

Anna, 12 January 2012 ①

14 January 2012

CCG ERROR BUDGET ($\omega_{cc} = 1.75 \cdot 10^{-4} \text{ Hz}$)

TILTS / HORIZONTAL ACCELERATIONS

(1) $\theta_{\text{tilt}} = \theta_{\text{terrain}} + \theta_{\text{ball bearings}}$

total tilt angle

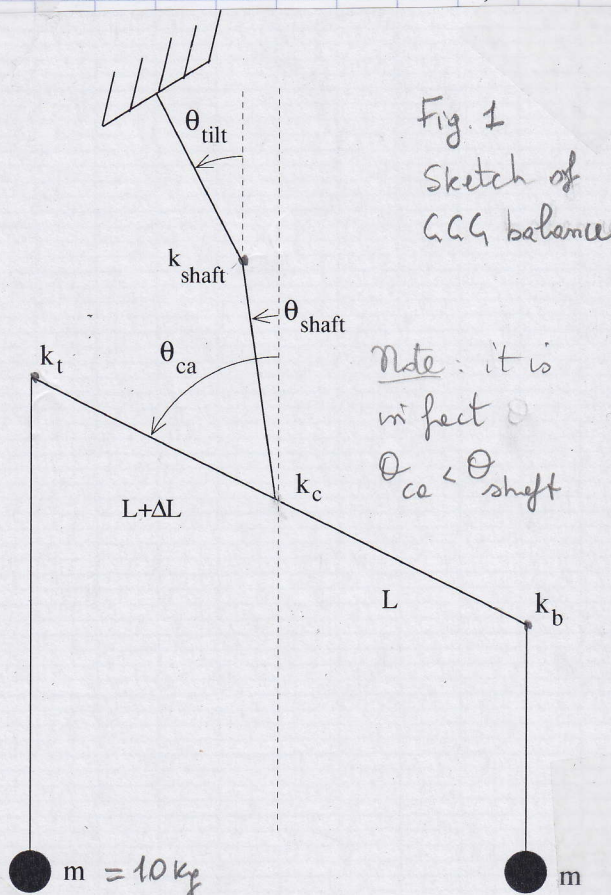
(2) 2D suspension below ball bearings on (rotating) shaft with elastic constant k_{shaft} (N.m/rad)

(3) $\theta_{\text{shaft}} = \theta_{\text{tilt}} \frac{k_{\text{shaft}}}{M_{\text{tot}} g L_{\text{tot}}}$

tilt of shaft due to θ_{tilt}

M_{tot} = total suspended mass

L_{tot} = from CM of suspended mass to suspension joint



L : half arm of balance

k_t, k_c, k_b : elastic contents of top, central and bottom joint

m : mass of each test cylinder

(4) $T_d^2 = \frac{4\pi^2}{\frac{k_t + k_c + k_b}{2mL^2} - \frac{g}{2L} \frac{\Delta L}{L}}$

Natural period of differential oscillations of test cylinders

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$$(5) \tau^2 = 4\pi^2 \frac{2mL^2}{k_c}$$

$$(6) \theta_{ca} = \frac{T_d^2}{\tau^2} \theta_{shaft}$$

tilt angle of the coupling arm
if shaft tilts by θ_{shaft}

$$(7) r_{d tilt} = 2L \theta_{ca}$$

differential displacement
caused by tilt θ_{ca} of the
coupling arm

$$a_{d tilt} = \frac{4\pi^2}{T_d^2} r_{d tilt}$$

differential acceleration

$$\frac{4\pi^2}{T_d^2} 2L \theta_{ca} = \frac{4\pi^2}{T_d^2} 2L \frac{T_d^2}{\tau^2} \theta_{shaft} =$$

$$= \frac{4\pi^2 \cancel{2L}}{4\pi^2 \cdot 2mL^2} \cdot k_c \theta_{shaft} = \frac{k_c}{mL} \theta_{shaft} =$$

$$(8) = \frac{k_c}{mL} \cdot \frac{k_{shaft}}{M_{tot} g L_{tot}} \theta_{tilt}$$

We don't know $\theta_{ball bearings}$ and assume:

$$\theta_{ball bearings} < \theta_{terrain} \quad i.e. \quad \theta_{tilt} \approx \theta_{terrain}$$

↓

$$(9) a_{d tilt} = \frac{k_c}{mL} \cdot \frac{k_{shaft}}{M_{tot} g L_{tot}} \theta_{terrain}$$

