

## CAN THE PIONEER ANOMALY BE INDUCED BY VELOCITY-DEPENDENT FORCES? TESTS IN THE OUTER REGIONS OF THE SOLAR SYSTEM WITH PLANETARY DYNAMICS

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We analyze the impact of some velocity-dependent forces recently proposed to explain the Pioneer anomaly on the orbital motions of the outer planets of the solar system from Jupiter to Pluto, and compare their predictions (secular variations of the longitude of perihelion  $\varpi$  or of the semimajor axis  $a$  and the eccentricity  $e$ ) with the latest observational determinations by E. V. Pitjeva with the EPM2006 ephemerides. It turns out that while the predicted centennial shifts of  $a$  are so huge that they would have been easily detected for all planets with the exception of Neptune, the predicted anomalous precessions of  $\varpi$  are too small, with the exception of Jupiter, so that they are still compatible with the estimated corrections to the standard Newton–Einstein perihelion precessions. As a consequence, we are inclined to discard those extra forces predicting secular variations of  $a$  and  $e$ , also for some other reasons, and to give a chance, at least observationally, to those models yielding perihelion precessions. Of course, adequate theoretical foundations for them should be found.

*Keywords:* Experimental tests of gravitational theories; modified theories of gravity; celestial mechanics.

### 1. Introduction

The Pioneer anomaly<sup>1</sup> (PA) consists of an unmodeled, almost constant and uniform acceleration approximately directed toward the Sun that is of magnitude

$$A_{\text{Pio}} = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2}, \quad (1)$$

detected<sup>2–5</sup> in the radiometric data from the Pioneer 10 (launched in March 1972) and Pioneer 11 (launched in April 1973) spacecraft after they passed the  $\approx 20$  AU threshold moving with speed  $v_{\text{Pio}} \approx 1.2 \times 10^4 \text{ m s}^{-1}$  along roughly antiparallel escape hyperbolic paths taken after their previous encounters with Jupiter ( $\approx 5$  AU) and Saturn ( $\approx 10$  AU), respectively. Concerning the possibility that the PA started

to manifest itself at shorter heliocentric distances,<sup>6,7</sup> efforts to retrieve and analyze early data from Pioneer 10/11 are currently being made.<sup>8,9</sup>

The Pioneer spacecraft were particularly well suited for radioscience celestial mechanics experiments, because they were spin-stabilized in practice, they could be regarded as gyroscopes so that only a few orientation maneuvers, easily modeled, were needed every year to keep the antenna pointed toward the Earth. On the contrary, three-axis stabilized spacecraft like Voyager 1/2 undergo continuous, semiautonomous, small gas jet thrusts to maintain the antenna facing the Earth; as a consequence, their navigation is not as precise as that of Pioneer 10/11.

The attempts performed so far to explain the PA in terms of known effects of gravitational<sup>3</sup> and/or nongravitational<sup>10,11</sup> origin were found to be not satisfactory,<sup>12,13</sup> so that a vast number of exotic explanations based on modified models of gravity were proposed (see e.g. Refs. 3, 14–16, and references therein). If the PA was due to some modifications of the known laws of gravity, this should be due to a radial extra force affecting the orbits of the planets as well, especially those moving in the region in which the PA manifested itself in its presently known form. The impact of a Pioneer-like additional acceleration on the motion of major and minor bodies in the outer regions of the solar system was recently studied by numerous authors with different approaches<sup>17–22</sup>: it turned out that a constant and uniform extra acceleration with the magnitude (1) would produce huge secular effects which are, instead, absent from the planetary data.

