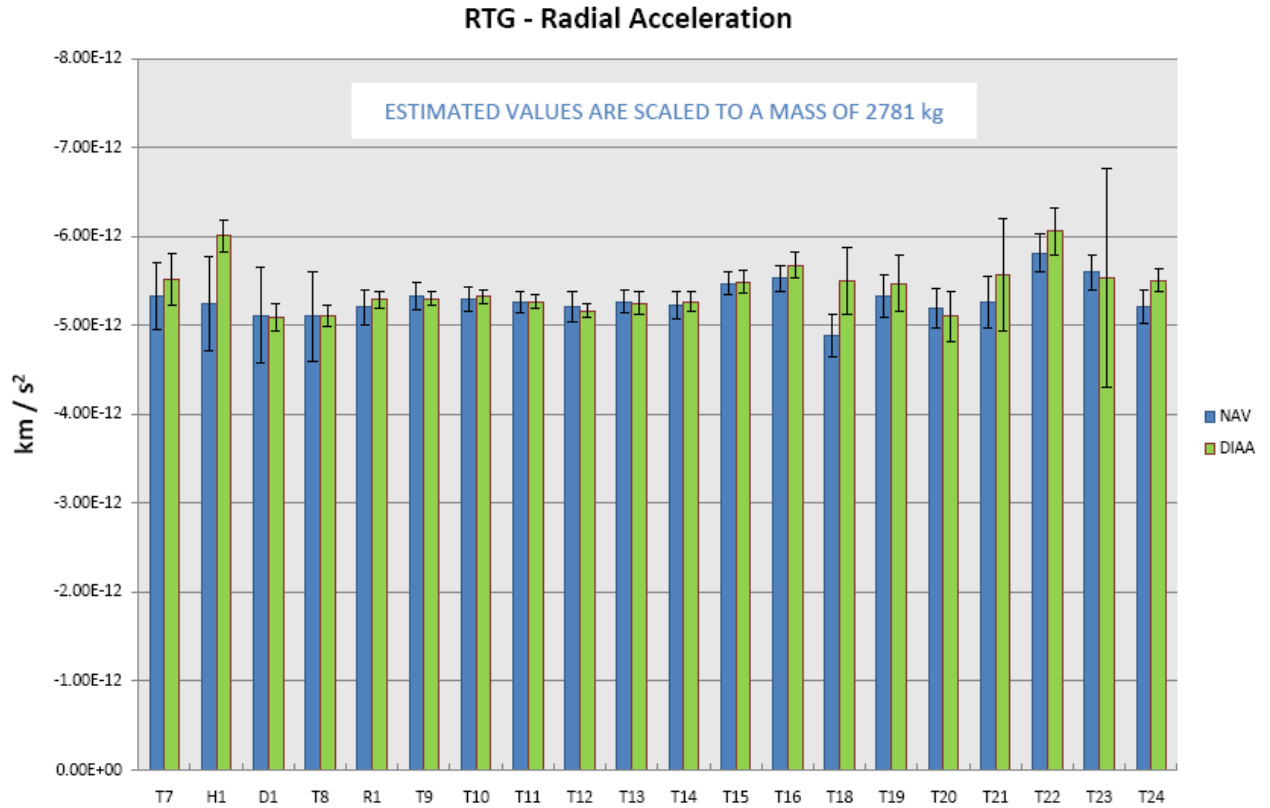


Fig. 1 RTG's radial accelerations. Blue solutions have been obtained by Cassini navigation team as a part of the orbit determination process while green ones are the multi-arc solutions obtained by DIAA.

<i>Global Parameter</i>	<i>A priori Sigma [km s⁻²]</i>	
GLAR	7×10^{-12}	
GLAX	2×10^{-12}	
GLAY	2×10^{-12}	

<i>Consider Parameter</i>	<i>A priori Value</i>	<i>A priori Sigma</i>	
NUF	0.215	0.15	
SPEC08	0.172	0.18	
DIFF08	0.180	0.18	
Saturn GM	37931207.2	km ³ s ⁻²	1.000 km ³ s ⁻²
Mimas GM	2.5024	km ³ s ⁻²	.0006 km ³ s ⁻²
Enceladus GM	7.2052	km ³ s ⁻²	.0125 km ³ s ⁻²
Thetys GM	41.2155	km ³ s ⁻²	.0038 km ³ s ⁻²
Dione GM	73.1148	km ³ s ⁻²	.0015 km ³ s ⁻²
Rhea GM	153.9426	km ³ s ⁻²	.0037 km ³ s ⁻²
Titan GM	8978.1393	km ³ s ⁻²	.0020 km ³ s ⁻²
Hyperion GM	.3722	km ³ s ⁻²	.0012 km ³ s ⁻²
Iapetus GM	120.4988	km ³ s ⁻²	.0080 km ³ s ⁻²
Phoebe GM	.5532	km ³ s ⁻²	.0006 km ³ s ⁻²

Tab. 2 A-priori uncertainties of global parameters and a-priori value and uncertainties of consider parameters. GLAR-X-Y are the RTG's accelerations coded names in the ODP along the Z-X-Y axis. NUF is the HGA diffuse reflectivity coefficient while SPEC08 and DIFF08 are, respectively, specular and diffuse reflectivity coefficients of magnetometer boom.