Anna, 12 January 2012 → 17 January 2012

①

GGonGround ERROR BUDGET & ROADMAP (@ $\nu_{GG} = 1.75 \cdot 10^{-4}$ Hz)

• TILTS / HORIZONTAL ACCELERATIONS

(1) $\theta_{tilt} = \theta_{terrain} + \theta_{ball\ bearings}$ total tilt angle

(2) 2D suspension below ball bearings on (rotating) shaft with elastic constant k_{shaft} (N·m/rad)

(3) $$\theta_{shaft} = \theta_{tilt} \frac{k_{shaft}}{M_{tot}\, g\, L_{tot}}$$ tilt of shaft due to θ_{tilt}

M_{tot} = total suspended mass

L_{tot} = from CM of suspended mass to suspension joint

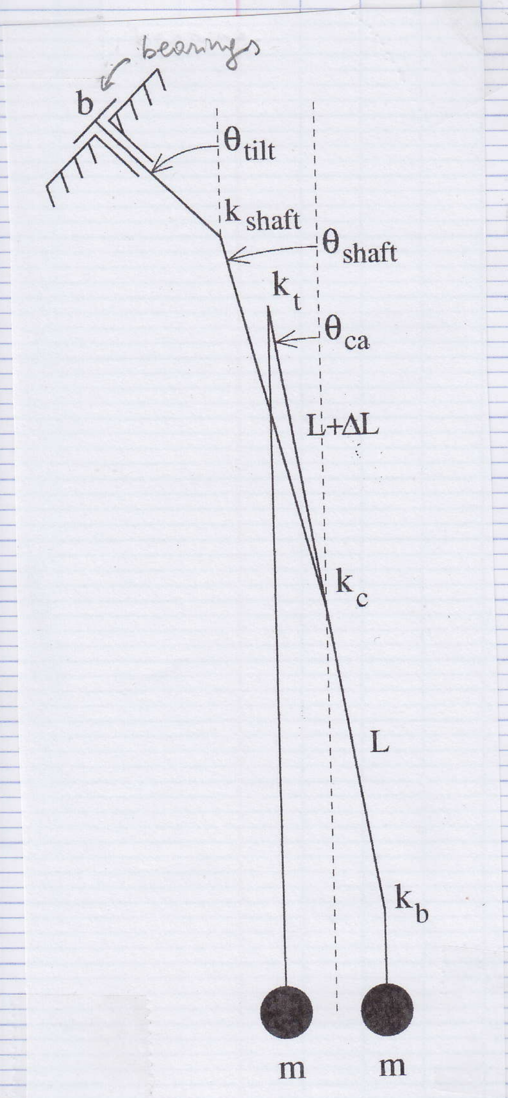

Fig. 1 Sketch of GGG balance

k_{shaft}: elastic constant of joint on shaft, below bearings

L: half arm of balance

k_t, k_c, k_b: elastic constants of top, central and bottom joint

m: mass of each test cylinder

(4) $$T_d^2 = \frac{4\pi^2}{\frac{k_t + k_c + k_b}{2mL^2} - \frac{g}{2L}\frac{\Delta L}{L}}$$ natural period of differential oscillations of test cylinders

②

(5) $\tau^2 \equiv 4\pi^2 \frac{2mL^2}{k_c}$

(6) $\theta_{ca} = \frac{T_d^2}{\tau^2}\,\theta_{shaft}$ tilt angle of the coupling arm if shaft tilts by θ_{shaft}

(7) $r_{d\,tilt} = 2L\,\theta_{ca}$ differential displacement caused by tilt θ_{ca} of the coupling arm