It was recently suggested<sup>23</sup> that, from a purely phenomenological point of view, test bodies moving in the (outer) solar system could experience velocity-dependent extra accelerations of the forms

$$A_v = -|v_r| \left( \frac{A_{\text{Pio}}}{v_{\text{Pio}}} \right), \quad A_v = -v_r \left( \frac{A_{\text{Pio}}}{v_{\text{Pio}}} \right), \quad (2)$$

$$A_{v^2} = -v_r^2 \left( \frac{A_{\text{Pio}}}{v_{\text{Pio}}^2} \right), \quad A_{v^2} = -|v_r| v_r \left( \frac{A_{\text{Pio}}}{v_{\text{Pio}}^2} \right), \quad (3)$$

where  $v_r$  is the radial component of the test particle's velocity  $\mathbf{v}$ ; Eqs. (2) and (3) would reduce to Eq. (1) for the Pioneer 10/11 spacecraft, whose velocities can be assumed to be entirely radial in the outer regions of the solar system in which the PA was detected. Standish<sup>22</sup> put to the test such a hypothesis by fitting huge planetary data sets with the dynamical force models of the latest Jet Propulsion Laboratory (JPL) DE ephemerides modified *ad hoc* according to Eqs. (2) and (3), and examining the results in terms of, for example, the reliability of the estimated parameters. His conclusion was that the existence of extra accelerations like those of Eqs. (2) and (3) at heliocentric distances  $\gtrsim 20$  AU cannot be ruled out by the present-day available data on the outer planets because Eq. (2) and especially Eq. (3) would induce orbital effects on them too small to be detected. Their existence in the inner regions of the solar system is, instead, ruled out.

In this paper we will follow a different approach by using the EPM2006 ephemerides produced by E. V. Pitjeva<sup>24</sup> at the Institute of Applied Astronomy

(IAA) of the Russian Academy of Sciences (RAS). First, we will analytically work out the secular effects of small perturbing accelerations like those of Eqs. (2) and (3) on the Keplerian orbital elements of a planet in order to gain as-clear-as-possible insights into the modifications which the orbits would undergo if Eqs. (2) and (3) were real; should some implausible physical feature turn up, it would be more difficult to trust such proposed anomalous forces. Then, we will compare some of such predictions with the latest observational determinations for the outer planets estimated by Pitjeva with the EPM2006 ephemerides in a purely phenomenological way as corrections to the known effects due to usual Newton–Einstein laws, without modeling any additional force. In Table 1 we quote some quantities which we will use. They are the outcome of a global fit of more than 400,000 data points (1913–2006) performed by Pitjeva<sup>25,24</sup> with the EPM2006 ephemerides; about 230 parameters were estimated. It should be noted that the uncertainties  $\delta\Delta\dot{\varpi}$  in the estimated corrections to the perihelion precessions are the formal ones rescaled by a factor 10 in order to obtain realistic evaluations for them.

## 2. The Orbital Effects of Velocity-Dependent Perturbing Forces Yielding Pioneer-Type Accelerations

### 2.1. Forces linear in velocity

According to the classification of Ref. 22, the first two kinds of extra forces are linear in  $v_r$  being

$$A_v^{(2)} = -|v_r|\mathcal{K}, \quad (4)$$

$$A_v^{(3)} = -v_r\mathcal{K}, \quad (5)$$

with

$$\mathcal{K} = \frac{A_{\text{Pio}}}{v_{\text{Pio}}} = 7.3 \times 10^{-14} \text{ rad s}^{-1} = 47.4 \text{ arcsec cy}^{-1}. \quad (6)$$

The radial acceleration of Eq. (4) is constantly inward, i.e. directed toward the Sun, while that of Eq. (5) is directed toward the Sun when  $v_r > 0$ , i.e. when the planet

Table 1. Second column: Formal standard deviations  $\delta a$ , in m, of the semimajor axes of the outer planets from a fit of 400,000 data points spanning almost one century with the EPM2006 ephemerides (from Table 3 of Ref. 24). Third column: Corrections to the standard Newton–Einstein secular precessions of the perihelia<sup>25</sup> in arcsec cy<sup>−1</sup>. Fourth column: Their formal errors, in arcsec cy<sup>−1</sup>, rescaled by a factor 10. For Neptune and Pluto no secular precessions have been estimated because the available modern data records for them do not yet cover an entire orbital revolution.