@ $\nu_{cg} = 1.75 \cdot 10^{-4}$ Hz, 30 days integration, we have: $\theta_{terrain}$
various ISA tiltmeters (by V. Iafolli)
2.48 · 10⁻¹⁰ rad IFSI, Rome Tor Vergata, underground tunnel
(similar @ Gran Sasso INFN lab)

$$6.2 \cdot 10^{-10} \text{ rad}$$

IFSI, Rome Tor Vergata ground floor lab

$$6.2 \cdot 10^{-9} \text{ rad}$$

Downtown Florence (old CGG lab @ LABEN)

③

and in San Piero

In Cesine (EGO/Virgo site) we assume for GAG a value of terrain tilt 20 times larger than at FSI tunnel (and Gran Sasso lab); this value is very close to the one measured in downtown Florence (both San Piero & Cesine are quite isolated locations):

$$(10) \quad \Theta_{\text{terrain } 30d} \approx 5 \times 10^{-9} \text{ rad}$$

Then:

$$(11) \quad a_{d \text{ tilt } 30d} \approx \frac{k_c}{m L} \cdot \frac{k_{\text{sheft}}}{M_{\text{tot}} g L_{\text{tot}}} \Theta_{\text{terrain } 30d}$$

We take: =

(12) {	$k_c = 0.04 \text{ Nm/rad}$	(now $k_c \approx 0.2 \text{ Nm/rad}$ no problem)
	$m = 10 \text{ Kg}$	same as now
	$L = 0.4 \text{ m}$	(now $L = 0.183 \text{ m}$ no problem)
	$k_{\text{sheft}} = 0.04 \text{ Nm/rad}$	(now $k_{\text{susp}} \approx 0.17 \text{ Nm/rad}$ no problem)
	$M_{\text{tot}} = 50 \text{ kg}$	(now $M_{\text{tot}} = 40.2 \text{ kg}$ no problem)
	$L_{\text{tot}} \approx 2L = 0.8 \text{ m}$	(now $L_{\text{tot}} = 2L = 2 \cdot 0.183 = 0.36 \text{ m}$ no problem)

↓

$$(13) \quad a_{d \text{ tilt } 30d} \approx \frac{0.04}{10 \cdot 0.4} \cdot \frac{0.04}{50 \cdot 9.81 \cdot 0.8} \cdot 5 \cdot 10^{-9} \text{ ms}^{-2} \approx 5.1 \cdot 10^{-15} \text{ ms}^{-2}$$

goal a_G (600 km altitude)

(14) $a_{d_{GG}} = 8 \cdot 10^{-17} \text{ ms}^{-2}$

@ $\nu_{a_G} = 1.75 \times 10^{-4} \text{ Hz}$

GGG goal for the Synergy Grant:

(15) $a_{d_{goal}} = 8 \cdot 10^{-15} \text{ ms}^{-2}$ in 30d

From (13) we see that tilts are not a problem having assumed $\theta_{\text{ball bearings}} < \theta_{\text{terrain}}$ (air bearings) (see p. 78)

NOTE: a horizontal acceleration a_{ha} $a = g \theta_{\text{tilt}}$ at the low frequency of our interest can be treated exactly like a tilt $\theta_{\text{tilt}} = a_{ha}/g$ as discussed above (just take into account the factor g)

• CAP BRIDGE MECHANICAL UNBALANCE

A tilt/horiz. acc. of the shaft - to which are rigidly connected the cap plates of the bridges which measure the relative displacements of the test cylinders - displaces the cap bridges w.r.t both test cylinders. So it is like a common mode displacement of the cylinders. The resulting signal of the bridges would be zero only if the plates are perfectly centred in between the test cylinders (the two gaps a, b should be exactly equal). If not:

(16) $\chi_{\text{bridge}} = \frac{d_1 - d_2}{d_1}$ mechanical unbalance of the cap bridge

and

(17) $r_{d_{\text{bridge}}} = \chi_{\text{bridge}} r_{\text{cm}}$
 $\uparrow$ resulting differential displacement $\swarrow$ common mode displacement

(5)

(18) $r_{cm} = L \theta_{\text{shaft}}$ common mode displacement as a result of a tilt of the shaft θ_{shaft}

(19) $r_{\text{dbridge}} = \chi_{\text{bridge}} L \theta_{\text{shaft}} = \chi_{\text{bridge}} L \frac{k_{\text{shaft}}}{M_{\text{tot}} g L_{\text{tot}}} \theta_{\text{tilt}}$