$a_{d\,tilt} = \frac{4\pi^2}{T_d^2}\, r_{d\,tilt}$ differential acceleration

$$\parallel$$

$$\frac{4\pi^2}{T_d^2}\,2L\theta_{ca} = \frac{4\pi^2}{\cancel{T_d^2}}\,2L\,\frac{\cancel{T_d^2}}{\tau^2}\,\theta_{shaft} =$$

$$= \frac{\cancel{4\pi^2}\,\cancel{2}\cancel{L}}{\cancel{4\pi^2}\cdot\cancel{2}mL^{\cancel{2}}}\cdot k_c\,\theta_{shaft} = \frac{k_c}{mL}\,\theta_{shaft} =$$

(8) $$= \frac{k_c}{mL}\cdot\frac{k_{shaft}}{M_{tot}\,g\,L_{tot}}\,\theta_{tilt}$$

We don't know $\theta_{ball\,bearings}$ and <u>assume</u>:

$\theta_{ball\,bearings} < \theta_{terrain}$ i.e. $\theta_{tilt} \simeq \theta_{terrain}$

$\Downarrow$

(9) $$a_{d\,tilt} = \frac{k_c}{mL}\cdot\frac{k_{shaft}}{M_{tot}\,g\,L_{tot}}\,\theta_{terrain}$$

@ $\nu_{GG} = 1.75\cdot 10^{-4}$ Hz, 30 days integration, we have: $\theta_{terrain}$

various ISA tiltmeters (by V. Iafolla)

$2.48\cdot 10^{-10}$ rad IFSI, Rome Tor Vergata, underground tunnel (similar @ Gran Sasso INFN lab)

$6.2\cdot 10^{-10}$ rad IFSI, Rome Tor Vergata ground floor lab

$6.2\cdot 10^{-9}$ rad Downtown Florence (old GGG lab @ LABEN) and in San Piero

In Cascina (EGO/Virgo site) we assume for GG a value of terrain tilt 20 times larger than at IFSI tunnel (and Gran Sasso lab); this value is very close to the one measured in downtown Florence (both San Piero & Cascina are quite isolated locations):

(10) $\theta_{terrain\,30d} \simeq 5 \times 10^{-9}$ rad

Then:

(11) $a_{d\,tilt\,30d} \simeq \frac{k_c}{mL} \cdot \frac{k_{shaft}}{M_{tot}\, g\, L_{tot}}\, \theta_{terrain\,30d}$.

We take:

(12)

$k_c = 0.04$ Nm/rad (now $k_c \simeq 0.2$ Nm/rad no problem)

$m = 10$ Kg same as now

$L = 0.4$ m (now $L = 0.183$ m no problem)

$k_{shaft} = 0.04$ Nm/rad (now $k_{susp} \simeq 0.17$ Nm/rad no problem)

$M_{tot} = 50$ kg (now $M_{tot} = 40.2$ kg no problem)

$L_{tot} \simeq 2L = 0.8$ m (now $L_{tot} \simeq 2L = 2 \cdot 0.18 = = 0.36$ m no problem)

$\Downarrow$

(13) $a_{d\,tilt\,30d} \simeq \frac{0.04}{10 \cdot 0.4} \cdot \frac{0.04}{50 \cdot 9.81 \cdot 0.8} \cdot 5 \cdot 10^{-9}\ ms^{-2}$

$\simeq 5.1 \cdot 10^{-15}\ ms^{-2}$

Goal GG (600 km altitude)

(14) $a_{d\,GG} = 8\cdot 10^{-17}\ ms^{-2}$ @ $\nu_{GG} = 1.75\times 10^{-4}\ Hz$

GGG goal for the Synergy Grant:

(15) $a_{d\,goal} = 8\cdot 10^{-15}\ ms^{-2}$ in 30d

From (13) we see that tilts are not a problem having assumed $\theta_{ball\ bearings} < \theta_{terrain}$ (air bearings see p. 7-8)

NOTE: a horizontal acceleration $a_{ha} = g\,\theta_{tilt}$ at the low frequency of our interest can be treated exactly like a tilt $\theta_{tilt} = a_{ha}/g$ as discussed above (just take into account the factor g)