Planet	$\delta a$ (m)	$\Delta\dot{\varpi}$ (arcsec cy <sup>−1</sup> )	$\delta\Delta\dot{\varpi}$ (arcsec cy <sup>−1</sup> )
Jupiter	615	0.0062	0.036
Saturn	4256	−0.92	2.9
Uranus	40,294	0.57	13.0
Neptune	463,307	N.A.	N.A.
Pluto	3,412,734	N.A.	N.A.

gets farther from the Sun, while it is directed away from the Sun when  $v_r < 0$ , i.e. when the planet gets closer to the Sun. Indeed, for an unperturbed Keplerian ellipse<sup>26</sup>

$$v_r = \frac{nae \sin f}{\sqrt{1-e^2}} = \frac{nae \sin E}{1-e \cos E}, \quad (7)$$

where  $a$  is the semimajor axis,  $e$  is the eccentricity,  $n = \sqrt{GM/a^3}$  is the mean motion,  $f$  is the true anomaly counted anticlockwise from the perihelion, and  $E$  is the eccentric anomaly.  $v_r > 0$  for  $0 < f < \pi$ , i.e. from the perihelion to the aphelion, and  $v_r < 0$  for  $\pi < f < 2\pi$ , i.e. from the aphelion back to the perihelion.

In view of the smallness of Eqs. (4) and (5) for the planets of the solar system, we will treat them perturbatively. Indeed, the radial velocities for the outer planets amount to  $10^1$ – $10^3$   $\text{ms}^{-1}$  only, so that  $A_r \approx 10^{-11} \text{ms}^{-2}$ , while the Newtonian attraction of the Sun is for them of the order of  $10^{-4}$ – $10^{-6} \text{ms}^{-2}$ .

Let us work out the secular precession of the longitude of perihelion  $\varpi$ . The Gauss equation for its variation due to an entirely radial perturbing acceleration  $A_r$  is<sup>26</sup>

$$\frac{d\varpi}{dt} = -\frac{\sqrt{1-e^2}}{nae} A_r \cos f. \quad (8)$$

By inserting Eq. (4) into Eq. (8), evaluating the r.h.s. over the unperturbed Keplerian ellipse characterized by

$$\begin{aligned} r &= a(1 - e \cos E), \\ dt &= \left( \frac{1 - e \cos E}{n} \right) dE, \\ \cos f &= \frac{\cos E - e}{1 - e \cos E}, \\ \sin f &= \frac{\sqrt{1-e^2} \sin E}{1 - e \cos E}, \end{aligned} \quad (9)$$

and averaging over one orbital period, we get

$$\langle \dot{\varpi} \rangle = -\frac{\mathcal{K}\sqrt{1-e^2}}{\pi} \left[ \frac{2e - (1-e^2) \ln \left( \frac{1+e}{1-e} \right)}{e^2} \right] < 0. \quad (10)$$

We have used the fact that

$$|v_r| = v_r, \quad 0 \leq f \leq \pi; \quad |v_r| = -v_r, \quad \pi \leq f \leq 2\pi. \quad (11)$$

Instead, Eq. (5) yields no perihelion precession. Indeed,

$$\langle \dot{\varpi} \rangle = -\frac{\mathcal{K}\sqrt{1-e^2}}{2\pi} \int_0^{2\pi} \frac{(\cos E - e) \sin E}{1 - e \cos E} dE = 0. \quad (12)$$

The Gauss equation for the variation of the semimajor axis due to the radial perturbing acceleration is<sup>26</sup>

$$\frac{da}{dt} = \frac{2e}{n\sqrt{1-e^2}} A_r \sin f. \quad (13)$$

By proceeding as before it turns out that Eq. (4) does not yield secular variations of  $a$ ; instead, Eq. (5) induces a secular decrease of  $a$  according to

$$\langle \dot{a} \rangle = 2\mathcal{K}a \left( \frac{1}{\sqrt{1-e^2}} - 1 \right). \quad (14)$$