(20) $a_{\text{dbridge}} = \frac{4\pi^2}{T_d^2} r_{\text{dbridge}} = \frac{4\pi^2}{T_d^2} \chi_{\text{bridge}} L \frac{k_{\text{shaft}}}{M_{\text{tot}} g L_{\text{tot}}} \theta_{\text{tilt}}$

The bridge unbalance χ_{bridge} is not a problem if:

(21) $a_{\text{dbridge}} \ll a_{\text{dtilt}}$
 $\Downarrow$
 $\rightarrow$ given by (8)

(22) $\chi_{\text{bridge}} \ll \frac{T_d^2}{4\pi^2} \frac{k_c}{mL^2}$

The differential period T_d is given by (4). In case of perfect balancing, i.e. $\frac{\Delta L}{L} = 0$ (note: in GSG we balance in fact the masses and not the masses of the balance).

(23) $T_{\text{d perfect balance}}^2 = 4\pi^2 \frac{2mL^2}{k_t + k_c + k_b}$

differential

This gives the longest period - unless we use gravity as negative spring - provided the balance is balanced sufficiently well, that is:

(24) $\frac{g}{2L} \frac{\Delta L}{L} \ll \frac{k_t + k_c + k_b}{2mL^2} \Rightarrow \frac{\Delta L}{L} \ll \frac{k_t + k_c + k_b}{mgL} \approx \frac{3k_c}{mgL}$

With the numbers (12) and

(25) $k_c = k_t = k_b \approx 0.04 \text{ N/m/read}$

if we have:

$$(26) \quad \frac{\Delta L}{L} < \frac{3 \cdot 0.04}{10 \cdot 9.81 \cdot 0.4} = 3 \times 10^{-3} = \frac{1}{327}$$

[which is possible because we balance the mass, with each test cylinder of 10 kg, and $1/327 \cdot 10 \text{ kg} \approx 30 \text{ grams}$ (no problem)], then the differential period is given by the value for the perfect balance. From (23)

$$(27) \quad T_{\text{perfect balance}}^2 = 4\pi^2 \cdot \frac{2 \cdot 10 \cdot 0.4^2}{0.04 + 0.04 + 0.04} = 1.05 \times 10^3 \text{ s}^2$$

↓

$$(28) \quad \text{(feasible)} \quad T_{\text{perfect balance}} = 32.4 \text{ s}$$

With the value (28) for the differential period, the requirement (22) becomes:

$$(29) \quad \chi_{\text{bridge}} < \frac{(32.4)^2}{4\pi^2} \cdot \frac{0.04}{10 \cdot 0.4^2} = 0.67$$

with the gap $d_1 = 1 \text{ mm}$

With this value the GGG goal in displacement from (15) is

$$\text{so } \sigma_{d30d} = \frac{8 \cdot 10^{-15} \cdot 32.4^2}{4\pi^2} = 2 \times 10^{-13} \text{ m} \quad (28')$$

$$\chi_{\text{bridge}} = \frac{d_1 - d_2}{d_1} < 0.67 \Rightarrow d_1 - d_2 < 0.67 \text{ mm} \quad (30)$$

(feasible)

If this condition is fulfilled, the effect of the bridge mechanical unbalance is below the effect of the terrain tilts and the goal (15) is feasible (see (13)).

• CAP BRIDGE READ-OUT ELECTRONICS NOISE

(Feb 2012)

The new electronics that Raffaelli is about to complete will have a noise (realistic estimate)

$$(31) \quad \frac{\tau_{\text{cap}}}{\sqrt{\text{Hz}}} = 3 \cdot 10^{-9} \frac{\text{m}}{\sqrt{\text{Hz}}}$$

@ 0.2 Hz

(better if GGG spins at higher frequency)

In 30 days this capacitance read out noise gives a displacement noise

$$(32) \quad r_{\text{cap 30d}} = 1.9 \times 10^{-12} \text{ m}$$

which is 10 times larger than the goal (28')

If we build and implement a laser gauge on GCG having:

- a 1 ± 2 cm gap between the test cylinders

- a higher spin frequency ($\approx 1 \text{ Hz}$)

such that its noise is

$$(33) \quad r_{\text{GCG eg}} = \frac{10^{-10} \text{ m}}{\sqrt{1 \text{ Hz}}} \quad @ 1 \text{ Hz}$$

we get

$$(34) \quad r_{\text{GCG eg 30d}} = 6.2 \times 10^{-14} \text{ m}$$

which is 3.2 times smaller than the target (28'), so that we can be sure that the result is not limited by read-out noise.