• CAP BRIDGE MECHANICAL UNBALANCE

A tilt/horiz. acc. of the shaft – to which are rigidely connected the cap plates of the bridges which measure the relative displacements of the test cylinders – displaces the cap bridges w.r.t both test cylinders. So it is like a common mode displacement of the cylinders. The resulting signal of the bridges would be zero only if the plates are perfectly centred in between the test cylinders (the two gaps a, b should be exactly equal). If not:

(16) $\chi_{bridge} = \frac{d_1 - d_2}{d_1}$ mechanical unbalance of the cap bridge

and

(17) $r_{d\,bridge} = \chi_{bridge}\, r_{cm}$

common mode displacement

resulting differential displacement

(18) $r_{cm} = L\,\theta_{shaft}$ common mode displacement as a result of a tilt of the shaft θ_{shaft}

(19) $r_{d\,bridge} = \chi_{bridge} L\,\theta_{shaft} = \chi_{bridge} L \dfrac{k_{shaft}}{M_{tot}\, g\, L_{tot}}\, \theta_{tilt}$

(20) $a_{d\,bridge} = \dfrac{4\pi^2}{T_d^2}\, r_{d\,bridge} = \dfrac{4\pi^2}{T_d^2}\, \chi_{bridge} L \dfrac{k_{shaft}}{M_{tot}\, g\, L_{tot}}\, \theta_{tilt}$

The bridge unbalance χ_{bridge} is not a problem if:

(21) $a_{d\,bridge} \ll a_{d\,tilt}$ ↳ given by (8)

$\Downarrow$

(22) $\chi_{bridge} \ll \dfrac{T_d^2}{4\pi^2}\, \dfrac{k_c}{mL^2}$

The differential period T_d is given by (4). In case of perfect balancing, i.e. $\frac{\Delta L}{L} = 0$ (note: in GG we balance in fact the masses and not the masses of the balance):

(23) $T^2_{d\,perfect\ balance} = 4\pi^2 \cdot \dfrac{2mL^2}{k_t + k_c + k_b}$

This gives the longest differential period - unless we use gravity as negative spring - provided the balance is balanced sufficiently well, that is:

(24) $\dfrac{g}{\cancel{2}\cancel{L}} \dfrac{\Delta L}{\cancel{L}} \ll \dfrac{k_t + k_c + k_b}{\cancel{2}mL^{\cancel{2}}} \Rightarrow \dfrac{\Delta L}{L} \ll \dfrac{k_t + k_c + k_b}{mgL} \simeq \dfrac{3k_c}{mgL}$

With the numbers (12) and

(25) $k_c = k_t = k_b \simeq 0.04$ Nm/rad

if we have:

(26) $$\frac{\Delta L}{L} < \frac{3 \cdot 0.04}{10 \cdot 9.81 \cdot 0.4} = 3 \times 10^{-3} = \frac{1}{327}$$

[which is possible because we balance the mass, with each test cylinder of 10 kg, and $1/327 \cdot 10\,kg \simeq 30$ grams (no problem)], then the differential period is given by the value for the perfect balance. From (23)

(27) $$T^2_{d\,perfect\,balance} = 4\pi^2 \cdot \frac{2 \cdot 10 \cdot 0.4^2}{0.04 + 0.04 + 0.04} = 1.05 \times 10^3\ s^2$$

$$\Downarrow$$

(28) (feasible) $$T_{d\,perfect\,balance} = 32.4\ s$$

With the value (28) for the differential period, the requirement (22) becomes:

(29) $$\chi_{bridge} < \frac{(32.4)^2}{4\pi^2} \cdot \frac{0.04}{10 \cdot 0.4^2} = 0.67$$

with the gap $d_1 = 1$ mm

$$\chi_{bridge} = \frac{d_1 - d_2}{d_1} < 0.67 \Rightarrow d_1 - d_2 < 0.67\ mm \qquad (30)$$

(feasible)

With this value the GGG goal in displacement from (15) is $r_{d\,30d} = \frac{8 \cdot 10^{-15} \cdot 32.4^2}{4\pi^2} = 2 \times 10^{-13}\ m$ (28')

If this condition is fulfilled, the effect of the bridge mechanical unbalance is below the effect of the terrain tilts and the goal (15) is feasible (see (13)).