Note that for circular orbits, i.e.  $v_r = 0$ ,  $\langle \dot{a} \rangle = 0$ . Since

$$\frac{de}{dt} = \left( \frac{1-e^2}{2ae} \right) \frac{da}{dt} \quad (15)$$

when  $A = A_r$ , the eccentricity also decreases:

$$\langle \dot{e} \rangle = \frac{\mathcal{K}(1-e^2)}{e} \left( \frac{1}{\sqrt{1-e^2}} - 1 \right). \quad (16)$$

As is expected for a central force, the orbital angular momentum  $L = \sqrt{GMa(1-e^2)}$  is conserved, on average: indeed, Eqs. (14) and (16) yield

$$\left\langle \frac{dL^2}{dt} \right\rangle = GM[\langle \dot{a} \rangle (1-e^2) - 2ae\langle \dot{e} \rangle] = 0. \quad (17)$$

Instead, the energy  $\mathcal{E} = -GM/2a$  is not conserved; according to Eq. (14),

$$\langle \dot{\mathcal{E}} \rangle = \frac{GM}{2a^2} \langle \dot{a} \rangle = \frac{\mathcal{K}}{a} \left( \frac{1}{\sqrt{1-e^2}} - 1 \right) < 0. \quad (18)$$

Such a result is certainly suspect from a physical point of view.

In order to independently check the results obtained analytically, we performed two numerical integrations of the equations of motion adding to the Newtonian monopole term the perturbing accelerations of Eqs. (4) and (5). The qualitative features of the resulting motions are depicted in Figs. 1 and 2.

Let us now consider the problem of the existence of the accelerations of Eqs. (4) and (5) from a phenomenological point of view, according to the present-day planetary data available. In Table 2 we quote the predictions for the outer planets of the centennial shifts in  $m$  of the semimajor axis, according to Eq. (14), and of the secular perihelion precessions in  $\text{arcsec cy}^{-1}$ , according to Eq. (11). Such predictions must be compared with the observationally determined parameters quoted in Table 1. Concerning the semimajor axis, the present-day accuracy in determining them would clearly allow one to detect shifts as large as those of Table 2 for all planets from Jupiter to Pluto with the exception of Neptune, even by rescaling the values of  $\delta a$  of Table 1 by a factor of 10 or more. The situation is less neat for the perihelion precessions. Indeed, it turns out that the present-day accuracy in determining them does not allow one to rule out Eq. (11), with the exception of Jupiter. Thus, we conclude that the acceleration of Eq. (5) proportional to  $-v_r$  is to be considered

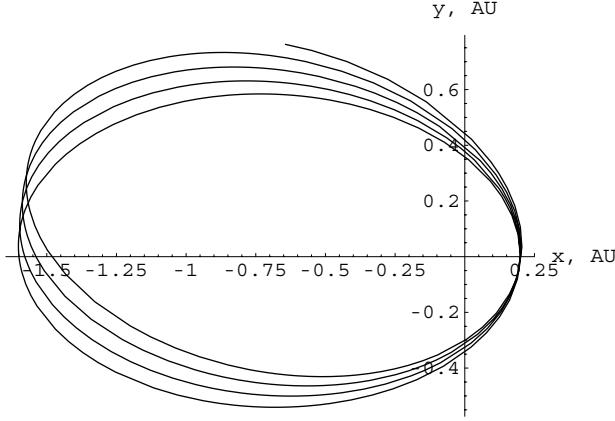


Fig. 1. Numerically integrated trajectory for the radial acceleration of Eq. (4) proportional to  $-|v_r|$ . The longitude of perihelion  $\varpi$  undergoes a retrograde precession, while neither the semi-major axis  $a$  nor the eccentricity  $e$  experience secular variations.