• BEARINGS NOISE

From the current ongoing run we conclude that the tilt noise of the present ball bearings (@ $\nu = 1.75 \times 10^{-4} \text{ Hz}$) is

$$(35) \quad \theta_{\text{bb 30d}} \approx 150 \theta_{\text{termin 30d}} \quad \rightarrow \text{given by (10),}$$

If we can set up air bearings (+ ferro fluid feed through) such that its tilt noise is:

$$(36) \quad \Theta_{a.b.30d} \leq \frac{1}{2} \Theta_{\text{term}} = \frac{1}{200} \Theta_{bb} \quad (\text{realistic})$$

then we have $1/2$ the acceleration noise (13) and by summing up the terrain and air bearings tilt/hoiz acceler. @ $1.75 \times 10^{-4} \text{ Hz}$ we have an acceleration noise below the GGG goal of $8 \cdot 10^{-15} \text{ ms}^{-2}$ in 30 d

Since the laser gauge read-out is a factor 3 below the target (having considered correctly the target in displacement - see (34)) we conclude that the result is limited by terrain/air bearings noise, neither of which is present in space

- See Table I - GGG Ground Error Budget (page 19)

Estimate (35) of the ball bearings noise is obtained as follows. From the current run we have:

$$(37) \quad a_{\text{now 30d}} \approx 8.49 \cdot 10^{-11} \text{ ms}^{-2}$$

both terrain tilts & ball bearings noise, and

Let us use (8) with the current measured numbers:

$$(38) \quad a_{\text{d tilt now}} = \frac{k_c}{mL} \frac{k_{\text{shaft}}}{M_{\text{rot}} g L_{\text{rot}}} (\Theta_{\text{terrain}} + \Theta_{bb}) =$$

$$= \frac{0.3 \cdot 0.17}{10 \cdot 0.18 \cdot 40 \cdot 9.81 \cdot 0.4} (\Theta_{\text{terrain}} + \Theta_{bb}) = 1.8 \cdot 10^{-4} (\Theta_{\text{terrain}} + \Theta_{bb}) \text{ ms}^{-2}$$

If it is $\Theta_{bb} \approx 100 \Theta_{\text{terrain}}$, we have (in 30d):

$$\Theta_{bb} \approx 5 \cdot 10^{-7} \text{ rad and } a_{\text{d tilt now 30d}} \approx 9 \cdot 10^{-11} \text{ ms}^{-2},$$

which is roughly what we measure with (37)

The ^{requirement} (36) for air bearings 200 times less noisy than ball bearings is very conservative, since air bearings are considered to be several orders of magnitude less noisy than ball bearings.

Table I - GG on Ground: Error budget

Test masses	a (m s^{-2})	ϵ (m)	Integration time (d)
Differential acceleration $a_{\text{GG}} = 1.75 \times 10^{-4} \text{ Hz}$			
1. GG goal $a_{\text{GG}} = \eta g(h)$ upconverted to 1 Hz	$8 \cdot 10^{-17}$ $\eta = 10^{-17} \quad h \approx 600 \text{ km}$	$6 \cdot 10^{-13}$ $T_d = 540 \text{ s}$	1
2. GG on Ground goal $a_{\text{GGG}} = 10^2 a_{\text{GG}}$ upconverted to $0.2 \pm 3 \text{ Hz}$	$8 \cdot 10^{-15}$	$2 \cdot 10^{-13}$ $T_d = 32.4 \text{ s}$	30
3. Tilts/horizontal accelerations $a_{\text{tilt}} = \frac{k_c}{mL} \frac{k_{\text{shaft}}}{M_{\text{tot}} g L_{\text{tot}}} \theta_{\text{tilt}}$	$10^{-6} \theta_{\text{tilt}}$ $\theta_{\text{tilt}} = \theta_{\text{terrain}} + \theta_{\text{bearings}}$ $(k_c = k_{\text{shaft}} = 0.04 \text{ Nm})$ $m = 10 \text{ kg}, L = 0.4 \text{ m}$ $M_{\text{tot}} = 50 \text{ kg}$ $L_{\text{tot}} = 0.8 \text{ m}$ $\theta_{\text{terrain}} \approx 5 \cdot 10^{-9} \text{ rad}$ $\theta_{\text{bb}} \approx 150 \theta_{\text{terrain}}$ $\theta_{\text{ab}} \approx \frac{1}{2} \theta_{\text{terrain}}$ $10^{-6} (\theta_{\text{terrain}} + \theta_{\text{ab}}) =$ $= 7.5 \times 10^{-15}$	$1.99 \cdot 10^{-13}$	30
4. Capacitance bridge unbalance $a_{\text{bridge}} \ll a_{\text{tilt}}$ if $\chi_{\text{bridge}} \ll \frac{T_d^2}{4\pi^2} \frac{k_c}{mL}$ satisfied $\chi_{\text{bridge}} = (d_1 - d_2)/d_1$			
5. Capacitance read out noise $\left(\frac{r}{\sqrt{\text{Hz}}} = 3 \cdot 10^{-9} \frac{\text{m}}{\sqrt{\text{Hz}}} \right)$ $1.3 \cdot 10^{-12}$			30
6. Laser gauge noise $\left(\frac{r}{\sqrt{\text{Hz}}} = 10^{-10} \frac{\text{m}}{\sqrt{\text{Hz}}} \right)$ $6.2 \cdot 10^{-14}$			30
7. Thermal noise	negligible		30