- CAP BRIDGE READ-OUT ELECTRONICS NOISE (Feb 2012)

The new electronics that Raffaella is about to complete will have a noise (realistic estimate)

(31) $$\frac{r_{cap}}{\sqrt{Hz}} = 3 \cdot 10^{-9}\ \frac{m}{\sqrt{Hz}} \quad @\ 0.2\ Hz$$

(better if GGG spins at higher frequency)

In 30 days this capacitance read out noise gives a displacement noise

(32) $r_{cap\,30d} = 1.9 \times 10^{-12}$ m

which is 10 times larger than the goal (28')

If we build and implement a laser gauge on GGG having:

- a $1 \div 2$ cm gap between the test cylinders
- a higher spin frequency ($\simeq 1$ Hz)

such that its noise is

(33) $r_{GGG\,lg} = 10^{-10} \dfrac{m}{\sqrt{Hz}}$ @ 1 Hz

we get

(34) $r_{GGG\,lg\,30d} = 6.2 \times 10^{-14}$ m

which is 3.2 times smaller than the target (28'), so that we can be sure that the result is not limited by read-out noise.

- BEARINGS NOISE

From the current ongoing run we conclude that the tilt noise of the present ball bearings (@ $\nu = 1.75 \times 10^{-4}$ Hz) is

(35) $\theta_{bb_{30d}} \simeq 150\, \theta_{terrain_{30d}}$ given by (10).

If we can set up air bearings (+ ferro fluid feed through) such that its tilt noise is:

(36) $\theta_{ab\,30d} \lesssim \frac{1}{2}\theta_{terr} \simeq \frac{1}{200}\theta_{bb}$ (realistic)

then we have 1/2 the acceleration noise (13) and by summing up the terrain and air bearings tilt/horiz accelerat. @ 1.75×10^{-4} Hz we have an acceleration noise below the GGG goal of $8\cdot10^{-15}$ ms^{-2} in 30 d

Since the laser gauge read-out is a factor 3 below the target (having considered correctly the target in displacement - see (34)) we conclude that the result is limited by terrain/air bearings noise, neither of which is present in space

- See Table I - GGG on Ground Error Budget (page (9))

Estimate (35) of the ball bearings noise is obtained as follows. From the current run we have:

(37) $a_{now\,30d} \simeq 8.49\cdot10^{-11}$ ms^{-2}

Let us use (8) with both terrain tilts & ball bearings noise, and the current measured numbers:

(38) $$a_{d\,tilt\,now} = \frac{k_r}{mL}\frac{k_{shaft}}{M_{rot}\,g\,L_{bof}}\left(\theta_{terr}+\theta_{bb}\right) =$$

$$= \frac{0.3\cdot0.17}{10\cdot0.183\cdot40\cdot9.81\cdot0.4}\left(\theta_{terr}+\theta_{bb}\right) \simeq 1.8\cdot10^{-4}\left(\theta_{ter}+\theta_{bb}\right) \text{ ms}^{-2}$$

If it is $\theta_{bb} \simeq 100\,\theta_{terr}$, we have (in 30d):

$\theta_{bb} \simeq 5\cdot10^{-7}$ rad and $a_{d\,tilt\,now\,30d} \simeq 9\cdot10^{-11}$ ms^{-2}, which is roughly what we measure with (37)

The requirement (36) for air bearings 200 times less noisy than ball bearings is very conservative, since air bearings are considered to be several orders of magnitude less noisy than ball bearings.