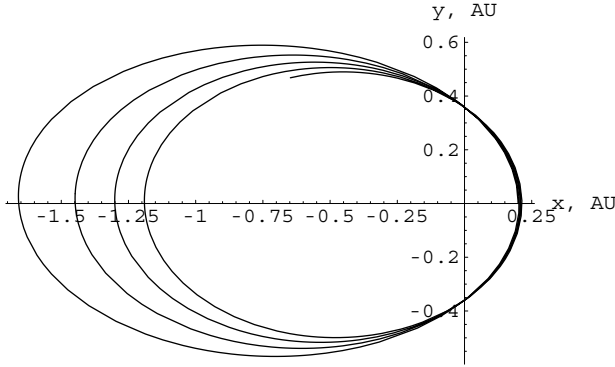


Fig. 2. Numerically integrated trajectory for the radial acceleration of Eq. (5) proportional to  $-v_r$ . Both the semimajor axis  $a$  and the eccentricity  $e$  secularly decrease, while the longitude of perihelion  $\varpi$  remains fixed.

ruled out by observations; it is true that, in principle, adjusting the ephemerides (without modifying their dynamical force models, as is done with EPM2006) may absorb the exotic signatures, but we do not believe that this could occur because of their huge size. Instead, the effects induced by Eq. (4), proportional to  $-|v_r|$ , are still compatible with data. Of course, in drawing such conclusions we are tacitly assuming that the Pioneer-type anomalous accelerations of Eqs. (4) and (5) exist since one century at least. Let us assume that they act since a much longer time, say 500 Myr; Eqs. (5) and (14) tell us that, in this case, 500 Myr ago the semimajor axes of the outer planets were equal to 19 AU (Jupiter), 44 AU (Saturn), 68 AU (Uranus), 32 AU (Neptune) and 2983 AU (Pluto). We have used the simple formula

$$a_0 = a - \dot{a}\Delta t, \quad (19)$$

Table 2. Second column: Shift  $\Delta a$ , in m, of the semimajor axis of the outer planets over 1 cy, according to Eq. (14) induced by the acceleration of Eq. (5) proportional to  $-v_r$ . Third column: Secular precessions  $\dot{\varpi}$  of the perihelia of the outer planets, in arcsec cy $^{-1}$ , according to Eq. (11) due to the acceleration proportional to  $-|v_r|$ . Such values are to be compared with those in Table 1.

Planet	$\Delta a$ (m)	$\dot{\varpi}$ (arcsec cy $^{-1}$ )
Jupiter	-419,726	-0.973
Saturn	-1,030,797	-1.1
Uranus	-1,470,544	-0.9
Neptune	-76,220	-0.1
Pluto	-88,154,057	-4.9

in which  $a_0$  represents the semimajor axis in the past while  $a$  denotes its current value. Concerning the eccentricities, they would have been larger than 1 according to Eq. (16) and  $e_0 = e - \dot{e}\Delta t$ . With regard to the future evolution of the orbits of the outer planets, the time required to circularize their orbits with respect to the present-day values of the eccentricities is of the order of  $8 \times 10^5$  yr, provided that the Pioneer-type forces considered here will continuously act on the planets for so long a time span. Of course, issues concerning a theoretical justification for Eqs. (4) and (5) remain: suffice it to say that they are not, in general, Lorentz-invariant, as can be straightforwardly shown by using

$$\mathbf{r}' = \Gamma(\mathbf{r} - \mathbf{V}t) + (\Gamma - 1)\frac{(\mathbf{r} \times \mathbf{V}) \times \mathbf{V}}{V^2}, \quad t' = \Gamma\left(t - \frac{\mathbf{r} \cdot \mathbf{V}}{c^2}\right), \quad (20)$$

$$\mathbf{v}' = \frac{1}{1 - \frac{\mathbf{v} \cdot \mathbf{V}}{c^2}} \left[ \mathbf{v} - \mathbf{V} + \left( \frac{\Gamma - 1}{\Gamma V^2} \right) (\mathbf{v} \times \mathbf{V}) \times \mathbf{V} \right], \quad (21)$$

with  $\Gamma = 1/\sqrt{1 - V^2/c^2}$ . The conclusions by Standish<sup>22</sup> are that Eqs. (4) and (5) cannot exist for planets up to Jupiter and Saturn, while their existence at heliocentric distances  $\gtrsim 20$  AU is virtually undetectable from the motion of Uranus, Neptune and Pluto.