aiming at $a_{\text{acc}} = 10 a_{\text{as}}$

Table I shows that there is no fundamental limitation to improving the G.G. on ground sensitivity by another order of magnitude provided we can improve the laser gauge by 1 order of magnitude with line 6 in Table I, namely to a performance

From line 3, in Table I, we can reduce

$$(39) a_{\text{tilt}} = \frac{k_c}{m L} \frac{k_{\text{shaft}}}{M_{\text{tot}} g L_{\text{tot}}} \Theta_{\text{tilt}}$$

by a factor 10 by slightly increasing L , L_{tot} and M_{tot}

$$(40) L M_{\text{tot}} L_{\text{tot}} = 0.4 \cdot 50 \cdot 0.8 = 16 \text{ kg m}^2 \quad \text{in Table I}$$

↓

$0.5 \text{ m} \cdot 80 \text{ kg} \cdot 4 \text{ m} = 160 \text{ kg m}^2$

the G.G. balance will have a coupling arm of 1m.
 ~~no problem~~ with cube top joint (similar to the one we have just mounted) there is a very high exertion load (and even higher breaking load). No problem

$\uparrow$

the suspension joint of the rotor can be placed 4m away from its center of mass

Thus, with the same $\Theta_{\text{tilt}} = \Theta_{\text{torque}} + \Theta_{\text{ab}}$ (i.e. same air bearing), the same k_c , k_{shaft} and the same T_d as in Table I and the same integration time of 30d, we would have a differential acceleration of $7.5 \cdot 10^{-16} \text{ ms}^{-2}$ (in line 3), i.e. $\leq 10 a_{\text{ff}} = 8 \cdot 10^{-16} \text{ ms}^{-2}$, which is the same as of tower balances (though, we are @ $1.75 \cdot 10^{-4} \text{ Hz}$, they are @ $1.16 \cdot 10^{-5} \text{ Hz}$).

With the same T_d the displacement noise is $1.98 \times 10^{-14} \text{ m}$ and for the laser gauge noise to be below it we need $\frac{\epsilon_g}{\sqrt{\text{Hz}}} \approx 10^{-11} \frac{\text{m}}{\sqrt{\text{Hz}}} @ 1-3 \text{ Hz}$ on GGL.

Since JPL has demonstrated $10^{-12} \frac{\text{m}}{\sqrt{\text{Hz}}} @ 1 \text{ Hz}$, and this is what we require in space, we should be able to demonstrate it on GGL.

- I think we should aim at

$$(41) \quad \alpha_{\text{GGL good } 30s} \lesssim 8 \cdot 10^{-16} \text{ m s}^{-2} @ 1.75 \times 10^{-4} \text{ Hz}$$

We would have a balance with the same sensitivity as the torque balance which in space should do only a factor 10 better in order to test EP to 10^{-17} . And it will be able to do that because of the reduced thermal noise thanks to rotation.

Instead, the torque balance has reduced thermal noise and cannot gain by rotation.

→ (terrain tilt noise and bearings noise is close to in space)

- The main issue in space not tested on ground is

Drop Free Control (TASI-To & GOCF)

+ better common mode rejection than on ground (but we have arguments...)