Table I - GGonGround: Error budget

	Test masses Differential acceleration a @ 1.75×10^{-4} Hz	a $(m\,s^{-2})$	$\simeq$ (m)	Integration time (d)
1. GG goal	$a_{GG} = \eta g(h)$ upconverted to 1 Hz	$8\cdot10^{-17}$ $\eta = 10^{-17}$ $h \simeq 600\,km$	$6\cdot10^{-13}$ $T_d = 540\,s$	1
2. GGonGround goal	$a_{GGG} = 10^2 a_{GG} \to 10^1 a_{GG}$ upconverted to $0.2 \div 3$ Hz	$8\cdot10^{-15}$	$2\cdot10^{-13}$ $T_d = 32.4\,s \downarrow 40.5\,s$	30
3. Tilts/horizontal accelerations	$a_{tilt} = \frac{k_c}{mL}\frac{k_{shaft}}{M_{tot}\,g\,L_{tot}}\theta_{tilt}$ 4m $7.65\cdot10^{-16}$	$10^{-6}\,\theta_{tilt}$ $\theta_{tilt} = \theta_{terrain} + \theta_{bearings}$ ($k_c = k_{shaft} = 0.04$ Nm $m = 10\,kg$, $L = 0.4\,m$ $M_{tot} = 50\,kg$ $L_{tot} = 0.8\,m$) $\theta_{terre} \simeq 5\cdot10^{-9}$ rad $\theta_{bb} \simeq 150\,\theta_{terre}$ $\theta_{ab} \lesssim \frac{1}{2}\theta_{terrein}$ $10^{-6}(\theta_{terre} + \theta_{ab}) =$ $= 7.5\times10^{-15}$	$1.99\cdot10^{-13}$	30
4. Capacitance bridge unbalance	$a_{bridge} \ll a_{tilt}$ if $\chi_{bridge} \ll \frac{T_d^2}{4\pi^2}\frac{k_c}{mL}$ $\chi_{bridge} = (d_1 - d_2)/d_1$		satisfied	
5. Capacitance read out noise			$\left(\frac{r}{\sqrt{Hz}} = 3\cdot10^{-9}\frac{m}{\sqrt{Hz}}\right)$	30
6. Laser gauge noise		$2.95\cdot10^{-12}\frac{m}{\sqrt{Hz}}$ $(r = 3.18\cdot10^{-14}\,m)$	$\left(\frac{r}{\sqrt{Hz}} = 10^{-10}\frac{m}{\sqrt{Hz}}\right)$	30
7. Thermal noise	Internal damping + Gas damping + Eddy currents (worst case, no μmetal shield)		: below target	30

AIMING AT $a_{GGG} = 10\, a_{GG}$

Table I shows that there is no fundamental limitation to improving the GG on ground sensitivity by another order of magnitude provided we can improve the laser gauge by 1 order of magnitude w.r.t line 6 in Table I, namely to a performance

From line 3. in Table I, we can reduce

$$(39)\quad a_{tilt} = \frac{k_c}{mL}\,\frac{k_{shaft}}{M_{tot}\, g\, L_{tot}}\,\theta_{tilt}$$

by a factor 10 by slighthly increasing L, L_{tot} and M_{tot}:

$$(40)\quad L M_{tot} L_{tot} = 0.4 \cdot 50 \cdot 0.8 = 16 \text{ kg m}^2 \quad \text{in Table I}$$

$$\downarrow$$

$$\underset{m}{0.5} \cdot \underset{kg}{80} \cdot \underset{m}{4} = 160 \text{ kg m}^2$$

the GGG balance will have a coupling arm of 1 m. No problem

with CuBe top joint (similar to the one we have just mounted) there is a very high enervation load (and even higher breaking load). No problem

the suspension joint of the rotor can be placed 4 m away from its center of mass

Thus, with the same $\theta_{tilt} = \theta_{terrain} + \theta_{ab}$ (i.e. same air bearing), the same k_c, k_{shaft} and the same T_d as in Table I and the same integration time of 30d, we would have a differential acceleration of $7.5 \cdot 10^{-16}\ ms^{-2}$ (in line 3), i.e $\lesssim 10\, a_{gg} = 8 \cdot 10^{-16}\ ms^{-2}$, which is the same as of torsion balances (though, we are @ $1.75 \cdot 10^{-4}$ Hz, they are @ $1.16 \cdot 10^{-5}$ Hz).