## 2.2. Forces quadratic in velocity

The other two anomalous radial accelerations examined in Ref. 22, quadratic in the radial velocity, are

$$A_{v^2}^{(4)} = -v_r^2 \mathcal{H}, \quad (22)$$

$$A_{v^2}^{(5)} = -|v_r| v_r \mathcal{H}, \quad (23)$$

with

$$\mathcal{H} = \frac{A_{\text{Pio}}}{v_{\text{Pio}}^2} = 6.07 \times 10^{-18} \text{ m}^{-1}. \quad (24)$$

The acceleration of Eq. (22) is always directed toward the Sun, while that of Eq. (23) is inward when the planet moves away from the Sun, and it is directed outward when the planet approaches the Sun, as in the case of Eq. (4).

An acceleration like Eq. (22) was theoretically obtained by Jaekel and Reynaud in the framework of their linear<sup>27,28</sup> and nonlinear<sup>29</sup> metric extensions of general relativity. Its orbital effects were worked out in Ref. 19: neither the semimajor axis nor the eccentricity undergoes secular variations, while the longitude of perihelion precesses according to

$$\langle \dot{\varpi} \rangle = \frac{\mathcal{H}na\sqrt{1-e^2}}{e^2}(-2 + e^2 + 2\sqrt{1-e^2}) < 0. \quad (25)$$

In Fig. 3 we show the results of the numerical integration of the equations of motion with Eq. (22) added to the Newtonian monopole: the results obtained analytically in Ref. 19 are confirmed.

In Ref. 19 it was shown that the inner planets' perihelion precessions predicted by Eq. (25) are neatly ruled out by the corrections to the perihelion precessions estimated by Pitjeva<sup>30</sup> with the EPM2004 ephemerides. In Table 3 we quote the predictions for the outer planets; it turns out that they are compatible with the results of Table 1, apart from Jupiter. Note that it is true also by considering the formal uncertainties in the estimated corrections to the perihelion precessions, i.e. the values of  $\delta\Delta\dot{\varpi}$  in Table 1 reduced by 10 times. Such a conclusion substantially agrees with that of Standish.<sup>22</sup>

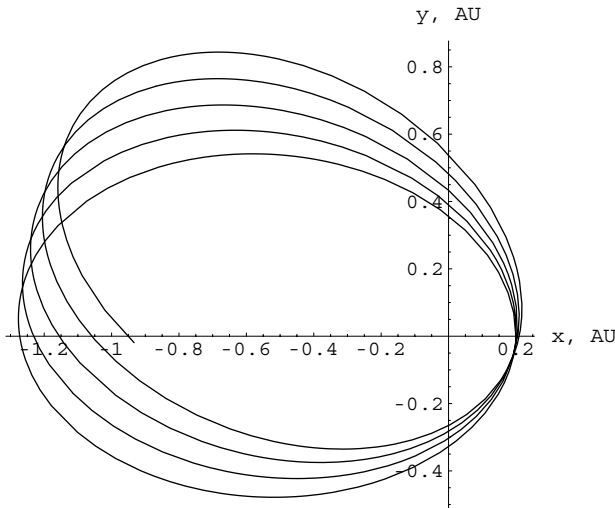


Fig. 3. Numerically integrated trajectory for the radial acceleration of Eq. (22) proportional to  $-v_r^2$ . Both the semimajor axis  $a$  and the eccentricity  $e$  remain unchanged, while the longitude of perihelion  $\varpi$  experiences a retrograde secular precession.



Table 3. Secular precessions  $\dot{\varpi}$  of the perihelia of the outer planets, in arcsec  $\text{cy}^{-1}$ , according to Eq. (25) due to the acceleration of Eq. (22) proportional to  $-v_r^2$ . Such values are to be compared with those in Table 1.

Planet	$\dot{\varpi}$ (arcsec $\text{cy}^{-1}$ )
Jupiter	-0.030
Saturn	-0.029
Uranus	-0.015
Neptune	-0.0004
Pluto	-0.308

Equation (23), contrary to Eq. (22), induces no secular perihelion precession and a secular variation of the semimajor axis and the eccentricity, which decrease according to

$$\langle \dot{a} \rangle = \frac{4\mathcal{H}na^2}{\pi} \left[ 2e + \ln \left( \frac{1-e}{1+e} \right) \right], \quad (26)$$

$$\langle \dot{e} \rangle = \frac{2\mathcal{H}na(1-e^2)}{\pi e} \left[ 2e + \ln \left( \frac{1-e}{1+e} \right) \right]. \quad (27)$$

Note that  $\langle \dot{a} \rangle = 0$  for circular orbits. Thus, also, Eq. (23) does not conserve the total energy. Such analytical results are confirmed by a numerical integration of the equations of motion showed in Fig. 4.

In Table 4 we quote the predictions for the centennial semimajor shifts according to Eq. (26). A comparison with Table 1 shows that the formal uncertainties  $\delta a$  are

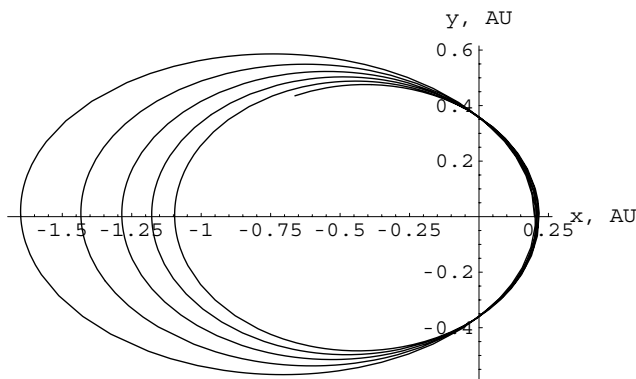


Fig. 4. Numerically integrated trajectory for the radial acceleration of Eq. (23) proportional to  $-|v_r|v_r$ . Both the semimajor axis  $a$  and the eccentricity  $e$  secularly decrease, while the longitude of perihelion  $\varpi$  remains unchanged.

Table 4. Shifts of the semimajor axes  $a$  of the outer planets, in m, according to the acceleration of Eq. (23) proportional to  $-|v_r|v_r$ . Such values are to be compared with those in Table 1.

Planet	$\Delta a$ (m)
Jupiter	-18,753
Saturn	-39,362
Uranus	-33,347
Neptune	-251
Pluto	-7,283,499

always rather smaller than such anomalous shifts, apart from Neptune. However, it must taken into account that realistic errors may be up to one order of magnitude larger: if so, it would not be possible to rule out the results of Table 4, apart from Jupiter. In this case, our conclusions would agree with those of Standish.<sup>22</sup> Of course, serious issues concerning theoretical justifications of Eq. (23) and the temporal extent of its existence remain open. Indeed, given the present-day values of the planetary semimajor axes and eccentricities and assuming that Eq. (23) existed unchanged in the deep past, about 100 Myr–1 Gyr ago  $e = 1$  for the planets from Jupiter to Pluto. Since, instead, the semimajor axes would have remained almost unchanged, this means that the perihelion distances vanished.

### 3. Conclusions

An ingenious attempt recently proposed to explain the Pioneer anomaly as being due to a modification of the usual Newton–Einstein laws of gravitation consists of postulating the existence of some velocity-dependent extra forces linear or quadratic in the radial component  $v_r$  of the velocity of a test body. We put to the test such empirical models in the outer regions of the solar system in which the Pioneer anomaly manifested itself in its presently known form with the latest observational determinations of the planetary motions obtained by Pitjeva with the EPM2006 ephemerides. It turns out that those models yielding anomalous perihelion precessions cannot yet be ruled out, at least phenomenologically, for heliocentric distances larger than 5 AU. On the contrary, those models predicting secular variations of the semimajor axis  $a$  and the eccentricity  $e$  are much more difficult to trust, not only because they would violate the conservation of energy but also because the centennial shifts for  $a$  predicted by them are so large that they should have been likely to be detected given the present-day accuracy in determining such orbital elements. However, it must be considered that sound theoretical justifications for such models must be given.

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