With the same T_d the displacement noise is 1.99×10^{-14} m and for the laser gauge noise to be below it we need $\frac{r_{eg}}{\sqrt{Hz}} \simeq 10^{-11} \frac{m}{\sqrt{Hz}}$ @ 1÷3 Hz on GGG.

Since JPL has demonstrated $10^{-12} \frac{m}{\sqrt{Hz}}$ @ 1Hz, and this is what we require in space, we should be able to demonstrate it on GGG

- I think we should aim at

(41) $a_{GGG\,goal_{30d}} \lesssim 8 \cdot 10^{-16}\ m\,s^{-2}$ @ 1.75×10^{-4} Hz

We would have a balance with the same sensitivity as the torsion balance which in space should do only a factor 10 better in order to test EP to 10^{-17}. And it will be able to do that because of the reduced thermal noise thanks to rotation

Instead, the torsion balance has reached thermal noise and cannot gain by rotation

(terrain tilt noise and bearings noise is absent in space)

- The main issue in space not tested on ground is Drag Free Control (TASI-To & GOCE)

+ better common mode rejection than on ground (but we have arguments...)

GG on Ground Roadmap		
Time (Months)		
		GG on Ground achieved performance
	t_0	$a_0 = 8.5 \cdot 10^{-11}\ ms^{-2}$ (in INFN lab San Piero a Grado, Pisa)
		First 18–month period targets
6	$t_0 + 6$	$a_1 = 2.8 \cdot 10^{-12}\ ms^{-2}$ ($T_d = 14.8\ s$ $r_{cap} = 1.45 \cdot 10^{-8} m/\sqrt{Hz}$; can be done with capacitance readout and ball bearings, requires weaker joints by a factor 4)
12	$t_0 + 12$	$a_2 = 7.7 \cdot 10^{-14}\ ms^{-2}$ ($T_d = 40\ s$ $r_{cap} = 3 \cdot 10^{-9} m/\sqrt{Hz}$; can be done with capacitance readout and ball bearings, requires 10 times longer suspension shaft)
18	$t_0 + 18 = t_1$	$a_3 = 5.6 \cdot 10^{-15}\ ms^{-2}$ ($T_d = 40\ s$ $r_{laser} = 220\ pm/\sqrt{Hz}$; requires preliminary version of air bearings and laser metrology)
		Second 18–month period targets
24	$t_1 + 6$	
30	$t_1 + 12$	
36	$t_1 + 18 = t_2$	$a_4 = 7.7 \cdot 10^{-16}\ ms^{-2}$ ($T_d = 40\,s$ $r_{laser} = 30\ pm/\sqrt{Hz}$; requires air bearings with full performance and improved laser metrology)
		Third 18–month period targets
42	$t_2 + 6$	Install rotating whirl control (as required in GG)
48	$t_2 + 12$	Demonstrate on bench laser gauge noise to $r_{laser} = 1\ pm/\sqrt{Hz}$ @ $1 - 2\ Hz$
54	$t_2 + 18 = t_3$	Optimize test masses different composition, manufacture test masses, measure their quadrupole moments and confirm sensitivity (test coating)
		Fourth 18–month period targets
60	$t_3 + 6$	Measure patch effects and demonstrate that they are not relevant; Phase Sensitive Detection @ 24 h in preparation for GG in space data analysis
66	$t_3 + 12$	Manufacture suspensions required for GG, meassure elastic constants and quality factors and confirm requirements of GG
72	$t_3 + 18 = t_4$	Test PZTs and inchworms to demonstrate feasibility of balancing in GG

Table 2: GG on Ground Roadmap